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Calculus Honors: Required Problems

Solve all problems on notebook and/or graph paper. Show ALL your work, skipping a line between each problem.

Functions

For problems 1 – 6, perform the indicated function evaluations.

1. $f(x) = 10x - 3$

- a. $f(-5)$ b. $f(0)$ c. $f(7)$
d. $f(t^2 + 2)$ e. $f(12 - x)$ f. $f(x + h)$

4. $f(x) = \sqrt{4x + 5}$

- a. $f(0)$ b. $f(-4)$ c. $f(-2)$
d. $f(5 - 12x)$ e. $f(2x^2 + 8)$ f. $f(x + h)$

2. $f(x) = 4x^2 - 7x + 1$

- b. $f(-5)$ b. $f(0)$ c. $f(7)$
d. $f(6z)$ e. $f(1 - 3y)$ f. $f(x + h)$

5. $f(x) = \frac{\sqrt{x^2+9}}{4x+8}$

- a. $f(-4)$ b. $f(4)$ c. $f(1)$
d. $f(2 - 7x)$ e. $f(\sqrt{3x+4})$ f. $f(x + h)$

3. $f(x) = \frac{x+5}{1-x}$

- c. $f(4)$ b. $f(0)$ c. $f(-7)$
d. $f(x^2 - 5)$ e. $f(4\sqrt{x} + 9)$ f. $f(x + h)$

6. $f(x) = \sqrt{3-x} - \frac{x}{2x+5}$

- a. $f(7)$ b. $f(0)$ c. $f(-4)$
d. $f(x^2 - 10)$ e. $f(5 - x)$ f. $f(6x - x^2)$

Difference Quotient

The **difference quotient** of a function $f(x)$ is defined by: $\frac{f(x+h)-f(x)}{h}$, compute the difference quotient of the following functions.

7. $f(x) = 4 - 7x$	8. $f(x) = 42$	9. $f(x) = 2x^2 + 9$
10. $f(x) = 3 - 8x - x^2$	11. $f(x) = \sqrt{4 + 3x}$	12. $f(x) = \frac{-4}{1-2x}$

Finding Roots

For the following problems, determine all the roots of the given function.

13. $f(x) = 40 + 3x - x^2$	14. $f(x) = 6x^4 - 5x^3 - 4x^2$	15. $f(x) = -x^2 - 11x + 6$
16. $f(x) = 4x^6 + 10x^5 + x^4$	17. $f(x) = x^7 + 6x^4 - 16x$	18. $f(x) = x^{\frac{1}{2}} - 8x^{\frac{1}{4}} - 15$

Finding Domain and Range

For the following problems, determine the domain and range of the given function.

19. $f(x) = x^2 - 8x + 3$	20. $f(x) = 4 - 7x - x^2$	21. $f(x) = 5 - 2\sqrt{x}$
22. $f(x) = 10 + \sqrt{9 + 7x^2}$	23. $f(x) = 1 + \sqrt{6 - 7x}$	24. $f(x) = -6 5 - x $
25. $f(x) = 12 + 9 x^2 - 1 $	26. $f(x) = \frac{8x^2 - 12x + 4}{16x + 9}$	27. $f(x) = \frac{x^3 - 27}{4 - 17x}$
28. $f(x) = \frac{x^3 - x^2 + x - 1}{35x^3 + 2x^4 - x^5}$	29. $f(x) = \frac{x^2 + x}{x^3 - 9x^2 + 2x}$	30. $f(x) = \frac{3 - x^4}{4x^2 + 10x + 2}$
31. $f(x) = \sqrt{x^2 - 16}$	32. $f(x) = \sqrt{36 - 9x^2}$	33. $f(x) = \sqrt{4x^3 - 4x^2 + x}$

Function Composition

For the following problems, compute $(f \circ g)(x)$ and $(g \circ f)(x)$.

