

MATH: Key Words and Converting Words to Equations/Fractions  
**GENERAL MATH**

**KEY WORDS AND CONVERTING WORDS TO EQUATIONS**

Sometimes math questions use key words to indicate what operation to perform. Becoming familiar with these key words will help you determine what the question is asking for.

OPERATION	OTHER WORDS WHICH INDICATE THE OPERATION
<b>Addition</b>	increased by; more than; combined together; total of; sum; added to. The symbol + means add
<b>Subtraction</b>	decreased by; minus; less; difference between/of; less than; fewer than. The symbol - means subtract
<b>Multiplication</b>	of; times; multiplied by; product of (For example: 4 + 4 + 4 equals 4 x 3). The symbols x and • both mean multiply.
<b>Division</b>	per; a; out of; ratio of; quotient of; percent (divide by 100). The symbol ÷ means divide.
<b>Equal</b>	is; are; was; will be; gives; yields; sold for The symbol = means equal.
<b>Per</b>	divided by
<b>Percent</b>	divide by 100 The symbol % means percent.

Here are some examples of words converted to equations.

WORDS	EQUATIONS
What is the sum of 8 and y?	$8 + y$
4 less than y	$y - 4$
y multiplied by 13	$13y$
The quotient of y and 3	$y / 3$
The difference of 5 and y	$5 - y$
The ratio of 9 more than y to y	$(y + 9) / y$
Nine less than the total of a number (y) and two	$(y + 2) - 9$ or $y - 7$

**FRACTIONS**

In order to accurately solve fraction problems, it is important to distinguish between the numerator and denominator.

**Numerator:** top number

**Denominator:** bottom number

**ADDING OR SUBTRACTING FRACTIONS**

Adding or subtracting fractions with the same denominator is straightforward.

<b>SAMPLE</b>	$\frac{5}{13} + \frac{6}{13}$
<i>The denominator for both is common so they remain the same. Add the top numbers</i>	$\frac{5 + 6}{13}$
<b>Answer</b>	$\frac{11}{13}$

If you do not have a common denominator (see *SIMPLIFYING FRACTIONS*, below), make one by multiplying the first denominator and the second denominator together.

<b>SAMPLE</b>	$\frac{3}{5} + \frac{2}{7}$
<i>Find the common denominator by multiplying five by seven</i>	$5 \cdot 7 = 35$
<i>To get new numerators, multiply the numerator by the same number as the denominator was multiplied by</i>	$3 \cdot 7 = 21$ $2 \cdot 5 = 10$
<i>Insert the new numbers into the numerator and add the fractions</i>	$\frac{21}{35} + \frac{10}{35}$
<b>Answer</b>	$\frac{31}{35}$

### MULTIPLYING FRACTIONS

Multiply the numerator times the numerator and the denominator by the denominator.

<b>SAMPLE</b>	$\frac{1}{4} \cdot \frac{3}{5}$
	$\frac{1 \cdot 3}{4 \cdot 5}$
<b>Answer</b>	$\frac{3}{20}$

Simplify the fraction (see *SIMPLIFYING FRACTIONS*, below) before and after you multiply; this will simplify the problem. (The problem may be calculated without simplifying the fractions, but it will be harder to simplify at the end.)

<b>SAMPLE</b>	$\frac{12}{15} \cdot \frac{5}{6}$
<i>Simplify <math>\frac{12}{15}</math> by dividing both numbers by 3</i>	$\frac{12 \div 3}{15 \div 3} \cdot \frac{5}{6}$
<i>Multiply both numerators and both denominators</i>	$\frac{4}{5} \cdot \frac{5}{6}$
	$\frac{20}{30}$
<i>Simplify by dividing by 10</i>	$\frac{20 \div 10}{30 \div 10}$
<b>Answer</b>	$\frac{2}{3}$

### DIVIDING FRACTIONS

Since division is the opposite of multiplication, first invert (flip over) one fraction, then multiply.

<b>SAMPLE</b>	$\frac{1}{5} \div \frac{2}{3}$
<i>Invert <math>\frac{2}{3}</math> and multiply</i>	$\frac{1 \cdot 3}{5 \cdot 2}$
<b>Answer</b>	$\frac{3}{10}$

**SIMPLIFYING FRACTIONS**

Try dividing both the numerator and the denominator by each prime number.

- Use the rules of divisibility.
- Start with 2: Even numbers (ones that end with 2, 4, 6, 8, or 0) can be divided by two without a remainder.
- Then go to 3: Find the sum of the digits (add the digits together). If the sum can be divided by three, then the number is divisible by 3.
- Next try 5: Numbers that end with 5 or 0 are divisible by five.
- Go on to 7, 11, 13, 17, and so on: Unfortunately, there is no easy way to determine whether the number will be divisible by these – you just have to try dividing by each. But you can stop trying when the number is smaller than the divisor.

