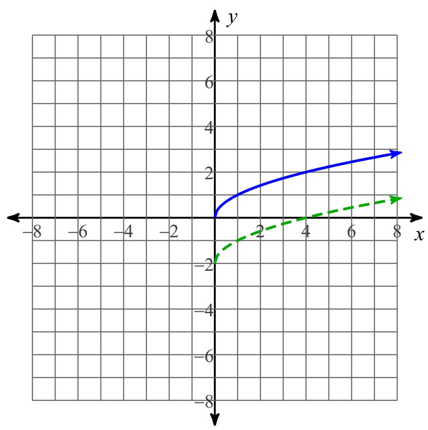
HL1 Name____________________________________ ID: 1

Summer Work

Date________________ Period____

Write $g(x)$ (dashed line) in terms of $f(x)$ (solid line).

1)



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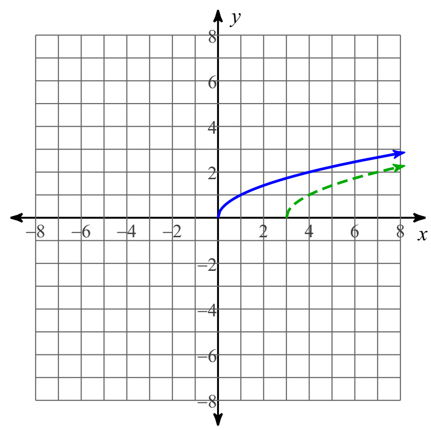

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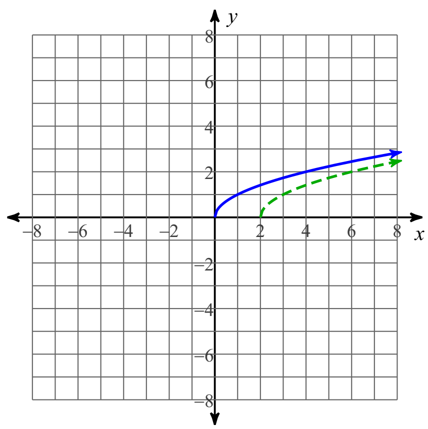

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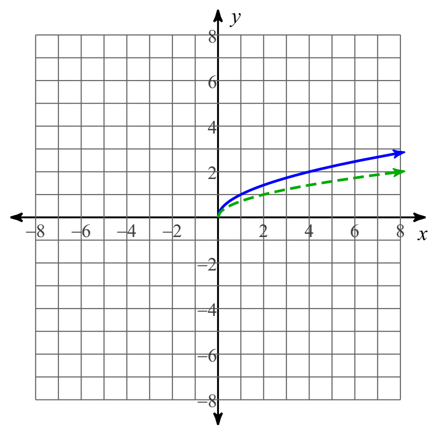


Describe the transformations necessary to transform the graph of $f(x)$ into that of $g(x)$.

5) $f(x) = x^3$
$g(x) = -x^3 + 1$

6) $f(x) = \frac{1}{x}$
$g(x) = -\frac{1}{x} + 3$

7) $f(x) = |x|$
$g(x) = \left|\frac{1}{3}x\right| + 1$

8) $f(x) = x^2$
$g(x) = (x + 3)^2 - 3$

Transform the given function $f(x)$ as described and write the resulting function as an equation.

9) $f(x) = \sqrt{x}$
compress horizontally by a factor of 3
reflect across the y-axis
reflect across the x-axis
translate right 2 units
translate up 1 unit

10) $f(x) = |x|$
expand vertically by a factor of 3
reflect across the x-axis
translate right 1 unit
translate down 2 units

11) $f(x) = \frac{1}{x}$
compress horizontally by a factor of 3
reflect across the x-axis
translate right 3 units
translate down 1 unit

12) $f(x) = \sqrt{x}$
expand horizontally by a factor of 3
reflect across the y-axis
reflect across the x-axis
translate left 2 units
translate down 2 units

Sketch the graph of each function.

13) $g(x) = -\left(\frac{1}{3}(x - 2)\right)^2 - 2$

14) $g(x) = -\frac{2}{x + 2} + 3$

15) $g(x) = -\left|2(x + 3)\right| + 3$

16) $g(x) = -(3(x + 1))^3 + 3$

Solve each equation.

17) $2^{-2a} = 2^{-2a}$

18) $27^{-2b - 3} = 9^{-3b - 3}$

19) $5^{-2r - 2} \cdot 5^{-2r} = 125$

20) $\frac{1}{216} \cdot 36^{3a} = 216$

21) $64^{-3k} \cdot 16^{-3k} = 4^3$

22) $25^{-3b} \cdot 25^{-3b} = 625^{-2b}$

23) $e^{k+1} + 7 = 41$

24) $-e^{b+2} = -71$

25) $-7e^{-9n} = -5$

Solve each equation. Round your answers to the nearest ten-thousandth.

26) $3e^{6-r} + 7.2 = 12$

27) $5.8e^{-0.3x-7.6} - 2 = 66$

28) $4e^{8m-7} - 1 = 75$

Solve each equation.

