SL2 Name____________________ ID: 1

Summer work

Date________________ Period____

Describe the transformations necessary to transform the graph of $f(x)$ into that of $g(x)$.

1) $f(x) = \frac{1}{x}$

$g(x) = \frac{2}{x} - 3$

2) $f(x) = |x|$

$g(x) = -|x| + 1$

3) $f(x) = x^3$

$g(x) = -3x^3$

4) $f(x) = x^3$

$g(x) = (2(x - 2))^3$

5) $f(x) = \frac{1}{x}$

$g(x) = \frac{2}{x + 3}$

6) $f(x) = \frac{1}{x}$

$g(x) = -\frac{1}{x - 3}$

Transform the given function $f(x)$ as described and write the resulting function as an equation.

7) $f(x) = x^2$

expand horizontally by a factor of 3
reflect across the x-axis
translate left 2 units
translate up 1 unit

8) $f(x) = x^3$

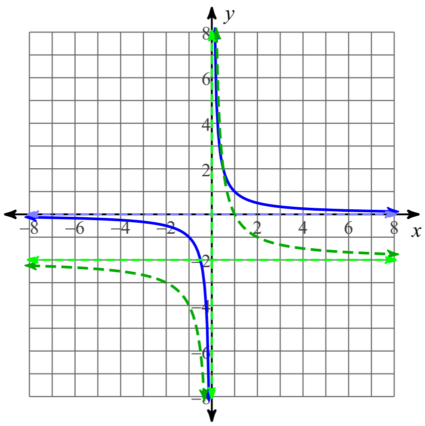
compress horizontally by a factor of 2
reflect across the x-axis
translate left 2 units
translate down 1 unit

9) $f(x) = x^2$

compress horizontally by a factor of 2
reflect across the x-axis
translate left 2 units
translate down 1 unit

Describe the transformations necessary to transform the graph of $f(x)$ (solid line) into that of $g(x)$ (dashed line).

10)



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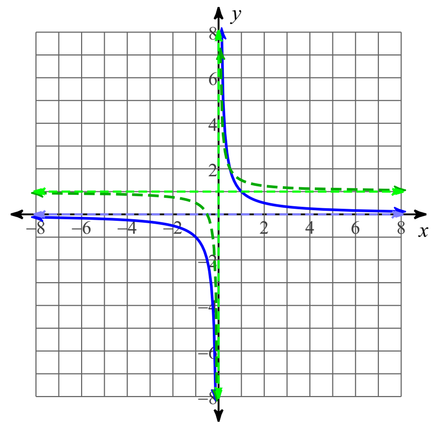

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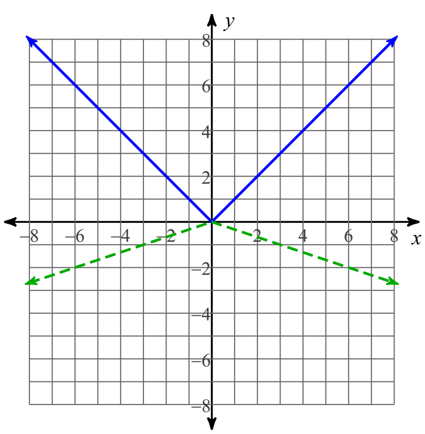

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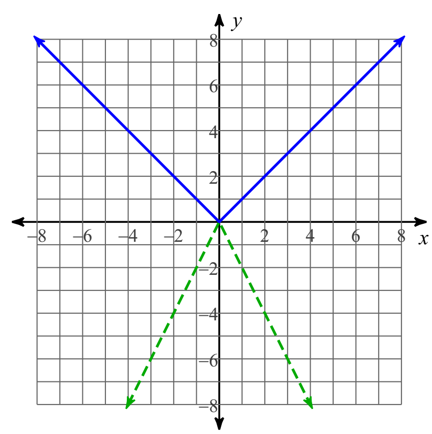

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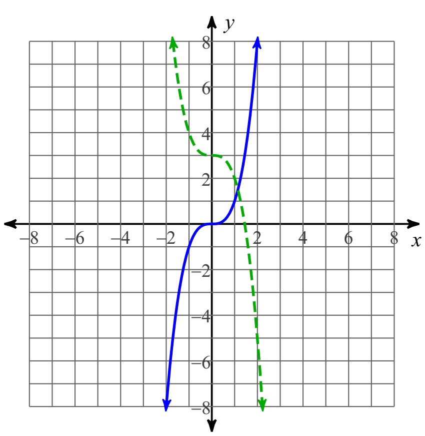
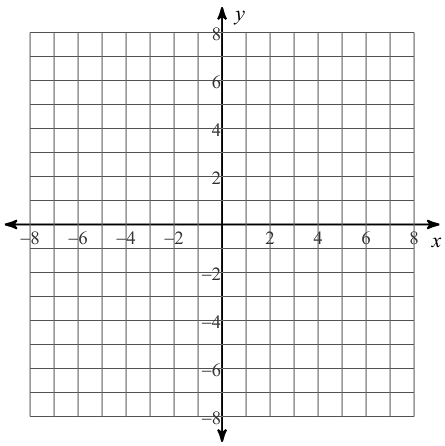
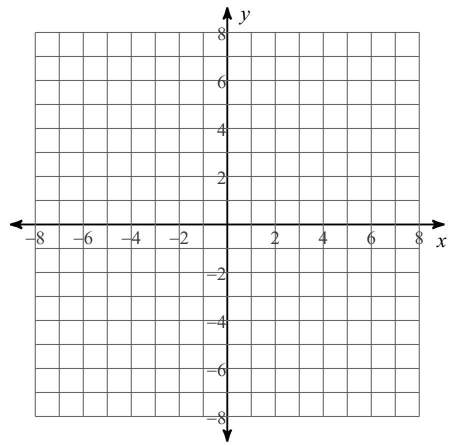


For each function: (1) state the maximum number of turns the graph could make, (2) determine the real zeros and state the multiplicity of any repeated zeros, (3) list the x-intercepts where the graph crosses the x-axis and those where it does not cross the x-axis, (4) describe the end behavior, and (5) sketch the graph.

