SPSG Summer Math Reinforcement Packet Created for Students Entering into SPSG 8th Geometry in the Fall 2023

Dear Students and Parents: The purpose of a summer math packet is to review math concepts that are often forgotten during the long break from school. Most of the concepts in this packet were covered in 7th grade. **It is imperative that you pace yourself and not complete all of the packet in the first few weeks or the last few weeks of summer. Please work on a little bit at a time throughout the whole summer.**

- Your completed packet will be checked for effort and completion during the first week of school.
- Please show all your work (when possible) to earn full effort credit.
- PLEASE DO NOT USE A CALCULATOR TO COMPLETE ANY OF THE WORK.
- A parent's signature will be required to earn a full effort/completion grade.
- Please pace yourself...completing 10 - 15 problems per week is a perfect pace.
- An additional optional Extra Fun Challenges section is included at the end of the packet.

(Another optional review tool is to use the IXL online math program and focus on skills for the upcoming grade level. 30-minute practice sessions each week are very beneficial.)

Have a wonderful summer!

The SPSG Math Department

Student's First Name:_____ Last Name:_____

The work in this packet was completed independently (without a calculator) by my daughter.

Parent Signature_____ Date _____

Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
   - Parentheses/Brackets (inside to outside)
   - Exponents
   - Multiplication/Division (left to right)
   - Addition/Subtraction (left to right)

ex: $9x^2 - 4(y + 3z)$
for $x = -3, y = 2, z = 5$

$9(-3)^2 - 4(2 + 3 \cdot 5)$

$9(-3)^2 - 4(2 + 15)$

$9(-3)^2 - 4 \cdot 17$

$9 \cdot 9 - 4 \cdot 17$

$81 - 4 \cdot 17$

$81 - 68 = \boxed{13}$

The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex: $5(8x - 3)$

$5(8x - 3)$

$5(8x) - 5(3)$

$\boxed{40x - 15}$

Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex: $2(3x - 4) - 12x + 9$

$2(3x - 4) - 12x + 9$

$6x - 8 - 12x + 9$

$\boxed{-6x + 1}$

Evaluate each expression for $a = 9$, $b = -3$, $c = -2$, $d = 7$. Show your work.

1. $a - cd$	2. $2b^3 + c^2$	3. $\frac{a + d - c}{b}$	4. $(a - b)^2 + d(a + c)$
5. $4c - (b - a)$	6. $\frac{a}{b} - 5a$	7. $2bc + d(12 - 5)$	8. $b + 0.5[8 - (2c + a)]$

Simplify each expression using the Distributive Property.

9. $5(2g - 8)$	10. $7(y + 3)$	11. $-3(4w - 3)$	12. $(6r + 3)2$

Simplify each expression, showing all work.

13. $8(x + 1) - 12x$	14. $6w - 7 + 12w - 3z$	15. $9n - 8 + 3(2n - 11)$	16. $3(7x + 4y) - 2(2x + y)$
17. $(15 + 8d)(-5) - 24d + d$	18. $9(b - 1) - c + 3b + c$	19. $20f - 4(5f + 4) + 16$	20. $8(h - 4) - h - (h + 7)$

Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex: $-18 = 6j$

$\frac{-18}{6} = \frac{6j}{6}$

$-3 = j \rightarrow \boxed{j = -3}$

Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex: $\frac{a}{7} - 12 = -9$

$\frac{a}{7} - 12 = -9$

$+12 \quad +12$

$7 \times \frac{a}{7} = 3 \times 7$

$\boxed{a = 21}$

Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex: $5(2x - 1) = 3x + 4x - 1$

$10x - 5 = 3x + 4x - 1$

$10x - 5 = 7x - 1$

$-7x \quad -7x$

$3x - 5 = -1$

$+5 \quad +5$

$\frac{3x}{3} = \frac{4}{3}$

$\boxed{x = \frac{4}{3}}$

Solve each equation, showing all work.

