This document is a draft and was created in collaboration with middle school and high school math specialists and the curriculum and instruction specialist. The approach detailed here will also be adjusted through our partnership with Michigan Math and Science Learning Network (MMSLN).

Comparison of Algebra 1 and Math 8 Standards:

Algebra 1 Standards	Math 8 Standards	Analysis
A-CED Create equations that describe numbers or relationships *"Create equations and inequalities in one variable and use them to solve problems."*	8.EE Analyze and solve linear equations *"Solve linear equations in one variable"*	In Math 8 students are solving only linear equations In Algebra 1 students are solving both linear equations and inequalities.
A-REI Reasoning with equations and Inequalities *"Solve systems of equations"*	8.EE Analyze and solve linear equations and pairs of simultaneous linear equations. *"Solve systems of two linear equations"*	In Math 8 the focus is the solution to a linear system of equations is the intersection of the linear equations, specifically representing this graphically. Students also solve systems algebraically. In Algebra 1 students prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A-SSE Seeing Structure in expressions	Not addressed in Math 8	

F-BF Building functions "*Build a function that models a relationship between two quantities"*	8.F Use functions to model relationships between quantities.	In Math 8 the focus is on linear models In Algebra 1 students construct and compare, linear, quadratic, and exponential functions.
F-IF Interpreting Functions "*Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x)."* *"Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima."*	8.F Define and evaluate and compare functions "*Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.1"* 8.EE Understand the connections between proportional relationships, lines, and linear equations. "*Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b."*	In Math 8 students define a function. In Algebra 1 use the definition of a function to introduce a new notation for functions In Math 8 students extend their understanding of proportional relationships established in Math 7 to define slope intercept form. In Algebra 1 students define slope intercept form, and are introduced to equivalent forms (i.e. standard and point-slope) to reveal and explain different properties.
F-LE Linear, Quadratic, and exponential models	Not addressed in Math 8	

N-Q Reason quantitatively and use units to solve problems.	Not addressed in Math 8	
N-RN Extend the properties of exponents to rational exponents Use properties of rational and irrational numbers.	8.EE Work with radicals and integer exponents. 8.NS Know that there are numbers that are not rational, and approximate them by rational numbers.	In Math 8 students establish integer exponents. In Algebra 1 students review integer exponents and establish rational exponents.
S-ID Summarize, represent, and interpret data on two categorical and quantitative variables Interpret linear models *"Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data."* Summarize, represent, and interpret data on two categorical and quantitative variables *"Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal,*	8.SP Investigate patterns of association in bivariate data. *"Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept."* *"Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe*	Standards in Math 8 and Algebra 1 in the domain of statistics and probability are extremely similar.

and conditional relative frequencies). Recognize possible associations and trends in the data."	*possible association between the two variables."*	
Not addressed	8.G Understand congruence and similarity using physical models, transparencies, or geometry software. Understand and apply the Pythagorean Theorem. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	The Math 8 geometry standards are closely aligned to the standards in high school geometry. Math 8 is the initial exposure but there is significantly more instructional time devoted to these concepts in the high school geometry course.

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7.3: It Doesn't Work!

Noah is having trouble solving two equations. In each case, he took steps that he thought were acceptable but ended up with statements that are clearly not true.

Analyze Noah's work on each equation and the moves he made. Were they acceptable moves? Why do you think he ended up with a false equation?

Discuss your observations with your group and be prepared to share your conclusions. If you get stuck, consider solving each equation.

1.

$x + 6 = 4x + 1 - 3x$	original equation
$x + 6 = 4x - 3x + 1$	apply the commutative property
$x + 6 = x + 1$	combine like terms
$6 = 1$	subtract x from each side

2.

$2(5 + x) - 1 = 3x + 9$	original equation
$10 + 2x - 1 = 3x + 9$	apply the distributive property
$2x - 1 = 3x - 1$	subtract 10 from each side
$2x = 3x$	add 1 to each side
$2 = 3$	divide each side by x

Are you ready for more?

1. We can't divide the number 100 by zero because dividing by zero is undefined.
   a. Instead, try dividing 100 by 10, then 1, then 0.1, then 0.01. What happens as you divide by smaller numbers?

   b. Now try dividing the number -100 by 10, by 1, by 0.1, 0.01. What is the same and what is different?

An example activity from Algebra 1, standard A-CED

Illustrative Mathematics

6.3: Which Would You Rather Solve?

