<u>Entering Calculus – Summer Packet</u>

<u>This summer packet is for students ENTERING AP CALCULUS AND DUAL CREDIT CALCULUS.</u>

Welcome to the world of Calculus! There is a major shift in expectation for students to be able to recall many skills at a moment's notice. Certain concepts that have been taught to you over the previous years are assumed to be mastered. If you do not have these skills, you will find that you will consistently get problems incorrect next year as you make mistakes. It is frustrating for students who spend much of their homework time relearning algebra concepts in addition to learning how to tie the concepts together. This summer packet is intended for you to brush up and possibly relearn these topics.

On the following pages, you have assorted problems related to specific topics. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the formulas and techniques necessary to solve the problem, there are a few websites that are listed below that have full instructions and brief refreshers at the beginning of some topics. Don't fake your way through these problems. You are only setting yourself up for a future struggle!

Realize also that many concepts are interrelated. This will be the focus throughout next year as we examine the mathematical relationships between topics numerically, algebraically, and graphically. While you may be strong in one of these approaches, you must learn to view each topic from the other two approaches as well to achieve full understanding. Success on your tests and quizzes throughout the year will depend on you being able to do so.

We want you to get off to a good start so spend some quality time on this packet this summer. Work needs to be shown when needed. Also, do not rely solely on the calculator to work through but you are expected to have strong calculator skills.

<u>This packet is to be completed by the first day of school.</u> This summer packet is a study guide for your first assessment in the course. You will work through any questions you have with your teachers in the first couple of days so come prepared to lead the discussion! You will also be expected to efficiently work through the problems under time constraints. Many students are not prepared for this expectation and find they do not have the time to check their answers like they are used to. Prepare accordingly.

<u>It is a mistake to do this now. Let it go until mid-summer</u>. We want these techniques to be relatively fresh in your mind in the fall. But, do not wait to do them at the very last minute. These take time. If you do a few concepts a day, the whole packet will take you about a week to complete.

We hope you take this seriously as we sincerely wish for you to be successful throughout this next year. Your preparation over the summer will be rewarded in unexpected ways during the school year.

Here are some helpful websites to use, if needed:
Use www.khanacademy.org to find specific math-related topics with accompanying videos.
Use www.patrickjmt.com to find specific math-related topics with accompanying videos.
Use www.youtube.com to find specific math-related topics with accompanying videos.

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Logarithms:
$y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$,
then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Fractional exponent: $\sqrt[b]{x^e} = x^{\frac{e}{b}}$

Negative Exponents: $x^{-n} = 1/x^n$

The Zero Exponent: $x^0=1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:
$(x^a)(x^b) = x^{(a+b)}$

Multiplying Powers of Different Bases:
$(xy)^a = (x^a)(y^a)$

Dividing Powers

Dividing Two Powers of the Same Base:
$(x^a)/(x^b) = x^{(a-b)}$

Dividing Powers of Different Bases:
$(x/y)^a = (x^a)/(y^a)$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\frac{\frac{25}{a}-a}{5+a}$

2. $\frac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\frac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\frac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\frac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Function

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________

7. $g(-3) =$ ____________

8. $f(t+1) =$ ____________

9. $f\left[g(-2)\right] =$ ____________

10. $g\left[f(m+2)\right] =$ ____________

11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ____________

13. $f\left(\dfrac{2\pi}{3}\right) =$ ____________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

14. $h\left[f(-2)\right] =$ ______

15. $f\left[g(x-1)\right] =$ ______

16. $g\left[h(x^3)\right] =$ ______

Find $\frac{f(x+h)-f(x)}{h}$ for the given function *f*.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int.}\ (Let\ y=0)$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1\ or\ x=3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

$y-\text{int.}\ (Let\ x=0)$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Systems

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug x=3 *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0 \qquad 5^2 - y^2 - 9 = 0$

$-y^2 = 0 \qquad 16 = y^2$

$y = 0 \qquad y = \pm 4$

Points of Intersection (5,4), (5,–4) *and* (3,0)

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as*

$x^2 - 8x + 15 = 0$ *previous example*)

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

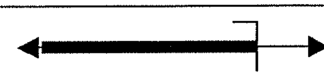
23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

25. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
$16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1,7)$	
		

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Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$g(f(x)) = 4\left(\frac{x-9}{4}\right)+9$$

$$= x-9+9$$

$$= x$$

$$f(g(x)) = \frac{(4x+9)-9}{4}$$

$$= \frac{4x+9-9}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

36. $f(x) = \frac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2, x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

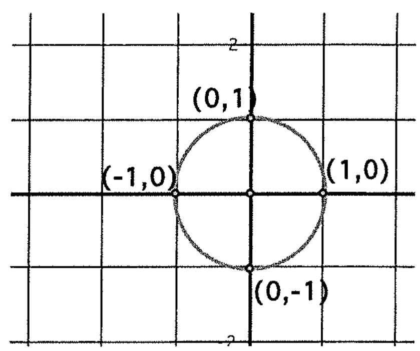
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

