Incoming AP Calculus BC Summer Work

Welcome! The Math Department is so excited that you are taking on the challenge of AP Calculus BC! AP Calculus BC is a wonderful expansion and growth on the knowledge that you built up in AP Calc AB. In order to jump into new material as soon as possible, you will need to review topics in the following packet.

A few guidelines:

- Try to complete this assignment without the use of your calculator. In the instance that you need to use a graphing calculator, please mark these problems and plan to check them on your own.
- For this class, **you will need a calculator, TI-83 or TI-84.**
- If at any time, you need assistance with the topics included here, you may use online resources, such as Khan Academy. Any sources that you use should be cited.
- **This assignment will be due the first full day of school**. There will be a **graded assessment** on these topics within the first two weeks of school.
- **Most of the questions on this assignment are covering limits**. We will not spend class time reviewing limits because you should be fluent with these topics from last year.
- Below, you will find a suggested breakdown of the assignment into manageable pieces.
    - Week 1: June 19 – June 23
        - Trig Review
    - Week 2: June 26 – June 30
        - Trig Graphs Review
    - Week 3: July 3 – July 7
        - Limits
    - Week 4: July 10 – July 14
        - Continuity
    - Week 5: July 17 – July 21
        - Limits at Infinity
    - Week 6: July 24 – July 28
        - Infinite Limits
    - Week 7: July 31 – August 4
        - Intermediate Value Theorem

If you have any questions, please also feel free to email Mrs. Hain at mhain@stpaulsmd.org. Please know it may take 48 hours to receive a response over the summer, and I will not be available to answer email from August 1 – August 7.

Thank you! Go Gators!

Week 1: Trig Review Worksheet

Without a calculator, evaluate each trig function at the given angle:

1. $\cos\frac{7\pi}{6}$
2. $\csc\frac{2\pi}{3}$
3. $\tan\frac{3\pi}{4}$
4. $\sin\frac{3\pi}{2}$
5. $\cos 240^\circ$
6. $\cot 2\pi$
7. $\cos\frac{-2\pi}{3}$
8. $\sin\frac{11\pi}{6}$
9. $\sec\frac{5\pi}{4}$
10. $\tan(-\pi)$
11. $\csc(-330^\circ)$
12. $\cot\frac{4\pi}{3}$

Without a calculator, solve for each angle(s) that makes the given equation true on the interval $[0, 2\pi]$.

13. $\sin\theta = \frac{\sqrt{2}}{2}$
14. $\cos\theta = -\frac{\sqrt{3}}{2}$
15. $\sec\theta = \sqrt{2}$
16. $\csc\theta = -1$
17. $\tan\theta = -\frac{\sqrt{3}}{3}$
18. $\cot\theta = \varnothing$

Use a calculator to find each value: (round to 3 decimal places if necessary)

19. $\sin 81^\circ$

20. $\tan -135^\circ$

21. $\cos \frac{4\pi}{9}$

22. $\sec -270^\circ$

23. $\cot \frac{3\pi}{4}$

24. $\cot 90^\circ$

Verify each of the following identities:

25. $\cos^3\theta + \sin^2\theta \cos\theta = \cos\theta$

26. $\frac{\sec^2\theta - 1}{\tan\theta} = \tan\theta$

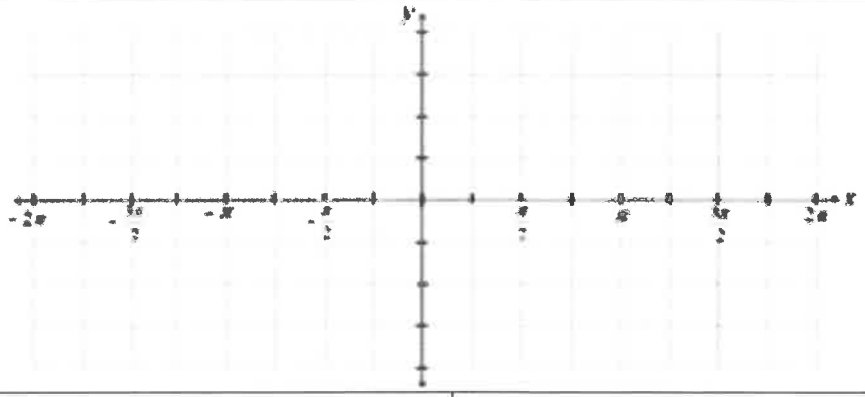
27 $(1 + \tan\theta)^2 = \sec^2\theta + 2\tan\theta$

