

Summer 2023 Honors Geometry Summer Packet

Completion of this packet is required for all students enrolled in Honors Geometry for the upcoming school year. **It looks long because it has review examples – you only need to do the circled problems.** In addition to this packet, students are expected to have memorized the following:

- Multiplication facts to 12×12
- Perfect squares and square roots up to 15×15

The expectation for Geometry Honors is that you do your work **in pencil** on separate lined paper, copying each problem and showing all of your work neatly. **In this class, no work = no credit.** Clearly number each topic and problem. **Calculators are NOT permitted.**

Each page of the packet summarizes a concept with examples. Use those examples if you have trouble remembering how to do any of the problems. **When you have finished the packet, check your answers against the answer key provided. Rework incorrect problems in a red or green pen until you are able to reach the correct answer.** If you get stuck, use an old textbook, the internet, or one of the following websites:

www.purplemath.com

www.khanacademy.org

www.mathbitsnotebook.com

www.mathantics.com

This packet should be completed **without a calculator.** Calculators will not be used until at least second quarter of the school year in this course.

On the first day of school, this packet will be checked for completion, which includes neat and complete work on lined paper, and corrections made with a colored pen. **A test on the material in this packet will be given during the first few days of school as your first test grade of the year. The score on this test is a strong indicator of success in this class, and the math department reserves the right to re-evaluate your placement if you do not score at least 75% on this test. Content from the packet will not be reviewed during class time.**

For Geometry Honors, the following materials will be required:

- Two thin and two chisel tip black Expo markers (to be with student in class at all times)
- Graphing calculator (TI-84 Plus or TI-84 Plus CE). Casio calculators are not permitted.
- 1" binder plus a spiral or composition notebook dedicated to math. Notebooks and binders may be collected from time to time for grading, so your binder and notebook must be dedicated to math.
- Pencils (pens are not permitted for math work, even if erasable), mechanical recommended
- Highlighters (2 colors)
- 6-inch or 12-inch ruler
- Protractor
- Graph paper and loose leaf paper

Good luck, pace yourself, and have a great summer!

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

Example 1 Solve each literal equation for y .

a. $3x + 5y = 45$

$$3x - 3x + 5y = 45 - 3x$$
 Subtract $3x$ from each side.

$$5y = 45 - 3x$$
 Simplify.

$$\frac{5y}{5} = \frac{45 - 3x}{5}$$
 Divide each side by 5.

$$y = 9 - \frac{3}{5}x$$
 Simplify.

► The rewritten literal equation is $y = 9 - \frac{3}{5}x$.

b. $2xy + 5y = 7$

$$y(2x + 5) = 7$$
 Distributive Property

$$\frac{y(2x + 5)}{2x + 5} = \frac{7}{2x + 5}$$
 Divide each side by $2x + 5$.

$$y = \frac{7}{2x + 5}$$
 Simplify.

► The rewritten literal equation is $y = \frac{7}{2x + 5}$.

c. $2x = \frac{3 + y}{y}$

$$2x \cdot y = \frac{3 + y}{y} \cdot y$$
 Multiply each side by y .

$$2xy = 3 + y$$
 Simplify.

$$2xy - y = 3 + y - y$$
 Subtract y from each side.

$$2xy - y = 3$$
 Simplify.

$$y(2x - 1) = 3$$
 Distributive Property

$$\frac{y(2x - 1)}{2x - 1} = \frac{3}{2x - 1}$$
 Divide each side by $2x - 1$.

$$y = \frac{3}{2x - 1}$$
 Simplify.

► The rewritten literal equation is $y = \frac{3}{2x - 1}$.

Practice

Check your answers at BigIdeasMath.com.

Solve the literal equation for y .

1. $x + 3y = 9$

2. $4x - 2y = 16$

3. $2x + 7y = 5$

4. $2x + 3y = 6$

5. $5x - 4y = 10$

6. $x - 2y = 8$

7. $2xy - 6 = 8x$

8. $4x = 9y + xy$

9. $4yz = 3y - 8x$

10. $2xy = 3z + 4y$

11. $\frac{2 + 7y}{y} = x$

12. $3x = \frac{5 + y}{y}$

Solving Linear Equations

To determine whether a value is a solution of an equation, substitute the value into the equation and simplify.

Example 1 Determine whether (a) $x = 1$ or (b) $x = -2$ is a solution of $5x - 1 = 4$.

a. $5x - 1 = -2x + 6$

$$5(1) - 1 \stackrel{?}{=} -2(1) + 6$$

$$4 = 4 \quad \checkmark$$

Substitute.

Simplify.

► So, $x = 1$ is a solution.

b. $5x - 1 = -2x + 6$

$$5(-2) - 1 \stackrel{?}{=} -2(-2) + 6$$

$$-11 \neq 10 \quad \times$$

Substitute.

Simplify.

► So, $x = -2$ is *not* a solution.

To solve a linear equation, isolate the variable.

Example 2 Solve each equation. Check your solution.

a. $4x - 3 = 13$

$$4x - 3 + 3 = 13 + 3$$

Add 3.

$$4x = 16$$

Simplify.

$$\frac{4x}{4} = \frac{16}{4}$$

Divide by 4.

$$x = 4$$

Simplify.

Check

$$4x - 3 = 13$$

$$4(4) - 3 \stackrel{?}{=} 13$$

$$13 = 13 \quad \checkmark$$

b. $2(y - 8) = y + 6$

$$2y - 16 = y + 6$$

Distributive Property

$$2y - y - 16 = y - y + 6$$

Subtract y .

$$y - 16 = 6$$

Simplify.

$$y - 16 + 16 = 6 + 16$$

Add 16.

$$y = 22$$

Simplify.

Check

$$2(y - 8) = y + 6$$

$$2(22 - 8) \stackrel{?}{=} 22 + 6$$

$$28 = 28 \quad \checkmark$$

Practice

Check your answers at BigIdeasMath.com.

Determine whether (a) $x = -1$ or (b) $x = 3$ is a solution of the equation.

1. $5x + 7 = 2$

2. $-4x + 8 = -4$

3. $2x - 1 = 3x - 4$

Solve the equation. Check your solution.

4. $x - 9 = 24$

5. $n + 14 = 0$

6. $-16 = 4y$

7. $-\frac{5}{6}t = -15$

8. $81 = 46 - x$

9. $4x + 5 = 1$

10. $x + 5 = 11x$

11. $9(y - 3) = 45$

12. $6 = 7k + 8 - k$

13. $6n + 3 = -4n + 7$

14. $2c + 5 = 3(c - 8)$

15. $18m + 3(2m + 8) = 0$

16. $\frac{w - 6}{5} = 8$

17. $\frac{15 + h}{3} = 10$

18. $\frac{8 - 3x}{5} = x$

19. $(8r + 6) + (4r - 1) = 14$

20. $\frac{2}{3}y - 3 = 9$

21. $\frac{1}{2}x - \frac{3}{10} = \frac{5}{2}x + \frac{7}{10}$

22. **MONEY** You have a total of \$3.25 in change made up of 25 pennies, 6 nickels, 2 dimes, and x quarters. How many quarters do you have?

REVIEW: Pythagorean Theorem

Name _____

Key Concept and Vocabulary

In any right triangle,

a and b are leg lengths.

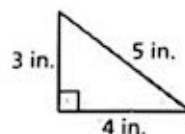
$$a^2 + b^2 = c^2$$

c is the length of the hypotenuse.

Pythagorean Theorem



Visual Model



$$3^2 + 4^2 = 5^2$$

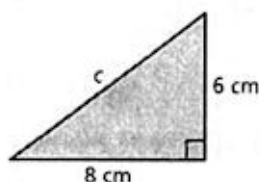
Skill Examples

1. $a^2 + b^2 = c^2$

$$6^2 + 8^2 = c^2$$

$$100 = c^2$$

$$10 \text{ cm} = c$$

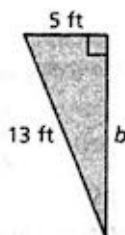


2. $a^2 + b^2 = c^2$

$$5^2 + b^2 = 13^2$$

$$b^2 = 144$$

$$b = 12 \text{ ft}$$



Application Example

3. The base of a ladder is 9 feet from a building. The ladder extends 12 feet up the side of the building. How long is the ladder?

