

**Multivariable Calculus
Summer Review Packet**

1. Given vectors $\vec{u} = \langle 4, 7, -3 \rangle$ and $\vec{v} = \langle -4, -10, 2 \rangle$, find...

a) $\vec{u} \cdot \vec{v}$

$$\begin{array}{r} 4 \cdot -4 = -16 \\ + \\ 7 \cdot -10 = -70 \\ + \\ -3 \cdot 2 = -6 \\ \hline \boxed{-92} \end{array}$$

b) $\vec{u} \times \vec{v}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 7 & -3 \\ -4 & -10 & 2 \end{vmatrix}$$

$$\begin{aligned} & \langle 14 - 30, -(8 - 12), -40 + 28 \rangle \\ & \boxed{\langle -16, 4, -12 \rangle} \end{aligned}$$

2. What is the significance of $\vec{u} \cdot \vec{v} = 0$? What about $\vec{u} \times \vec{v} = 0$?

Dot product = 0 means orthogonal
(perpendicular)
vectors

Cross product = 0 means parallel vectors

3. If $\vec{r}(t) = (\cos t)\vec{i} + (\sin^2 t)\vec{j}$, find...

a) $\vec{r}'(t)$

$$(-\sin t)\vec{i} + (2\sin t \cos t)\vec{j}$$

- b) the arc length of $\vec{r}(t)$ from $t = 0$ to $t = 2\pi$

$$\int_0^{2\pi} \sqrt{(-\sin t)^2 + (2\sin t \cos t)^2} dt$$

3 cont. If $\vec{r}(t) = (\cos t)\vec{i} + (\sin^2 t)\vec{j}$, find...

$$\cos 2t = 1 - 2\sin^2 t$$

$$\sin^2 t = \frac{1 - \cos 2t}{2}$$

c) $\int \vec{r}(t) dt$

$$\langle \int \cos t dt, \int \sin^2 t dt \rangle$$

$$\langle \sin t, \frac{t}{2} - \frac{\sin 2t}{4} \rangle$$

$$\frac{1}{2} \int (1 - \cos 2t) dt$$

$$\left(\frac{1}{2} \left(t - \frac{\sin 2t}{2} \right) \right)$$

d) $\lim_{t \rightarrow \infty} \vec{r}(t)$

Does not exist Oscillates btwn -1 and 1 for $\vec{i}$ and 0 and for $\vec{j}$

4. How do you parameterize the following? Change this from rectangular to parametric form.

a) $x^2 + y^2 = 3$ $r = \sqrt{3}$

$$x = r \cos t$$

$$y = r \sin t$$

$$\boxed{\begin{matrix} x = \sqrt{3} \cos t \\ y = \sqrt{3} \sin t \end{matrix}}$$