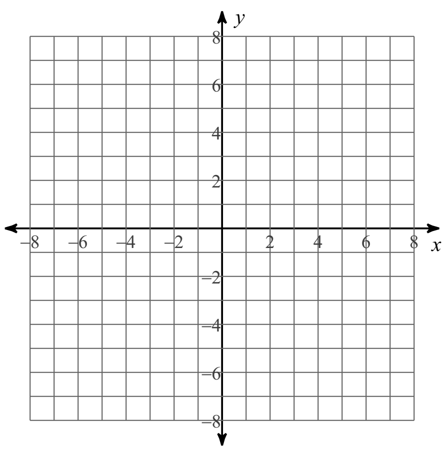
15) $f(x) = -x^2 - x$



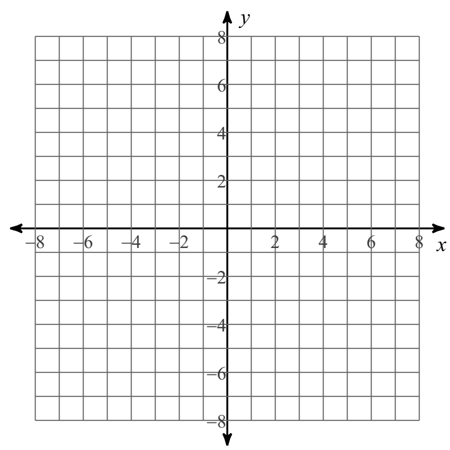
16) $f(x) = x^2 - 7x + 12$



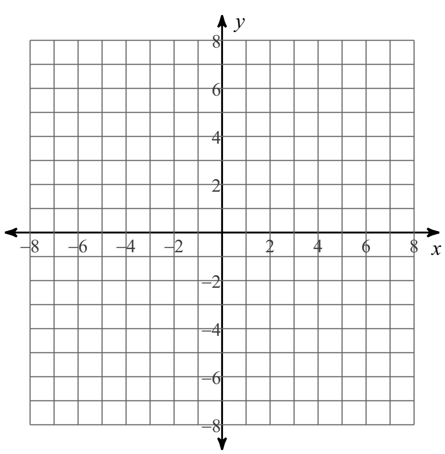
17) $f(x) = -x^2$



18) $f(x) = x^2 + x$



19) $f(x) = x^2$



Solve each equation.

20) $243^{2x} = \left(\frac{1}{81}\right)^{2x - 1}$

21) $32^{2p + 2} = 4$

22) $2^{3n} = 2^{3n + 2}$

23) $6^{-2r} = 6^3$

24) $3^{-b} = 9$

Solve each equation. Round your answers to the nearest ten-thousandth.

25) $11^{n + 9} - 6 = 47$

26) $20^{-8.4p} - 10 = 39$

27) $16^{r + 5} - 2.6 = 3$

28) $16^{2x} - 4 = 44$

29) $2^{n + 6} + 6.5 = 83.4$

Solve each equation.

30) $\log_{11} (5v - 4) = \log_{11} v$

31) $\log_{12} -3r = \log_{12} (-2r + 10)$

32) $\ln 2a = \ln (a + 9)$

33) $\log_7 (2r - 4) = \log_7 3r$

34) $\log_{12} (-5m - 7) = \log_{12} (7 - 4m)$

35) $\log_4 5 - \log_4 (x - 4) = 2$

36) $\log_8 5 + \log_8 (x - 8) = \log_8 48$

37) $\log_7 (x - 2) - \log_7 x = 2$

38) $\log_7 (x - 5) - \log_7 4 = \log_7 5$

39) $\log_5 (x^2 + 9) - \log_5 2 = 1$

40) $2^{-b} = 2^{3b}$

41) $125^{3b} = \left(\frac{1}{25}\right)^{-b}$

42) $3^{-x} = 9$

43) $4^{-3a-3} = 64$

44) $6^{-x} = 36$

Solve each equation. Round your answers to the nearest ten-thousandth.

45) $18^{n-4} + 4 = 70.3$

46) $18^{x+6} - 5 = 58$

47) $10^{p-8} - 1.1 = 36$

48) $14^{8r} - 9 = 10$

49) $-8 \cdot 3^{m-1} = -26$

Condense each expression to a single logarithm.

50) $\log_5 7 + 5\log_5 3 + 5\log_5 8$

51) $\log_8 11 + \frac{4\log_8 3}{3} + \frac{\log_8 2}{3}$

52) $6\ln u + 3\ln v + \ln w$

53) $16\log_5 12 - 4\log_5 7 - 4\log_5 11$

54) $3\log_9 11 + 6\log_9 7 + 3\log_9 8$

55) $\log_3 7 + \frac{4\log_3 5}{3} + \frac{\log_3 6}{3}$

Solve each equation.

56) $\log_{19}(3x+6) = \log_{19}(2-4x)$

57) $\log_8(5n+4) = \log_8(2n+7)$

58) $\log_{20}(-x+4)=\log_{20}(-5x-8)$

59) $\log_{20}(3r+4)=\log_{20}(-3r-2)$

60) $\log_{13} -x=\log_{13} 5x$

61) $\log_{17}(x+7)=\log_{17} 2x$

62) $\log_7 5+\log_7 -4x=\log_7 78$

63) $\log_6(x+7)-\log_6 2=2$

64) $\log(x+5)-\log x=\log 20$

65) $\log_9 x-\log_9(x-4)=1$

66) $\log_9 2x^2-\log_9 2=2$

67) $\log_9 10-\log_9 3x=1$

Evaluate each expression.

68) $\log_5 \frac{1}{25}$

69) $\log_4 4$

70) $\log_2 16$

71) $\log_7 \frac{1}{343}$

72) $\log_7 49$

73) $\log_3 \frac{1}{243}$

Expand each logarithm.