SAMPLE	Simplify $\frac{26}{65}$	
<i>Twenty-six can be divided by two without a remainder (because it is even), but 65 can't</i>	$26 \div 2 = 13$ $65 \div 2 = 32.5$	YES NO
<i>The digits do not add up to three</i>	$2 + 6 = 8 \div 3$ $6 + 5 = 11 \div 3$	NO NO
<i>Sixty-five can be divided by five without a remainder, but 26 can't</i>	$65 \div 5 = 13$ $26 \div 5 = 5.2$	YES NO
<i>Try 7</i>	$26 \div 7 = 3.7$ $65 \div 7 = 9.3$	NO NO
<i>Try 11</i>	$26 \div 11 = 2.4$ $65 \div 11 = 5.9$	NO NO
<i>Try 13 - and it works!</i>	$26 \div 13 = 2$ $65 \div 13 = 5$	YES YES
<b>Answer</b>	$\frac{2}{5}$	

**WRITING A DECIMAL AS A FRACTION**

SAMPLE	Decimal	Fraction	Final Answer
<i>For a number in the tenths place, remove the decimal, divide by ten, and simplify</i>	.5	$\frac{5}{10}$	$\frac{1}{2}$
<i>For a number in the hundredths place, remove the decimal, divide by 100, and simplify</i>	.05	$\frac{5}{100}$	$\frac{1}{20}$
<i>For a number in the thousandths place, remove the decimal, divide by 1,000, and simplify</i>	.005	$\frac{5}{1,000}$	$\frac{1}{200}$

**MIXED NUMBERS & IMPROPER FRACTIONS**

- A mixed number contains a whole number and a fraction. When the numerator is more than the denominator, it is an improper fraction. Solving equations containing mixed numbers is easier when all mixed numbers are converted to improper fractions.
- A whole number can be converted to an improper fraction by simply making the denominator one.

SAMPLE	$5 = \frac{5}{1}$
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- When working with improper fractions, all the same rules of working with fractions apply

**CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION**

- Multiply the whole number by the fraction's denominator
- Add that total to the numerator
- That result is the new numerator and the denominator remains the same.

<b>SAMPLE</b>	<b>Convert to Improper Fraction</b>	<b><math>3 \frac{4}{5}</math></b>
<i>Multiply the whole number and the fraction's denominator</i>	$3 \cdot 5 = 15$	
<i>Add the numerator</i>	$15 + 4 = 19$	
<i>Put the total above the denominator</i>	$\frac{19}{5}$	
<i>Improper Fraction</i>	$\frac{19}{5}$	

**CONVERTING AN IMPROPER FRACTION TO A MIXED NUMBER**

- Divide the numerator by the denominator and calculate the whole number including the remainder
- The whole number will be the mixed number's whole number
- The remainder will be the mixed number's numerator

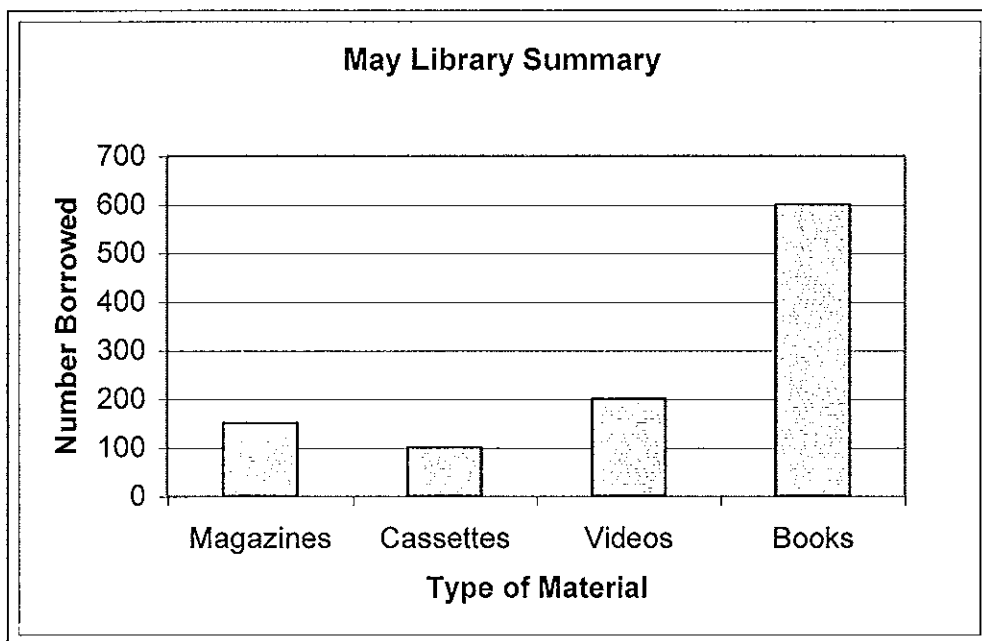
<b>SAMPLE</b>	<b>Convert to Whole Number</b>	<b><math>\frac{19}{5}</math></b>
<i>Divide the numerator by the denominator</i>	$19 \div 5 = \text{Whole number} = 3$ Remainder = 4	
<i>Use the whole number as the mixed number's whole number and the remainder as the mixed number's numerator</i>	$3 \frac{4}{5}$	
<i>Mixed Number</i>	$3 \frac{4}{5}$	