29) $\log_6 x - \log_6 (x - 4) = 1$

30) $\log 2 - \log -x = \log 5$

31) $\log_8 4 + \log_8 (x + 4) = 3$

32) $\log_4 3 - \log_4 (x - 2) = 3$

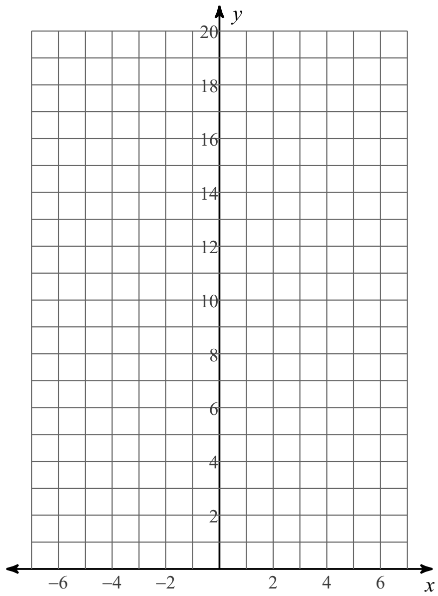
33) $\log_4 6 - \log_4 (x + 5) = 2$

34) $\log_2 (x + 9) + \log_2 6 = 1$

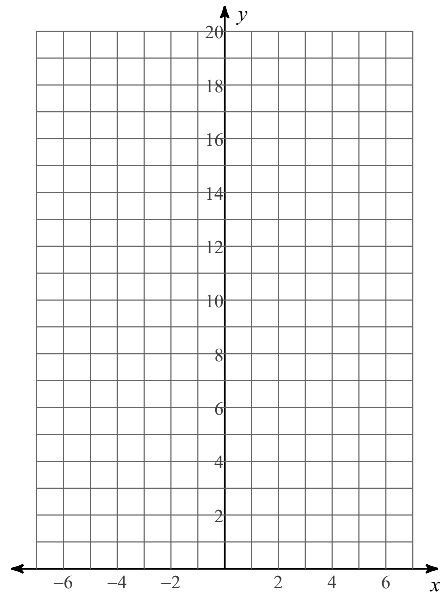
35) $\log_5 4x^2 + \log_5 3 = \log_5 27$

Sketch the graph of each function.

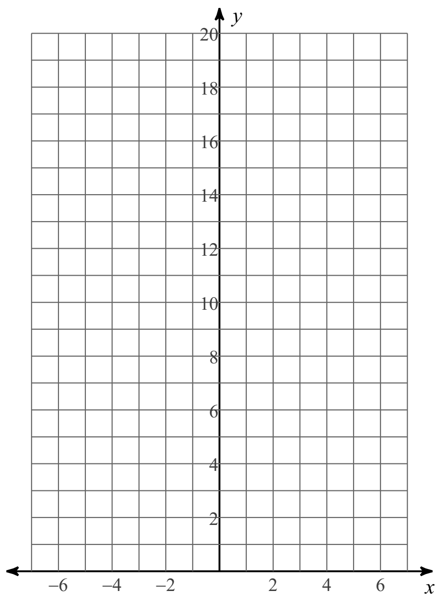
36) $y = \frac{1}{3} \cdot 5^x$



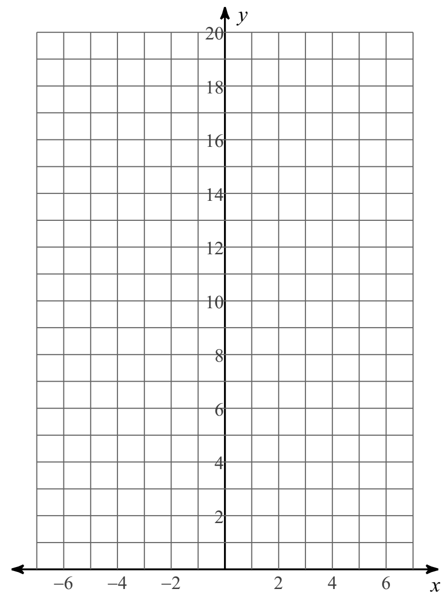
37) $y = \frac{1}{3} \cdot 7^x$



38) $y = \frac{1}{4}e^x$

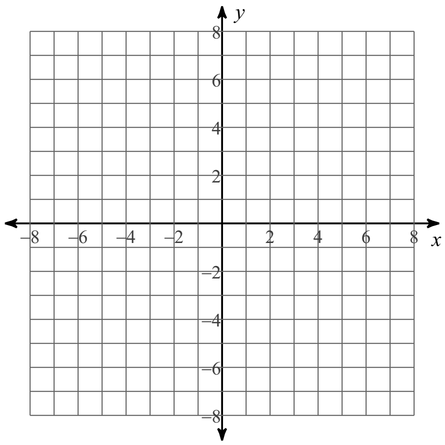


39) $y = 5 \cdot 2^x$

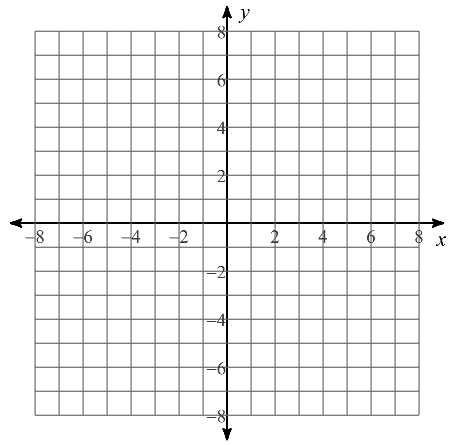


Sketch the graph of each function using endbehaviour.