74) $\log_2\left(w\sqrt[3]{u\cdot v}\right)$

75) $\log_3\left(z\sqrt{x\cdot y}\right)$

76) $\log_6\left(5\cdot 12^5\right)^2$

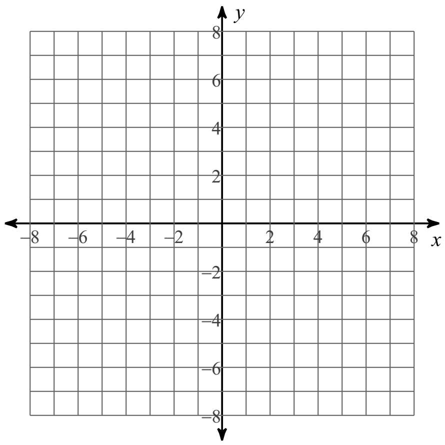
77) $\log_8\left(x^2\cdot y\right)^5$

78) $\log_8 \frac{12^4}{11^3}$

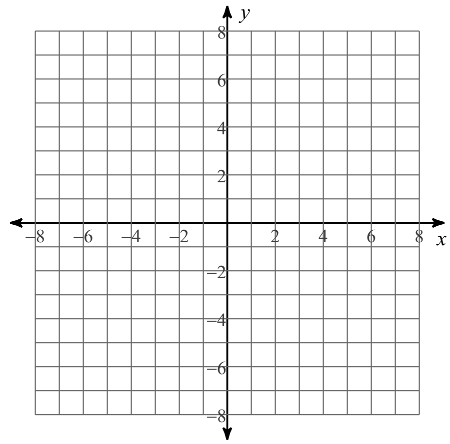
79) $\log_3\left(11^3\cdot 7^3\right)$

Identify the domain and range of each. Then sketch the graph.

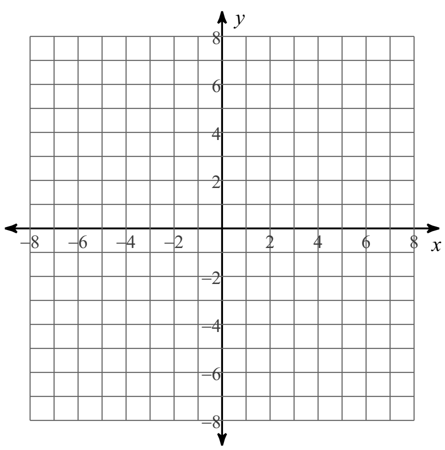
80) $y = \log \left(x + 4\right) - 5$



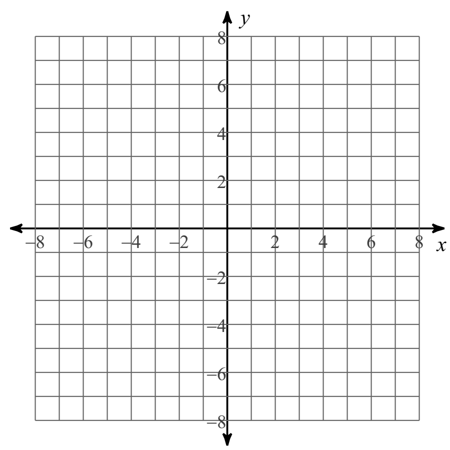
81) $y = \log_2 \left(x - 1\right) - 5$



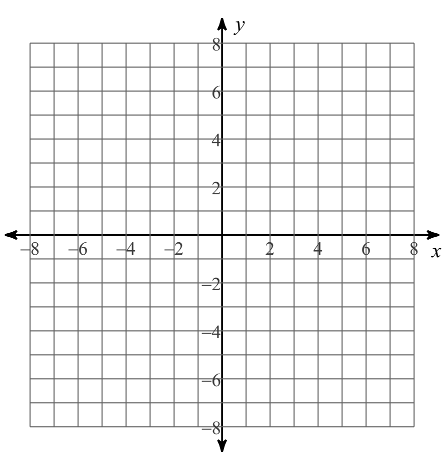
82) $y = \log_3 \left(x - 1\right) + 3$



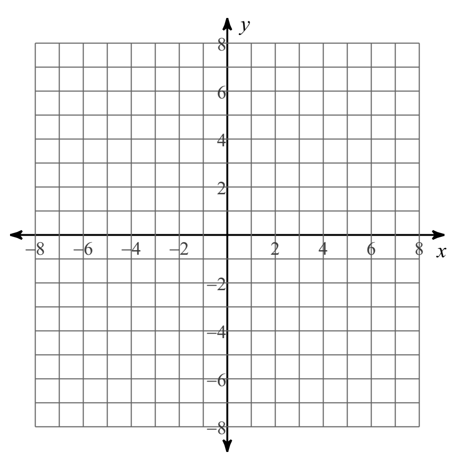
83) $y = \log_5 \left(x + 4\right) + 5$



84) $y = \log_4 \left(x - 1\right) - 1$



85) $y = \log_6 \left(x - 1\right) - 2$



Find the inverse of each function.

86) $y = -8\log_x 3$

87) $y = -9\log_5 x$

88) $y = \log_4 (3x)$

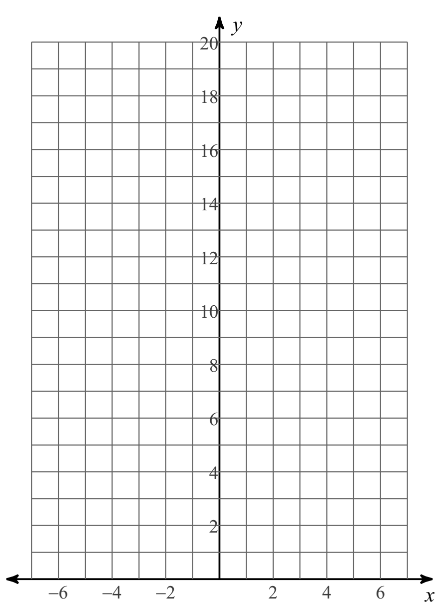
89) $y = \log_4 (x - 9)$

90) $y = \log_5 (x - 5)$

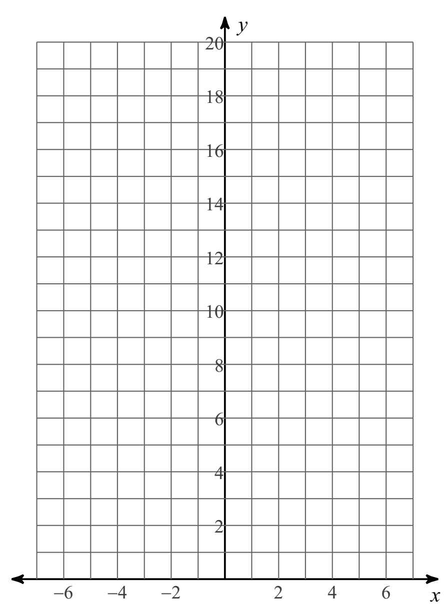
91) $y = -3\log_4 x$

Sketch the graph of each function.