**READING TABLES AND CHARTS**

Some questions test the ability to understand, interpret, and use information in tables and charts. Often answering these questions depends on looking in the correct places for information. It is important to know that the horizontal row at the bottom is the x-axis and the vertical column on the left side is the y-axis.

**SAMPLES**

Use this chart to answer the questions that follow.



According to the May Library Summary chart, what was the number of videos borrowed in May?

- a. 100
- b. 150
- c. 200
- d. 300

**Solution:** This question requires you to extract information from the chart.

- Begin by determining what information the question asks for.
- Look for the number borrowed (on the left vertical column or y axis) for videos (a type of material found on the horizontal row at the bottom or x axis).
- Find the videos column, then look to the left and read the number that lines up with the top of the videos column.
- Choice a is the number of cassettes borrowed in May; therefore, this choice is incorrect.
- Choice b is the number of magazines that was borrowed in May; therefore, this choice is incorrect.
- Choice c is correct because the top of the "videos" bar meets the line for 200.
- Choice d does not correspond to any of the numbers on the chart; therefore, this choice is incorrect.

According to the May Library Summary chart, what percent of the total number of items borrowed were cassettes and videos?

- a. 9.52 %
- b. 19.05 %
- c. 28.57 %
- d. 35.00 %

**Solution:** This question requires you to extract information from the chart and then calculate a percentage.

- Begin by determining what information the question asks for: cassettes plus videos compared to the total number of materials borrowed.
- Add up the number borrowed in each column to obtain the total ( $150 + 100 + 200 + 600 = 1,050$ ).
- Find the cassettes and videos total ( $100 + 200 = 300$ ).
- Divide the cassettes and videos total by the total number borrowed and multiply by 100 ( $300 \div 1,050 = .2857 \cdot 100 = 28.57\%$ ).
- Choice a is only the percentage of cassettes borrowed in May; therefore, this choice is incorrect.
- Choice b is only the percentage of videos borrowed in May; therefore, this choice is incorrect.
- Choice c is correct because the number of cassettes and videos (300) divided by the total (1,050) and multiplied by 100 is equal to 28.57.
- Choice d is only the answer if, in the last step, 1,050 is divided by 300 and then multiplied by 10; therefore this choice is incorrect.

**STATISTICS**

Find the average (also known as the mean) by adding the sum of the data and dividing that sum by the number of data elements.

<b>SAMPLE</b>	<b>What is the average of 79, 67, 81, 99, 88, and 72?</b>
<i>The six data elements</i>	79, 67, 81, 99, 88, and 72
<i>Add the numbers</i>	$79 + 67 + 81 + 99 + 88 + 72 = 486$
<i>Divide by the number of data elements</i>	$486 \div 6$
<b>Answer</b>	<b>81</b>

**EXPONENTS**

An exponent is a superscript, or small number written at the top right corner of a number, variable, or parenthesis (for example:  $3^4$ ). This tells you to multiply 1 by the number as many times as the exponent says.

<b>SAMPLE</b>	<b>Simplify <math>3^4</math></b>
<i>Multiply one by three multiplied by itself four times</i>	$1 (3 \cdot 3 \cdot 3 \cdot 3)$
	$1 (81)$
<b>Answer</b>	<b>81</b>

When multiplying exponents, add the superscripts

<b>SAMPLE</b>	<b>Simplify <math>x^{16}x^2</math></b>
<i>Add the superscripts <sup>16</sup> and <sup>2</sup></i>	$x^{16+2}$
<b>Answer</b>	$x^{18}$

When dividing exponents, subtract the superscripts:

<b>SAMPLE</b>	<b>Simplify</b>	$\frac{x^6}{x^2}$
<i>Subtract the superscripts <sup>6</sup> and <sup>2</sup></i>	$x^{6-2}$	
<b>Answer</b>	$x^4$	

