

Summer Review Packet for Students Entering Calculus – PART 1

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1, which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x - 7 - 6}{5} = \frac{-7x - 13}{5}$$

$$\frac{\frac{-2}{x} + \frac{3x}{x-4}}{5 - \frac{1}{x-4}} = \frac{\frac{-2}{x} + \frac{3x}{x-4}}{5 - \frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4) + 3x(x)}{5(x)(x-4) - 1(x)} = \frac{-2x + 8 + 3x^2}{5x^2 - 20x - x} = \frac{3x^2 - 2x + 8}{5x^2 - 21x}$$

Simplify each of the following.

1) $\frac{\frac{25}{a} - a}{5 + a}$

2) $\frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}}$

3) $\frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}$

4) $\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$

5) $\frac{1 - \frac{2x}{3x-4}}{x + \frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read “ f of g of x ” Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned}f(g(x)) &= f(x - 4) \\ &= 2(x - 4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33\end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

6) $f(2) =$ _____ 7) $g(-3) =$ _____ 8) $f(a+1) =$ _____

9) $f[g(-2)] =$ _____ 10) $g[f(m+2)] =$ _____ 11) $\frac{f(x+h) - f(x)}{h} =$ _____

Let $f(x) = \sin x$ Find each exactly.

12) $f\left(\frac{\pi}{2}\right) =$ _____ 13) $f\left(\frac{2\pi}{3}\right) =$ _____

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

14) $h[f(-2)] =$ _____ 15) $f[g(x-1)] =$ _____ 16) $g[h(x^3)] =$ _____

Find $\frac{f(x+h)-f(x)}{h}$ for the given function f .

17) $f(x) = 9x + 3$

18) $f(x) = 5 - 2x$

Intercepts and Points of Intersection

To find the x-intercepts, let $y = 0$ in your equation and solve.

To find the y-intercepts, let $x = 0$ in your equation and solve.

Example: $y = x^2 - 2x - 3$

x - int. (Let $y = 0$)

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x - intercepts $(-1, 0)$ and $(3, 0)$

y - int. (Let $x = 0$)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y - intercept $(0, -3)$

Find the x and y intercepts for each.

19) $y = 2x - 5$

20) $y = x^2 + x - 2$

21) $y = x\sqrt{16 - x^2}$

22) $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.

Example:

$$x^2 + y - 16x + 39 = 0$$

$$x^2 - y^2 - 9 = 0$$

Elimination Method

$$2x^2 - 16x + 30 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ and } x = 5$$

Plug $x=3$ and $x=5$ into one original

$$3^2 - y^2 - 9 = 0 \quad 5^2 - y^2 - 9 = 0$$

$$-y^2 = 0 \quad 16 = y^2$$

$$y = 0 \quad y = \pm 4$$

Points of Intersection $(5, 4)$, $(5, -4)$ and $(3, 0)$

Substitution Method

Solve one equation for one variable.

$$y^2 = -x^2 + 16x - 39 \quad (\text{1st equation solved for } y)$$

$$x^2 - (-x^2 + 16x - 39) - 9 = 0 \quad \text{Plug what } y^2 \text{ is equal to into second equation.}$$

$$2x^2 - 16x + 30 = 0 \quad (\text{The rest is the same as previous example})$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ or } x = 5$$

Find the point(s) of intersection of the graphs for the given equations.


23) $x + y = 8$
 $4x - y = 7$

24) $x^2 + y = 6$
 $x + y = 4$

25) $x^2 - 4y^2 - 20x - 64y - 172 = 0$
 $16x^2 + 4y^2 - 320x + 64y + 1600 = 0$
 (CALCULATOR ALLOWED #25 only)

Interval Notation

26) Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27) $2x - 1 \geq 0$

28) $-4 \leq 2x - 3 < 4$

29) $\frac{x}{2} - \frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30) $f(x) = x^2 - 5$

31) $f(x) = -\sqrt{x+3}$

32) $f(x) = 3\sin x$

33) $f(x) = \frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new “ y ” value.

Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

34) $f(x) = 2x + 1$

35) $f(x) = \frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
 $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ and $g(x) = 4x+9$ show $f(x)$ and $g(x)$ are inverses of each other.

$$f(g(x)) = 4\left(\frac{x-9}{4}\right) + 9$$

$$= x - 9 + 9$$

$$= x$$

$$g(f(x)) = \frac{(4x+9)-9}{4}$$

$$= \frac{4x+9-9}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses
of each other.