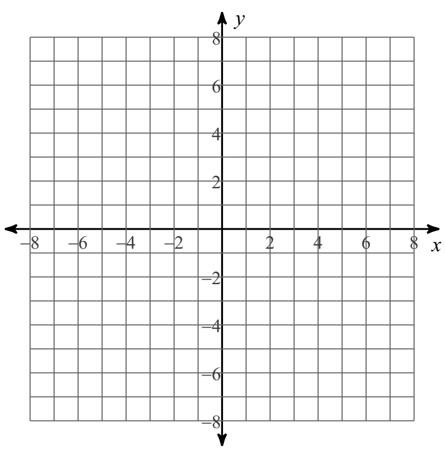
40) $f(x) = x^4 + x^3 - 3x^2 - 2$



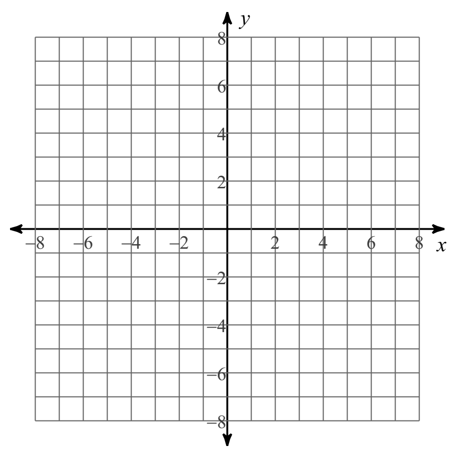
41) $f(x) = x^5 - 4x^3 + 2x + 1$



42) $f(x) = x^2 - 4x - 1$

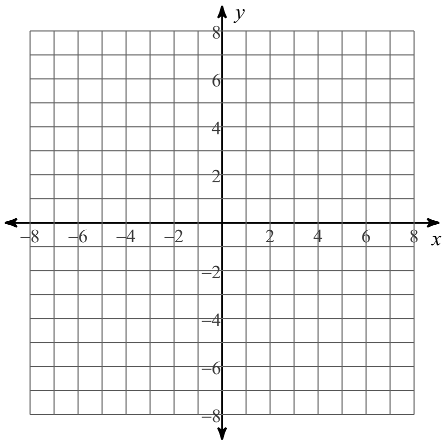


43) $f(x) = x^5 - 4x^3 + 4x$

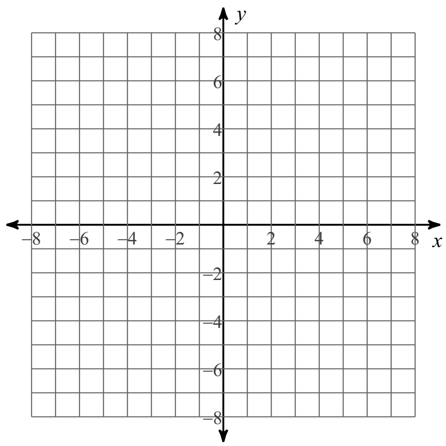


For each function, identify the holes, intercepts, and horizontal asymptote. Then sketch the graph.

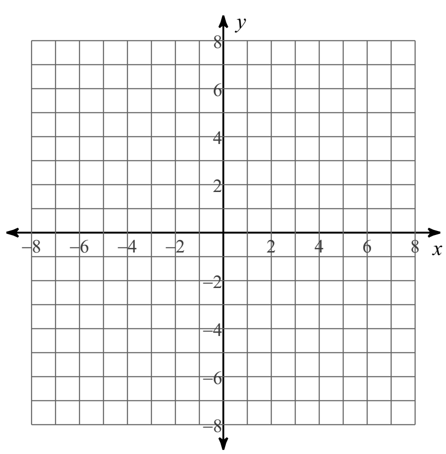
44) $f(x) = \dfrac{4}{x+1} - 1$



45) $f(x) = \frac{1}{x - 4} - 1$

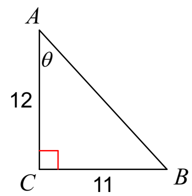


46) $f(x) = \frac{3}{x} - 2$



Find the measure of each angle indicated. Round to the nearest tenth.

47)



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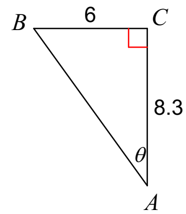
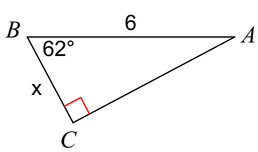


Find the measure of each side indicated. Round to the nearest tenth.

49)



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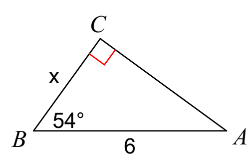


Answers to Summer Work (ID: 1)

1) $g(x) = f(x) - 2$

2) $g(x) = f(x - 3)$

3) $g(x) = f(x - 2)$

4) $g(x) = f\left(\frac{1}{2}x\right)$

5) reflect across the x-axis
translate up 1 unit

6) reflect across the x-axis
translate up 3 units

7) expand horizontally by a factor of 3
translate up 1 unit

8) translate left 3 units
translate down 3 units

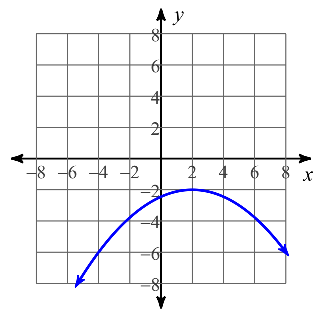
9) $g(x) = -\sqrt{-3(x - 2)} + 1$

10) $g(x) = -3\left|x - 1\right| - 2$

11) $g(x) = -\dfrac{1}{3(x - 3)} - 1$

12) $g(x) = -\sqrt{-\dfrac{1}{3}(x + 2)} - 2$

13)



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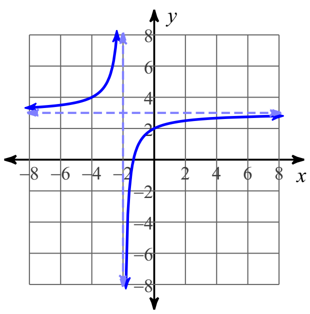

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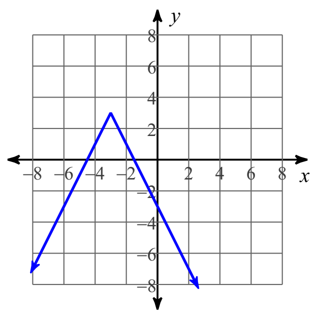

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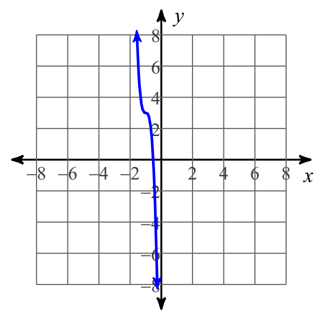


17) { All real numbers. }

18) No solution.

19) $\left\{-\frac{5}{4}\right\}$

20) $\{1\}$

21) $\left\{-\frac{1}{5}\right\}$

22) $\{0\}$

23) $\ln 34 - 1$

24) $\ln 71 - 2$

25) $-\dfrac{\ln \frac{5}{7}}{9}$

26) 5.53

27) −33.5388

28) 1.2431

29) $\left\{\frac{24}{5}\right\}$

30) $\left\{-\frac{2}{5}\right\}$

31) $\{124\}$

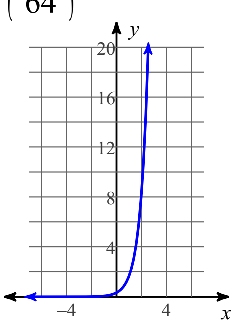
32) $\left\{\frac{131}{64}\right\}$

33) $\left\{-\frac{37}{8}\right\}$

34) $\left\{-\frac{26}{3}\right\}$

35) $\left\{\frac{3}{2}, -\frac{3}{2}\right\}$

36)



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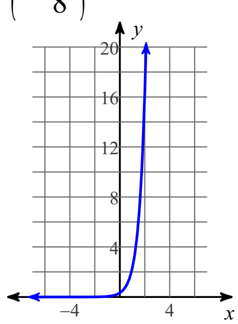