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b) $2x + 3y = 24$

$$x\text{-int: } (12, 0) \quad x = 12t$$

$$y\text{-int: } (0, 8) \quad y = -8t + 8$$

c) the left side only of $y = x^2 + 4$

$$\boxed{\begin{matrix} x = t \\ y = t^2 + 4 \\ t, (-\infty, 0] \end{matrix}}$$



d) $\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$

ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $\cos^2 t + \sin^2 t = 1$

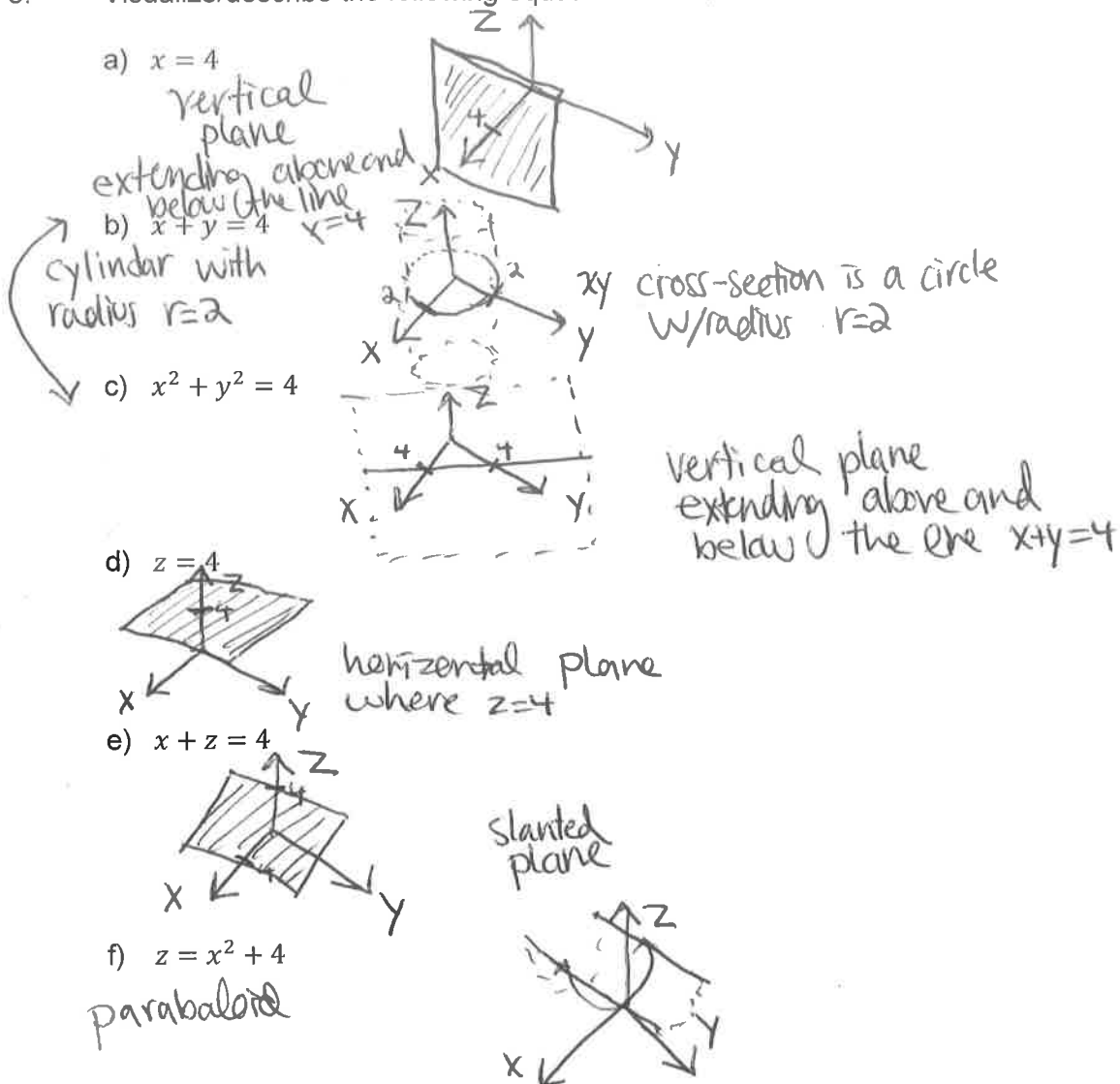
$$\frac{x^2}{9} = \cos^2 t$$

$$\boxed{x = 3 \cos t}$$

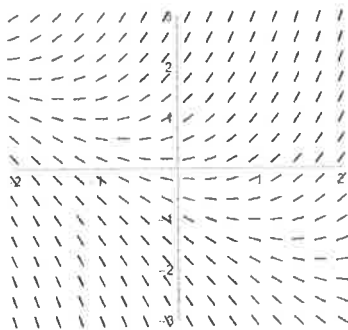
$$\frac{y^2}{4} = \sin^2 t$$

$$\boxed{y = 2 \sin t}$$

5. Visualize/describe the following equations in 3D.



6. Which of the following differential equations could be a match to the slope field below?



a) $\frac{dy}{dx} = x^2 + y$

b) $\frac{dy}{dx} = y^2 + y$

c) $\frac{dy}{dx} = xy$

d) $\frac{dy}{dx} = \frac{y}{x}$

e) none of the above

7. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

a) $y = a \cdot \sin(cx) + \cos^2(dx)$

$$\frac{dy}{dx} = ac \cos(cx) + 2\cos(dx)(-d\sin(dx))$$

$$\frac{dy}{dx} = ac \cdot \cos(cx) - 2d \sin(2 \cdot dx)$$

$$\frac{d^2y}{dx^2} = -ac^2 \sin(cx) - 2d^2 \cos(2 \cdot dx)$$

c) $y = \int_a^{x^2} \ln(3t^2) dt$

$$\frac{dy}{dx} = 2x \ln(3x^4)$$

$$\frac{d^2y}{dx^2} = 2 \ln(3x^4) + \frac{2x}{3x^4} (12x^3)$$

$$\frac{d^2y}{dx^2} = 2 \ln(3x^4) + 8, x \neq 0$$

8. Find each indefinite integral.

a) $\int x \cdot \ln x dx$

$$u = \ln x \quad dv = x$$

$$du = \frac{dx}{x} \quad v = \frac{1}{2}x^2$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2} \int x dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$$x = e^\theta \cos^2 \theta$$