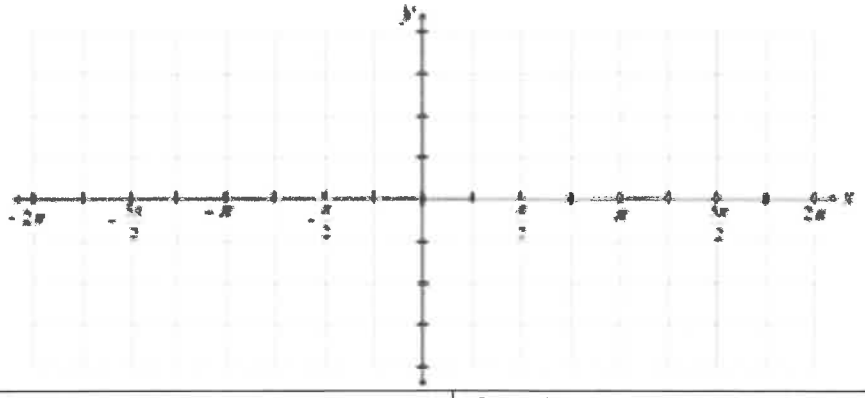
28. $\frac{1 - 2\csc\theta}{\cot\theta} = \tan\theta - 2\sec\theta$

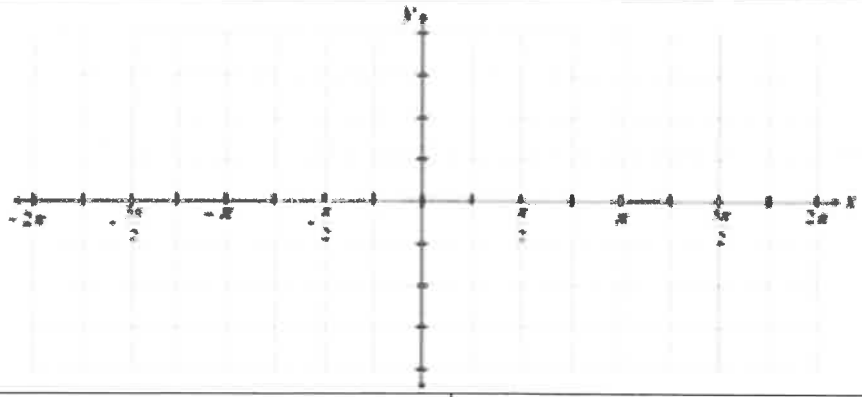
29. $\csc^4\theta - \cot^4\theta = 2\csc^2\theta - 1$

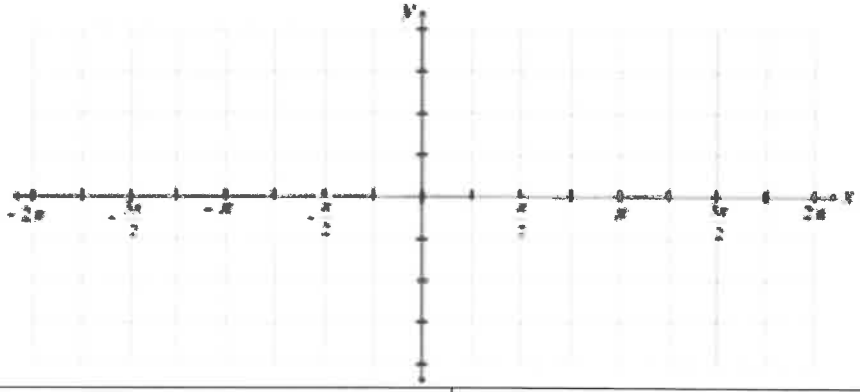
30. $\frac{1 + \sec\theta}{\tan\theta + \sin\theta} = \csc\theta$