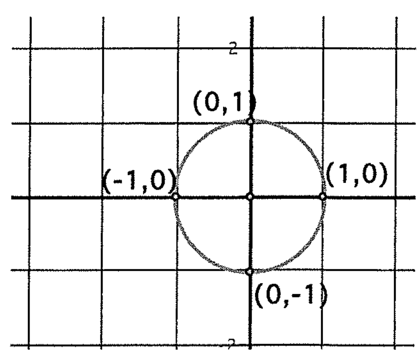
Example: $\sin 90^\circ = 1$ $\cos\frac{\pi}{2} = 0$



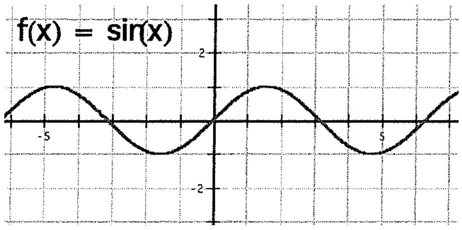
50. *a.*) $\sin 180^\circ$ *b.*) $\cos 270^\circ$

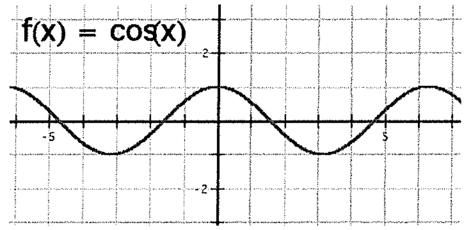
c.) $\sin(-90^\circ)$ *d.*) $\sin \pi$

e.) $\cos 360^\circ$ *f.*) $\cos(-\pi)$



Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51. $f(x) = 5\sin x$

52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the beginning of the packet.)

55. $\sin x = -\frac{1}{2}$

56. $2\cos x = \sqrt{3}$

57. $\cos 2x = \frac{1}{\sqrt{2}}$

58. $\sin^2 x = \frac{1}{2}$

59. $\sin 2x = -\frac{\sqrt{3}}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

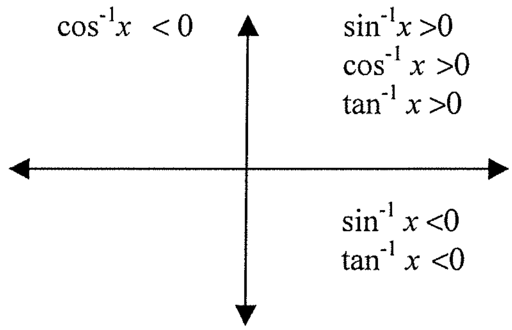
62. $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad \sin^{-1}(x)$$

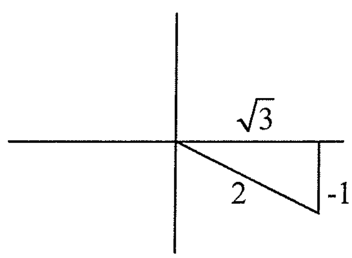
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: $y = -\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

62.2 $y = \arcsin\frac{-\sqrt{3}}{2}$

62.4 $y = \arccos(-1)$

62.6 $y = \arctan(-1)$

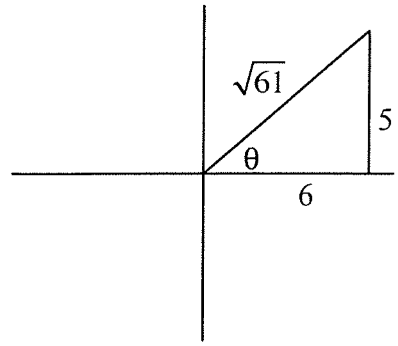
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

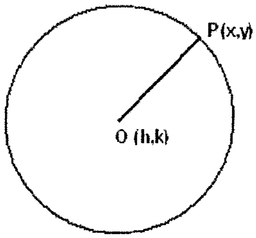
63. $\tan\left(\arccos\frac{2}{3}\right)$

64. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

65. $\sin\left(\arctan\frac{12}{5}\right)$

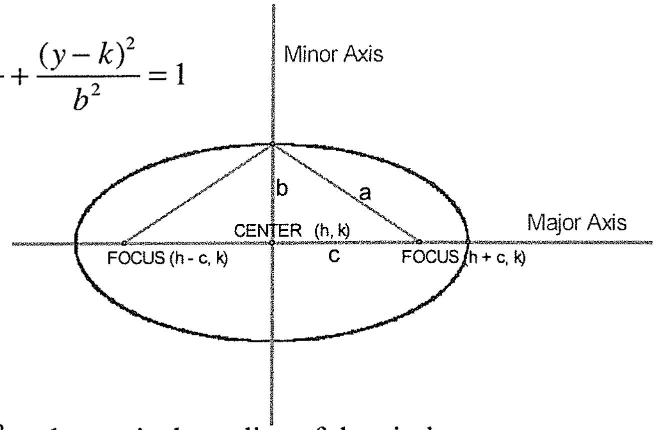
66. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