## PRE-ALGEBRA AND ALGEBRA

### SPECIAL NOTATION FOR MULTIPLICATION AND DIVISION WITH VARIABLES

Here are some examples of special notations and what they mean:

- $2b$  means  $2 \cdot b$
- $2(a + 5)$  means twice the sum of a number ( $a$ ) and five (add a number, represented by the letter  $a$ , to 5, then multiply by 2) or  $(2 \cdot a) + (2 \cdot 5)$
- $bc$  means  $b \cdot c$
- $4bc$  means  $4 \cdot b \cdot c$
- $d/5$  means  $d \div 5$

### ALGEBRA WORD PROBLEMS

In algebra you solve problems by essentially making two groups, one for each side of an equation. An unknown number or value is represented by a letter (for example:  $x$ ).

#### BASIC STEPS

- 1) Define the variable
- 2) Translate the problem into an equation and plug in known values
- 3) Set all know values equal to  $x$
- 4) Solve the equation
- 5) Go back to the problem and plug in the new value to obtain the answer

<b>SAMPLE #1</b>	<b>A car dealership has 15 new cars and 12 used cars. How many cars do they have?</b>
<i>Define the unknown variable</i>	Let $x$ = Total Cars
<i>Translate the problem into an equation and insert known values</i>	$15 + 12 = x$
<i>All know values are already equal to <math>x</math> so solve the equation</i>	$27 = x$
<b>Answer</b>	<b>There are 27 Total Cars.</b>

<b>SAMPLE #2</b>	<b>Tickets to the concert are \$20 each. If you spent \$200, how many tickets did you buy?</b>
<i>Define the unknown variable</i>	Let $x$ = The number of tickets you purchased
<i>Translate the problem into an equation and insert known values</i>	$\$20x = \$200$
<i>Set all known values equal to <math>x</math> by dividing \$20 from both sides of the equation</i>	$\$20x \div \$20 = \$200 \div \$20$
<i>Setting all known values equal to <math>x</math> has also solved the equation</i>	$x = 10$
<b>Answer</b>	<b>10 tickets were purchased</b>

**ORDER OF OPERATIONS**

1. Parenthesis and Brackets from the inside out.
2. Exponents of numbers or parenthesis.
3. Multiplication and Division in the order they appear.
4. Addition and Subtraction in the order they appear.

<b>SAMPLE</b>	<b>Simplify the following expression: <math>2 + (3 - 1)3^2</math></b>
<i>Simplify parenthesis &amp; brackets from the inside out (subtract 1 from 3)</i>	$2 + (3 - 1) 3^2 = 2 + (2) 3^2$
<i>Simplify exponents (<math>3^2</math> becomes 9)</i>	$2 + (2) 3^2 = 2 + (2) 9$
<i>Simplify multiplication and division (multiply 2 and 9)</i>	$2 + (2) 9 = 2 + 18$
<i>Simplify addition and subtraction (combine like terms)</i>	$2 + 18 = 20$
<b>Answer</b>	<b>20</b>

**SIMPLIFYING EXPRESSIONS**

1. Combine like terms; 2. Simplify multiplication; 3. Distribute a number or sign in to parenthesis;
4. Use the FOIL Method to multiply two or more parenthesis; 5. Simplify Exponents of a number

**1. Combine Like Terms**

- Combine or add up all of the like terms.

Examples of like terms because they are all:

- x with a coefficient
  - 2x, 45x, x, 0x, -26x, -x
- Constants
  - 15, -2, 27, 9043, 0.6
- $y^2$  with a coefficient
  - $3y^2$ ,  $y^2$ ,  $-y^2$ ,  $26y^2$

For comparison, below are a few examples of unlike terms because they:

- are different letter variables
  - 17x, 17z
- are different powers or exponents
  - 15y,  $19y^2$ ,  $31y^5$
- both have the letter x but the second term has another variable in it
  - 19x, 14xy

<b>SAMPLE</b>	<b><math>5x + 7x</math></b>
<i>Add like terms</i>	$5x + 7x = 12x$
<b>Answer</b>	<b>12x</b>

<b>SAMPLE</b>	<b><math>14a + 7 + 21a</math></b>
<i>Organize like terms together</i>	$14a + 21a + 7$
<i>Add like terms</i>	$14a + 21a + 7 = 35a + 7$
<b>Answer</b>	<b><math>35a + 7</math></b>



## 2. Simplify Multiplication

### Same Variables

When multiplying same letter variables, keep the letter and add exponents.