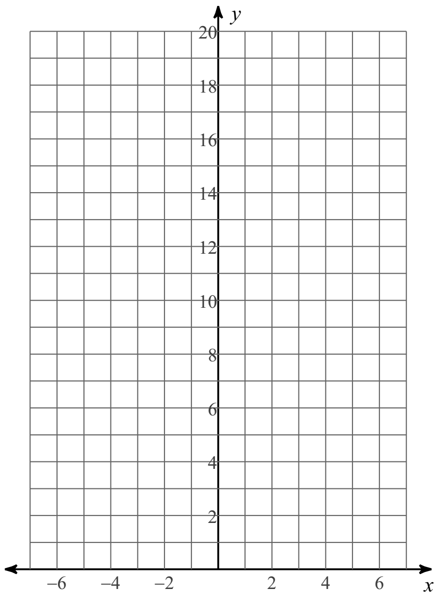
92) $y = 5 \cdot \left(\frac{1}{2}\right)^x$



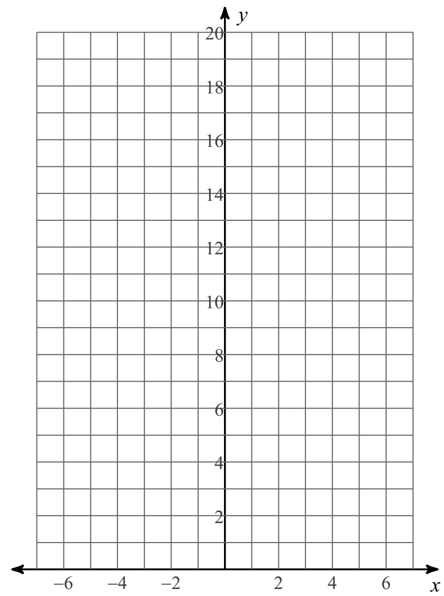
93) $y = \frac{1}{4} \cdot \left(\frac{1}{7}\right)^x$



94) $y = \frac{1}{4} \cdot 7^x$

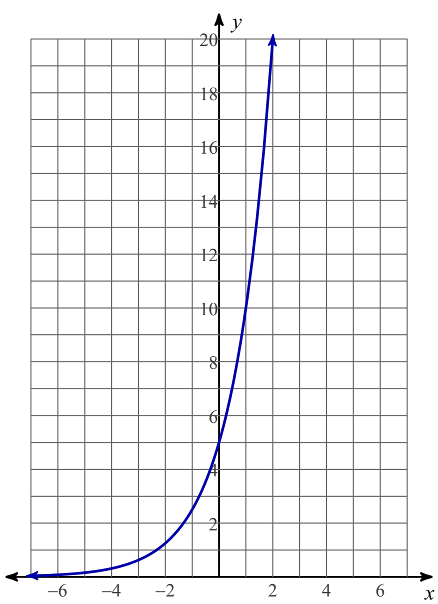


95) $y = 4 \cdot 2^x$

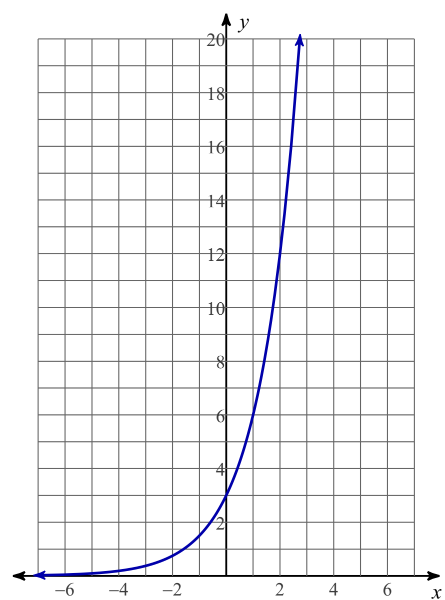


Write an equation for each graph.

96)



97)



Solve each equation. Round your answers to the nearest ten-thousandth.

98) $5^{1.3m} - 8 = 19$

99) $8^{6x} - 9 = 74$

100) $10^{-2x} + 4 = 52$

101) Carlos invests $4,089 in a retirement account with a fixed annual interest rate of 9% compounded 3 times per year. What will the account balance be after 20 years?

102) Willie invests $7,220 in a retirement account with a fixed annual interest rate of 2% compounded 4 times per year. What will the account balance be after 13 years?

103) Jasmine invests $7,035 in a savings account with a fixed annual interest rate of 7% compounded 12 times per year. What will the account balance be after 4 years?

104) Ted invests $8,965 in a retirement account with a fixed annual interest rate of 9% compounded continuously. What will the account balance be after 19 years?

105) Arjun invests $2,034 in a retirement account with a fixed annual interest rate of 6% compounded continuously. What will the account balance be after 18 years?

106) Ming invests $1,549 in a retirement account with a fixed annual interest rate of 5% compounded continuously. What will the account balance be after 15 years?

107) Asanji invests $7,223 in a savings account with a fixed annual interest rate of 8% compounded continuously. How long will it take for the account balance to reach $17,413.93?

108) Arjun invests $1,368 in a retirement account with a fixed annual interest rate of 8% compounded continuously. How long will it take for the account balance to reach $5,329.99?

109) Danielle invests $1,303 in a retirement account with a fixed annual interest rate of 8% compounded continuously. How long will it take for the account balance to reach $4,326.11?

110) Sarawong invests $7,116 in a savings account with a fixed annual interest rate of 4% compounded 6 times per year. How long will it take for the account balance to reach $9,039.01?

111) Shanice invests $3,004 in a retirement account with a fixed annual interest rate of 2% compounded 12 times per year. How long will it take for the account balance to reach $4,479.95?