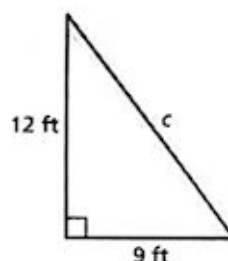
The ladder is the hypotenuse of a right triangle.

$$a^2 + b^2 = c^2$$

$$9^2 + 12^2 = c^2$$

$$225 = c^2$$

$$15 = c$$



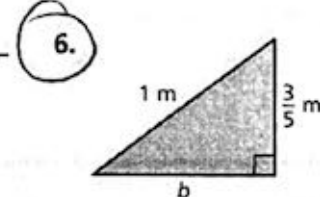
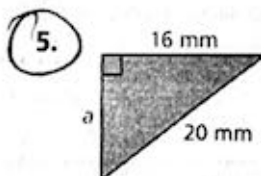
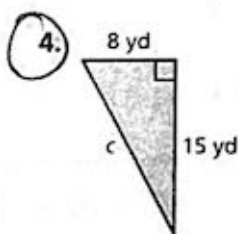
∴ The ladder is 15 feet long.

PRACTICE MAKES PURR-FECT®

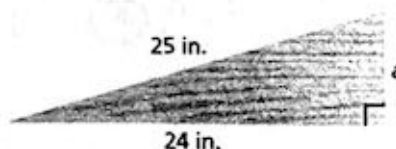


Check your answers at BigIdeasMath.com.

Find the missing length of the triangle.



7. **RAMP** You make the skateboarding ramp shown. Find the height of the ramp. _____



Properties of Square Roots

A **radical expression** is an expression that contains a radical. A radical expression involving square roots is in **simplest form** when these three conditions are met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

You can use the properties below to simplify radical expressions involving square roots.

| Product Property of Square Roots | Quotient Property of Square Roots |
|---|--|
| $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$ | $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$ |

Example 1 Simplify (a) $\sqrt{75}$ and (b) $\sqrt{\frac{13}{25}}$.

- a. $\sqrt{75} = \sqrt{25 \cdot 3}$ Factor using the greatest perfect square factor.
 $= \sqrt{25} \cdot \sqrt{3}$ Product Property of Square Roots
 $= 5\sqrt{3}$ Simplify.
- b. $\sqrt{\frac{13}{25}} = \frac{\sqrt{13}}{\sqrt{25}}$ Quotient Property of Square Roots
 $= \frac{\sqrt{13}}{5}$ Simplify.

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

Example 2 Simplify $\frac{10}{\sqrt{7}}$ by rationalizing the denominator.

$$\begin{aligned} \frac{10}{\sqrt{7}} &= \frac{10}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} && \text{Multiply by } \frac{\sqrt{7}}{\sqrt{7}} \\ &= \frac{10\sqrt{7}}{\sqrt{49}} && \text{Product Property of Square Roots} \\ &= \frac{10\sqrt{7}}{7} && \text{Simplify.} \end{aligned}$$

Practice

Check your answers at BigIdeasMath.com.

Simplify the expression.

1. $\sqrt{12}$

2. $\sqrt{92}$

3. $\sqrt{500}$

4. $\sqrt{112}$

5. $\sqrt{216}$

6. $\sqrt{\frac{10}{49}}$

7. $-\sqrt{\frac{8}{25}}$

8. $\frac{\sqrt{48}}{\sqrt{81}}$

9. $\frac{3}{\sqrt{5}}$

10. $-\frac{14}{\sqrt{10}}$

11. $\sqrt{\frac{3}{8}}$

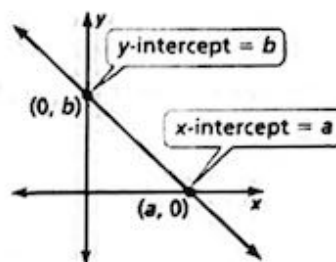
12. $\sqrt{\frac{7}{32}}$

Slope-Intercept Form

The **x-intercept** of a line is the x-coordinate of the point where the line crosses the x-axis. It occurs when $y = 0$.

The **y-intercept** of a line is the y-coordinate of the point where the line crosses the y-axis. It occurs when $x = 0$.

A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m , and the y-intercept of the line is b .



$$y = mx + b$$

slope
y-intercept

Example 1 Identify the slope and the y-intercept of the graph of each linear equation.

a. $y = -3x - 8$

$y = -3x + (-8)$ Write in slope-intercept form.

► The slope is -3 , and the y-intercept is -8 .

b. $y - 4 = \frac{1}{3}x$

$y = \frac{1}{3}x + 4$ Add 4 to each side.

► The slope is $\frac{1}{3}$, and the y-intercept is 4 .

Example 2 Find the x-intercept and the y-intercept of the graph of $2x + y = 4$.

To find the x-intercept, substitute 0 for y and solve for x.

$$\begin{aligned} 2x + y &= 4 \\ 2x + (0) &= 4 \\ x &= 2 \end{aligned}$$

► The x-intercept is 2, and the y-intercept is 4.

To find the y-intercept, substitute 0 for x and solve for y.

$$\begin{aligned} 2x + y &= 4 \\ 2(0) + y &= 4 \\ y &= 4 \end{aligned}$$

Practice

Check your answers at BigIdeasMath.com.

Identify the slope and the y-intercept of the graph of the linear equation.

1. $y = 4x + 7$

2. $y = -\frac{1}{3}x + 8$

3. $y = \frac{1}{2}x - 6$

4. $y + 9 = -5x$

5. $y - 2x = -6$

6. $7 + y = -\frac{2}{3}x$

Find the x-intercept and the y-intercept of the graph of the equation.

7. $y = 2x$

8. $y = x + 8$

9. $y = 3x + 6$

10. $3x + y = 9$

11. $2x + 3y = 12$

12. $2x - 5y = 10$

13. **SHOPPING** The amount of money you spend on x books and y movies is given by the equation $8x + 12y = 96$. Find the intercepts of the graph of the equation. What do these values represent?

Name _____ Date _____

Writing Linear Equations

Given a point on a line and the slope of the line, you can write an equation of the line.

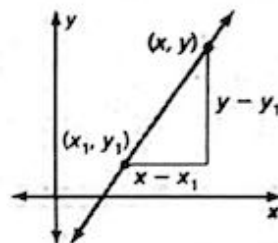
Example 1 Write an equation in slope-intercept form of the line that passes through the point $(-5, 6)$ and has a slope of $\frac{3}{5}$.

$$\begin{aligned} y &= mx + b && \text{Write the slope-intercept form.} \\ 6 &= \frac{3}{5}(-5) + b && \text{Substitute } \frac{3}{5} \text{ for } m, -5 \text{ for } x, \text{ and } 6 \text{ for } y. \\ 6 &= -3 + b && \text{Simplify.} \\ 9 &= b && \text{Solve for } b. \end{aligned}$$

► So, the equation is $y = \frac{3}{5}x + 9$.

A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1) , and the slope of the line is m .

$$\begin{array}{c} \text{slope} \\ \downarrow \\ y - y_1 = m(x - x_1) \\ \uparrow \quad \uparrow \\ \text{passes through } (x_1, y_1) \end{array}$$



Example 2 Write an equation in point-slope form of the line that passes through the point $(-8, 3)$ and has a slope of $\frac{3}{4}$.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Write the point-slope form.} \\ y - 3 &= \frac{3}{4}[x - (-8)] && \text{Substitute } \frac{3}{4} \text{ for } m, -8 \text{ for } x_1, \text{ and } 3 \text{ for } y_1. \\ y - 3 &= \frac{3}{4}(x + 8) && \text{Simplify.} \end{aligned}$$

► So, the equation is $y - 3 = \frac{3}{4}(x + 8)$.

Practice

Check your answers at BigIdeasMath.com.