34. $f(x) = 2x + 5$ $g(x) = 8 - 23x$	35. $f(x) = \sqrt{2 - x}$ $g(x) = 2x^2 - 9$
36. $f(x) = 2x^2 + x - 4$ $g(x) = 7x - x^2$	37. $f(x) = \frac{x}{2x + 3}$ $g(x) = 5x + 8$

Inverse Functions

For each of the following functions, find the inverse of the function. Verify your inverse by computing $(f \circ g)(x)$ or $(g \circ f)(x)$, which will equal x if they are indeed inverses of each other.

38. $f(x) = 11x - 8$	39. $f(x) = 4 - 10x$	40. $f(x) = 7 + (2x + 1)^3$	41. $f(x) = \frac{2x+14}{6x+1}$
42. $f(x) = 2x^7 - 9$	43. $f(x) = \frac{7}{\sqrt[7]{15x+2}}$	44. $f(x) = \sqrt[3]{6-18x}$	45. $f(x) = \frac{1-x}{9-12x}$

Trig Functions

Without using a calculator, determine the exact value of each of the following.

46. $\tan\left(\frac{3\pi}{4}\right)$	47. $\sin\left(\frac{7\pi}{6}\right)$	48. $\sin\left(-\frac{3\pi}{4}\right)$	49. $\cos\left(\frac{4\pi}{3}\right)$
50. $\cot\left(\frac{5\pi}{4}\right)$	51. $\sin\left(-\frac{5\pi}{6}\right)$	52. $\sec\left(-\frac{\pi}{6}\right)$	53. $\cos\left(\frac{11\pi}{6}\right)$
54. $\cot\left(-\frac{4\pi}{3}\right)$	55. $\cos\left(-\frac{\pi}{4}\right)$	56. $\csc\left(\frac{2\pi}{3}\right)$	57. $\cos\left(\frac{5\pi}{4}\right)$
58. $\tan\left(\frac{31\pi}{6}\right)$	59. $\cos\left(-\frac{15\pi}{4}\right)$	60. $\sec\left(-\frac{23\pi}{4}\right)$	61. $\cot\left(\frac{11\pi}{4}\right)$

Solving Trig Equations

Without using a calculator find the solution(s) to the following equations. If an interval is given then find only those solutions that are in the interval. If no interval is given then find all solutions to the equation.

62. $10 \cos(8x) = -5$	63. $10 \cos(8x) = -5; [-\frac{\pi}{4}, \frac{\pi}{4}]$	64. $2 \sin\left(\frac{2x}{3}\right) + \sqrt{2} = 0; [0, 5\pi]$
65. $2 \sin\left(\frac{x}{4}\right) = \sqrt{3}$	66. $2 \sin\left(\frac{x}{4}\right) = \sqrt{3}; [0, 16\pi]$	67. $\sqrt{6} = -\sqrt{8} \cos(3x); [0, \frac{5\pi}{3}]$

With the aid of a calculator find the solution(s) to the following equations. If an interval is given then find only those solutions that are in the interval. If no interval is given then find all solutions to the equation.

68. $2 - 14 \sin\left(\frac{t}{3}\right) = 5$	69. $4 \cos(4x) + 8 = 10 - \cos(4x)$	70. $1 = 3 + 8 \cos\left(\frac{w}{2}\right); [-20, 5]$
71. $2 \tan(3w) + 3 = 25$	72. $2 \sin\left(\frac{3x}{5}\right) - \frac{7}{5} = \frac{1}{5}; [0, 15]$	73. $45 \sin\left(\frac{x}{2}\right) - 9 = 7 \sin\left(\frac{x}{2}\right) + 17; [-10, 20]$

With the aid of a calculator find all of the solutions to the following equations. You may have to do crazy things, **like factor**, in order to get the solution. Use at least 4 decimal places.

74. $4 \tan\left(\frac{x}{3}\right) \sin(2x) - \tan\left(\frac{x}{3}\right) = 0$	75. $3 \tan(4x) \sec(2x - 1) + \sec(2x - 1) = 0$
76. $4 \cos^2(x) - 4 \cos(x) = -1$	77. $2 - \sin(2x) = 3 \sin^2(2x)$

Exponential Functions

Sketch the graphs of each of the following functions.