21. $f - 64 = -23$	22. $-7 = 2d$	23. $\frac{b}{-12} = -6$	24. $13 = m + 21$
25. $5x - 3 = -28$	26. $\frac{w + 8}{-3} = -9$	27. $-8 + \frac{h}{4} = 13$	28. $22 = 6y + 7$
29. $8x - 4 = 3x + 1$	30. $-2(5d - 8) = 20$	31. $7r + 21 = 49r$	32. $-9g - 3 = -3(3g + 2)$
33. $5(3x - 2) = 5(4x + 1)$	34. $3d - 4 + d = 8d - (-12)$	35. $f - 6 = -2f + 3(f - 2)$	36. $-2(y - 1) = 4y - (y + 2)$

Scientific Notation

Standard Form to Scientific Notation: move the decimal after the first non-zero digit and eliminate any trailing zeros. Multiply by 10 to the power equal to the number of places you moved the decimal point. If the original number was greater than 1, the exponent is positive. If the number was less than 1, the exponent is negative.

ex: 0.0000571

0.0000571

Original number < 1, so negative exponent

$= \boxed{5.71 \times 10^{-5}}$

Scientific Notation to Standard Form: move the decimal point the number of places indicated by the exponent. If the exponent is positive, move the decimal right. If negative, move left.

ex: 3.5×10^3

Positive exponent, so move decimal right

$3.500. = \boxed{3,500}$

Negative Exponents & Simplifying Monomials

Zero Exponent: Any number raised to the zero power equals 1

ex: $y^0 = 1$

Negative Exponent: Move the base to the opposite side of the fraction line and make the exponent positive

ex: $x^{-4} = \frac{1}{x^4}$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases

ex: $(4x^3)(2x^5) = 8x^8$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases

ex: $\frac{a}{a^6} = a^{-5} = \frac{1}{a^5}$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex: $(-2fg^5)^3 = -8f^3g^{15}$

Power of a Quotient: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex: $\left(\frac{5d^3}{c}\right)^2 = \frac{25d^6}{c^2}$

Convert each number to Scientific Notation.

37. 67,000,000,000	38. 0.0009213	39. 0.00000000004	40. 3,201,000,000,000,000

Convert each number to Standard Form.

41. 5.92×10^{-5}	42. 1.1×10^{7}	43. 6.733×10^{-8}	44. 3.27×10^{2}

Simplify each expression. Write your answers using only positive exponents.

45. w^{-9}	46. $\frac{m^5}{m^2}$	47. $f^5 \cdot f^3$	48. $\left(\frac{h^2}{g}\right)^3$
49. $(a^5)^2$	50. $\frac{1}{b^{-3}}$	51. z^0	52. $4r^6 \cdot 3r \cdot 2r^2$
53. $\frac{9p^{-2}}{3q^{-3}}$	54. $\frac{8d^3}{2cd^{-2}}$	55. $(g^4h)^2 \cdot (2g^3h^{-1})^2$	56. $(6a)^0$
57. $(-3n^2k^4)^2$	58. $\left(\frac{w^5x^{-2}y}{w^2xy^4}\right)^3$	59. $\frac{6 \cdot 10^7}{2 \cdot 10^3}$	60. $(1.5 \cdot 10^{-6}) \cdot (4 \cdot 10^9)$

Slope & Rate of Change

Finding the Slope Given Two Points: Use the coordinates from the points in the slope formula:

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

ex: $(4, -2)$, $(-3, 8)$ — x_1 y_1 x_2 y_2

$$m = \frac{8 - (-2)}{-3 - 4} = \frac{10}{-7} = \boxed{-\frac{10}{7}}$$

Finding the Rate of Change From a Table: Determine the amount the dependent variable (y) is changing and the amount the independent variable (x) is changing.

$$\text{Rate of Change} = \frac{\text{change in y}}{\text{change in x}}$$

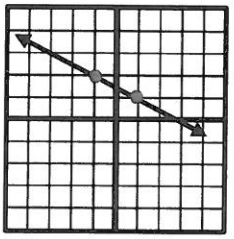
ex:

x	# months	3	5	7	9
y	Cost ($)	80	130	180	230

(+2, +2, +2 across the x row; +50, +50, +50 across the y row)

$$m = \frac{50}{2} = \boxed{25 \text{ dollars/month}}$$

Finding the Slope From a Graph: Choose 2 points on the graph. Find the vertical change (rise) and horizontal change (run) between the 2 points and write it as a fraction $\frac{\text{rise}}{\text{run}}$. (Up is positive, down is negative, right is positive, and left is negative).