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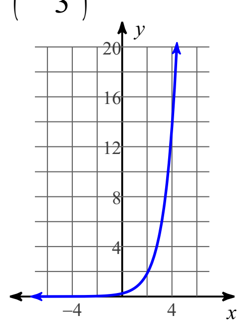

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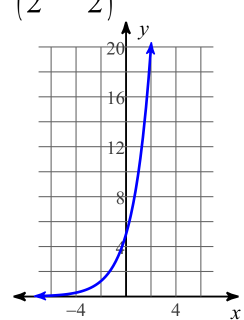
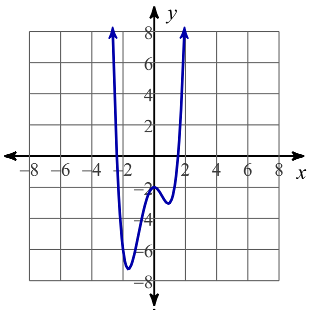
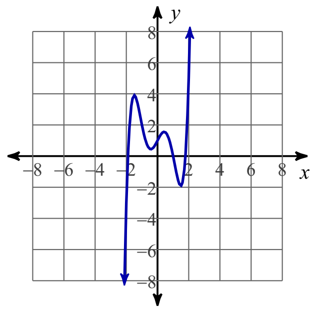
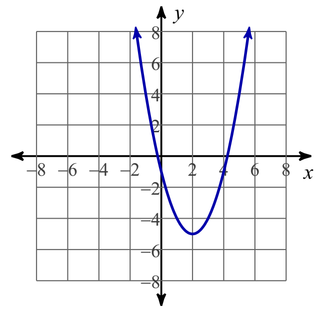
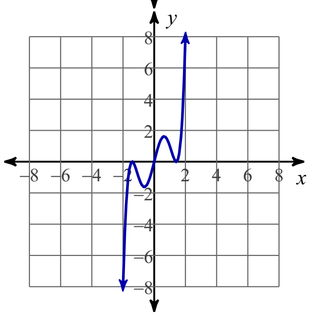
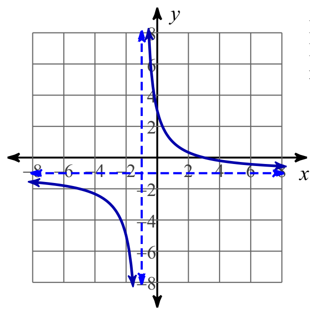


40) 

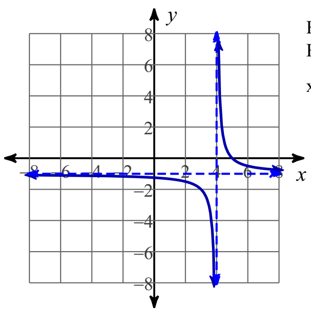
41) 

42) 

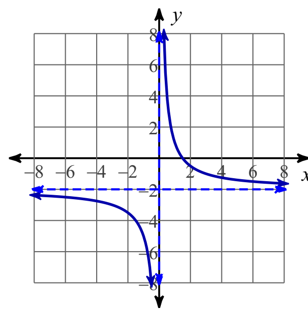
43) 

44) 

Holes: None
Horz. Asym.: $y = -1$
x-intercepts: 3, y-intercept: 3

45) 

Holes: None
Horz. Asym.: $y = -1$
x-intercepts: 5, y-intercept: $-\frac{5}{4}$

46) 

Holes: None
Horz. Asym.: $y = -2$
x-intercepts: $\frac{3}{2}$, y-intercept: None

47) 42.5°

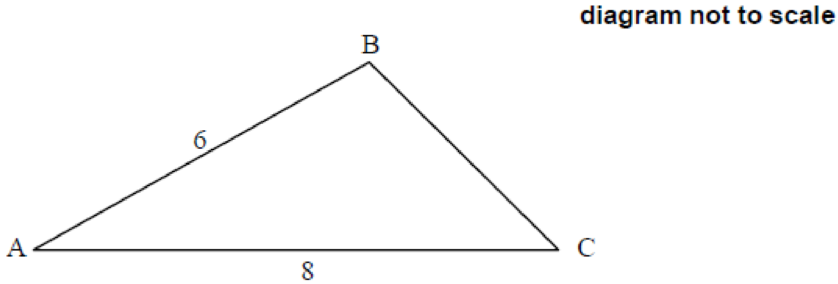
48) 35.9°

49) 2.8

50) 3.5

HL1 IB Questions *[102 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

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1b. Find the area of triangle ABC. *[2 marks]*

2a. Show that $\log_9 (\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$. *[3 marks]*

2b. Hence or otherwise solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. *[5 marks]*

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The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

3a. Show that $u_1 = 4$. *[2 marks]*

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3b. Prove that the sum of the first n terms of this arithmetic sequence is a square number. *[4 marks]*

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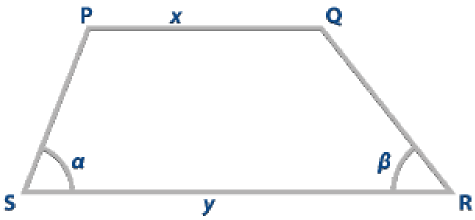
4. Consider quadrilateral PQRS where $[\mathrm{PQ}]$ is parallel to $[\mathrm{SR}]$. *[5 marks]*



In PQRS, $\mathrm{PQ} = x$, $\mathrm{SR} = y$, $\mathrm{R\widehat{S}P} = \alpha$ and $\mathrm{Q\widehat{R}S} = \beta$.

Find an expression for PS in terms of $x,\ y,\ \sin\beta$ and $\sin(\alpha+\beta)$.

5a. Show that $2x - 3 - \frac{6}{x-1} = \frac{2x^2-5x-3}{x-1},\ x \in \mathbb{R},\ x \neq 1.$ *[2 marks]*

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5b. Hence or otherwise, solve the equation $2\sin 2\theta - 3 - \frac{6}{\sin 2\theta - 1} = 0$ for $0 \leq \theta \leq \pi,\ \theta \neq \frac{\pi}{4}$. *[5 marks]*

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6. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3\left(4x^3\right)$, where $x > 0$. *[5 marks]*

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

7a. Show that $m = 4$. *[6 marks]*

The function f can be expressed in the form $f(x) = 4(x-p)(x-q)$, where $p,\ q \in \mathbb{R}$.