78. $f(x) = 7^{3-\frac{x}{2}}$	79. $f(x) = 3 - 5^{4x+1}$
80. $f(x) = 6e^{2x-1} - 3$	81. $f(x) = 7 + 9e^{2-\frac{3t}{5}}$

Logarithmic Functions

Without using a calculator, determine the exact value of each of the following.

82. $\log_7 343$	83. $\log_4 1024$	84. $\log_{\frac{3}{8}} \frac{27}{512}$
85. $\log_{11} 121$	86. $\ln \frac{1}{\sqrt[5]{e}}$	87. $\log 10000$

Write each of the following in terms of simpler logarithms.

88. $\log_7(10a^7b^3c^{-8})$	89. $\log[z^2(x^2 + 4)^3]$	90. $\ln\left(\frac{w^2\sqrt[4]{t^3}}{\sqrt{t+x}}\right)$
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Combine each of the following into a single logarithm with a coefficient of one.

91. $7 \ln t - 6 \ln s + 5 \ln w$	92. $\frac{1}{2} \log(z + 1) - 2 \log x - 4 \log y - 3 \log z$	93. $2 \log_3(x + y) + 6 \log_3 x - \frac{1}{3}$
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Use the change of base formula and a calculator to find the value of each of the following.

94. $\log_7 100$	95. $\log_{\frac{5}{7}} \frac{1}{8}$
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Exponential and Logarithm Equations

Find all the solutions to the given equations. If there is no solution to the equation clearly explain why.

96. $15 = 12 + 5e^{10x-7}$	97. $4e^{2x+x^2} - 7 = 2$	98. $3e^{7x} - 12e^{8x+5} = 0$
99. $2 \log(w) - \log(3w + 7) = 1$	100. $x \log(6x + 1) - 3x^2 \log(6x + 1) = 0$	
101. $\ln(3x + 1) - \ln(x) = -2$	102. $1 + 3^{z^2-2} = 5$	

Compound Interest

103. We have \$2,500 to invest for 80 months. How much money will we have if we put the money into an account that has an annual interest rate of 9% and interest is compounded:

(a) quarterly

(b) monthly

(c) continuously

104. Starting with \$60,000 and putting it into an account that earns an annual interest rate of 7.5%. How long will it take for the money to reach \$100,000 if the interest is compounded:

(a) quarterly

(b) monthly

(c) continuously

Exponential Growth/Decay

105. A population of bacteria initially has 90,000 present and in 2 weeks there will be 200,000 bacteria present.

(a) Determine the exponential growth equation for this population. $Q = Q_0 e^{kt}$

(b) How long will it take for the population to grow from its initial population of 90,000 to a population of 150,000?

106. We initially have 2kg of some radioactive element and in 7250 years there will be 1.5kg left.

(a) Determine the exponential decay equation for this element. $Q = Q_0 e^{kt}$

(b) How long will it take for half the element to decay?

(c) How long will it take until 250 grams of the element left?

Common Graphs

Without using a graphing calculator or utility, sketch the graph of each of the following.

107. $y = -2x + 7$	108. $f(x) = x + 4 $	109. $f(x) = \sqrt{x} - 5$
110. $f(x) = x + 2 - 4$	111. $f(x) = \sqrt{x - 6} + 3$	112. $f(x) = x^2 + 8x - 1$
113. $f(x) = (x + 5)^2 + 3$	114. $f(x) = \ln x$	115. $f(x) = 2^x$
116. $x^2 - 6x + y^2 + 8y + 24 = 0$	117. $\frac{(x+4)^2}{25} + \frac{(y+2)^2}{25} = 1$	118. $f(x) = \tan\left(x + \frac{\pi}{3}\right)$
119. $f(x) = \sec(x) + 2$	120. $f(x) = 2^{-x}$	121. $f(x) = 2 \sin\left(2x - \frac{\pi}{6}\right) - 1$