rise = +1
run = -2

$$m = \frac{1}{-2} = \boxed{-\frac{1}{2}}$$

Graphing Linear Equations

Slope-Intercept Form: $y = mx + b$ (m: slope, b: y-intercept)

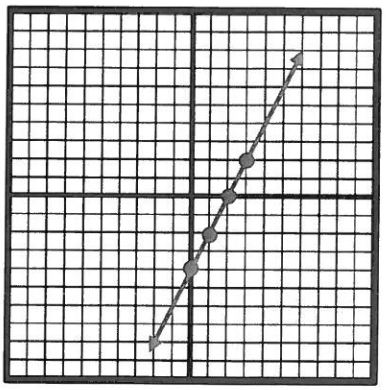
ex: $y = 2x - 4$

y-intercept: -4

slope: $2 = \frac{2}{1}$ ← rise, ← run

How To Graph:

1. Make a point on the y-axis at the y-intercept.
2. Use the slope to determine where to make the next point. The numerator tells you the rise (how far up/down) and the denominator tells you the run (how far right/left) to make the next point.
3. Repeat to make more points and then connect the points with a line.



Find the slope of the line that passes through the points. Show your work.

61. (-5, 3), (2, 1)

62. (8, 4), (11, 6)

63. (9, 3), (9, -1)

64. (-4, -2), (-6, 4)

Find the rate of change. Show your work.

65.

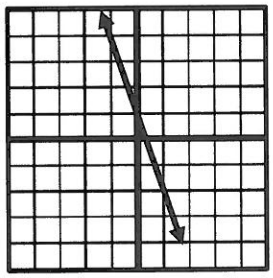
Number of Hours	3	6	9	12
Distance (in miles)	135	270	405	540

66.

Number of Weeks	1	3	5	7
Pounds	173	169	165	161

Find the slope of the line.

67.



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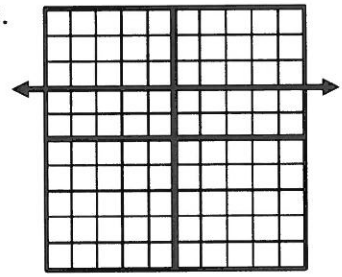

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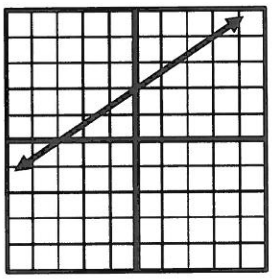


Graph the line.

70. $y = -x - 3$

71. $y = \frac{1}{3}x + 2$

72. $y = -3x - 1$

73. $y = -\frac{3}{2}x - 2$

74. $y = 2x + 1$

75. $y = \frac{1}{4}x$

Solving Proportions

1. Set the two cross-products equal to each other
2. Solve the equation for the variable

ex: $\frac{m}{4} = \frac{3}{5}$

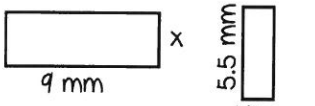
$\frac{5m}{5} = \frac{12}{5}$

$m = 2.4$

Similar Figures

1. To find a missing side length, set up a proportion, matching up corresponding sides.
2. Solve the proportion using the steps above.

ex:



$\frac{x}{1.5} = \frac{9}{5.5}$

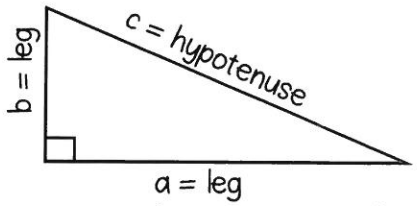
$x = 2.\overline{45}$ mm

The Pythagorean Theorem

*** The Pythagorean Theorem applies to right triangles only **

The sides next to the right angle (a & b) are legs

The side across from the right angle (c) is the hypotenuse

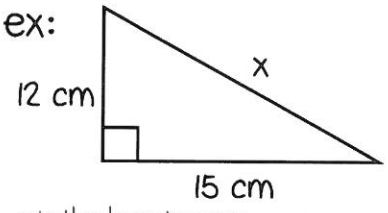


Pythagorean Theorem: $a^2 + b^2 = c^2$

To find the hypotenuse: add the squares of the legs and then find the square root of the sum