Week 2: Graphs of Trigonometric Functions Review

$y = \sin x$		
Domain: Range:		
x-intercepts: y-intercepts:	Local max: Local min:	Continuous: Discontinuous:
Even/Odd/Neither:	End Behavior:	

$y = \cos x$		
Domain: Range:		
x-intercepts: y-intercepts:	Local max: Local min:	Continuous: Discontinuous:
Even/Odd/Neither:	End Behavior:	

$y = \tan x$		
Domain: Range:		
x-intercepts: y-intercepts:	Local max: Local min:	Continuous: Discontinuous:
Even/Odd/Neither:	End Behavior:	

$y = \arctan x$		
Domain: Range:		
x-intercepts: y-intercepts:	Local max: Local min:	Continuous: Discontinuous:
Even/Odd/Neither:	End Behavior:	

1.7 Selecting Procedures for Determining Limits

Calculus Name: _____ Week 3: Limits

Evaluate each limit.

1. $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$

2. $\lim_{x\to 3} \sqrt{2x+4}$

3. $\lim_{x\to 0} \frac{x}{\frac{1}{x+3}-\frac{1}{3}}$

4. $\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1}$

5. $\lim_{x\to -1} \frac{x+1}{x^2+2x+2}$

6. $\lim_{x\to 0} \frac{\frac{1}{x+2}-\frac{1}{2}}{x}$

7. $\lim_{x\to -1} \frac{x+1}{x^2-2x-3}$

8. $\lim_{x\to 13} \frac{\sqrt{x+12}-5}{x-13}$

9. $\lim_{x\to 0} \frac{\frac{1}{x-6}+\frac{1}{6}}{x}$

10. $\lim_{x\to 3} \frac{x^2-5x+6}{x-3}$

1.11 Defining Continuity at a Point

Calculus Name: _____

State whether the function is continuous at the given x values. Justify your answers!

1. $f(x) = \begin{cases} 2^x + 1, & x \le -1 \\ 2 + \frac{x}{2}, & -1 < x \le 4 \\ x^2 - 3x, & x > 4 \end{cases}$

Continuous at $x = -1$?

Continuous at $x = 4$?

2. $g(x) = \begin{cases} x^2 + 3x - 7, & x < -3 \\ 2x - 1, & -3 \le x < 1 \\ \ln x, & x \ge 1 \end{cases}$

Continuous at $x = -3$?

Continuous at $x = 1$?

For each function identify the type of each discontinuities and where they are is located.

3. $h(x) = \begin{cases} 4^x, & x < -1 \\ 2, & x = -1 \\ -\frac{x}{4}, & -1 < x \le 4 \\ \sqrt{x} & x > 4 \end{cases}$

4. $f(x) = \begin{cases} \cos\left(\frac{x}{3}\right), & x < \pi \\ \frac{\sqrt{2}}{2}, & x = \pi \\ \sin(x), & \pi < x < \frac{\pi}{2} \\ \sin\left(\frac{x}{2}\right) & x \ge \frac{\pi}{2} \end{cases}$

For each function find the value k that makes the function continuous.

5. $g(x) = \begin{cases} x^2 + 6, & x \le 2 \\ \frac{x}{3} + k, & x > 2 \end{cases}$

6. $h(x) = \begin{cases} (k - x)(k + 1), & x \le -1 \\ -13x - 2k, & x > -1 \end{cases}$

1.15 Limits at Infinity and Horizontal Asymptotes

Week 5: Limits at Infinity

Calculus Name: _____

Identify the horizontal asymptotes of each function.