Prove f and g are inverses of each other.

$$36) f(x) = \frac{x^3}{2} \quad g(x) = \sqrt[3]{2x}$$

$$37) f(x) = 9 - x^2, x \geq 0 \quad g(x) = \sqrt{9 - x}$$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

38) Use slope-intercept form to find the equation of the line having a slope of 3 and a y -intercept of 5.

39) Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40) Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41) Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $2/3$.

42) Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43) Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44) Find the equation of a line passing through the points (-3, 6) and (1, 2).

45) Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi \text{ radians}}$ to get rid of radians and convert to degrees.

Use $\frac{\pi \text{ radians}}{180^\circ}$ to get rid of degrees and convert to radians.

46) Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ (CALC) c. 2.63 radians (CALC)

47) Convert to radians: a. 45° b. -17° (CALC) c. 237° (CALC)

Angles in Standard Position

48) Sketch the angle in standard position.

a. $\frac{11\pi}{6}$

b. 230°

c. $-\frac{5\pi}{3}$

d. 1.8 radians

Reference Triangles

49) Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

c. $-\frac{\pi}{4}$

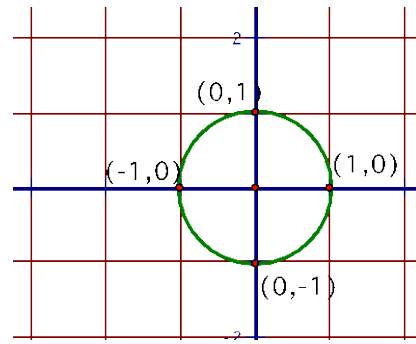
d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

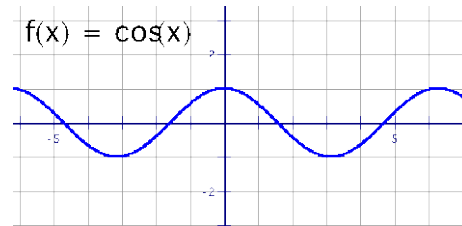
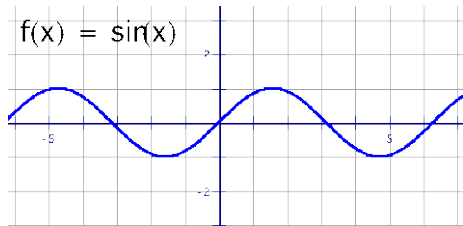
Example: $\sin 90^\circ = 1$

$$\cos \frac{\pi}{2} = 0$$



- 50) a.) $\sin 180^\circ$ b.) $\cos 270^\circ$ c.) $\sin(-90^\circ)$ d.) $\sin \pi$ e.) $\cos 360^\circ$ f.) $\cos(-\pi)$

Graphing Trig Functions



$y = \sin x$ and $y = \cos x$ have a period of 2π and an amplitude of 1. Use the parent graphs above to help you

sketch a graph of the functions below. For $f(x) = A \sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period,

$\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51) $f(x) = 5 \sin x$

52) $f(x) = \sin 2x$

53) $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54) $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \leq x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \leq x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

55) $\sin x = -\frac{1}{2}$

56) $2 \cos x = \sqrt{3}$

57) $\cos 2x = \frac{1}{\sqrt{2}}$

58) $\sin^2 x = \frac{1}{2}$

59) $\sin 2x = -\frac{\sqrt{3}}{2}$

60) $2 \cos^2 x - 1 - \cos x = 0$

61) $4 \cos^2 x - 3 = 0$

62) $\sin^2 x + \cos 2x - \cos x = 0$

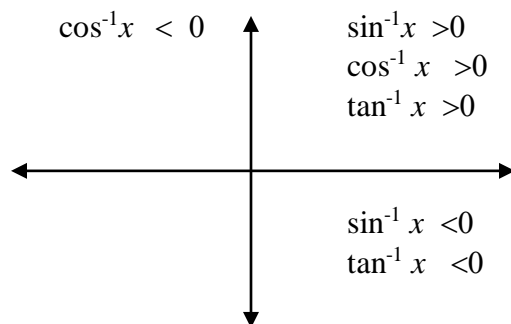
Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x)$$

$$\sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.

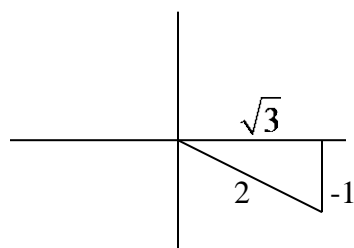


Example:

Express the value of “y” in radians.

$$y = \arctan \frac{-1}{\sqrt{3}}$$

Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$\text{Answer: } y = -\frac{\pi}{6}$$

For each of the following, express the value for “y” in radians.

76) $y = \arcsin \frac{-\sqrt{3}}{2}$

77) $y = \arccos(-1)$

78) $y = \arctan(-1)$

Example: Find the value without a calculator.