To find a leg: subtract the square of the given leg from the square of the hypotenuse and then find the square root of the difference

ex:



x is the hypotenuse

$12^2 + 15^2 = x^2$

$144 + 225 = x^2$

$369 = x^2$

$x = \sqrt{369}$ ≈ 19.2 cm

ex: a = ?, b = 3, c = 6

a is a leg

$a^2 + 3^2 = 6^2$

$a^2 + 9 = 36$

$a^2 = 36 - 9 = 27$

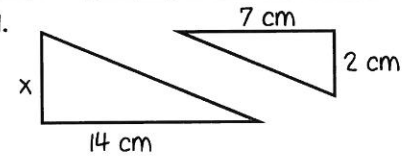
$a = \sqrt{27}$ ≈ 5.2

Solve each proportion, showing all work.

76. $\frac{6}{7} = \frac{4}{m}$	77. $\frac{12}{5} = \frac{k}{3}$	78. $\frac{h}{7} = \frac{8}{2}$	79. $\frac{22}{n} = \frac{9}{36}$	80. $\frac{4}{21} = \frac{3}{c}$

Assume each pair of figures is similar. Find the missing side length, showing all work.

81.



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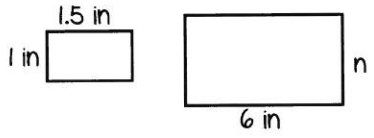

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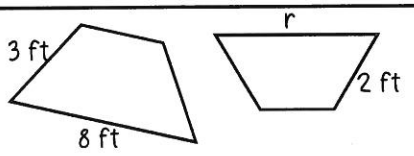

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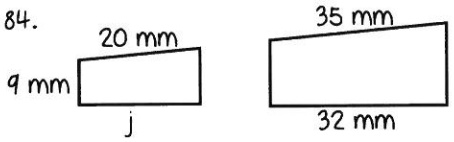

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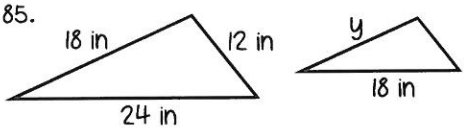

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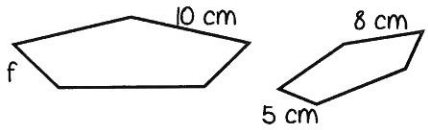
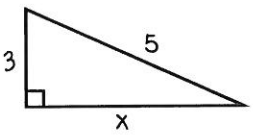


Find the missing side length in each right triangle to the nearest tenth. Show your work!

87. a = 6, b = 8, c = ?	88. a = ?, b = 9 cm, c = 13 cm	89. a = 7, b = ?, c = 14	90. a = 14 in, b = 14 in, c = ?

91.



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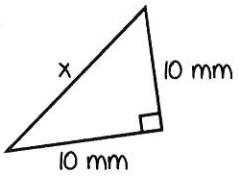

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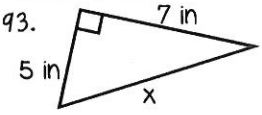

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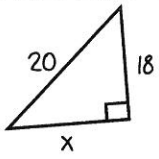

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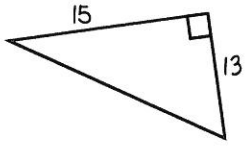

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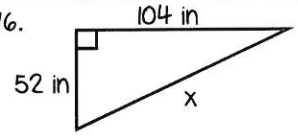

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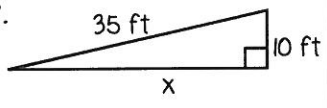

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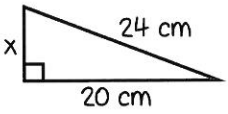


Determine whether or not you can form a right triangle from the given side lengths. Explain.