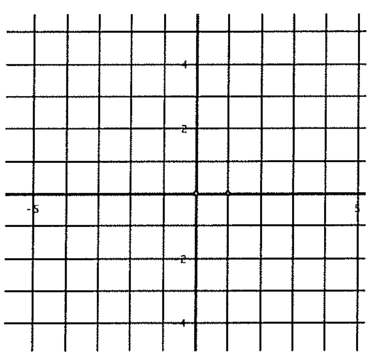


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

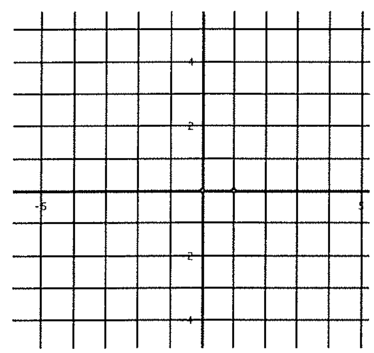
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the y-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

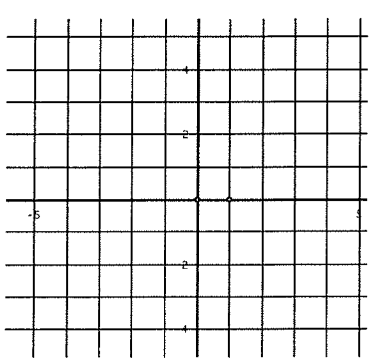
67. $x^2 + y^2 = 16$



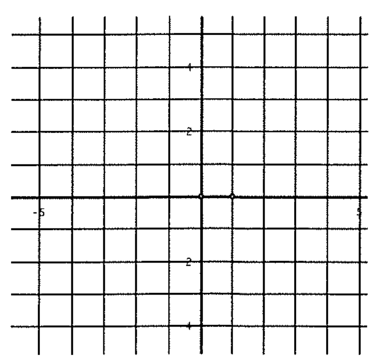
68. $x^2 + y^2 = 5$



69. $\frac{x^2}{1} + \frac{y^2}{9} = 1$



70. $\frac{x^2}{16} + \frac{y^2}{4} = 1$



Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

71. $f(x)=\frac{1}{x^2}$

72. $f(x)=\frac{x^2}{x^2-4}$

73. $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

74. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

75. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

76. $f(x)=\frac{4x^5}{x^2-7}$

Laws of Exponents

Write each of the following expressions in the form ca^pb^q where c, p and q are constants (numbers).

75. $\dfrac{(2a^2)^3}{b}$

76. $\sqrt{9ab^3}$

(Hint: $\sqrt{x} = x^{1/2}$)

77. $\dfrac{a(2/b)}{3/a}$

78. $\dfrac{ab-a}{b^2-b}$

79. $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$

80. $\left(\dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\dfrac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

Laws of Logarithms

Simplify each of the following:

81. $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

82. $2\log_2 9 - \log_2 3$

83. $3^{2\log_3 5}$

84. $\log_{10}(10^{1/2})$

85. $\log_{10}(\frac{1}{10^x})$

86. $2\log_{10}\sqrt{x} + \log_{10} x^{1/3}$

Solving Exponential and Logarithmic Equations

Solve for x. (DO NOT USE A CALCULATOR)

87. $5^{(x+1)} = 25$

88. $\frac{1}{3} = 3^{2x+2}$

89. $\log_2 x^2 == 3$

90. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Factor Completely

91. $x^6 - 16x^4$

92. $4x^3 - 8x^2 - 25x + 50$

93. $8x^3 + 27$

94. $x^4 - 1$

Solve the following equations for the indicated variables:

95. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1,\ \textit{for } a.$

96. $V = 2(ab + bc + ca),\ \textit{for } a.$

97. $A = 2\pi r^2 + 2\pi rh,\ \textit{for positive } r.$

Hint: use quadratic formula

98. $A = P + xrP,\ \textit{for } P$

99. $2x - 2yd = y + xd,\ \textit{for } d$

100. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0,\ \textit{for } x$

Solve the equations for x:

101. $4x^2 + 12x + 3 = 0$

102. $2x + 1 = \frac{5}{x+2}$

103. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Polynomial Division

104. $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$

105. $(x^5 - x^4 + x^3 + 2x^2 - x + 4) \div (x^3 + 1)$