7b. Find the value of p and the value of q. *[2 marks]*

The function f can also be expressed in the form $f(x) = 4(x-h)^2 + k$, where $h,\ k \in \mathbb{R}$.

7c. Find the value of h and the value of k. *[3 marks]*

An arithmetic sequence has first term 60 and common difference -2.5.

8a. Given that the kth term of the sequence is zero, find the value of k. *[2 marks]*

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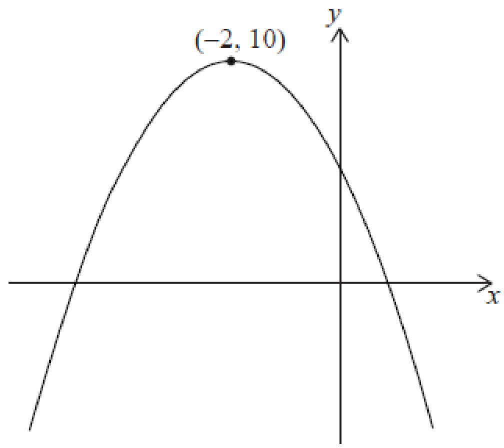
8b. Let S_n denote the sum of the first n terms of the sequence. *[3 marks]*

Find the maximum value of S_n.

The diagram shows the graph of the quadratic function $f(x) = ax^2 + bx + c$, with vertex $(-2, 10)$.



The equation $f(x) = k$ has two solutions. One of these solutions is $x = 2$.

9a. Write down the other solution of $f(x) = k$. *[2 marks]*

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9b. Complete the table below placing a tick (✔) to show whether the unknown parameters a and b are positive, zero or negative. The row for c has been completed as an example. *[2 marks]*

	positive	zero	negative
a			
b			
c	✓		

9c. State the values of x for which $f(x)$ is decreasing. *[2 marks]*

Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of $30\ \mathrm{cm}^3$. The fifth smallest slice has a volume of $240\ \mathrm{cm}^3$.

10a. Find the common ratio of the sequence. *[2 marks]*

10b. Find the volume of the smallest slice of pie. *[2 marks]*

10c. The apple pie has a volume of $61\ 425\ \mathrm{cm}^3$. *[2 marks]*

Find the total number of slices Mia can cut from this pie.

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

11a. Find the common difference. *[2 marks]*

11b. Find the first term. *[2 marks]*

11c. Find the sum of the first 20 terms. *[2 marks]*

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm^2.

12a. Show that $\sin\theta = \frac{\sqrt{15}}{4}$. *[1 mark]*

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12b. Find the two possible values for the length of the third side. *[6 marks]*

13a. Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. *[2 marks]*

13b. Show that $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$. *[4 marks]*

14. Solve $(\ln x)^2 - (\ln 2)(\ln x) < 2(\ln 2)^2$. *[6 marks]*

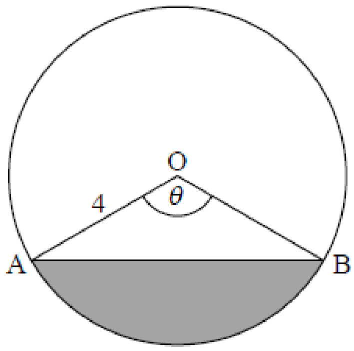
15. Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$. *[5 marks]*

Find, in terms of *b*, the solutions of $\sin 2x = -a,\ 0 \leqslant x \leqslant \pi$.

16. Show that $\log_{r^2} x = \frac{1}{2}\log_r x$ where $r,\ x \in \mathbb{R}^+$. *[2 marks]*

The diagram shows a circle, centre O, with radius 4 cm. Points A and B lie on the circumference of the circle and $A\hat{O}B = \theta$, where $0 \le \theta \le \pi$.

diagram not to scale



17a. Find the area of the shaded region, in terms of θ. *[3 marks]*

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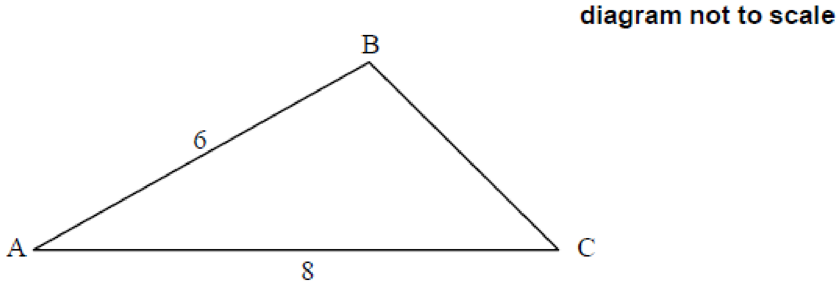
17b. The area of the shaded region is 12 cm^2. Find the value of θ. *[3 marks]*

HL1 IB Questions *[102 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

Markscheme

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

1b. Find the area of triangle ABC. *[2 marks]*

Markscheme

$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

area $= 4\sqrt{11}$ ***A1***

[2 marks]

2a. Show that $\log_9 \left(\cos 2x + 2\right) = \log_3 \sqrt{\cos 2x + 2}$. *[3 marks]*

Markscheme

attempting to use the change of base rule ***M1***

$\log_9(\cos 2x + 2) = \frac{\log_3(\cos 2x+2)}{\log_3 9}$ ***A1***

$= \frac{1}{2}\log_3(\cos 2x + 2)$ ***A1***

$= \log_3\sqrt{\cos 2x + 2}$ ***AG***

[3 marks]

2b. Hence or otherwise solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. *[5 marks]*

Markscheme

$\log_3(2\sin x) = \log_3\sqrt{\cos 2x + 2}$

$2\sin x = \sqrt{\cos 2x + 2}$ ***M1***

$4\sin^2 x = \cos 2x + 2$ (or equivalent) ***A1***

use of $\cos 2x = 1 - 2\sin^2 x$ ***(M1)***

$6\sin^2 x = 3$

$\sin x = (\pm)\frac{1}{\sqrt{2}}$ ***A1***

$x = \frac{\pi}{4}$ ***A1***

Note: Award ***A0*** if solutions other than $x = \frac{\pi}{4}$ are included.

