FAIRFIELD PUBLIC SCHOOLS

Grade(s):	**12**
Discipline/Course:	**Mathematics**
Course Title:	**Multivariable Calculus**
Prerequisite(s):	AP Calculus AB or BC
Course Description: *Program of Studies*	Multivariable Calculus is a rigorous second year course in college level calculus. This course provides an in-depth study of vectors and the calculus of several variables for the student who has successfully completed AP Calculus. The successful student will bring to the course a solid understanding of the concepts of first-year calculus as well as the ability to approach complex problems and applications with insight, imagination, and persistence. The critical areas of focus for this course are: 1. The dot product and cross product of vectors are given geometric definitions, motivated by work and torque, before the algebraic expressions are deduced. To facilitate the discussion of surfaces, functions of two variables and their graphs are introduced. 2. The calculus of vector functions is used to provide Kepler's First Law of planetary motion. In keeping with the introduction of parametric curves as introduced in prior courses, parametric surfaces are introduced, thus allowing for a discussion of tangent planes and areas of parametric surfaces. 3. Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. Directional derivatives are estimated from contour maps of temperature, pressure, and snowfall. 4. Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, areas of parametric surfaces, volumes of hyperspheres, and the volume of intersection of three cylinders.
Course Essential Questions:	● How can we use the concepts of multivariable calculus to understand the physical world around us?

Course Enduring Understandings:	● We live in a three dimensional world and in order to understand that world, physicist and mathematicians need models that involve multivariables.
Duration:	One Year
Course Materials/Resources:	Calculus: Concepts and Contexts, Single-Term (Stewart) WebAssign K12 Instant Access for Stewart's Calculus: Concepts and Contexts, Single-Term

***Note: Topics listed in the units may evolve over time based on adaptations to implementation. However, the overall content of the entire course will not change**

Academic Expectations The Fairfield Public Schools describe a variety of cross curricular expectations that all students should exemplify during their time within the schooling experience. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:		
Standards	Explanations	Example
1. Exploring and Understanding [MP1]	When students engage in problem solving situations, they should be able to understand the problem, determine relevant information, and ask relevant additional questions.	Students should be able to answer the following questions when approaching a problem: 1. Do you understand all the words used in stating the problem? 2. What are you asked to find or show? 3. Can you restate the problem in your own words? 4. Can you think of a picture or diagram that might help you understand the problem?
2. Synthesizing and Evaluating	Engaging in a problem solving situation, students should be able to analyze the most efficient approach, and reflect on the process used to solve the problem.	Students should be able to answer the following questions when analyzing how to approach a problem, and also reflect on the result: 1. Is there enough information to enable you to find a solution? If not, what additional information is needed? 2. Are there multiple ways to complete the task? Which approach do you think is most efficient, and why? 3. Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown. Can you use it? 4. Was your strategy effective? What worked? What didn't? 5. Was there another approach that could have been more efficient? 6. Is your answer reasonable? How do you know? 7. Was your presentation approach effective? If not, what would you change? 8. How did the communication tools allow you to get the message across to the intended audience?

3. Creating and Constructing	Engaged in a problem solving situation, students should implement a plan.	Students should be able to answer the following question to implementing their plan to solve a problem: 1. What strategy will you use to complete the task?
4. Conveying Ideas	Students should be able to use correct mathematical language, logically display their work for the desired problem.	Students should be able to answer the following questions to convey their mathematical thinking to solve a problem: 1. How will you present your information to your intended audience? 2. Does your response illustrate the correct terms and work to the problem?
5. Using Communication Tools	Students should be able to choose the correct tools to illustrate their mathematical work to solve a specific problem.	Students should be able to answer the following question to use specific communication tools to solve a problem: 1. If applicable, what communication tools will you use to convey your ideas and solution?
6. Collaborating Strategically	Students should be able to work collaboratively to solve problems.	Students should be able to answer the following question to collaboratively solve problems: 1. In what ways did you work together to help solve the desired problem?

Unit Number and Title:	Unit 1: Vectors and the Geometry of Space,
Duration:	9 Weeks
Resource(s):	Calculus: Concepts and Contexts, Single-Term
Learning Goals	
Standard(s):	N/A
Essential Question(s):	• What is a vector, what are its properties and how does one measure its magnitude and direction? • What operations can be performed using vectors? • What is a dot product and how can it be used to measure the work done by a force? • What is a cross product and what are its applications in physics and engineering? • What is a determinant? • How can determinants be used to solve systems of equations involving two or more variables (Cramer's Rule)? • What is Gaussian Elimination and how can it be used to solve systems of equations involving multiple variables? • How are lines and planes defined in three space? • How can you use vectors to determine equations of lines and planes in space? • How are functions in two variables defined and how do they relate to surfaces in three space? • How do the rectangular, cylindrical and spherical coordinate systems relate to each other in three-space.
Enduring Understanding(s):	• Vectors and coordinate systems for three-dimensional space. • Real-Value functions of two variables as a representation of a surface in space • Vectors as a way of describing lines and planes in space as well as velocities and accelerations of objects that move through space.

