Grade Six

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

Number Sense

1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, a to b, $a{:}b$).

1.3 Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $5/8 \div 15/16 = 5/8 \times 16/15 = 2/3$).

2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Algebra and Functions

1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

1.1 Write and solve one-step linear equations in one variable.

1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.

1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator.

2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

2.2 Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

2.3 Solve problems involving rates, average speed, distance, and time.

3.0 Students investigate geometric patterns and describe them algebraically:

3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2}bh$, $C = \pi d$—the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

3.2 Express in symbolic form simple relationships arising from geometry.

Measurement and Geometry

1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

1.1 Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.

1.2 Know common estimates of π (3.14; $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base × height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

2.0 Students identify and describe the properties of two-dimensional figures:

2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Statistics, Data Analysis, and Probability

1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1-P is the probability of an event not occurring.

3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5 Understand the difference between independent and dependent events.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.5 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.6 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.7 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.

Glossary

absolute value. A number's distance from zero on the number line. The absolute value of -4 is 4; the absolute value of 4 is 4.

algorithm. An organized procedure for performing a given type of calculation or solving a given type of problem. An example is long division.

arithmetic sequence. A sequence of elements, $a_1, a_2, a_3, \ldots$, such that the difference of successive terms is a constant $a_{i+1} - a_i = k$; for example, the sequence {2, 5, 8, 11, 14, . . .} where the common difference is 3.

asymptotes. Straight lines that have the property of becoming and staying arbitrarily close to the curve as the distance from the origin increases to infinity. For example, the x-axis is the only asymptote to the graph of $\sin(x)/x$.

axiom. A basic assumption about a mathematical system from which theorems can be deduced. For example, the system could be the points and lines in the plane. Then an axiom would be that given any two distinct points in the plane, there is a unique line through them.

binomial. In algebra, an expression consisting of the sum or difference of two monomials (see the definition of *monomial*), such as $4a–8b$.

binomial distribution. In probability, a binomial distribution gives the probabilities of k outcomes A (or $n–k$ outcomes B) in n independent trials for a two-outcome experiment in which the possible outcomes are denoted A and B.

binomial theorem. In mathematics, a theorem that specifies the complete expansion of a binomial raised to any positive integer power.

box-and-whisker plot. A graphical method for showing the median, quartiles, and extremes of data. A box plot shows where the data are spread out and where they are concentrated.

complex numbers. Numbers that have the form $a + bi$ where a and b are real numbers and i satisfies the equation $i^2 = -1$. Multiplication is denoted by $(a+bi)(c+di) = (ac–bd) + (ad+bc)i$, and addition is denoted by $(a+bi) + (c + di) = (a+c) + (b+d)i$.

congruent. Two shapes in the plane or in space are congruent if there is a rigid motion that identifies one with the other (see the definition of *rigid motion*).

conjecture. An educated guess.

coordinate system. A rule of correspondence by which two or more quantities locate points unambiguously and which satisfies the further property that points unambiguously determine the quantities; for example, the usual Cartesian coordinates x, y in the plane.

cosine. $\cos(\theta)$ is the x-coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of θ with the positive x-axis. When θ is an angle of a right triangle, then $\cos(\theta)$ is the ratio of the adjacent side with the hypotenuse.

dilation. In geometry, a transformation *D* of the plane or space is a dilation at a point *P* if it takes *P* to itself, preserves angles, multiplies distances from *P* by a positive real number *r*, and takes every ray through *P* onto itself. In case *P* is the origin for a Cartesian coordinate system in the plane, then the dilation *D* maps the point (x, y) to the point (rx, ry).

dimensional analysis. A method of manipulating unit measures algebraically to determine the proper units for a quantity computed algebraically. For example, velocity has units of the form length over time (e.g., meters per second [*m*/*sec*]), and acceleration has units of velocity over time; so it follows that acceleration has units $(m/sec)/sec = m/(sec^2)$.

expanded form. The expanded form of an algebraic expression is the *equivalent expression* without parentheses. For example, the expanded form of $(a + b)^2$ is $a^2 + 2ab + b^2$.

exponent. The power to which a number or variable is raised (the exponent may be any real number).

exponential function. A function commonly used to study growth and decay. It has the form $y = a^x$ with *a* positive.

factors. Any of two or more quantities that are multiplied together. In the expression 3.712×11.315, the factors are 3.712 and 11.315.

function. A correspondence in which values of one variable determine the values of another.

geometric sequence. A sequence in which there is a common ratio between successive terms. Each successive term of a geometric sequence is found by multiplying the preceding term by the common ratio. For example, in the sequence {1, 3, 9, 27, 81, . . . } the common ratio is 3.

histogram. A vertical block graph with no spaces between the blocks. It is used to represent frequency data in statistics.

inequality. A relationship between two quantities indicating that one is strictly *less than* or *less than or equal* to the other.

integers. The set consisting of the positive and negative whole numbers and zero; for example, {. . . -2, -1, 0, 1, 2 . . . }.

irrational number. A number that cannot be represented as an exact ratio of two integers. For example, the square root of 2 or π.

linear expression. An expression of the form $ax + b$, where *x* is variable and *a* and *b* are constants; or in more variables, an expression of the form $ax + by + c$, $ax + by + cz + d$, etc.