Here are a lot of equations:

A. $-\frac{5}{6}(8 + 5b) = 75 + \frac{5}{3}b$

B. $-\frac{1}{2}(t + 3) - 10 = -6.5$

C. $\frac{10-v}{4} = 2(v + 17)$

D. $2(4k + 3) - 13 = 2(18 - k) - 13$

E. $\frac{n}{7} - 12 = 5n + 5$

F. $3(c - 1) + 2(3c + 1) = -(3c + 1)$

G. $\frac{4m-3}{4} = -\frac{9+4m}{8}$

H. $p - 5(p + 4) = p - (8 - p)$

I. $2(2q + 1.5) = 18 - q$

J. $2r + 49 = -8(-r - 5)$

1. Without solving, identify 3 equations that you think would be least difficult to solve and 3 equations you think would be most difficult to solve. Be prepared to explain your reasoning.

2. Choose 3 equations to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

An example activity from Math 8, standard 8.EE

Conclusions:

The chart above illustrates that many of the Math 8 standards can be embedded into Algebra 1. Let's take a deep dive into Algebra 1 Unit 2 in particular, to see how we ensure Math 8 learning happens in Algebra 1. Our wondering is how much condensing of content will have to happen. We also recognize that some students have already explored some of these concepts in previous courses. Algebra 1 is where students are formalizing, and generalizing concepts.

In the following example we will show how Algebra 1 will address the essential learning from Math 8. In this unit narrative, provided by IM, we have highlighted learning that is identified as taking place in Math 8. We can see that the Math 8 concepts are embedded using a just-in-time model that can be changed and adapted to meet the needs of students. We illustrate the process of using the unit narrative, the pre-assessment (check your readiness assessment), and adaptation pack to ensure Math 8 concepts are not skipped.

IM Algebra 1 Unit Narrative Example:

Unit 2: Linear Equations, Inequalities, and Systems

In middle school, students began building an understanding of how variables, expressions, equations, and inequalities could be used to represent quantities and relationships. Students also made connections among different kinds of representations—algebraic, verbal, tabular, and graphical. In this unit, students further develop their capacity to create, manipulate, interpret, and connect these representations and to use them for modeling.

In the first few lessons, students learn to think of equations as a way to represent constraints or limitations on quantities. (For instance, if the cost of food, f, and the cost of drinks, d, for a party add up to \$80, we can write $f + d = 80$ to represent this constraint.) Students understand that when we solve equations, we are looking for values that satisfy the constraints and make the

equations true. (For example, $f = 53$ and $d = 27$ could be a pair of solutions to $f + d = 80$, but $f = 50$ and $d = 35$ could not be.) Students also see that graphs of equations can help us make sense of constraints and identify values that satisfy them.

Students then investigate different ways to express the same relationship or constraint—by analyzing and writing equivalent equations. They look at moves that can transform one equation to an equivalent equation, recognizing that these are the moves we make to solve equations. The focus here is not only on identifying acceptable moves for solving, but also on explaining why these moves keep each subsequent equation true and maintain the solutions of the original equation.

Along the way, students realize that some equations are more helpful than others, depending on what we want to know. In some equations, the quantity of interest is easy to pin down. In others, we may need to manipulate the equation and solve for a particular variable. Students also explore how the form and the parts of a linear equation in two variables are related to the features of its graph. They see that understanding the structure and connections across representations can give us deeper insights about the situation being studied.

Next, students encounter situations that involve two or more constraints. In those cases, we often want to find values that satisfy both or all constraints simultaneously. Systems of equations are helpful for representing these constraints. (The work here is limited to systems of linear equations in two variables.)

Students draw on their understanding of systems of linear equations from grade 8 to solve problems, but soon notice the limitations of solving systems by graphing and by substitution. They then learn to solve systems of equations by elimination, to explain why the steps taken to eliminate a variable are valid and productive, and to articulate how the process essentially entails writing a series of equivalent systems. Additionally, students reinforce their awareness that a system of equations could have one solution, no solutions, or infinitely many solutions.

In the last third of the unit, students rely on their understanding of equations to explore inequalities in one and two variables. They see that inequalities are a handy way to express constraints that involve an upper or lower limit, and can be satisfied by a range of values rather than a single value. (For instance, if the weekly work hours, h, of an employee must be at least 40 or $h \geq 40$, any value that is 40 or greater meets this constraint.)

Students see that a solution to an inequality (in one or two variables) is a value or a pair of values that makes the inequality true, and a solution to a system of inequalities in two variables is any pair of values that make both inequalities in the system true. The solution set of a system of inequalities, they learn, can be best represented by graphing."