[5 marks]

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

3a. Show that $u_1 = 4$. *[2 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1 + u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1 + (3u_1 + 8)}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

3b. Prove that the sum of the first n terms of this arithmetic sequence is a square number. *[4 marks]*

Markscheme

$d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

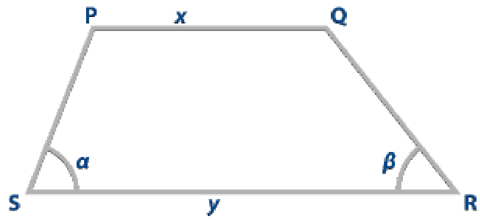
$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

[4 marks]

4. Consider quadrilateral PQRS where $[\mathrm{PQ}]$ is parallel to $[\mathrm{SR}]$. *[5 marks]*



In PQRS, $\mathrm{PQ} = x$, $\mathrm{SR} = y$, $\mathrm{R\hat{S}P} = \alpha$ and $\mathrm{Q\hat{R}S} = \beta$.

Find an expression for PS in terms of $x,\ y,\ \sin\beta$ and $\sin(\alpha+\beta)$.

Markscheme

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METHOD 1

from vertex P, draws a line parallel to $[\mathrm{QR}]$ that meets $[\mathrm{SR}]$ at a point X **(M1)**

uses the sine rule in $\Delta\mathrm{PSX}$ **M1**

$\frac{\mathrm{PS}}{\sin\beta} = \frac{y-x}{\sin(180^\circ-\alpha-\beta)}$ **A1**

$\sin(180^\circ - \alpha - \beta) = \sin(\alpha+\beta)$ **(A1)**

$\mathrm{PS} = \frac{(y-x)\sin\beta}{\sin(\alpha+\beta)}$ **A1**

METHOD 2

let the height of quadrilateral PQRS be h

$h = \mathrm{PS}\sin\alpha$ **A1**

attempts to find a second expression for h **M1**

$h = (y - x - \mathrm{PS}\cos\alpha)\tan\beta$

$\mathrm{PS}\sin\alpha = (y - x - \mathrm{PS}\cos\alpha)\tan\beta$

writes $\tan\beta$ as $\frac{\sin\beta}{\cos\beta}$, multiplies through by $\cos\beta$ and expands the RHS **M1**

$\mathrm{PS}\sin\alpha\cos\beta = (y-x)\sin\beta - \mathrm{PS}\cos\alpha\sin\beta$

$\mathrm{PS} = \frac{(y-x)\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$ **A1**

$\mathrm{PS} = \frac{(y-x)\sin\beta}{\sin(\alpha+\beta)}$ **A1**

[5 marks]

5a. Show that $2x - 3 - \frac{6}{x-1} = \frac{2x^2-5x-3}{x-1},\ x \in \mathbb{R},\ x \neq 1$. *[2 marks]*

Markscheme

METHOD 1

attempt to write all LHS terms with a common denominator of $x - 1$ ***(M1)***

$2x - 3 - \frac{6}{x-1} = \frac{2x(x-1)-3(x-1)-6}{x-1}$ OR $\frac{(2x-3)(x-1)}{x-1} - \frac{6}{x-1}$

$= \frac{2x^2-2x-3x+3-6}{x-1}$ OR $\frac{2x^2-5x+3}{x-1} - \frac{6}{x-1}$ ***A1***

$= \frac{2x^2-5x-3}{x-1}$ ***AG***

METHOD 2

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x - 3$ and remainder -6 ***A1***

$= 2x - 3 - \frac{6}{x-1}$ as required. ***AG***

[2 marks]

5b. Hence or otherwise, solve the equation $2\sin 2\theta - 3 - \frac{6}{\sin 2\theta - 1} = 0$ for $0 \leq \theta \leq \pi,\ \theta \neq \frac{\pi}{4}$. *[5 marks]*

Markscheme

consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$

EITHER

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ ***(M1)***

Note: Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

OR

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$

THEN

$\sin 2\theta = -\frac{1}{2}$ or $\sin 2\theta = 3$ ***(A1)***

Note: Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

Note: Award ***A0*** if additional answers given.

[5 marks]

6. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3(4x^3)$, where $x > 0$. *[5 marks]*

Markscheme

attempt to use change the base ***(M1)***

$\log_3 \sqrt{x} = \frac{\log_3 2}{2} + \log_3 \left(4x^3\right)$

attempt to use the power rule ***(M1)***

$\log_3 \sqrt{x} = \log_3 \sqrt{2} + \log_3 \left(4x^3\right)$

attempt to use product or quotient rule for logs, $\ln a + \ln b = \ln ab$ ***(M1)***

$\log_3 \sqrt{x} = \log_3 \left(4\sqrt{2}x^3\right)$

Note: The ***M*** marks are for attempting to use the relevant log rule and may be applied in any order and at any time during the attempt seen.

$\sqrt{x} = 4\sqrt{2}x^3$

$x = 32x^6$

$x^5 = \frac{1}{32}$ ***(A1)***

$x = \frac{1}{2}$ ***A1***

[5 marks]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

7a. Show that $m = 4$. *[6 marks]*

Markscheme

METHOD 1 (discriminant)

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

recognizing $\Delta = 0$ (seen anywhere) ***M1***

$\Delta = (-3m)^2 - 4(m)(9)$ (do not accept only in quadratic formula for x) ***A1***

valid approach to solve quadratic for m ***(M1)***

$9m(m-4)=0$ OR $m=\frac{36\pm\sqrt{36^2-4\times9\times0}}{2\times9}$

both solutions $m=0,\ 4$ ***A1***

$m\neq0$ with a valid reason ***R1***

the two graphs would not intersect OR $0\neq-9$

$m=4$ ***AG***

METHOD 2 (equating slopes)

$mx^2-2mx=mx-9$ (seen anywhere) ***(M1)***

$f'(x)=2mx-2m$ ***A1***

equating slopes, $f'(x)=m$ (seen anywhere) ***M1***

$2mx-2m=m$

$x=\frac{3}{2}$ ***A1***

substituting their x value ***(M1)***

$\left(\frac{3}{2}\right)^2m-2m\times\frac{3}{2}=m\times\frac{3}{2}-9$

$\frac{9}{4}m-\frac{12}{4}m=\frac{6}{4}m-9$ ***A1***

$\frac{-9m}{4}=-9$

$m=4$ ***AG***

METHOD 3 (using $\frac{-b}{2a}$)

$mx^2-2mx=mx-9$ ***(M1)***

$mx^2-3mx+9=0$

attempt to find x-coord of vertex using $\frac{-b}{2a}$ ***(M1)***

$\frac{-(-3m)}{2m}$ ***A1***

$x=\frac{3}{2}$ ***A1***

substituting their x value ***(M1)***

$\left(\frac{3}{2}\right)^2m-3m\times\frac{3}{2}+9=0$

$\frac{9}{4}m-\frac{9}{2}m+9=0$ ***A1***

$-9m=-36$

$m=4$ ***AG***

[6 marks]

The function f can be expressed in the form $f(x) = 4(x-p)(x-q)$, where $p,\ q \in \mathbb{R}$.