Write an equation in **slope-intercept form** of the line that passes through the given point and has the given slope.

1. $(1, 3); m = 2$

2. $(4, 2); m = 3$

3. $(-2, 3); m = \frac{1}{2}$

4. $(6, -5); m = \frac{2}{3}$

5. $(4, -2); m = -\frac{1}{4}$

6. $(-7, -3); m = -\frac{2}{7}$

Write an equation in **point-slope form** of the line that passes through the given point and has the given slope.

7. $(1, 1); m = 5$

8. $(-3, 4); m = 2$

9. $(6, -3); m = \frac{3}{2}$

10. $(5, 7); m = \frac{2}{5}$

11. $(-4, 5); m = -\frac{3}{4}$

12. $(-2, -3); m = -\frac{3}{8}$

Graphing Linear Equations

Example 1 Graph $x + 3y = -3$ using intercepts.

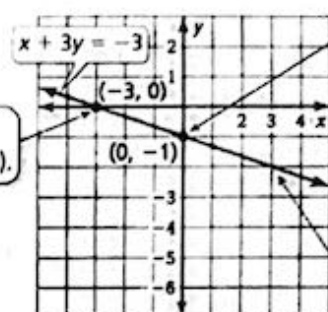
Step 1 To find the x -intercept, substitute 0 for y .

$$\begin{aligned}x + 3y &= -3 \\x + 3(0) &= -3 \\x &= -3\end{aligned}$$

To find the y -intercept, substitute 0 for x .

$$\begin{aligned}x + 3y &= -3 \\0 + 3y &= -3 \\y &= -1\end{aligned}$$

Step 2 Graph the equation.



The x -intercept is -3 . So, plot $(-3, 0)$.

The y -intercept is -1 . So, plot $(0, -1)$.

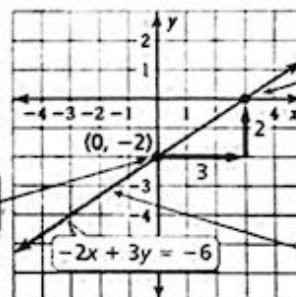
Draw a line through the points.

Example 2 Graph $y = \frac{2}{3}x - 2$ using the slope and y -intercept.

$$y = \frac{2}{3}x + (-2)$$

slope \uparrow \uparrow y -intercept

The y -intercept is -2 . So, plot $(0, -2)$.



Use the slope to plot another point, $(3, 0)$.

Draw a line through the points.

Practice

Check your answers at BigIdeasMath.com.

Graph the linear equation using intercepts.

1. $x - 4y = -8$

2. $-18x + 9y = 72$

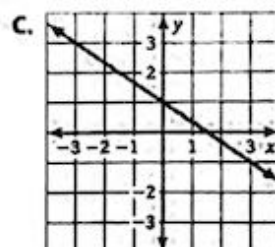
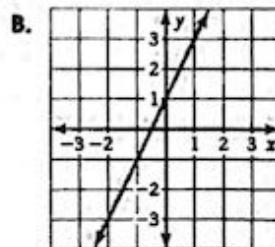
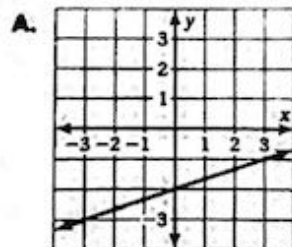
3. $2x - 3y = 12$

Match the equation with its graph. Identify the slope and y -intercept.

4. $y = 2x + 1$

5. $y = \frac{1}{3}x - 2$

6. $y = -\frac{2}{3}x + 1$



Solving Systems of Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown at the right. A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

$$\begin{array}{ll} x + 2y = 5 & \text{Equation 1} \\ x - y = -1 & \text{Equation 2} \end{array}$$

Example 1 Solve the system of linear equations above by (a) graphing, (b) substitution, and (c) elimination.

- a. Graph each equation. The graphs appear to intersect at (1, 2). Check this point.

Equation 1 $x + 2y = 5$

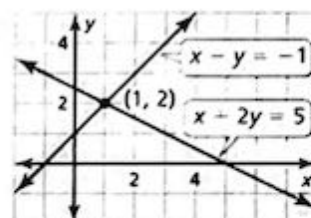
$$1 + 2(2) \stackrel{?}{=} 5$$

$$5 = 5 \quad \checkmark$$

Equation 2 $x - y = -1$

$$1 - 2 \stackrel{?}{=} -1$$

$$-1 = -1 \quad \checkmark$$



► The solution is (1, 2).

- b. Solve for x in Equation 2. $x - y = -1$
 $x = y - 1$

Substitute $y - 1$ for x in Equation 1 and solve for y . $x + 2y = 5$

$$(y - 1) + 2y = 5$$

$$y = 2$$

Substitute 2 for y in Equation 2 and solve for x . $x - y = -1$

$$x - 2 = -1$$

$$x = 1$$

► The solution is (1, 2).

- c. Multiply Equation 2 by 2. Then add the equations and solve the resulting equation.

$$\begin{array}{rcl} x + 2y = 5 & \Rightarrow & x + 2y = 5 \\ x - y = -1 & \Rightarrow & 2x - 2y = -2 \\ \hline 3x = 3 & & \\ x = 1 & & \end{array}$$

Substitute 1 for x in Equation 2 and solve for y . $x - y = -1$

$$1 - y = -1$$

$$2 = y$$

► The solution is (1, 2).

Practice

Check your answers at BigIdeasMath.com.

Solve the system of linear equations by graphing.

1. $y = x - 3$
 $y = -x + 1$

2. $-x + 2y = -1$
 $x + y = 4$

3. $2x + y = 5$
 $4x - 2y = 6$

4. $9x - 3y = 3$
 $3x + y = 1$

Solve the system of linear equations by substitution.

5. $y = 1 - x$
 $-2x + y = 4$

6. $x = y + 3$
 $5x - y = 7$

7. $3x - y = 5$
 $2x - y = -3$

8. $x - 2y = -3$
 $7x - 2y = 15$

Solve the system of linear equations by elimination.

9. $-2x + 2y = -2$
 $2x + y = 5$

10. $x - 4y = -3$
 $4x + y = 5$

11. $x + 5y = -2$
 $5x + y = 14$

12. $2x + 3y = 5$
 $4x + 6y = -10$

Parallel and Perpendicular Lines

Parallel lines are coplanar lines that do not intersect. Nonvertical parallel lines have the same slope. Two lines that intersect to form a right angle are **perpendicular lines**. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1 .

Example 1 Determine which of the lines are parallel and which are perpendicular.

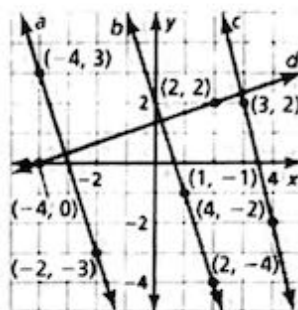
Find the slope of each line.

$$\text{Line } a: m = \frac{3 - (-3)}{-4 - (-2)} = -3$$

$$\text{Line } b: m = \frac{-1 - (-4)}{1 - 2} = -3$$

$$\text{Line } c: m = \frac{2 - (-2)}{3 - 4} = -4$$

$$\text{Line } d: m = \frac{2 - 0}{2 - (-4)} = \frac{1}{3}$$

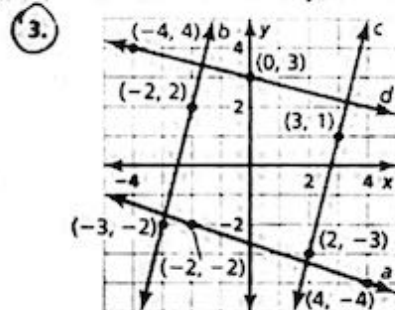
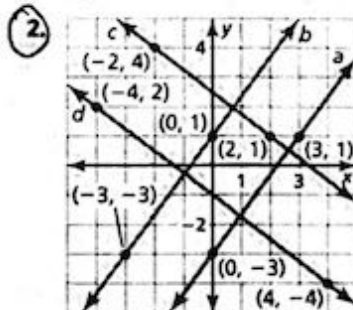
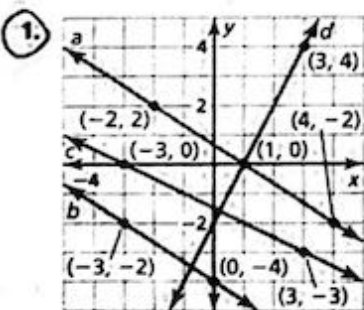


- Because lines a and b have the same slope, lines a and b are parallel. Because $\frac{1}{3}(-3) = -1$, lines a and d are perpendicular and lines b and d are perpendicular.