112) Nicole invests $6,081 in a retirement account with a fixed annual interest rate of 7% compounded 4 times per year. How long will it take for the account balance to reach $22,729.55?

Perform the indicated operation.

113) $g(t) = 2t - 2$
$f(t) = 4t + 2$
Find $(g + f)(t)$

114) $g(n) = n^2 - 4n$
$f(n) = 3n + 2$
Find $(3g + 5f)(n)$

115) $g(x) = -2x - 3$
$h(x) = x^3 + 5x$
Find $(g \circ h)(x)$

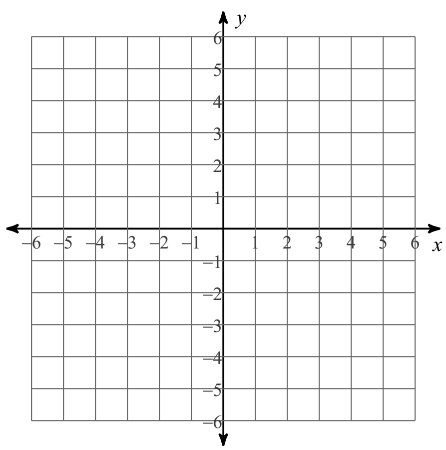
116) $h(x) = -2x^3 + 3x$
$g(x) = -2x + 4$
Find $(h \cdot g)(x)$

117) $g(a) = 4a + 5$
$f(a) = 2a^2 + 4$
Find $(g \cdot f)(a)$

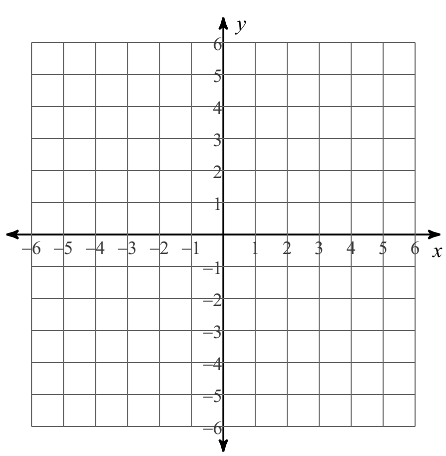
118) $g(n) = n + 2$
$f(n) = 3n + 3$
Find $(g - f)(n)$

Find the inverse of each function. Then graph the function and its inverse.

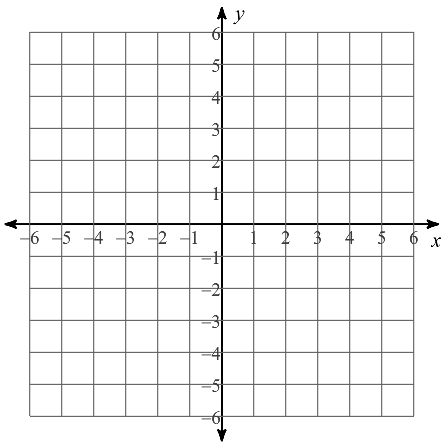
119) $g(x) = \sqrt[3]{x+3} - 1$



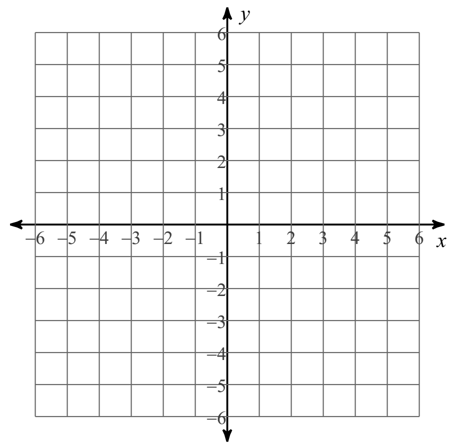
120) $g(n) = \dfrac{1}{n-1} - 1$



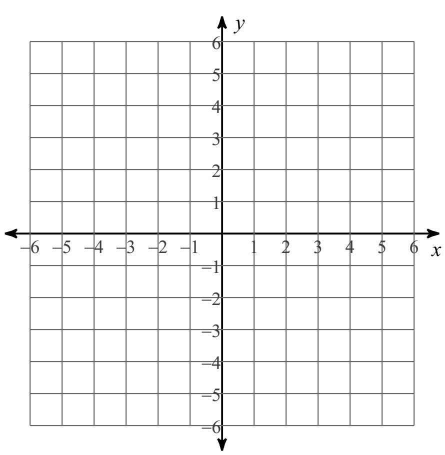
121) $f(n) = 7n + 2$



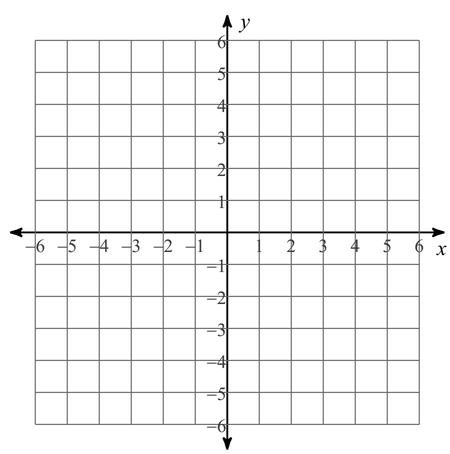
122) $f(x) = \dfrac{1}{x - 3} + 1$



123) $g(n) = \sqrt[3]{n + 1}$

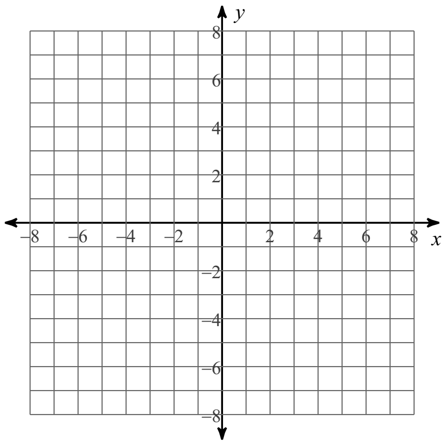


124) $h(x) = \dfrac{1}{x} - 1$

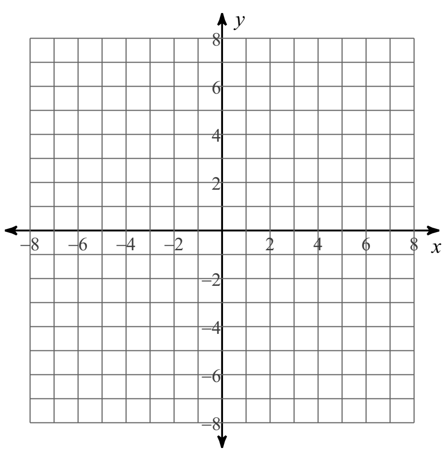


For each function, identify the holes, intercepts, and horizontal asymptote. Then sketch the graph.