linear equation. An equation containing linear expressions.

logarithm. The inverse of exponentiation; for example, $a^{\log_a x} = x$.

mean. In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

median. In statistics, the quantity designating the middle value in a set of numbers.

mode. In statistics, the value that occurs most frequently in a given series of numbers.

monomial. In the variables *x*, *y*, *z*, a monomial is an expression of the form $ax^m y^n z^k$, in which *m*, *n*, and *k* are nonnegative integers and *a* is a constant (e.g., $5x^2$, $3x^2y$ or $7x^3yz^2$).

nonstandard unit. Unit of measurement expressed in terms of objects (such as paper clips, sticks of gum, shoes, etc.).

parallel. Given distinct lines in the plane that are infinite in both directions, the lines are parallel if they never meet. Two distinct lines in the coordinate plane are parallel if and only if they have the same slope.

permutation. A permutation of the set of numbers {1, 2, . . . *n*} is a reordering of these numbers.

polar coordinates. The coordinate system for the plane based on *r*θ, the distance from the origin and θ, and the angle between the positive *x*-axis and the ray from the origin to the point.

polar equation. Any relation between the polar coordinates (*r*, θ) of a set of points (e.g., $r = 2\cos\theta$ is the polar equation of a circle).

polynomial. In algebra, a sum of monomials; for example, $x^2 + 2xy + y^2$.

prime. A natural number *p* greater than 1 is prime if and only if the only positive integer factors of *p* are 1 and *p*. The first seven primes are 2, 3, 5, 7, 11, 13, 17.

quadratic function. A function given by a polynomial of degree 2.

random variable. A function on a probability space.

range. In statistics, the difference between the greatest and smallest values in a data set. In mathematics, the image of a function.

ratio. A comparison expressed as a fraction. For example, there is a ratio of three boys to two girls in a class (3/2, 3:2).

rational numbers. Numbers that can be expressed as the quotient of two integers; for example, 7/3, 5/11, -5/13, 7 = 7/1.

real numbers. All rational and irrational numbers.

reflection. The reflection through a line in the plane or a plane in space is the transformation that takes each point in the plane to its mirror image with respect to the line or its mirror image with respect to the plane in space. It produces a mirror image of a geometric figure.

rigid motion. A transformation of the plane or space, which preserves distance and angles.

root extraction. Finding a number that can be used as a factor a given number of times to produce the original number; for example, the fifth root of 32 = 2 because $2 \times 2 \times 2 \times 2 \times 2 = 32$).

rotation. A rotation in the plane through an angle θ and about a point *P* is a rigid motion *T* fixing *P* so that if *Q* is distinct from *P*, then the angle between the lines *PQ* and *PT(Q)* is always θ. A rotation through an angle θ in space is a rigid motion *T* fixing the points of a line *l* so that it is a rotation through θ in the plane perpendicular to *l* through some point on *l*.

scalar matrix. A matrix whose diagonal elements are all equal while the nondiagonal elements are all 0. The identity matrix is an example.

scatterplot. A graph of the points representing a collection of data.

scientific notation. A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (e.g., $7000 = 7 \times 10^3$ or $0.0000019 = 1.9 \times 10^{-6}$).

similarity. In geometry, two shapes *R* and *S* are similar if there is a dilation *D* (see the definition of *dilation*) that takes *S* to a shape congruent to *R*. It follows that *R* and *S* are similar if they are congruent after one of them is expanded or shrunk.

sine. Sin(θ) is the *y*-coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of θ with the positive *x*-axis. When θ is an angle of a right triangle, then *sin*(θ) is the ratio of the opposite side with the hypotenuse.

square root. The square roots of *n* are all the numbers *m* so that $m^2 = n$. The square roots of 16 are 4 and -4. The square roots of -16 are 4 *i* and -4 *i*.

standard deviation. A statistic that measures the dispersion of a sample.

symmetry. A symmetry of a shape *S* in the plane or space is a rigid motion *T* that takes *S* onto itself $(T(S) = S)$. For example, reflection through a diagonal and a rotation through a right angle about the center are both symmetries of the square.

system of linear equations. Set of equations of the first degree (e.g., $x + y = 7$ and $x - y = 1$). A solution of a set of linear equations is a set of numbers *a*, *b*, *c*, . . . so that when the variables are replaced by the numbers all the equations are satisfied. For example, in the equations above, $x = 4$ and $y = 3$ is a solution.

translation. A rigid motion of the plane or space of the form *X* goes to $X + V$ for a fixed vector *V*.

transversal. In geometry, given two or more lines in the plane a transversal is a line distinct from the original lines and intersects each of the given lines in a single point.

unit fraction. A fraction whose numerator is 1 (e.g., $1/\pi$, $1/3$, $1/x$). Every nonzero number may be written as a unit fraction since, for *n* not equal to 0, $n = 1/(1/n)$.

variable. A placeholder in algebraic expressions; for example, in $3x + y = 23$, *x* and *y* are variables.

vector. Quantity that has magnitude (length) and direction. It may be represented as a directed line segment.

zeros of a function. The points at which the value of a function is zero.