99. 18, 22, 26	100. 5, 12, 13

2023

Optional Summer
Extra Fun Challenges

Incoming 8 Grade Geometry

The pages that follow are optional summer work for students who like puzzles and other challenges. Good luck!

CARNIVAL CONUNDRUM LOGIC PUZZLE

NAME: _____

DATE: _____

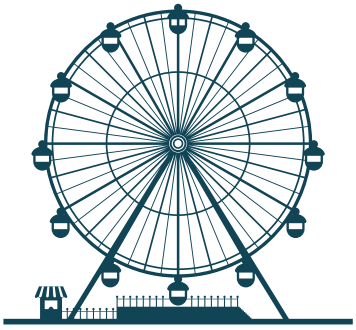
Camille and three of her friends spent an afternoon at the Cobb County Carnival. Each friend had a different ride that they liked best, and each friend bought a different snack. At the end of the afternoon, some of the friends had tickets left over to use another day.

DIRECTIONS: **Use the clues and the grid below to help you match each friend to their favorite ride, the snack they bought, and the number of tickets they had left over!**

CLUES:

- The friend who liked the Tilt-a-Whirl best had more leftover tickets than Erik but fewer leftover tickets than Zaria.
- Neither Erik nor Camille bought a corn dog.
- The Zany Zipper was Camille's favorite ride.
- The friend who ate popcorn did not like the Cyclone or the SkyScraper.
- The friend with the fewest leftover tickets liked the SkyScraper best.
- Zaria had one more ticket left than Kai.
- The friend whose favorite ride was the Tilt-a-Whirl bought nachos at the carnival.
- Zaria did not eat nachos.

		Favorite Ride				Snack				Leftover Tickets			
		Zany Zipper	Cyclone	Tilt-a-Whirl	SkyScraper	Cotton Candy	Corn Dog	Popcorn	Nachos	Zero	Two	Three	Five
Friend	Camille												
	Erik												
	Kai												
	Zaria												
Leftover Tickets	Zero												
	Two												
	Three												
	Five												
Snack	Cotton Candy												
	Corn Dog												
	Popcorn												
	Nachos												



Name _____ Date _____

FUR FAMILIES: LOGIC PUZZLE

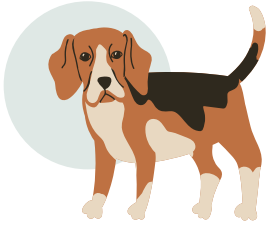
Yesterday, five different people adopted dogs from Fur Family Animal Sanctuary. The dogs were all different breeds and different ages, and each dog received a unique name.

DIRECTIONS: Use the clues and the grid below to help you match each person to their dog, including the breed of dog they adopted, the age of their dog, and the name of their dog.

CLUES:

- Ashley did not name her dog Duke, nor is her dog five years old.
- The four-year-old dog was not named Otto.
- Gus, who is one year old, is not a collie.
- Vijay named his dog Jax, and Lamar named his dog Duke.
- Neither Ashley nor Lamar adopted a dachshund.
- Russell adopted a beagle.
- Otto, who is a pug, is not two years old.
- The bulldog is the oldest dog, and the dachshund is the youngest dog.
- Lamar's dog is two years old.

		BREED OF DOG					AGE OF DOG					NAME OF DOG				
		Beagle	Bulldog	Collie	Dachshund	Pug	1 year old	2 years old	3 years old	4 years old	5 years old	Duke	Gus	Jax	Otto	Ruby
PERSON	Ashley															
	Kimi															
	Lamar															
	Russell															
	Vijay															
NAME OF DOG	Duke															
	Gus															
	Jax															
	Otto															
	Ruby															
AGE OF DOG	1 year old															
	2 years old															
	3 years old															
	4 years old															
	5 years old															



Name _____ Date _____

Multi-Step Equations: TIC-TAC-TOE

Find the row, column, or diagonal where all of the equations have the same solution.