125) $f(x) = \dfrac{3}{x + 1}$

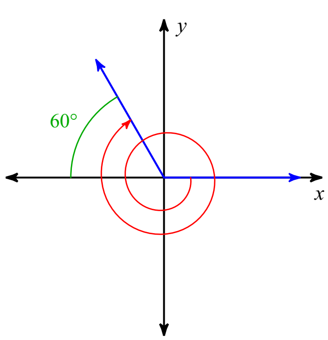


126) $f(x) = \dfrac{2}{x - 2} - 3$



Find the measure of each angle.

127)



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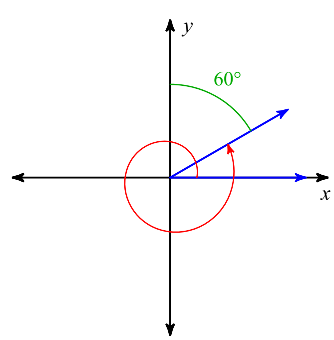
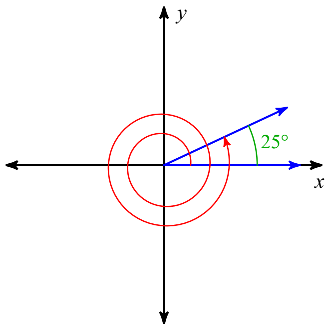


129)



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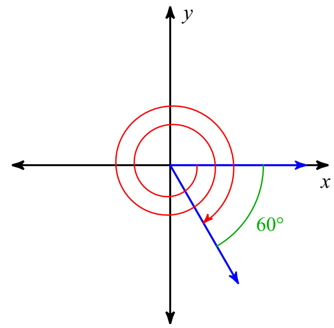

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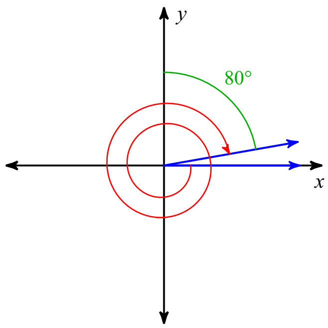

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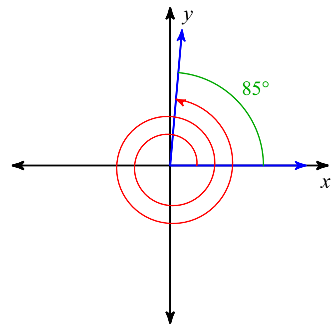
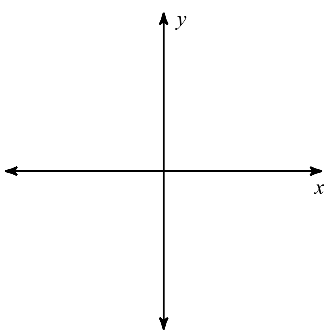
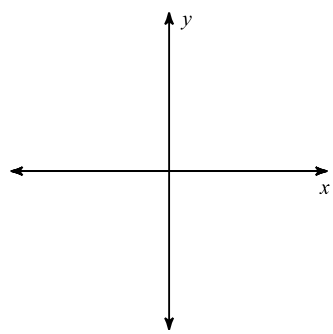


Draw an angle with the given measure in standard position.

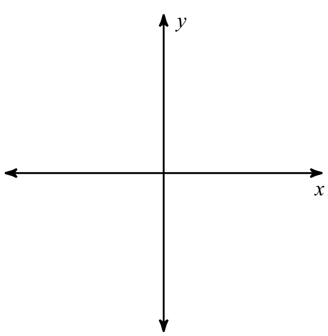
133) $-700°$



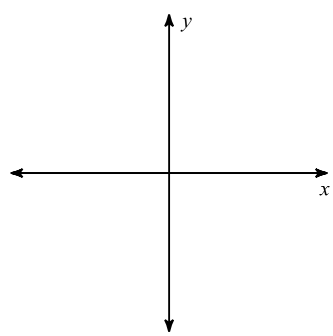
134) $-70°$



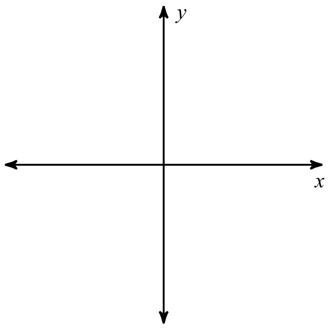
135) 50°



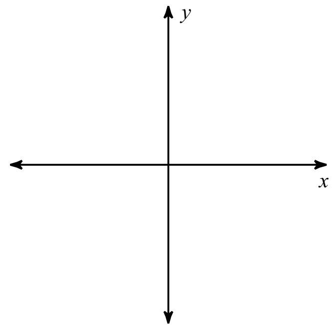
136) 200°



137) $-290°$



138) $-130°$



Find the reference angle.

139) $\frac{13\pi}{6}$

140) $-\frac{8\pi}{3}$

141) $-\frac{47\pi}{18}$

142) $\frac{11\pi}{3}$

143) $\frac{7\pi}{9}$

144) $-\frac{11\pi}{6}$

Solve each equation for $0 \le \theta < 2\pi$.

145) $\cos\theta = -\frac{\sqrt{2}}{2}$

146) $\frac{\sqrt{3}}{2} = \cos\theta$

147) $0 = \sin\theta$

148) $\tan\theta = -\sqrt{3}$

149) $\sin\theta = -1$

150) $\cos\theta = 0$

151) $\sin\theta = -\frac{\sqrt{3}}{2}$

152) $1 = \cos\theta$

153) $\cos\,\theta = -1$

154) $-\frac{1}{2} = \sin\,\theta$

155) $\cos\,\theta = -\frac{1}{2}$

Convert each degree measure into radians and each radian measure into degrees.