<b>SAMPLE</b>	$a \times a$
<i>Neither a has a visible exponent, so their exponents are both 1</i>	$a^1 \cdot a^1$
<i>Add the exponents</i>	$a^{1+1} = a^2$
<b>Answer</b>	$a^2$

### Different Variables

<b>SAMPLE</b>	$y^5 \cdot a^2$
<i>The terms cannot be multiplied by simply adding the exponents because each multiplier is a different letter</i>	$y^5 \cdot a^2$
<b>Answer</b>	$y^5 a^2$

<b>SAMPLE</b>	$a^2 \cdot a^3 y^2$
<i>Add the exponents of <math>a^2</math> and <math>a^3</math></i>	$a^5 \cdot y^2$
<b>Answer</b>	$a^5 y^2$

## 3. Distribute a Number or Sign in to Parenthesis

<b>SAMPLE</b>	$6(2 + 4a)$
<i>Remove parentheses and multiply each term by six</i>	$(6 \cdot 2) + (6 \cdot 4a)$
<b>Answer</b>	$12 + 24a$

## 4. Use the FOIL Method to multiply two or more parenthesis

- First; Outer; Inner; Last

<b>SAMPLE</b>	$(3 + 7x)(6 + 2x)$
<i>Multiply the first term</i>	$(3 + 7x)(6 + 2x) = 6 \cdot 3 = 18$ 18
<i>Multiply the outer terms</i>	$(3 + 7x)(6 + 2x) = 3 \cdot 2x = 6x$ 18 + 6x
<i>Multiply the inner terms</i>	$(3 + 7x)(6 + 2x) = 7x \cdot 6 = 42x$ 18 + 6x + 42x
<i>Multiply the last terms</i>	$(3 + 7x)(6 + 2x) = 7x \cdot 2x = 14x^2$ 18 + 6x + 42x + 14x <sup>2</sup>
<i>Combine like terms</i>	$18 + 6x + 42x + 14x^2 = 18 + 48x + 14x^2$
<b>Answer</b>	$18 + 48x + 14x^2$

## 5. Simplify Exponents of a number.

- See the EXPONENTS section for a review

**PRIME FACTORIZATION****WAYS TO OBTAIN THE PRIME FACTOR**

- Repeatedly divide by prime numbers. A prime number is a positive integer greater than one that can only be divided by itself and one. Some examples are 2,3,5,7,11,13,17, and 19. 1 is NOT a prime number
- Choose any pair of factors and split these factors until all the factors are prime.
- Work backwards from the answers, seeing which one is BOTH only prime numbers, *and* produces the correct product

<b>SAMPLE</b>	<b>What is the prime factorization for 68?</b>
<i>Divide by 2 (a prime number)</i>	$68 \div 2 = 34$
<i>The correct way to represent prime factorization</i>	$2 \times 34$
<i>Divide 34 by 2 and you are left with 17 (a prime number)</i>	$34 \div 2 = 17$
<b>Answer</b>	$2 \times 2 \times 17$

**GREATEST COMMON FACTOR (GCF/GCD)**

The **greatest common factor** is the largest integer that is a common factor of all the given integers.

**FIND THE GCF BY:**

- Finding the prime factorization of each integer.
- The GCF is the product of all prime factors common to every number.

<b>SAMPLE</b>	<b>What is the greatest common factor of 8 and 44?</b>
<i>Find the prime factorization of each integer</i>	$8 = 2 \times 2 \times 2$ $44 = 2 \times 2 \times 11$
<i>Identify the common prime factors</i>	$8 = 2 \times 2 \times 2$ $44 = 2 \times 2 \times 11$
<i>Multiply the common prime factors</i>	$2 \cdot 2 = 4$
<b>Answer</b>	<b>4</b>

**LEAST COMMON DENOMINATOR (LCD/LCM) (sample on next page)**

The **least common denominator** (multiple) is the smallest integer that is a common multiple (denominator) of the given integers.

**FIND THE LCD BY:**

- Finding the prime factorization of each integer.
- Take the greatest power on each prime and multiply them to obtain the LCD.

SAMPLE	What is the least common denominator of 12, 50, and 90?
<i>Find the prime factorization of each integer</i>	12 = $2 \times 2 \times 3$ 50 = $2 \times 5 \times 5$ 90 = $2 \times 3 \times 3 \times 5$
<i>Identify the prime factor that appears most frequently within the prime factorization</i>	12 = $2 \times 2 \times 3$ 50 = $2 \times 5 \times 5$ 90 = $2 \times 3 \times 3 \times 5$
<i>List the most frequent prime factors with their exponent as the number of times it appeared</i>	12: $2^2$ 50: $5^2$ 90: $3^2$
<i>Multiply the prime factors</i>	$2^2 \cdot 5^2 \cdot 3^2$
<b>Answer</b>	<b>900</b>

## FACTORING

Factoring is writing a math expression as a product of factors. For example: writing 14 as (2)(7), where 2 and 7 are factors of 14. Factoring can also be done with trinomial and polynomial expressions.