$6x + 3 = 9x + 24$	$4x + 20 = -2(3x - 5)$	$9x - 5x - 6 = 2(x + 12)$
$4x + 23 + 3x = 2x - 22$	$7x - 5 = 2(x + 35)$	$9x - 8 - x = 5(x - 12) + 7$
$10(x - 7) = 4(x + 5)$	$30 - 18x = -6x + 9(8 - 2x)$	$5(x + 5) - 8x = -x - 5$

A Trip With Two-Step Equations

Tyler and his family took a train into Philadelphia for the day. Write and solve an equation for each problem.

1 Tyler's mom had a coupon for $4 off each of the train tickets. If they bought 5 train tickets and paid a total of $145, what was the original cost of each ticket?

2 When they arrived in Philadelphia, Tyler's family took a taxi to a museum. The taxi charged $2.70 per mile, and they paid the driver a $3.00 tip. If they paid a total of $16.50 for the taxi ride, about how far was the museum from the train station?

3 When they arrived at the museum, Tyler's dad went to the ticket counter. For each of the 5 members of the family, he purchased a general admission ticket for $8.75 and a ticket to see a special exhibit. He paid a total of $70. How much did each ticket to the special exhibit cost?

4 Tyler and his family had lunch at the museum cafeteria. Each person ordered a sandwich that cost the same amount, and they got $5 off their entire purchase with proof of admission to the special exhibit. If they paid a total of $35 after the discount, how much did each sandwich cost?

A Trip With Two-Step Equations

Keep going! Write and solve an equation for each problem.

5 After lunch, Tyler's family went on a sightseeing trolley tour. They purchased 2 adult tickets and 3 child tickets. Tyler's mom was pleased to find that the combined cost for the 3 child tickets was only $27. If they paid $62.50 in total for the trolley tour, what was the price of each adult ticket?

6 Tyler's family spent 90 minutes on the trolley sightseeing tour. If they spent 36 minutes riding the trolley and about 9 minutes stopped at each sight, how many sights did they stop at?

7 After the tour, Tyler's mom bought a pretzel and a bottle of water for each family member. The bottles of water cost $1.50 each. If she paid a total of $30, how much did each pretzel cost?

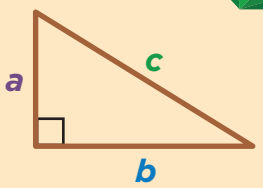
8 Before the trip, Tyler read 24 pages of his Philadelphia guidebook in an hour. The guidebook is 84 pages long. If he continued to read it at the same rate, how long did it take Tyler to finish reading the guidebook before his family's trip?

Name _____ Date _____

Pythagorean Theorem: WORD PROBLEMS

The Pythagorean theorem relates the side lengths of a right triangle. You can show the Pythagorean theorem using the following equation, where *a* and *b* represent the legs and *c* represents the hypotenuse:

$$a^2 + b^2 = c^2$$



Try it! Use the Pythagorean theorem to solve each word problem. Draw a picture of a right triangle to help you! Round your answer to the nearest tenth, if necessary.

1. A contractor is building a new room onto the back of Darnell's house. The contractor adds a diagonal brace to the frame of one of the walls. The frame is 15 feet long and 8 feet tall. How long is the diagonal brace?

2. Paola's dog, Rex, dug holes in opposite corners of Paola's rectangular yard. If the yard is 11 meters long and 4 meters wide, what is the distance between the holes?

3. Aika is setting up a volleyball net on the beach. There is a support cable from the top of one end of the net to the ground 6 feet away from the net. If the cable is 10 feet long, how high off of the ground is the net?

4. Feng is designing the layout for his rectangular garden. He plans to include a path that connects the opposite corners of the garden. The path will be 12 feet long, and the garden will be 9 feet long. How wide will the garden be?

5. Xavier wants to buy a new TV. Before he goes to the store, he measures his TV and notes that it is 56 inches long and 33 inches wide. When Xavier gets to the store, he realizes that TVs are measured by their diagonal length! What is the diagonal length of his current TV?

6. At marching band practice, Jada marches from one corner of her school's field to the opposite corner, covering 150 yards. If the field is 120 yards long, how wide is it?