Practice

Check your answers at BigIdeasMath.com.

Determine which of the lines are parallel and which are perpendicular. Show your work.



4. **GEOMETRY** The vertices of a quadrilateral are $A(-5, 3)$, $B(2, 2)$, $C(4, -3)$, and $D(-2, -2)$. Is the quadrilateral a parallelogram? Explain your reasoning.
5. **GEOMETRY** The vertices of a parallelogram are $J(-5, 0)$, $K(1, 4)$, $L(3, 1)$, and $M(-3, -3)$. Is the parallelogram a rectangle? Explain your reasoning.

Equations of Perpendicular Lines

You can use the slope-intercept form or the point-slope form to write equations of perpendicular lines.

Example 1 Write an equation of the line passing through $(-3, 8)$ that is perpendicular to the line $y = -3x + 4$.

Step 1 Find the slope of the perpendicular line. The graph of the given equation has a slope of -3 . Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line passing through $(-3, 8)$ is $\frac{1}{3}$.

Step 2 Use the slope $m = \frac{1}{3}$ and the slope-intercept form to write an equation of the perpendicular line passing through $(-3, 8)$.

$$y = mx + b$$

Write the slope-intercept form.

$$8 = \frac{1}{3}(-3) + b$$

Substitute $\frac{1}{3}$ for m , -3 for x , and 8 for y .

$$9 = b$$

Solve for b .

► So, an equation of the perpendicular line is $y = \frac{1}{3}x + 9$.

Example 2 Write an equation of the line passing through $(1, -2)$ that is perpendicular to the line $2x - 3y = -9$.

Step 1 Find the slope of the perpendicular line.

$$2x - 3y = -9$$

Write original equation.

$$y = \frac{2}{3}x + 3$$

Solve for y .

The graph of the given equation has a slope of $\frac{2}{3}$. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line passing through $(1, -2)$ is $-\frac{3}{2}$.

Step 2 Find the slope $m = -\frac{3}{2}$ and the point-slope form to write an equation of the perpendicular line passing through $(1, -2)$.

$$y - y_1 = m(x - x_1)$$

Write the point-slope form.

$$y - (-2) = -\frac{3}{2}(x - 1)$$

Substitute $-\frac{3}{2}$ for m , 1 for x_1 , and -2 for y_1 .

$$y + 2 = -\frac{3}{2}(x - 1)$$

Simplify.

► So, an equation of the perpendicular line is $y + 2 = -\frac{3}{2}(x - 1)$.

Practice

Check your answers at BigIdeasMath.com.

Write an equation of the line passing through point P that is perpendicular to the line.

1. $P(-4, 5); y = -4x + 2$

2. $P(6, 2); y = -\frac{1}{3}x + 1$

3. $P(-3, 7); 2x + y = -5$

4. $P(4, -5); y + 2 = \frac{4}{3}(x - 5)$

5. $P(1, 0); y = 8$

6. $P(-6, -1); x = 3$

Name _____ Date _____

Ratios and Proportions

A **proportion** is an equation stating that two ratios are equivalent. Two quantities that form a proportion are **proportional**.

$$\frac{3}{4} = \frac{6}{8}$$

The proportion is read "3 is to 4 as 6 is to 8."

Example 1 Tell whether the ratios form a proportion.

a. $\frac{4}{12}, \frac{6}{18}$

b. $\frac{27}{18}, \frac{30}{21}$

a. Compare the ratios in simplest form, or use the cross products property.

$$\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$$

$$\frac{6}{18} = \frac{6 \div 6}{18 \div 6} = \frac{1}{3}$$

The ratios are equivalent.

► So, $\frac{4}{12}$ and $\frac{6}{18}$ form a proportion.

b. Compare the ratios in simplest form.

$$\frac{27}{18} = \frac{27 \div 9}{18 \div 9} = \frac{3}{2}$$

$$\frac{30}{21} = \frac{30 \div 3}{21 \div 3} = \frac{10}{7}$$

The ratios are *not* equivalent.

► So, $\frac{27}{18}$ and $\frac{30}{21}$ do *not* form a proportion.

?
72 = 72 Yes, so
 $\frac{4}{12}$ and $\frac{6}{18}$ form
a proportion

Practice

Check your answers at BigIdeasMath.com.

Tell whether the ratios form a proportion.

1. $\frac{2}{5}, \frac{3}{15}$

2. $\frac{6}{8}, \frac{15}{20}$

3. $\frac{4}{10}, \frac{2}{6}$

4. $\frac{9}{12}, \frac{21}{28}$

5. $\frac{6}{24}, \frac{7}{28}$

6. $\frac{6}{15}, \frac{9}{36}$

7. $\frac{72}{10}, \frac{36}{8}$

8. $\frac{38}{14}, \frac{57}{21}$

9. $\frac{30}{25}, \frac{16}{12}$

10. $\frac{45}{27}, \frac{75}{45}$

11. $\frac{64}{36}, \frac{56}{38}$

12. $\frac{72}{32}, \frac{63}{28}$

13. **FITNESS** You can do 62 push-ups in 2 minutes. Your friend can do 93 push-ups in 3 minutes. Do these rates form a proportion? Explain.

14. **KAYAKS** You and your friend rent kayaks. Are the rates for renting a kayak proportional? Explain your reasoning.

| | Cost | Hours |
|--------|------|-------|
| You | \$23 | 2 |
| Friend | \$30 | 3 |

Name _____ Date _____

Solving Proportions

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. To solve proportions, use the Cross Products Property.

| Cross Products Property | |
|--|--|
| Words The cross products of a proportion are equal. | |
| Numbers | Algebra |
| $\frac{2}{3} = \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$ | $\frac{a}{b} = \frac{c}{d}$ $ad = bc,$ where $b \neq 0$ and $d \neq 0$ |

Example 1 Solve each proportion.

a. $\frac{x}{6} = \frac{5}{2}$

$x \cdot 2 = 6 \cdot 5$

$2x = 30$

$x = 15$

Cross Products Property

Multiply.

Divide.

► The solution is 15.

b. $\frac{8}{y} = \frac{4}{9}$

$8 \cdot 9 = y \cdot 4$

$72 = 4y$

$18 = y$

► The solution is 18.

Practice

Check your answers at BigIdeasMath.com.

Solve the proportion.

1. $\frac{1}{3} = \frac{x}{6}$

2. $\frac{2}{5} = \frac{y}{10}$

3. $\frac{z}{9} = \frac{2}{3}$

4. $\frac{2}{7} = \frac{j}{14}$

5. $\frac{4}{9} = \frac{k}{36}$

6. $\frac{m}{24} = \frac{3}{8}$

7. $\frac{11}{3} = \frac{p}{6}$

8. $\frac{n}{54} = \frac{8}{3}$

9. $\frac{14}{a} = \frac{7}{2}$

10. $\frac{15}{b} = \frac{3}{5}$

11. $\frac{21}{2} = \frac{42}{d}$

12. $\frac{9}{16} = \frac{27}{g}$

13. $\frac{21}{r} = \frac{7}{5}$

14. $\frac{25}{q} = \frac{5}{2}$

15. $\frac{9}{8} = \frac{36}{s}$

16. $\frac{4}{15} = \frac{20}{t}$

17. $\frac{x}{2.4} = \frac{3.1}{1.2}$

18. $\frac{4.8}{1.5} = \frac{m}{4.5}$

19. $\frac{3.3}{y} = \frac{1.1}{1.6}$

20. $\frac{2.8}{5.4} = \frac{1.4}{c}$

21. **PENCILS** Thirty-six pencils are packaged in 6 boxes. How many pencils are packaged in 10 boxes?
22. **TICKETS** Two tickets cost \$15. How much does it cost to buy seven tickets?
23. **SALADS** Three salads cost \$6.50. How much does it cost to buy six salads?
24. **FIELD TRIP** There are 108 students on a field trip. The ratio of girls to boys is 5 to 4. How many are girls?