Learning Goal(s): *Students will be able to use their learning to:*	1. Investigate the relationship between points, lines, and planes in three-dimensions.
	    a. Represent equations of lines in space using vectors.
	    b. Express analytic geometry of three dimensions (equations of planes, parallelism, perpendicularity, angles) in terms of the dot product and cross product of vectors.
	2. Recognize and apply properties of matrices.
	    a. Find the determinant of 2-by-2 and 3-by-3 matrices.
	    b. Represent a 3-by-3 system of linear equations as a matrix and solve the system in multiple ways: the inverse matrix, row operations, and Cramer's Rule.
	    c. Apply properties of similar and orthogonal matrices to prove statements about matrices.
	    d. Find and apply the eigenvectors and eigenvalues of a 3-by-3 matrix.

Unit Number and Title:	Unit 2: Vector Functions
Duration:	9 weeks
Resource(s):	Calculus: Concepts and Contexts, Single-Term
Learning Goals	
Standard(s):	N/A
Essential Question(s):	● What is a vector function? ● What is a derivative/integral of a vector function? ● What is a space curve and how do we measure its length and curvature? ● What are tangent, bi-normal, and normal vectors as well as the normal and osculating planes and how do they relate to a space curve? ● How can the ideas of a tangent and normal vectors and curvature be used in physics to study the motion of an object, including its velocity and acceleration, along a space curve? ● What is a parametric surface and how are they defined by parametric vector functions?
Enduring Understanding(s):	● Vector-valued functions as a description of curves and surfaces in space. ● Vector-valued functions as a description of the motion of objects through space.
Learning Goal(s): *Students will be able to use their learning to:*	1. Explore functions of two independent variables of the form z = f(x, y) and implicit functions of the form f(x, y, z) = 0. a. Evaluate such functions at a point in the plane. b. Graph the level curves of such functions. c. Determine points or regions of discontinuity of such functions.

Unit Template

Unit Number and Title:	Unit 3: Partial Derivatives,
Duration:	9 weeks
Resource(s):	Calculus: Concepts and Contexts, Single-Term
Learning Goals	
Standard(s):	N/A
Essential Question(s):	• What is a contour map and what are level curves and level surfaces? • What does it mean for a function in three-space to be continuous? • How can you determine if a limit exists in three-space? • What is a partial derivative and how is it interpreted? • What is Laplace's equation and the wave equation and what are there significance? • What is a linear or tangent plane approximation of a function at a point and what is it used for? • How is the chain rule applied when taking derivatives of functions of two variables? • How does one implicitly differentiate a function of three variables? • What is the Implicit Function Theorem? • What is a directional derivative? • What is a gradient vector and what meaning does it have? • How does one calculate the minima and maxima values of a function of two variables? • What applications are there for maximizing or minimizing the value of a function?
Enduring Understanding(s):	• The concepts of differential calculus as they relate to various real and vector valued functions. • Various functions will be explored and understood verbally, numerically, algebraically and visually
Learning Goal(s):	1. Explore the continuity of functions of two independent variables in terms of the limits of such

Students will be able to use their learning to:	functions as (x, y) approaches a given point in the plane. a. Determine the continuity of a function given a point on the plane. 2. Explore, find, use, and apply partial differentiation of functions of two independent variables of the form z = f(x, y) and implicit functions of the form f(x, y, z) = 0. a. Approximate the partial derivatives at a point of a function defined by a table of data. b. Find expressions for the first and second partial derivatives of a function. c. Explore the continuity of functions of two independent variables in terms of the limits of such functions as (x, y) approaches a given point in the plane. d. Determine the continuity of a function given a point on the plane. e. Explore, find, use, and apply partial differentiation of functions of two independent variables of the form z = f(x, y) and implicit functions of the form f(x, y, z) = 0. f. Approximate the partial derivatives at a point of a function defined by a table of data. g. Find expressions for the first and second partial derivatives of a function.

Unit Template

Unit Number and Title:	Unit 4: Integration
Duration:	9 weeks
Resource(s):	Calculus: Concepts and Contexts, Single-Term
Learning Goals	
Standard(s):	N/A
Essential Question(s):	● What is a double integral and how can it be used to find the volume of a solid? ● How do you express a double integral as an iterated integral so that we can use standard integration methods to evaluate the expression? ● How do we integrate a function of a general, non-rectangular, region? ● How do you use polar coordinates to simplify the integration of solids over circular regions? ● How can double integrals be used to calculate mass, electrical charge, center of mass, moment of inertia and other physical attributes of a solid? ● How can double integrals be used to calculate the surface area of a solid? ● What is a triple integral and how can it be used to evaluate functions of three variables? ● How can triple integrals be used to calculate various physical attributes of a function of three variables, such as density? ● How do we evaluate the triple integral of certain solids using cylindrical or spherical coordinates? ● How can the method of substitution, change of variables, be used to simplify an integral? ● What is a Jacobian?
Enduring Understanding(s):	● The idea of the definite integral is extended to double and triple integrals of functions of two and three variables ● Double and Triple integrals are used to calculate volumes, surface areas, masses, electrical

	charges and other variable characteristics of a surface.
Learning Goal(s): *Students will be able to use their learning to:*	1. Integrate functions of the form z = f(x, y) or w = f(x, y, z). a. Define, use, and interpret double and triple integrals in terms of volume and mass. b. Represent integrals of vectors as double and triple integrals. c. Integrate functions through various techniques such as changing the order of integration, substituting variables, or changing to polar coordinates. 2. Apply and interpret the theorems of Green, Stokes, and Gauss. a. Apply line and surface integrals to functions representing real-world phenomena. b. Recognize, understand, and use line integrals that are independence of path.\ c. Define and apply the gradient, the divergence, and the curl in terms of integrals of vectors.