Unit 2 Check your Readiness (pre-unit diagnostic assessments)

At the start of each unit is a pre-unit diagnostic assessment that is titled *Check Your Readiness*. These assessments vary in length. Most of the problems address prerequisite concepts and skills for the unit. Teachers can use these problems to identify students with particular below-grade needs, or topics to carefully address during the unit. *Check Your Readiness* also may include problems that assess what students already know of the upcoming unit's key ideas, which teachers can use to pace or tune instruction; in rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.

What if a large number of students can't do the same pre-unit assessment problem? Teachers are encouraged to address below-grade skills while continuing to work through the on-grade tasks and concepts of each unit, instead of abandoning the current work in favor of material that only addresses below-grade skills. Look for opportunities within the upcoming unit where the target skill could be addressed in context. For example, an upcoming activity might require solving an equation in one variable. Some strategies might include:

- ask a student who can do the skill to present their method
- add additional questions to the warm-up with the purpose of revisiting the skill
- add to the activity launch a few related equations to solve, before students need to solve an equation while working on the activity
- pause the class while working on the activity to focus on the portion that requires solving an equation

Then, attend carefully to students as they work through the activity. If difficulty persists, add more opportunities to practice the skill, by adapting tasks or practice problems.

What if all students do really well on Check Your Readiness? That means they are ready for the work ahead, and special attention doesn't likely need to be paid to below-grade skills.

Zooming In - #3 on the Check your Readiness assessment:

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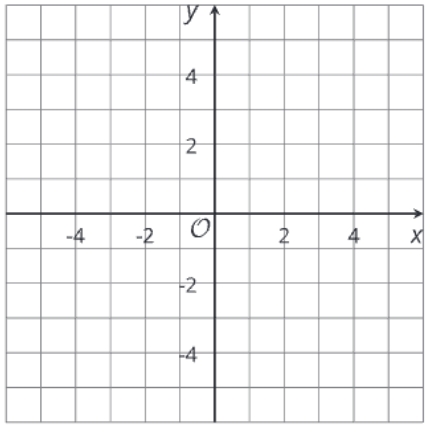
3. Graph the equations $y = 4x + 2$ and $4x - 3y = 12$ on the grid. Label each graph.



Embedded Algebra 1 Supports to ensure learning for all:

See supports for question #3 below.

Problem	Standard(s)	Content is first encountered in...	If students struggle...	If most students do well...
1	8.EE.C.7	Lesson 6: Equivalent Equations	• Plan to take time in the Lesson 6 Activity 2 synthesis to discuss basic moves for solving one-step equations. • For instance, $a - 4 = 6$ is equivalent to $a = 10$, and $\frac{1}{2}t = 7$ is equivalent to $t = 14$.	
2	6.EE.A.3	Lesson 8: Which Variable to Solve for? (Part 1)	• Plan to monitor during Lesson 8 Activity 3 for student errors and use of any equivalent forms when solving for a variable. • During the synthesis, propose alternate ways of writing answers based on these errors or equivalent forms. • For example: $\frac{1}{9}(p - 124)$ and $\frac{p}{9} - 124$ for $\frac{p-124}{9}$. If there is disagreement, substitute particular values of p to settle the issue. • Additionally, the Lesson 11 warmup is explicitly about expressions of this form. • If students still need extra practice, plan to spend extra time on this warmup before going on to convert linear equations from standard form into slope-intercept form.	
3	8.EE.B	Lesson 10: Connecting Equations to Graphs (Part 1)	• Plan to spend as much time as needed on each question in the Activity 10.2 synthesis, which covers both conceptual and procedural aspects of graphing lines in detail.	• Plan to have the Lesson 10 Activity 2 synthesis go quickly, and perhaps skip some questions. • Students may still need the review of the real-life interpretation of slope.
4	8.EE.C.8.a	Lesson 12: Writing and Graphing Systems of Linear Equations		• If most students do well with this item, plan to emphasize the aspects of Lesson 12 that have to do with writing equations. • Also spend time in Activity 2 discussing the connection between the tables of values and the graphs.

Example Adaptation Pack for just-in-time support:

Using the data from the Check your Readiness assessment and from formative and summative assessments in previous units, teachers may decide to implement parts, or all, of the adaptation pack modified plan to address unfinished learning according to the Math 8 standards.