- **Always factor as much as you can!** Often all terms in an expression have a common factor. FIRST group the like terms, and then find the greatest common factor and extract it (this is like the distributive law in reverse).

SAMPLE	Factor $5x + 7x$
<i>x is a factor of both 5x and 7x, extract x and add the contents of the parentheses</i>	$x(5 + 7)$
<b>Answer</b>	<b><math>12x</math></b>

SAMPLE	Factor $14a + 7 + 21a$
<i>Organize like terms</i>	$14a + 21a + 7$
<i>Since a is a factor of both 14a and 21a, extract a and add the content of the parentheses</i>	$a(14 + 21) + 7$
<b>Answer</b>	<b><math>35a + 7</math></b>

## TRINOMIALS

- **Always factor as much as you can!** Often all terms in an expression have a common factor, first group the like terms and then find the greatest common factor and extract it (this is like the distributive law in reverse).
- Next Reverse the FOIL method to get the factored form:
  1. Set up a product of two expressions, where parentheses hold each of the two expressions.
  2. Find the factors that go in the first positions.
  3. Look at the signs before the second and third terms in the trinomial:
    - two negative signs (for example:  $x^2 - 2x - 3$ ): the signs in each expression are opposite with the larger number being negative
    - two positive signs (for example:  $x^2 + 4x + 3$ ): the signs are both positive
    - negative then a positive (for example:  $x^2 - 4x + 3$ ): the signs are both negative
    - positive and negative (for example:  $x^2 + 2x - 3$ ): the signs are opposite and the larger number is positive
  4. Find the factors that go in the last positions.
- Check your work

<b>SAMPLE</b>	<b>Factor the trinomial:</b> $x^2 - 4x - 32$
Reverse the FOIL method to get the factored form	$x^2 - 4x - 32$
$x$ multiplied $x$ equals $x^2$ thus one $x$ in each parenthesis	$(x +/- \underline{\quad})(x +/- \underline{\quad})$
Since the signs on the 2 <sup>nd</sup> and 3 <sup>rd</sup> trinomial terms are both negative, the signs of the second term in each factor must be opposite	$(x - \underline{\quad})(x + \underline{\quad})$
What two numbers multiplied by one another would equal 32?	Possibilities: 32 and 1 16 and 2 8 and 4
Since the signs on the 2 <sup>nd</sup> and 3 <sup>rd</sup> trinomial terms are both negative, the larger multiple will have a negative sign	Possibilities: -32 and 1 -16 and 2 - 8 and 4
Out of the possibilities, which pair added to one another equals -4?	- 8 and 4
Plug these into the equation	$(x - 8)(x + 4)$
Check your work by using FOIL on the two factors	$x^2 - 8x + 4x - 32$
Simplify	$x^2 - 4x - 32$
<b>Answer</b>	$(x - 8)(x + 4)$

### POLYNOMIALS STRATEGIES

- Always factor as much as you can! Often all terms in an expression have a common factor, first group the like terms and then find the greatest common factor and extract it (this is like the distributive law in reverse).
- Look for perfect squares:  
 $a^2 + 2ab + b^2 = (a + b)^2$   
 $a^2 - 2ab + b^2 = (a - b)^2$
- Look for the difference of squares:  
 $a^2 - b^2 = (a + b)(a - b)$
- Others  
 $(a + b) \times c = ac + bc$   
 $(a - b) \times c = ac - bc$   
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$   
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Factor by trial and error
- Reverse the FOIL method to get the factored form:
  1. Set up a product of two expressions, where parentheses hold each of the two expressions.
  2. Find the factors that go in the first positions.
  3. Find the factors that go in the last positions.
- Check your work

<b>SAMPLE</b>	<b>Factor <math>a^2 - 81</math></b>
Since $9^2 = 81$ , this looks like a difference of squares	$(a + 9)(a - 9)$
Check your work by using FOIL	$a^2 + 9a - 9a - 81$
The positive and negative $9a$ cancel each other out	$a^2 - 81$
<b>Answer</b>	$(a + 9)(a - 9)$

<b>SAMPLE</b>	<b>Factor <math>3x^2 + 9x + 6</math></b>
Factor out 3	$3(x^2 + 3x + 2)$
Factor by trial and error, since the second and third term in the parentheses are positive, the signs in each factor must be positive	$3(x + \underline{\quad})(x + \underline{\quad})$