Solve each proportion. Leave your answer as a fraction in simplest form.

$$13) \frac{9}{8} = \frac{k+6}{6}$$

$$8(k+6) = 9 \cdot 6$$

$$8k + 48 = 54$$

$$8k = 6$$

$$k = \frac{3}{4}$$

$$15) \frac{10}{p+2} = \frac{4}{3}$$

$$14) \frac{2}{10} = \frac{4}{a-3}$$

$$16) \frac{4}{6} = \frac{8}{x-1}$$

$$17) \frac{m}{8} = \frac{m+7}{9}$$

$$18) \frac{n}{n+1} = \frac{3}{5}$$

$$19) \frac{9}{4} = \frac{r-10}{r}$$

$$20) \frac{x+6}{x} = \frac{10}{7}$$

$$21) \frac{n-9}{n+5} = \frac{7}{4}$$

$$22) \frac{6}{b+9} = \frac{4}{b+5}$$

$$23) \frac{8}{3} = \frac{v-9}{7v+4}$$


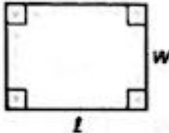
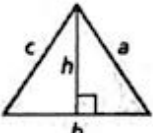
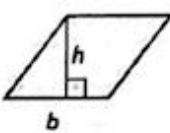
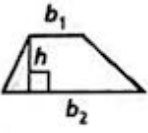
$$24) \frac{8}{5x-4} = \frac{6}{x+5}$$

Perimeter and Area of Figures

Perimeter and Area of Polygons

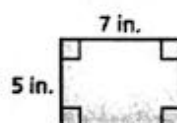
The **perimeter** P of a figure is the distance around the figure. The **area** A of a figure is the number of square units enclosed by the figure.

Perimeter and Area

| Square | Rectangle | Triangle | Parallelogram | Trapezoid |
|---|---|---|--|---|
|  |  |  |  |  |
| $P = 4s$ | $P = 2\ell + 2w$ | $P = a + b + c$ | $A = bh$ | $A = \frac{1}{2}h(b_1 + b_2)$ |
| $A = s^2$ | $A = \ell w$ | $A = \frac{1}{2}bh$ | | |

Example 1 Find the perimeter and area of the figure.

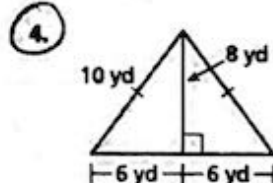
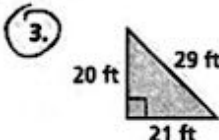
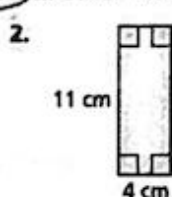
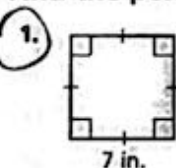
$$\begin{aligned}
 P &= 2\ell + 2w \\
 &= 2(7) + 2(5) \\
 &= 24 \text{ in.}
 \end{aligned}
 \qquad
 \begin{aligned}
 A &= \ell w \\
 &= 7(5) \\
 &= 35 \text{ in.}^2
 \end{aligned}$$



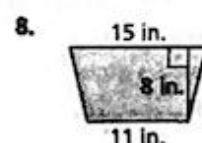
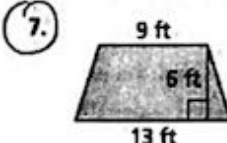
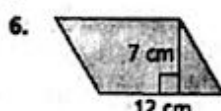
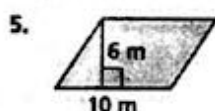
Practice

Check your answers at BigIdeasMath.com.

Find the perimeter and area of the figure.



Find the area of the figure.



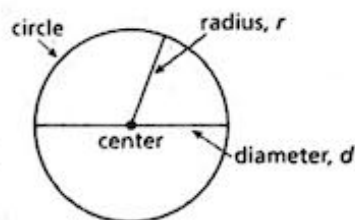
Use a geometric formula to solve the problem.

- A triangle has a base of 7 feet and an area of 63 square feet. Find the height.
- A rectangle has a length of 6 inches and a perimeter of 28 inches. Find the width.

Circumference and Area of a Circle

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**. The distance from the center to any point on the circle is the **radius**. The distance across the circle through the center is the **diameter**. The diameter is twice the radius.

The **circumference** of a circle is the distance around the circle. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for every circle and is represented by the Greek letter π , called **pi**. Pi is an irrational number whose value is approximately 3.14 or $\frac{22}{7}$.



| Circumference of a Circle | Area of a Circle |
|---|---|
| The circumference C of a circle is equal to π times the diameter d or π times twice the radius r . $C = \pi d$ or $C = 2\pi r$ | The area A of a circle is the product of π and the square of the radius. $A = \pi r^2$ |

Example 1 The diameter of a circle is 8.5 meters. Find the radius.

$$\begin{aligned} r &= \frac{d}{2} && \text{Radius of a circle} \\ &= \frac{8.5}{2} && \text{Substitute 8.5 for } d. \\ &= 4.25 && \text{Divide.} \end{aligned}$$

► The radius is 4.25 meters.

Example 2 The radius of a circle is $5\frac{3}{4}$ feet. Find the diameter.

$$\begin{aligned} d &= 2r && \text{Diameter of a circle} \\ &= 2\left(5\frac{3}{4}\right) && \text{Substitute } 5\frac{3}{4} \text{ for } r. \\ &= 11\frac{1}{2} \end{aligned}$$

► The diameter is $11\frac{1}{2}$ feet.

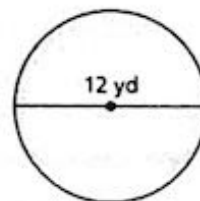
Example 3 Find (a) the circumference C and (b) the area A of the circle.

$$\begin{aligned} \text{a. } C &= \pi d \\ &= \pi(12) \\ &\approx 37.7 \end{aligned}$$

► The circumference is about 37.7 yards.

$$\begin{aligned} \text{b. } A &= \pi r^2 \\ &= \pi \cdot (6)^2 \\ &= 36\pi \\ &\approx 113.1 \end{aligned}$$

► The area is about 113.1 square yards.



Practice

Check your answers at BigIdeasMath.com.

- The radius of a circle is 4.6 millimeters. Find the diameter.
- The diameter of a circle is $2\frac{1}{4}$ miles. Find the radius.

Find the circumference and area of the circle with the given radius or diameter.

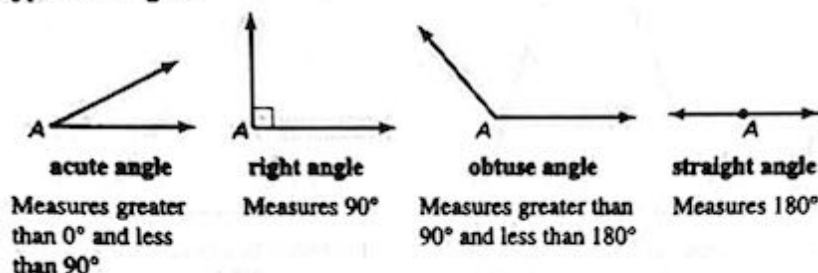
- $r = 16$ inches
- $d = 10$ centimeters
- $r = 7$ meters
- $d = 2.4$ yards

- The area of a circle is 81π square feet. Find the radius.

Measuring and Classifying Angles

A protractor helps you approximate the measure of an angle. You can classify angles according to their measures.