156) $250°$

157) $-\frac{43\pi}{12}$

158) $-\frac{4\pi}{9}$

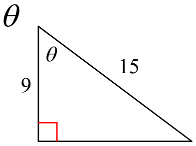
159) $20°$

160) $210°$

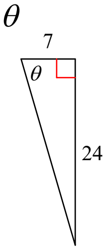
161) $-255°$

Find the value of the trig function indicated.

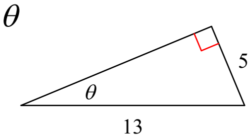
162) $\csc\,\theta$



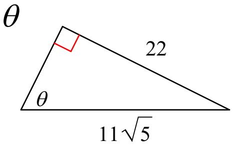
163) $\sec\,\theta$



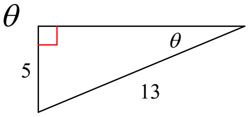
164) $\sec\,\theta$



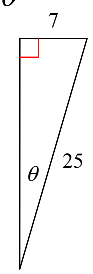
165) $\tan\,\theta$



166) $\csc\,\theta$

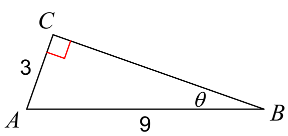


167) csc θ

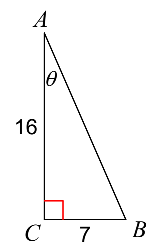


Find the measure of each angle indicated. Round to the nearest tenth.

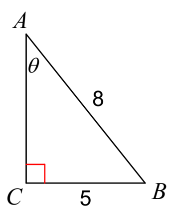
168)



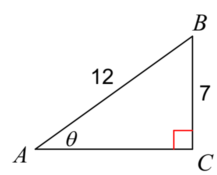
169)



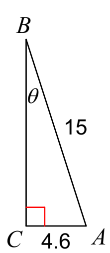
170)



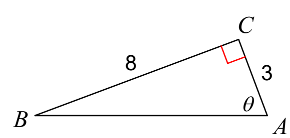
171)



172)



173)



Find the exact value of each trigonometric function.

174) $\cos -\frac{5\pi}{4}$

175) $\cos -\frac{16\pi}{3}$

176) $\sin 30°$

177) $\sec -\frac{2\pi}{3}$

178) $\tan 945°$

179) $\cot 390°$

180) $\tan \frac{15\pi}{4}$

181) $\cos -\frac{13\pi}{6}$

182) $\cos \frac{5\pi}{2}$

183) $\csc -675°$

184) $\cos -480°$

185) $\tan -870°$

186) $\cot -\frac{35\pi}{6}$

187) $\csc -585°$

188) $\sin -\frac{7\pi}{6}$

189) $\cot -870°$

Find all roots.

190) $x^6 - 64 = 0$

191) $x^6 + 124x^3 - 125 = 0$

192) $x^3 + 64 = 0$

193) $x^4 + 6x^2 + 8 = 0$

194) $x^4 + 15x^2 + 54 = 0$

195) $x^6 - x^4 - x^2 + 1 = 0$

196) $x^4 + x^2 - 12 = 0$

197) $x^4 + x^2 - 30 = 0$

198) $x^5 - 14x^3 + 48x = 0$

199) $x^6 - 1 = 0$

200) $x^4 - 3x^2 - 54 = 0$

Answers to Summer work (ID: 1)

1) expand vertically by a factor of 2
translate down 3 units

2) reflect across the x-axis
translate up 1 unit

3) expand vertically by a factor of 3
reflect across the x-axis

4) compress horizontally by a factor of 2
translate right 2 units

5) expand vertically by a factor of 2
translate left 3 units

6) reflect across the x-axis
translate right 3 units

7) $g(x) = -\left(\frac{1}{3}(x + 2)\right)^2 + 1$

8) $g(x) = -(2(x + 2))^3 - 1$

9) $g(x) = -(2(x + 2))^2 - 1$

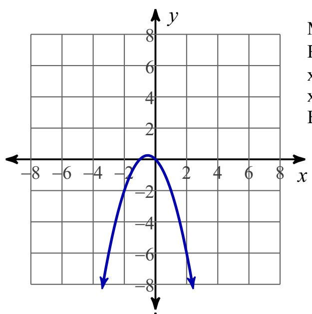
10) expand vertically by a factor of 2
translate down 2 units

11) compress horizontally by a factor of 2
translate up 1 unit

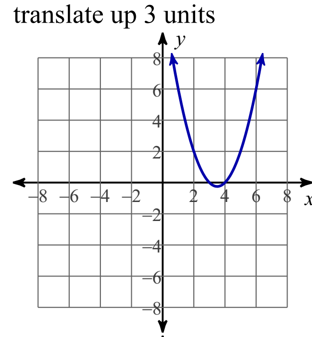
12) compress vertically by a factor of 3
reflect across the x-axis

13) compress horizontally by a factor of 2
reflect across the x-axis

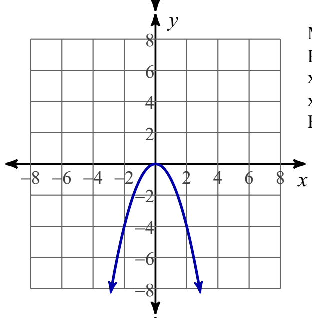
14) reflect across the x-axis
translate up 3 units

15)


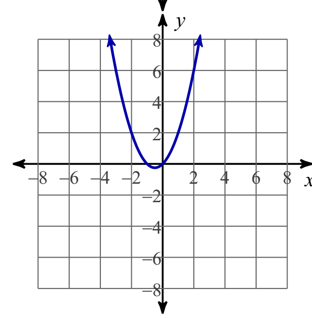
Max # turns: 1
Real zeros: $\{0, -1\}$
x-int, crosses: 0, −1
x-int, doesn't cross: None
End behavior:
$\lim_{x\to-\infty} f(x) = -\infty$
$\lim_{x\to\infty} f(x) = -\infty$