<i>What two numbers multiplied by one another would equal 2 and added to one another would equal 3? How about 2 and 1?</i>	$3(x + 2)(x + 1)$
<i>Check your work by using FOIL on the two factors in parentheses</i>	$3(x^2 + 2x + 1x + 2)$
<i>Simplify</i>	$3(x^2 + 3x + 2)$
<i>Distribute the 3</i>	$3x^2 + 9x + 6$
<b>Answer</b>	$3(x + 2)(x + 1)$

**SAMPLE ALGEBRA PROBLEMS**

<b>SAMPLE</b>	Name the like terms in $7s + 9y + y$
<b>Answer</b>	$9y, y$

<b>SAMPLE</b>	Explain why $7a + 8z - 9x$ is in simplest form.
<b>Answer</b>	It has no like terms and no parentheses.

<b>SAMPLE</b>	Explain why $6 + 2(x - 4)$ is not in simplest form.
<b>Answer</b>	The two has not been distributed to the terms in the parentheses, and then simplified by combining like terms.

<b>SAMPLE</b>	Simplify $r + 3(s + 7r)$
<i>Distribute the 3 to the contents of the parenthesis</i>	$r + 3(s) + 3(7r) = r + 3s + 21r$
<i>Organize like terms</i>	$1r + 21r + 3s$
<i>Combine like terms</i>	$1r + 21r + 3s = 22r + 3s$
<b>Answer</b>	$22r + 3s$

<b>SAMPLE</b>	Simplify $8 + (-7)$
<i>Adding a negative number is the same as subtracting that number</i>	$8 - 7$
<b>Answer</b>	1

<b>SAMPLE</b>	If $14 = j - (-20)$ , what is the value of $j$ ?
	$14 = j - (-20)$
<i>Subtracting a negative number is the same as adding a positive number</i>	$14 = j + 20$
<i>Set all known values equal to <math>j</math> by subtracting 20 from both sides</i>	$-20 + 14 = j + 20 - 20$
<b>Answer</b>	$-6 = j$

<b>SAMPLE</b>	How is the product $3 \cdot 3 \cdot 3$ expressed in exponential notation?
<i>Set up the equation</i>	$3^1 \cdot 3^1 \cdot 3^1$
<i>When multiplying, add the exponents</i>	$3^{1+1+1}$
<b>Answer</b>	$3^3$

<b>SAMPLE</b>	What is the value of $3t^5$ if $t = 2$ ?
<i>Replace <math>t</math> with 2</i>	$3(2^5)$
<i>Simplify exponents</i>	$3(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
	$3(32)$
<b>Answer</b>	96

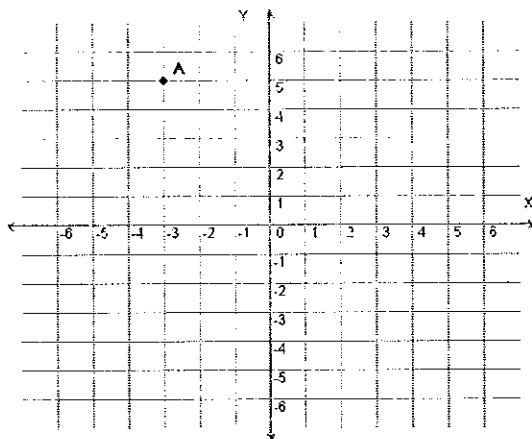
<b>SAMPLE</b>	<b>Simplify <math>(-4a^5b)(8a^2)</math></b>
<i>Multiply like terms beginning with -4 and 8</i>	$-32(a^5b)(a^2)$
<i>Multiply <math>a^5</math> and <math>a^2</math></i>	$-32(a^7)(b)$
<b>Answer</b>	$-32a^7b$

<b>SAMPLE</b>	<b>Solve <math>\frac{10}{y} = \frac{7}{y+3}</math></b>
<i>Cross multiply</i>	$10(y+3) = 7y$
<i>Solve for y, begin by distributing the 10</i>	$10y + 30 = 7y$
<i>Combine like terms, begin by subtracting 10y from both sides</i>	$-10y + 10y + 30 = 7y - 10y$
<i>After combining like terms</i>	$30 = -3y$
<i>Set all known values equal to y by dividing both sides by -3</i>	$30 \div -3 = -3y \div -3$
<b>Answer</b>	$-10 = y$

## THE COORDINATE SYSTEM

### GRID GRAPHS

The location of any point on a grid can be indicated by an **ordered pair** of numbers (x,y) where x represents the number of units on the horizontal line stemming away from zero (called the x-axis), and y represents the number of units on the vertical line stemming away from zero (called the y-axis). The x is always listed first, and the y is always listed second in an ordered pair. The numbers in an ordered pair are called **coordinates**. For example: if the x-coordinate is -3 and the y-coordinate is 5, the ordered pair for the point would be (-3,5).