Lessons to Add	Lessons to Remove or Modify
1. Combine 8.4.3 and 4, particularly Activity 3 in Lesson 3 and Activities 2 and 3 in Lesson 4. Focus on the idea of using the same operation with the expressions on each side of an equation or changing the form of one of the expressions through combining like terms, applying the distributive property, and similar operations 2. Combine 8.3.8 and 8.3.9. Introduce $y = mx + b$, and include an activity from Lesson 9 to introduce negative slope. 3. 8.3.10: Calculating slope 4. 8.3.11: Equations for horizontal and vertical lines 5. 8.4.12: Solving systems by graphing	1. Combine Lessons 1 and 2: Use an activity from Lesson 1 to introduce the idea of constraint. Remove Activities 1 and 4 from Lesson 2. 2. Combine Lessons 8 and 9. (Remove 8.2 and the cool down for Lesson 8, and 9.2). 3. Combine Lessons 15 and 16. 4. Remove Lesson 26.
Lessons added: 5	Lessons removed: 4

Modified plan

Check Your Readiness modified plan

Use the A1.2 Check Your Readiness Assessment to determine student needs for incorporating prior content.

Day-by-day modified plan

This modified unit plan is provided to show how the recommended lesson additions could be blended into the unit to provide review as it is needed.

Day	IM lesson	Notes
1	A1.2.1 A1.2.2	Use equations to represent situations, introduces constraint. Consolidate Lessons 1 and 2. The Pizza Party activity can be shortened to get at this idea, and the cool-down can be safely skipped. Activities 2 and 3 from Lesson 2 should be emphasized. The percent work from the warm-up and Activity 4 are good activities to think about percentages in context, but omitting them will not get in the way of learning in future lessons.
2	A1.2.3	Write equations to model relationships.
3	A1.2.4	Equations and their solutions
4	A1.2.5	Equations and their graphs
5	8.4.3 8.4.4	Solving equations. Choose activities from these lessons to activate prior knowledge of solving equations and focus on the concept of “balancing equations.” This will support student thinking in the Algebra 1 lessons where they move to more formal understandings of solving equations.

6	A1.2.6	Solve equations.
7	A1.2.7	Solve equations and explain the steps for solving.
8	A1.2.8 A1.2.9	Rearrange equations to solve for a specific variable.
9	8.3.8 8.3.9	Focus on introducing slope-intercept form and understanding negative slope.
10	8.3.10	Calculate slope.
11	8.3.11	Write equations of horizontal and vertical lines.
12	A1.2.10	Connect equations to graphs (emphasis on standard form of equations).
13	A1.2.11	Connect equations to graphs (emphasis on standard form of equations and rearranging the equation to move between standard and slope-intercept form).
14	8.4.12	Solve systems of equations by graphing
15	A1.2.12	Write and solve systems of equations by graphing.
16	A1.2.13	Solve systems of equations by substitution.
17	A1.2.14	Solve systems of equations by elimination.

18	A1.2.15 A1.2.16	Solve systems of equations by elimination. (Lesson 15 emphasizes multiplying one equation by a constant to eliminate. Lesson 16 emphasizes multiplying both equations by constants to eliminate.)
19	A1.2.17	Systems of linear equations and their solutions
20	MUA	Mid-Unit Assessment
21	A1.2.18	Represent situations with inequalities
22	A1.2.19	Solutions to inequalities in one variable
23	A1.2.20	Write and solve inequalities in one variable
24	A1.2.21	Graph linear inequalities in two variables.
25	A1.2.22	Graph linear inequalities in two variables
26	A1.2.23	Solve problems with inequalities in two variables
27	A1.2.24	Solutions to systems of linear inequalities
28	A1.2.25	Solve problems with systems of linear inequalities
29	EOU	End-of-Unit Assessment

Replicating this process for all units in Algebra 1 below are the implications for instruction based on full implementation of the adaptation pack. Please remember not all these standards may need to be addressed, it depends on the needs of the students.

Algebra 1 Unit	Math 8 standards addressed	Estimated number of instructional days added
1	Not applicable	
2	8.EE.C, 8.EE.C.7, 8.EE.B, 8.G.A.1, 8.EE.B, 8.EE.B.6, 8.EE.C.8, 8.EE.C.8.b	5
3	8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4,	3
4	8.F.A, 8.F.A.1, 8.F.B.4	2
5	8.EE.A.1, 8.EE.A.3, 8.EE.A.4	2
6	Not applicable	
7	Not applicable	

On-going work and considerations:

- Math 8 concepts that are not embedded into Algebra 1 can be addressed with just-in-time lessons.
- Illustrative Math's Algebra 1 course is 140 lessons (including assessments).
- In our previous curriculum Algebra 1 is 160 days (including assessments).
- Could some, or all, of the geometry standards move to Honors Geometry? What might that mean for the Honors Geometry course? Given that many of the Math 8 geometry standards are also addressed in Geometry.
- If the Math 8 geometry standards are added to Algebra 1 we anticipate an additional 2-3 weeks of instruction