Types of Angles



Example 1 Find the measure of each angle. Then classify the angle.

a. $\angle GHK$

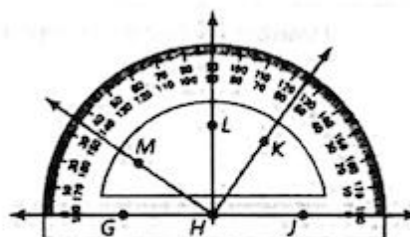
b. $\angle JHL$

c. $\angle LHK$

a. \overline{HG} lines up with 0° on the outer scale of the protractor. \overline{HK} passes through 125° on the outer scale. So, $m\angle GHK = 125^\circ$. It is an *obtuse angle*.

b. \overline{HJ} lines up with 0° on the inner scale of the protractor. \overline{HL} passes through 90° . So, $m\angle JHL = 90^\circ$. It is a *right angle*.

c. \overline{HL} passes through 90° . \overline{HK} passes through 55° on the inner scale. So, $m\angle LHK = |90 - 55| = 35^\circ$. It is an *acute angle*.



Practice

Check your answers at BigIdeasMath.com.

Use the diagram to find the angle measure. Then classify the angle.

1. $\angle BOC$

2. $\angle AOB$

3. $\angle DOB$

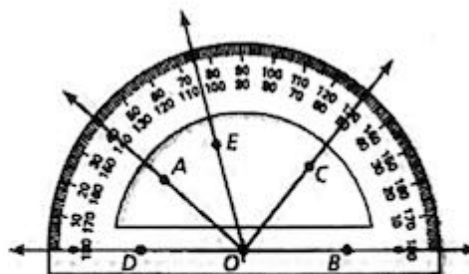
4. $\angle DOE$

5. $\angle AOC$

6. $\angle BOE$

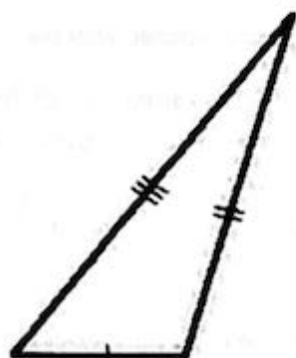
7. $\angle EOC$

8. $\angle COD$



Indicator marks for sides and angles in a triangle diagram

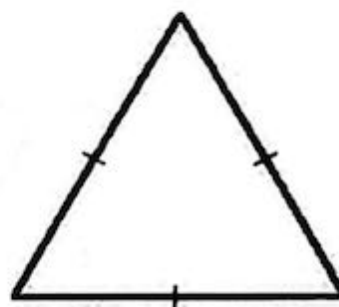
Indicator marks are used to show whether two or more sides are congruent (equal in measure).



Three sides of different lengths.

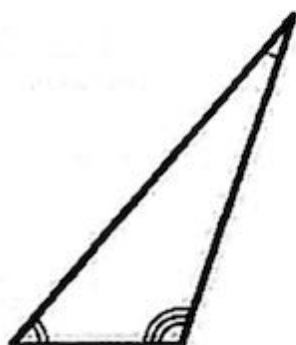


Two sides with the same length, the third side with a different length.

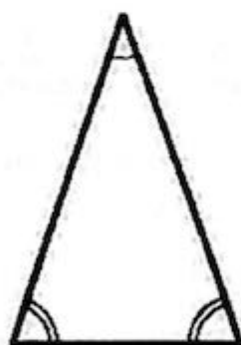


All three sides with the same length.

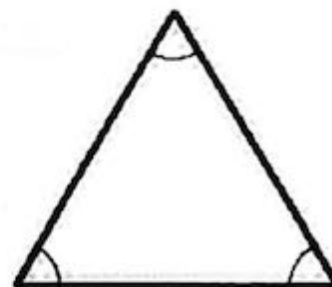
Indicator marks are also used to show whether two or more angles are congruent (equal in measure).



Three angles of different measures.



Two angles with the same measure, the third angle with a different measure.



All three angles with the same measure.

Name _____ Date _____

Classifying Triangles

You can use angle measures and side lengths to classify triangles.

Classifying Triangles Using Angles

acute triangle



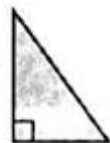
all acute angles

obtuse triangle



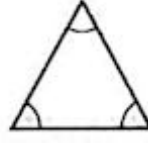
1 obtuse angle

right triangle



1 right angle

equiangular triangle



3 congruent angles

Classifying Triangles Using Sides

scalene triangle



no congruent sides

isosceles triangle



at least 2 congruent sides

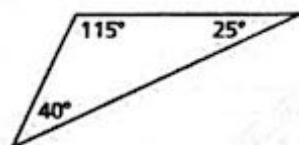
equilateral triangle



3 congruent sides

Example 1 Classify each triangle by its angles and by its sides.

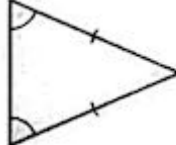
a.



The triangle has one obtuse angle and no congruent sides.

► So, the triangle is an obtuse scalene triangle.

b.



The triangle has all acute angles and two congruent sides.

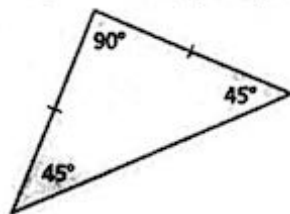
► So, the triangle is an acute isosceles triangle.

Practice

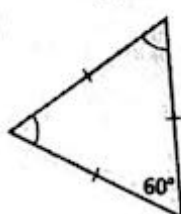
Classify the triangle by its angles and by its sides.

Check your answers at BigIdeasMath.com.

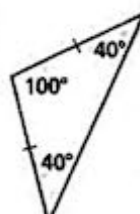
1.



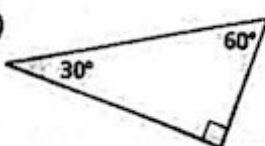
2.



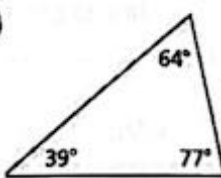
3.



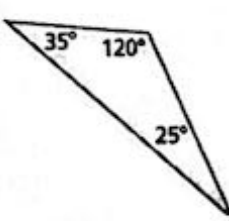
4.



5.



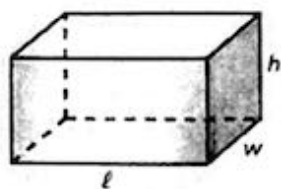
6.



REVIEW: Surface Areas of Prisms

Name _____

Key Concept and Vocabulary

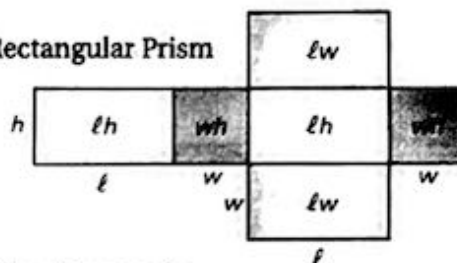


$$S = 2lw + 2lh + 2wh$$

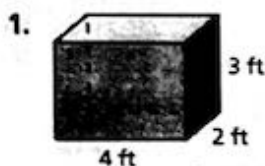


Visual Model

Net for a Rectangular Prism



Skill Example



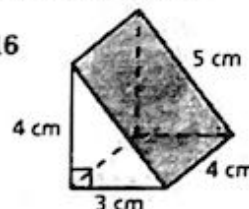
$$\begin{aligned} S &= 2(4 \cdot 2) + 2(4 \cdot 3) + 2(2 \cdot 3) \\ &= 16 + 24 + 12 \\ &= 52 \text{ ft}^2 \end{aligned}$$

Application Example

2. Find the surface area of the block.

$$\begin{aligned} S &= 2\left(\frac{1}{2} \cdot 3 \cdot 4\right) + 4 \cdot 5 + 3 \cdot 4 + 4 \cdot 4 \\ &= 12 + 20 + 12 + 16 \\ &= 60 \text{ cm}^2 \end{aligned}$$

∴ The area is 60 cm².