### SLOPE COORDINATES

- The **x-intercept** is the point where a line crosses the x-axis. It is found by setting  $y = 0$  and solving the resulting equation.
- The **y-intercept** is the point where a line crosses the y-axis. It is found by setting  $x = 0$  and solving the resulting equation.

<b>SAMPLE</b>	<b>What are the coordinates of the x-intercept of the line <math>4y - x = 5</math>?</b>
<i>Set up the equation</i>	$4y - x = 5$
<i>Set <math>y = 0</math> and solve for x</i>	$4(0) - x = 5$
	$-x = 5$
<i>Multiply both sides by -1</i>	$(-1) \cdot x = (-1) \cdot 5$ $x = -5$
<b>Answer</b>	<b><math>(-5, 0)</math></b>

## GEOMETRY

### BASICS

- The angles of any four sided figure always add up to  $360^\circ$
- Two lines are **perpendicular** ( $\perp$ ) when they meet at a  $90^\circ$  angle
- Two lines are **parallel** ( $\parallel$ ) when they never intersect
- **Bisect** means to cut in half

### SQUARES

- Each of the 4 sides are always equal in length
- Each of the 4 angles is always equal to  $90^\circ$
- The area (A) of a square is found by squaring the measurement of one side
  - $A = s^2$
- Find the perimeter by adding up the length of all the sides
  - Perimeter =  $4s$

### RECTANGLES

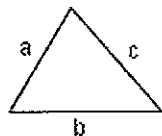
- Opposite sides are always equal
- Each of the 4 angles is always equal to  $90^\circ$
- The area of a rectangle is found by multiplying the rectangle's length by its width
  - $A = lw$
- Find the perimeter by multiplying the length by two and the width by two and adding those products
  - Perimeter =  $2l + 2w$

### CIRCLES

- There are  $360^\circ$  in a circle
- Radius = distance from the center to any point on the edge of the circle ( $r$ )
- Diameter = straight line distance from one point on the circle to another, passing through the center point ( $d$ )
- $\text{Pi} = 3.14$  ( $\pi$ )
- The area of a circle is found by multiplying Pi by the radius squared
  - $A = \pi \cdot r^2$
- Circumference is the distance around the outside of the circle, find it by multiplying two by Pi by the radius
  - Circumference =  $2\pi \cdot r$

### TRIANGLES

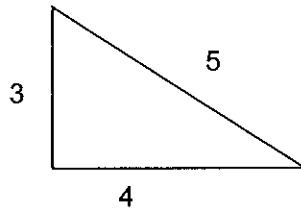
- Each of the 3 angles will always add up to  $180^\circ$
- On right triangles two sides intersect to form a  $90^\circ$  angle
- The area of a triangle is found by multiplying the triangle's base by its height and dividing the product in half
  - $A = \frac{1}{2}bh$
- Find the perimeter by adding up the length of all the sides



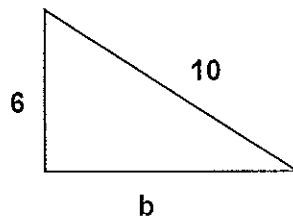
- A hypotenuse is the side of a right triangle that is opposite the right ( $90^\circ$ ) angle. By using the Pythagorean Theorem one can find the length of an unknown side of a right triangle.
  - The Pythagorean Theorem is:  $a^2 + b^2 = c^2$ , where  $c$  equals the hypotenuse.

**SAMPLE**

In the right triangle below, the length of side  $a = 3$ , the length of side  $b = 4$  and the hypotenuse (side  $c$ ), has a length of 5. Using the Pythagorean Theorem ( $a^2 + b^2 = c^2$ ), we see that  $3^2 + 4^2 = 5^2$ .

**SAMPLE**

Determine the length of side  $b$ , given that side  $a = 6$  and side  $c = 10$



<i>Use the Pythagorean Theorem</i>	$a^2 + b^2 = c^2$
<i>Plug in known values</i>	$6^2 + b^2 = 10^2$
<i>Combine like terms (subtract <math>6^2</math> from both sides)</i>	$b^2 = 10^2 - 6^2$
<i>Simplify exponents (<math>1 \cdot 10 \cdot 10</math>) - (<math>1 \cdot 6 \cdot 6</math>)</i>	$b^2 = 100 - 36$
	$b^2 = 64$
<i>Obtain the square root of 64 (<math>b^2 = 8^2</math>)</i>	$\sqrt{b^2} = \sqrt{64}$
<b>Answer</b>	<b><math>b = 8</math></b>