7b. Find the value of p and the value of q. *[2 marks]*

Markscheme

$4x(x-2)$ ***(A1)***

$p=0$ and $q=2$ OR $p=2$ and $q=0$ ***A1***

[2 marks]

The function f can also be expressed in the form $f(x) = 4(x-h)^2 + k$, where $h,\ k \in \mathbb{R}$.

7c. Find the value of h and the value of k. *[3 marks]*

Markscheme

attempt to use valid approach ***(M1)***

$\frac{0+2}{2}, \frac{-(-8)}{2\times4}, f(1), 8x-8=0$ OR $4\left(x^2-2x+1-1\right)\left(=4(x-1)^2-4\right)$

$h=1, k=-4$ ***A1A1***

[3 marks]

An arithmetic sequence has first term 60 and common difference -2.5.

8a. Given that the kth term of the sequence is zero, find the value of k. *[2 marks]*

Markscheme

attempt to use $u_1+(n-1)d=0$ ***(M1)***

$60-2.5(k-1)=0$

$k=25$ ***A1***

[2 marks]

8b. Let S_n denote the sum of the first n terms of the sequence. *[3 marks]*

Find the maximum value of S_n.

Markscheme

METHOD 1

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60 + 0), S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

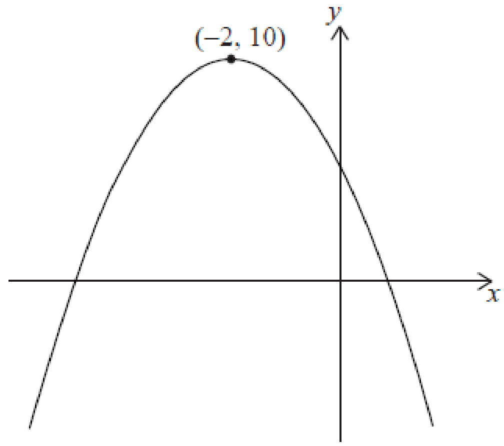
$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ ***(A1)***

THEN

$= 750$ ***A1***

[3 marks]

The diagram shows the graph of the quadratic function $f\left(x\right)=ax^2+bx+c$, with vertex $\left(-2, 10\right)$.



The equation $f(x)=k$ has two solutions. One of these solutions is $x=2$.

9a. Write down the other solution of $f(x)=k$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

$(x=)\ (-2)-4$ **OR** $(x=)\ (-2)-(2-(-2))$ ***(M1)***

Note: Award ***(M1)*** for correct calculation of the left symmetrical point.

$(x=)\ -6$ ***(A1)*** ***(C2)***

[2 marks]

9b. Complete the table below placing a tick (✔) to show whether the unknown parameters a and b are positive, zero or negative. The row for c has been completed as an example. *[2 marks]*

	positive	zero	negative
a			
b			
c	✓		

Markscheme

	positive	zero	negative
a			✓
b			✓

(A1)(A1) (C2)

Note: Award ***(A1)*** for each correct row.

[2 marks]

9c. State the values of x for which $f(x)$ is decreasing. *[2 marks]*

Markscheme

$x > -2$ **OR** $x \geq -2$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for -2 seen as part of an inequality, ***(A1)*** for completely correct notation. Award ***(A1)(A1)*** for correct equivalent statement in words, for example "decreasing when x is greater than negative 2".

[2 marks]

Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of $30\ \text{cm}^3$. The fifth smallest slice has a volume of $240\ \text{cm}^3$.

10a. Find the common ratio of the sequence. *[2 marks]*

Markscheme

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$u_1 r = 30$ **and** $u_1 r^4 = 240,$ ***(M1)***

Note: Award ***(M1)*** for both the given terms expressed in the formula for u_n.

OR

$30r^3 = 240 \quad \left(r^3 = 8\right)$ ***(M1)***

Note: Award ***(M1)*** for a correct equation seen.

$(r =)\ 2$ ***(A1)*** ***(C2)***

[2 marks]

10b. Find the volume of the smallest slice of pie. *[2 marks]*

Markscheme

$u_1 \times 2 = 30$ **OR** $u_1 \times 2^4 = 240$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in geometric sequence formula.

$(u_1 =)\ 15$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

10c. The apple pie has a volume of $61\ 425\ \mathrm{cm}^3$. *[2 marks]*

Find the total number of slices Mia can cut from this pie.

Markscheme

$\frac{15\left(2^{n}-1\right)}{2-1}=61425$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted geometric series formula equated to 61425.

$(n=)\ 12$ (slices) ***(A1)*(ft)** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

In an arithmetic sequence, $u_2=5$ and $u_3=11$.

11a. Find the common difference. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $11-5$, $11=5+d$

$d=6$ ***A1 N2***

[2 marks]

11b. Find the first term. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg u_2-d, $5-6$, $u_1+(3-1)(6)=11$

$u_1=-1$ ***A1 N2***

[2 marks]

11c. Find the sum of the first 20 terms. *[2 marks]*

Markscheme

correct substitution into sum formula

eg $\frac{20}{2}(2(-1)+19(6))$, $\frac{20}{2}(-1+113)$ ***(A1)***

$S_{20}=1120$ ***A1 N2***

[2 marks]

The lengths of two of the sides in a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm^2.