$$y = e^\theta \cos \theta \sin \theta = \frac{1}{2} e^\theta \sin 2\theta$$

b) $r(\theta) = e^\theta \cos \theta$

$$\frac{dy}{dx} = \frac{\frac{1}{2}e^\theta \sin 2\theta + e^\theta \cos 2\theta}{e^\theta \cos^2 \theta + e^\theta \cdot 2\cos \theta (-\sin \theta)}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2} \sin 2\theta + \cos 2\theta}{\cos^2 \theta - \sin 2\theta}$$

d) $r(t) = \langle e^{3t}, \tan(3t) \rangle$

$$\frac{dy}{dx} = \frac{3 \sec^2(3t)}{3e^{3t}} = \frac{\sec^2(3t)}{e^{3t}}$$

$$\frac{d^2y}{dx^2} = \frac{e^{3t} (2 \sec(3t) \cdot 3 \sec(3t) \tan(3t)) - 3e^{3t} \sec^2(3t)}{(e^{3t})^2}$$

$$\frac{d^2y}{dx^2} = \frac{2 \sec^2(3t) \tan(3t) - \sec^2(3t)}{(e^{3t})^2}$$

b) $\int \frac{2}{x^2 - x - 6} dx$

$$\frac{A}{x-3} + \frac{B}{x+2} = \frac{2}{(x-3)(x+2)}$$

$$= \frac{2}{5} \int \frac{dx}{x-3} - \frac{2}{3} \int \frac{dx}{x+2}$$

$$4x + 2A + Bx - 3B = 2$$

$$-2(A+B=0)$$

$$2A - 3B = 2$$

$$= \frac{2}{5} \ln|x-3| - \frac{2}{3} \ln|x+2| + C$$

$$-5B = 2$$

$$B = -2/5$$

$$A = 2/5$$

$$= \frac{2}{5} \ln \left| \frac{x-3}{x+2} \right| + C$$

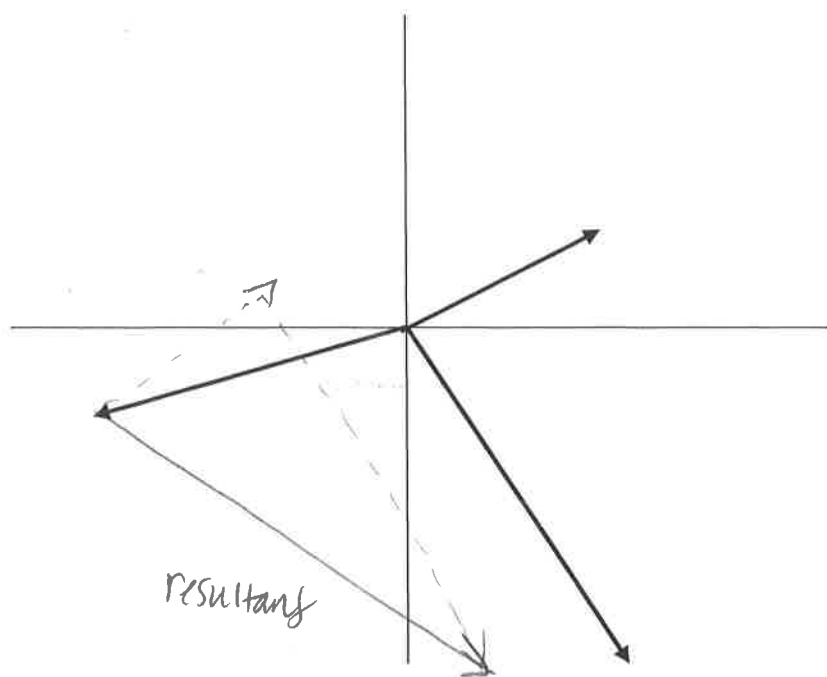
c) $\int (a^2 + 5a + 7) dx$

$$= (a^2 + 5a + 7)x + C$$

d) $\int e^{2x-5} dx$

$$= \frac{1}{2} e^{2x-5} + C$$

9. Given the vectors below, sketch the resultant.



- 10) Conic sections – circles, ellipses, parabolas and hyperbolas - will be discussed the first few days of class....