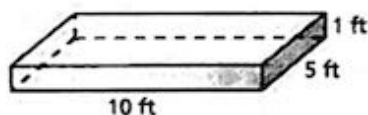


PRACTICE MAKES PURR-FECT™

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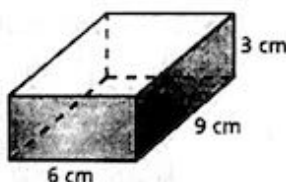
Find the surface area of the prism.

3. Rectangular Prism



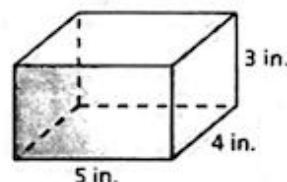
$$S = \underline{\hspace{2cm}}$$

4. Rectangular Prism



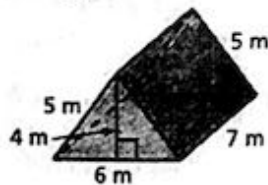
$$S = \underline{\hspace{2cm}}$$

5. Rectangular Prism



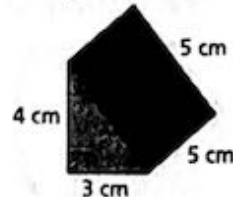
$$S = \underline{\hspace{2cm}}$$

6. Triangular Prism



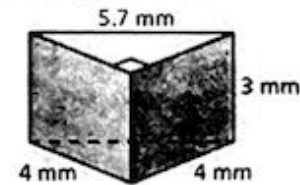
$$S = \underline{\hspace{2cm}}$$

7. Triangular Prism



$$S = \underline{\hspace{2cm}}$$

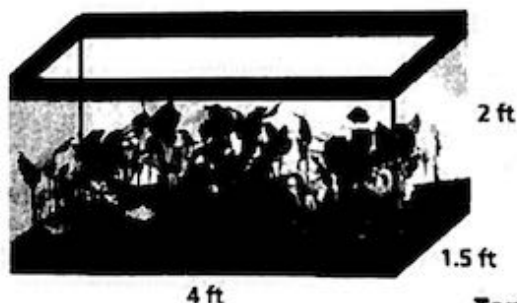
8. Triangular Prism



$$S = \underline{\hspace{2cm}}$$

9. **AQUARIUM** How much glass is used to make the four sides of the aquarium? _____

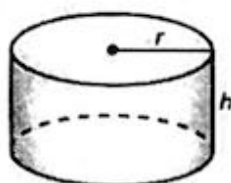
10. **AQUARIUM** How much glass is used to make the base of the aquarium? _____



REVIEW: Surface Areas of Cylinders

Name _____

Key Concept and Vocabulary

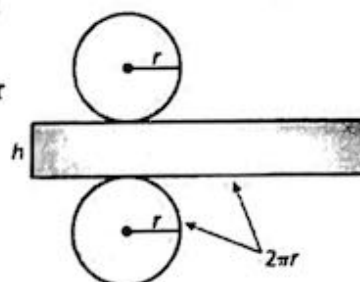


$$S = 2\pi r^2 + 2\pi rh$$

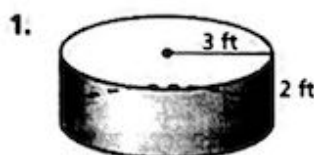


Visual Model

Net for a Circular Cylinder



Skill Example



$$\begin{aligned} S &= 2\pi \cdot 3^2 + 2\pi \cdot 3 \cdot 2 \\ &= 18\pi + 12\pi \\ &= 30\pi \text{ ft}^2 \end{aligned}$$

Application Example

2. Find the surface area of the soup can.

$$\begin{aligned} S &= 2\pi \cdot 1.5^2 + 2\pi \cdot 1.5 \cdot 5 \\ &= 4.5\pi + 15\pi \\ &= 19.5\pi \text{ in.}^2 \end{aligned}$$



✧ The area is 19.5π square inches.

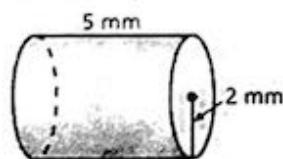
PRACTICE MAKES PURR-FECT™



Check your answers at BigIdeasMath.com.

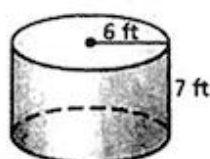
Find the surface area of the circular cylinder.

3. Circular Cylinder



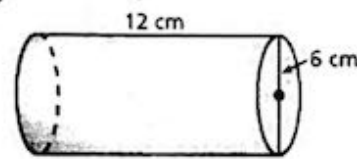
$S =$ _____

4. Circular Cylinder



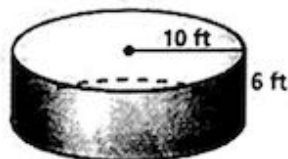
$S =$ _____

5. Circular Cylinder



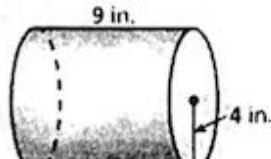
$S =$ _____

6. Circular Cylinder



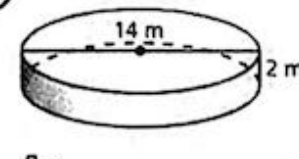
$S =$ _____

7. Circular Cylinder



$S =$ _____

8. Circular Cylinder



$S =$ _____

9. **OIL TANKER TRUCK** The truck's tank is a stainless steel cylinder. How many square feet of stainless steel are needed to make the tank? _____

10. **OIL TANKER TRUCK** What percent of the stainless steel in the tank is used to make the two ends? _____

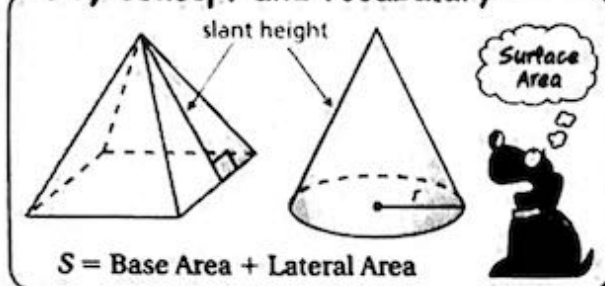


Length = 50 ft
Radius = 4 ft

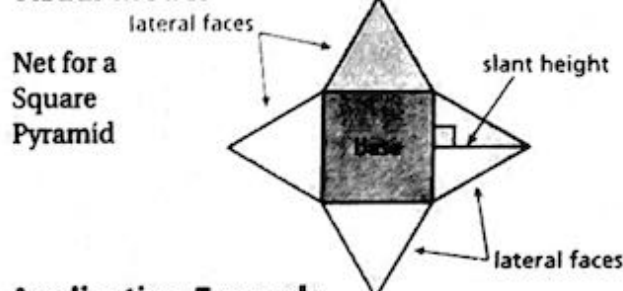
REVIEW: Surface Areas of Pyramids and Cones

Name _____

Key Concept and Vocabulary



Visual Model

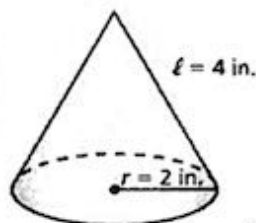


Skill Example

1.

Cone

$$\begin{aligned} S &= \pi r^2 + \pi r \ell \\ &= \pi \cdot 2^2 + \pi \cdot 2 \cdot 4 \\ &= 12\pi \text{ in.}^2 \end{aligned}$$



Application Example

2. Find the lateral surface area of the square pyramid.

$$\begin{aligned} S &= 4 \left(\frac{1}{2} \cdot 40 \cdot 35 \right) \\ &= 2800 \text{ m}^2 \end{aligned}$$



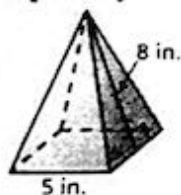
The area is 2800 square meters.

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Check your answers at BigIdeasMath.com.

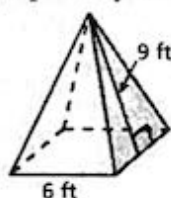
Find the surface area of the pyramid or cone.