1. $f(x) = \frac{\sqrt{9x^6+2x^2+x}}{x^3+9x^2}$

2. $f(x) = \frac{(3x+1)(x-4)}{(5x+1)^2}$

3. $f(x) = \frac{(4x+3)(6-3x)}{(3x-4)^2}$

4. $f(x) = \frac{5x^2}{\sqrt{5x^4-2x}}$

Evaluate each limit.

5. $\lim_{x\to\infty} \frac{2x^4+3x^3+4}{5x^7+2x^4+2x^3}$

6. $\lim_{x\to\infty} 4^{-x}+1$

7. $\lim_{x\to\infty} \frac{6x^6-3x^3+5}{2x^3+x^8+2x^4}$

8. $\lim_{x\to\infty} -x\cos x$

9. $\lim_{x\to\infty} \sin\left(\frac{4x+\pi x^2}{x^3}\right)$

10. $\lim_{x\to\infty} \sin(6x)$

11. $\lim_{x\to\infty} \frac{-5x^4-3x^2-6}{10x^4+6x+11}$

12. $\lim_{x\to\infty} \cos\left(\frac{x+4\pi x^2}{10-2x^2}\right)$

13. $\lim_{x\to\infty} \frac{2x^2+3x^4+1}{6x^2+5x-10}$

14. $\lim_{x\to\infty} \left(3\sin\frac{1}{x}\right)$

15. $\lim_{x\to\infty} \frac{e^3}{3^x}$

16. $\lim_{x\to\infty} \left(\frac{\sin x}{x}\right)$

1.14 Infinite Limits and Vertical Asymptotes

Calculus Name: _____

Identify the vertical asymptotes of each function.

1. $f(x) = \frac{x^2+4x+3}{x+1}$

2. $f(x) = \frac{x-5}{x^2-11x+30}$

3. $f(x) = \tan(2x)$ on the interval $[0, \pi]$

4. $f(x) = \frac{x-3}{x^2-10x+21}$

Evaluate the limit.

5. $\lim_{x\to 2^-} \frac{x}{2-x}$

6. $\lim_{x\to -2} \frac{x-1}{x^2+4x+4}$

7. $\lim_{x\to 1} \frac{x+2}{x^2+x-2}$

8. $\lim_{x\to 3^-} \frac{x-2}{x^2-5x+6}$

1.16 Intermediate Value Theorem (IVT)
Calculus

Week 7: Intermediate Value Theorem

Below is a table of values for a continuous function f.

x	0	2	4	7	10
$f(x)$	-100	-1	-3	2	-5

1. On the interval $0 \le x \le 2$, must there be a value of x for which $f(x) = -4$? Explain.

2. What is the minimum number of zeros f must have on the interval $0 \le x \le 10$?

3. For $2 \le x \le 7$, what is the fewest possible number of times $f(x) = 1$?

4. For $0 \le x \le 7$, what is the fewest possible number of times $f(x) = -2$?

Below is a table of values for a continuous function g.

x	-5	10	17	20	30
$g(x)$	2	6	-5	9	1

5. On the interval $10 \le x \le 17$, must there be a value of x for which $g(x) = 2$? Explain.

6. What is the minimum number of zeros g must have on the interval $-5 \le x \le 30$?

7. For $17 \le x \le 30$, what is the fewest possible number of times $f(x) = 10$?

8. For $-5 \le x \le 20$, what is the fewest possible number of times $f(x) = 3$?

Use the Intermediate Value Theorem to answer each problem.

9. If $f(x) = 10 - x^2$, will $f(x) = 5$ on the interval $[-1, 3]$? Explain.

10. If $g(x) = \frac{1}{x^2}$, will $g(x) = 2$ on the interval $\left[\frac{1}{2}, 3\right]$? Explain.