12a. Show that $\sin\theta=\frac{\sqrt{15}}{4}$. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

EITHER

$\frac{5\sqrt{15}}{2}=\frac{1}{2}\times 4\times 5\sin\theta$ ***A1***

OR

height of triangle is $\frac{5\sqrt{15}}{4}$ if using 4 as the base or $\sqrt{15}$ if using 5 as the base ***A1***

THEN

$\sin\theta=\frac{\sqrt{15}}{4}$ ***AG***

[1 mark]

12b. Find the two possible values for the length of the third side. *[6 marks]*

Markscheme

let the third side be x

$x^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\theta$ ***M1***

valid attempt to find $\cos\theta$ ***(M1)***

Note: Do not accept writing cos $\left(\arcsin\left(\frac{\sqrt{15}}{4}\right)\right)$ as a valid method.

$\cos\theta = \pm\sqrt{1 - \frac{15}{16}}$

$= \frac{1}{4},\ -\frac{1}{4}$ ***A1A1***

$x^2 = 16 + 25 - 2 \times 4 \times 5 \times \pm\frac{1}{4}$

$x = \sqrt{31}$ or $\sqrt{51}$ ***A1A1***

[6 marks]

13a. Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$ ***M1A1***

Note: Do not award the ***M1*** for just $\sin^2 x + \cos^2 x$.

Note: Do not award ***A1*** if correct expression is followed by incorrect working.

$= 1 + \sin 2x$ ***AG***

[2 marks]

13b. Show that $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$. *[4 marks]*

Markscheme

$\sec 2x + \tan 2x = \frac{1}{\cos 2x} + \frac{\sin 2x}{\cos 2x}$ ***M1***

Note: ***M1*** is for an attempt to change both terms into sine and cosine forms (with the same argument) or both terms into functions of $\tan x$.

$= \frac{1+\sin 2x}{\cos 2x}$

$= \frac{(\sin x+\cos x)^2}{\cos^2 x-\sin^2 x}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$= \frac{(\sin x+\cos x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ ***M1***

$= \frac{\cos x+\sin x}{\cos x-\sin x}$ ***AG***

Note: Apply MS in reverse if candidates have worked from RHS to LHS.

Note: Alternative method using $\tan 2x$ and $\sec 2x$ in terms of $\tan x$.

[4 marks]

14. Solve $(\ln x)^2 - (\ln 2)(\ln x) < 2(\ln 2)^2$. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\ln x)^2 - (\ln 2)(\ln x) - 2(\ln 2)^2 \, (= 0)$

EITHER

$\ln x = \frac{\ln 2 \pm \sqrt{(\ln 2)^2 + 8(\ln 2)^2}}{2}$ ***M1***

$= \frac{\ln 2 \pm 3 \ln 2}{2}$ ***A1***

OR

$(\ln x - 2\ln 2)(\ln x + 2\ln 2) \, (= 0)$ ***M1A1***

THEN

$\ln x = 2\ln 2$ or $-\ln 2$ ***A1***

$\Rightarrow x = 4$ or $x = \frac{1}{2}$ ***(M1)A1***

Note: ***(M1)*** is for an appropriate use of a log law in either case, dependent on the previous ***M1*** being awarded, **A1** for both correct answers.

solution is $\frac{1}{2} < x < 4$ ***A1***

[6 marks]

15. Let $a = \sin b, \; 0 < b < \frac{\pi}{2}$. *[5 marks]*

Find, in terms of *b*, the solutions of $\sin 2x = -a, \; 0 \leqslant x \leqslant \pi$.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

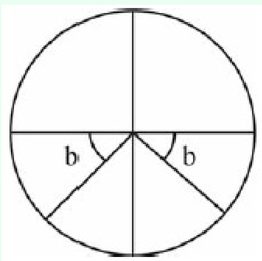
$\sin 2x = -\sin b$

EITHER

$\sin 2x = \sin(-b)$ or $\sin 2x = \sin(\pi + b)$ or $\sin 2x = \sin(2\pi - b)$ … ***(M1)(A1)***

Note: Award ***M1*** for any one of the above, ***A1*** for having final two.

OR



(M1)(A1)

Note: Award ***M1*** for one of the angles shown with b clearly labelled, ***A1*** for both angles shown. Do not award ***A1*** if an angle is shown in the second quadrant and subsequent ***A1*** marks not awarded.

THEN

$2x = \pi + b$ or $2x = 2\pi - b$ ***(A1)(A1)***

$x = \frac{\pi}{2} + \frac{b}{2},\ x = \pi - \frac{b}{2}$ ***A1***

[5 marks]

16. Show that $\log_{r^2} x = \frac{1}{2}\log_r x$ where $r,\ x \in \mathbb{R}^+$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

$\log_{r^2} x = \frac{\log_r x}{\log_r r^2} \left(= \frac{\log_r x}{2\log_r r}\right)$ ***M1A1***

$= \frac{\log_r x}{2}$ ***AG***

[2 marks]

METHOD 2

$\log_{r^2} x = \frac{1}{\log_x r^2}$ ***M1***

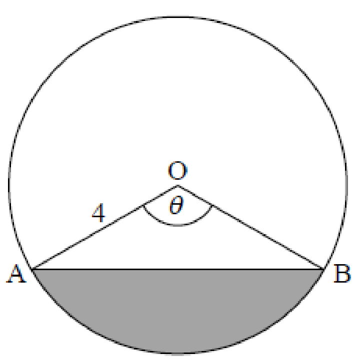
$= \frac{1}{2\log_x r}$ ***A1***

$= \frac{\log_r x}{2}$ ***AG***

[2 marks]

The diagram shows a circle, centre O, with radius 4 cm. Points A and B lie on the circumference of the circle and AÔB = θ , where $0 \le \theta \le \pi$.

diagram not to scale



17a. Find the area of the shaded region, in terms of θ. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach to find area of segment ***(M1)***

eg area of sector – area of triangle, $\frac{1}{2}r^2\left(\theta - \sin\theta\right)$

correct substitution ***(A1)***

eg $\frac{1}{4}(4)^2\theta - \frac{1}{2}(4)^2\sin\theta,\ \frac{1}{2} \times 16\left[\theta - \sin\theta\right]$

area = 80 – 8 sinθ, 8(θ – sinθ) ***A1 N2***

[3 marks]

17b. The area of the shaded region is 12 cm^2. Find the value of θ. *[3 marks]*

Markscheme

setting **their** area expression equal to 12 ***(M1)***

eg 12 = 8(θ – sinθ)

2.26717

θ = 2.27 (do not accept an answer in degrees) ***A2 N3***

[3 marks]