3. Square Pyramid



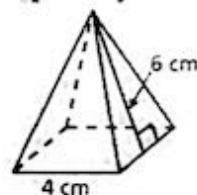
$S =$ _____

4. Square Pyramid



$S =$ _____

5. Square Pyramid



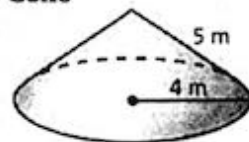
$S =$ _____

6. Cone



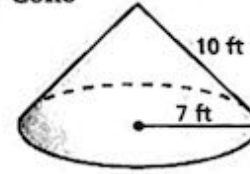
$S =$ _____

7. Cone



$S =$ _____

8. Cone

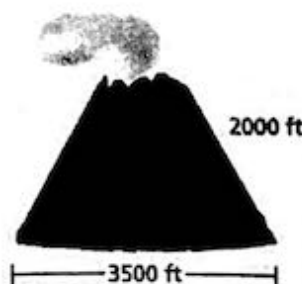


$S =$ _____

9. **VOLCANO** Find the lateral surface area of the volcano.

Use 3.14 for π . Round your answer to the nearest hundred square feet. _____

10. **VOLCANO** Find the area of the circular region covered by the base of the volcano. Use 3.14 for π . Round your answer to the nearest hundred square feet. _____



Geometry Honors Summer Packet

Answer Key

Rewriting Literal Equations

$$\textcircled{8} \ y = \frac{4x}{x+9}$$

$$\textcircled{10} \ y = \frac{3z}{2(x-2)}$$

$$\textcircled{9} \ y = \frac{8x}{3-4z}$$

$$\textcircled{11} \ y = \frac{2}{x-7}$$

Solving Linear Equations

$$\textcircled{2} \text{ a) No } \textcircled{b) \text{ Yes}}$$

$$\textcircled{15} \ m = -1$$

$$\textcircled{3} \text{ a) No } \textcircled{b) \text{ Yes}}$$

$$\textcircled{20} \ y = 18$$

$$\textcircled{14} \ c = 29$$

$$\textcircled{21} \ x = -\frac{1}{2}$$

Properties of Square Roots

$$\textcircled{1} \ 2\sqrt{3}$$

$$\textcircled{4} \ 4\sqrt{7}$$

$$\textcircled{2} \ 2\sqrt{23}$$

$$\textcircled{5} \ 6\sqrt{6}$$

$$\textcircled{3} \ 10\sqrt{5}$$

$$\textcircled{6} \ \frac{\sqrt{10}}{7}$$

Pythagorean Theorem

$$\textcircled{4} \ 17 \text{ yd}$$

$$\textcircled{6} \ 0.8 \text{ m}$$

$$\textcircled{5} \ 12 \text{ mm}$$

$$\textcircled{7} \ 7 \text{ in.}$$

Slope-Intercept Form

④ slope = -5
y-int (0, -9)

⑥ slope = $-\frac{2}{3}$
y-int (0, -7)

⑦ x-int (0, 0)
y-int (0, 0)

⑨ x-int (-2, 0)
y-int (0, 6)

⑪ x-int (6, 0)
y-int (0, 4)

Writing Linear Equations

① $y = 2x + 1$

⑤ $y = -\frac{1}{4}x - 1$

⑥ $y = -\frac{2}{7}x - 5$

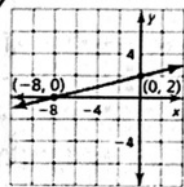
⑧ $y - 4 = 2(x + 3)$

⑪ $y - 5 = -\frac{3}{4}(x + 4)$

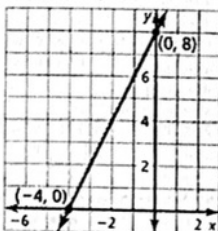
Graphing Linear Equations

Graph the linear equation using intercepts.

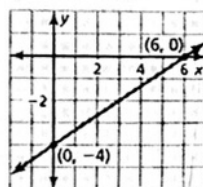
① $x - 4y = -8$



② $-18x + 9y = 72$

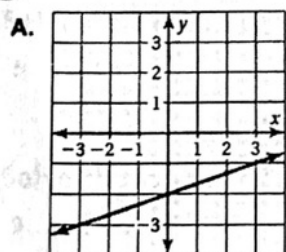


③ $2x - 3y = 12$



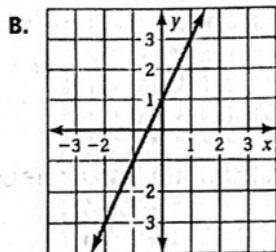
Match the equation with its graph. Identify the slope and y-intercept.

④ $y = 2x + 1$



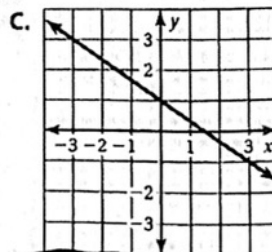
B; slope: 2, y-intercept: 1

⑤ $y = \frac{1}{3}x - 2$



A; slope: $\frac{1}{3}$, y-intercept: -2

⑥ $y = -\frac{2}{3}x + 1$



C; slope: $-\frac{2}{3}$, y-intercept: 1

Solving Systems of Equations

② (3, 1)

⑦ (8, 19)

⑫ No solution

③ (2, 1)

⑧ (3, 3)

⑥ (1, -2)

⑩ (1, 1)

Parallel + Perpendicular Lines

① $a \parallel b ; c \perp d$

② $a \parallel b ; c \parallel d ; a \perp c ; a \perp d ; b \perp c ; b \perp d$

③ $b \parallel c ; b \perp d ; c \perp d$

Equations of Perpendicular Lines

① ~~$y = 5 = \frac{1}{4}x + 6$~~ $y = \frac{1}{4}x + 6$

④ $y = -\frac{3}{4}x - 2$

⑤ $x = 1$

② $y = 3x - 16$

⑥ $y = -1$

③ $y = \frac{1}{2}x + \frac{17}{2}$

Ratios + Proportions

④ Yes

⑬ Yes, both can do
31 pushups per
minute

⑤ Yes

⑦ No

Solving Proportions

⑧ 144

⑪ 60

⑭ 60

⑩ 25

⑫ \$52.50

⑬ 14.4

⑮ \$13.00

Solving Proportions (continued)

⑭ $a = 23$

⑰ $m = 56$

⑮ $p = \frac{11}{2}$

⑳ $b = 3$

⑯ $x = 13$

Perimeter + Area of Figures

① $P = 28 \text{ in.}$
 $A = 49 \text{ in}^2$

④ $P = 32 \text{ yd.}; A = 48 \text{ yd}^2$

③ $P = 70 \text{ ft.}$
 $A = 210 \text{ ft}^2$

⑦ 66 ft^2

Circumference + Area

⑬ $C = 32\pi$ inches; $A = 256\pi \text{ in}^2$

⑭ $C = 10\pi \text{ cm}; A = 25\pi \text{ cm}^2$

⑰ $r = 9 \text{ ft.}$

Measuring + Classifying Angles

① 50° , acute

④ 75° , acute

⑦ 55° , acute

② 140° , obtuse

⑤ 90° , right

⑧ 130° , obtuse

③ 180° , straight

⑥ 105° , obtuse

Classifying Triangles

① right isosceles

④ right scalene

② ~~obtuse isosceles~~ equilateral/
equiangular

⑤ acute scalene

③ obtuse isosceles

⑥ obtuse scalene

Surface Areas of Prisms

$$\textcircled{4} S = 198 \text{ cm}^2$$

$$\textcircled{7} S = 72 \text{ cm}^2$$

$$\textcircled{6} S = 136 \text{ m}^2$$

$$\textcircled{8} S = 57.1 \text{ mm}^2$$

Surface Areas of Cylinders

$$\textcircled{4} S = 156\pi \text{ ft}^2$$

$$\textcircled{5} S = 90\pi \text{ cm}^2$$

$$\textcircled{8} S = 126\pi \text{ m}^2$$

Surface Area of Pyramids + Cones

$$\textcircled{4} S = 144 \text{ ft}^2$$

$$\textcircled{6} 27\pi \text{ in}^2$$