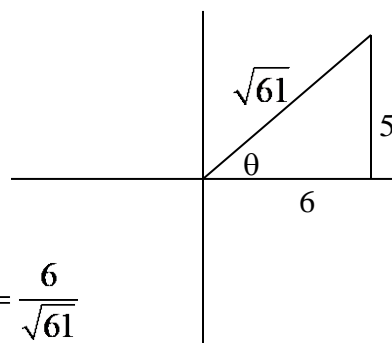
$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$



For each of the following give the value without a calculator.

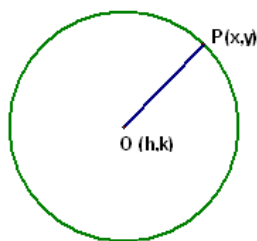
63) $\tan\left(\arccos\frac{2}{3}\right)$

64) $\sec\left(\sin^{-1}\frac{12}{13}\right)$

65) $\sin\left(\arctan\frac{12}{5}\right)$

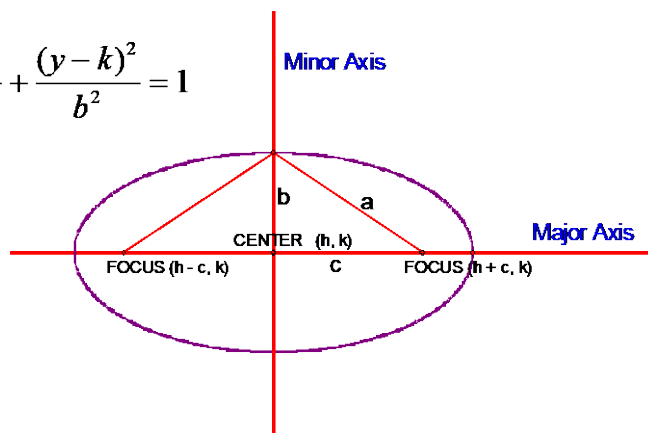
66) $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles & Ellipses NOTE: If you did not learn ellipses, please read the review & attempt to graph them.



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

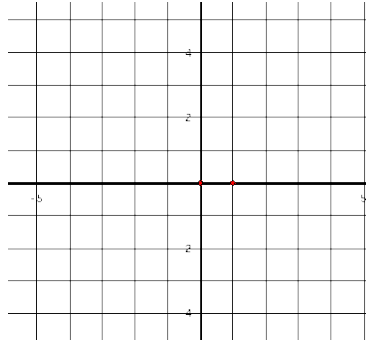


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where r is the radius of the circle.

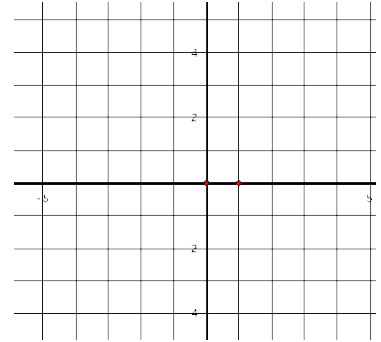
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the distance from the center to the ellipse along the x -axis and b is the distance from the center to the ellipse along the y -axis. If the larger number is under the y^2 term, the ellipse is elongated along the y -axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

67) $x^2 + y^2 = 16$

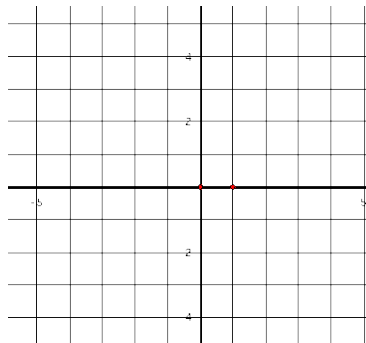


68) $x^2 + y^2 = 5$



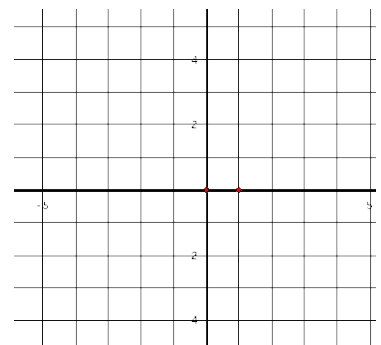
69) $\frac{x^2}{1} + \frac{y^2}{9} = 1$

(Calculator allowed to check work on this problem.)



70) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

(Calculator allowed to check work on this problem.)



Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

89) $f(x) = \frac{1}{x^2}$

90) $f(x) = \frac{x^2}{x^2 - 4}$

91) $f(x) = \frac{2 + x}{x^2(1 - x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

$$92) f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$$

$$93) f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$$

$$94) f(x) = \frac{4x^5}{x^2 - 7}$$

Useful Formulas for Trigonometry and Logarithms

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$
 $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $= 1 - 2 \sin^2 x$
 $= 2 \cos^2 x - 1$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$ Power property: $\log_b m^p = p \log_b m$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$ Property of equality: If $\log_b m = \log_b n$, then $m = n$

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \rightarrow \infty} \frac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$ Point-slope form: $y - y_1 = m(x - x_1)$ Standard form: $Ax + By + C = 0$

AP Calculus AB – Summer Assignment Part 2

These are additional algebra problems and skills that you need to be able to do before you start Calculus. Problems in this section are designed to be done **without using a calculator**, as a large part of Calculus involves solving problems without using a calculator. This section does not include review of how to do the problems (as the previous section of problems did.) You may need to do some research or review online to help you remember how to solve them.

- All work should be done on this packet OR on lined notebook paper if preferred,
- IF done on your own (lined) paper:
- Write the original problem and all work shown for full credit. No work = no credit.
- Label each section and problem.
- In either case (on this packet OR on your own (lined) paper):
- Show all work and highlight or circle your final answers.
- All problems should be completed without a calculator.
- Put name on all sheets and staple together in order.
- **Due the first day of class.**

Section 1: Special Factorization

Completely factor the following expressions.

1. $x^3 - 25x$

2. $3x^8 - 3$

3. $16x^4 - 24x^2y + 9y^2$

4. $144 + 32x^2 - x^4$

5. $x^3 - xy^2 + x^2y - y^3$

Section 2: Solving Quadratic Equations

Solve the following equations. If the equation will not factor, use the quadratic formula.

6. $x^2 + x + \frac{1}{4} = 0$

7. $2x^2 - 72 = 0$

8. $7x^2 - 7x + 2 = 0$

$$9. x^3 - 5x^2 + 5x - 25 = 0$$

$$10. x + \frac{1}{x} = \frac{17}{4}$$

Section 3: Negative and Fractional Exponents

Simplify the following expressions. Be sure to your final answers only contains positive exponents.

$$11. -12^2x^{-5}$$

$$12. (4x^{-1})^{-1}$$

$$13. \frac{(x^2-1)^{-\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}}$$

$$14. \frac{1}{4}(16x^2)^{-\frac{3}{4}}(32x)$$

$$15. (x^{-1} + y^{-1})^{-1}$$

Section 4: Rational Expressions – adding, subtracting, and solving

Add or subtract the following rational expressions.

$$16. \frac{1}{x-3} - \frac{1}{x+3}$$

$$17. \frac{2x-1}{x-1} - \frac{3x}{2x+1}$$

$$18. \frac{x^2-2x+3}{x^2+7x+12} - \frac{x^2-4x-5}{x^2+7x+12}$$

Solve the following rational equations.

$$19. \frac{5}{2x} - \frac{5}{3(x+5)} = \frac{5}{x}$$

$$20. \frac{2x-1}{x-1} - \frac{3x}{2x+1} = \frac{x^2+11}{2x^2-x-1}$$

Section 5: Solving Inequalities

Solve the following inequalities. For the inequalities involving rational functions, be sure to include your number line where you tested each interval. All final answers should be written in interval notation.

$$21. \frac{2x-7}{x-5} \leq 1$$

$$22. 4 - \frac{5x}{3} > -(2x + \frac{1}{2})$$

$$23. x^2 - 3x > 18$$

$$24. -7 \leq 6x - 1 < 11$$

$$25. x^3 < 4x^2$$

Section 6: Exponential Functions and Logarithms

Solve the following equations. Since you are not using a calculator, your answers may still contain logs or e .

$$26. \log_9(x^2 - x + 3) = \frac{1}{2}$$

$$27. \ln x^3 - \ln x^2 = \frac{1}{2}$$

$$28. e^{3x} + 5 = 6$$

Evaluate the following. Your answer will be a number.

$$29. \ln \frac{1}{\sqrt[3]{e^2}}$$

$$30. \log_{12} 2 + \log_{12} 9 + \log_{12} 8$$

Section 7: Unit Circle values

Without having a Unit Circle in front of you, give the simplified, exact, rationalized answer (if it exists) for each of the following. Write "DNE" if it does not exist.

$$31. \sin \frac{7\pi}{4}$$

$$32. \cot \frac{5\pi}{3}$$

$$33. \cos \frac{11\pi}{6}$$

$$34. \cos \frac{2\pi}{3}$$

$$35. \sec \frac{5\pi}{6}$$