16)


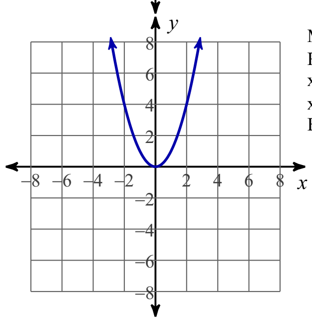
Max # turns: 1
Real zeros: $\{4, 3\}$
x-int, crosses: 4, 3
x-int, doesn't cross: None
End behavior:
$\lim_{x\to-\infty} f(x) = \infty$
$\lim_{x\to\infty} f(x) = \infty$

17)


Max # turns: 1
Real zeros: $\{0 \text{ mult. } 2\}$
x-int, crosses: None
x-int, doesn't cross: 0
End behavior:
$\lim_{x\to-\infty} f(x) = -\infty$
$\lim_{x\to\infty} f(x) = -\infty$

18)


Max # turns: 1
Real zeros: $\{0, -1\}$
x-int, crosses: 0, −1
x-int, doesn't cross: None
End behavior:
$\lim_{x\to-\infty} f(x) = \infty$
$\lim_{x\to\infty} f(x) = \infty$

19)


Max # turns: 1
Real zeros: $\{0 \text{ mult. } 2\}$
x-int, crosses: None
x-int, doesn't cross: 0
End behavior:
$\lim_{x\to-\infty} f(x) = \infty$
$\lim_{x\to\infty} f(x) = \infty$

20) $\left\{\frac{2}{9}\right\}$

21) $\left\{-\frac{4}{5}\right\}$

22) No solution.

23) $\left\{-\frac{3}{2}\right\}$

24) $\{-2\}$

25) −7.3443

26) −0.1547

27) −4.3786

28) 0.6981

29) 0.2649

30) $\{1\}$

31) $\{-10\}$

32) $\{9\}$

33) No solution.

34) $\{-14\}$

35) $\left\{\frac{69}{16}\right\}$

36) $\left\{\frac{88}{5}\right\}$

37) No solution.

38) $\{25\}$

39) $\{1, -1\}$

40) $\{0\}$

41) $\{0\}$

42) $\{-2\}$

43) $\{-2\}$

44) $\{-2\}$

45) 5.4511

46) −4.5666

47) 9.5694

48) 0.1395

49) 2.0729

50) $\log_5 (7 \cdot 8^5 \cdot 3^5)$

51) $\log_8 (33\sqrt[3]{6})$

52) $\ln (wv^3u^6)$

53) $\log_5 \frac{12^{16}}{11^4 \cdot 7^4}$

54) $\log_9 (11^3 \cdot 8^3 \cdot 7^6)$

55) $\log_3 (35\sqrt[3]{30})$

56) $\left\{-\frac{4}{7}\right\}$

57) $\{1\}$

58) $\{-3\}$

59) $\{-1\}$

60) No solution.

61) $\{7\}$

62) $\left\{-\frac{39}{10}\right\}$

63) $\{65\}$

64) $\left\{\frac{5}{19}\right\}$

65) $\left\{\frac{9}{2}\right\}$

66) $\{9, -9\}$

67) $\left\{\frac{10}{27}\right\}$

68) -2

69) 1

70) 4

71) -3

72) 2

73) -5

74) $\log_2 w + \frac{\log_2 u}{3} + \frac{\log_2 v}{3}$

75) $\log_3 z + \frac{\log_3 x}{2} + \frac{\log_3 y}{2}$

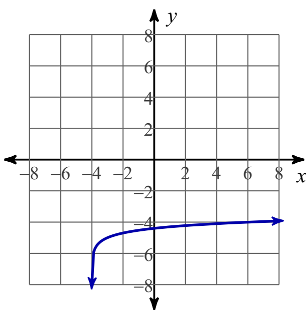
76) $2\log_6 5 + 10\log_6 12$

77) $10\log_8 x + 5\log_8 y$

78) $4\log_8 12 - 3\log_8 11$

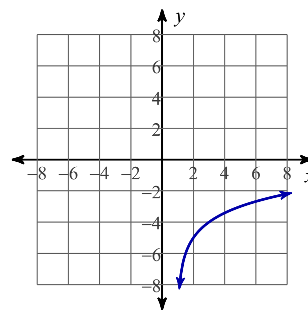
79) $3\log_3 11 + 3\log_3 7$

80)



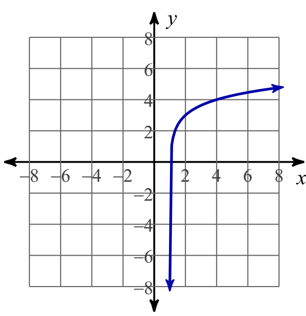
Domain: $x > -4$
Range: All reals

81)



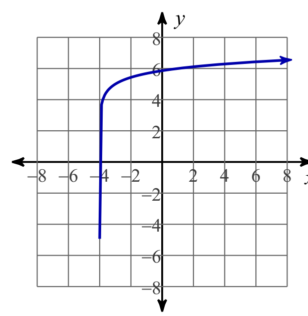
Domain: $x > 1$
Range: All reals

82)



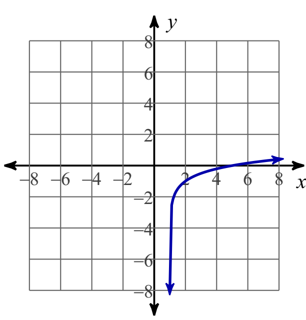
Domain: $x > 1$
Range: All reals

83)



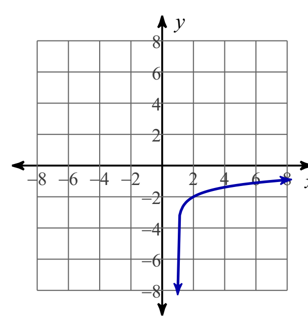
Domain: $x > -4$
Range: All reals

84)



Domain: $x > 1$
Range: All reals

85)



Domain: $x > 1$
Range: All reals

86) $y = 3^{-\frac{8}{x}}$

87) $y = 5^{-\frac{x}{9}}$

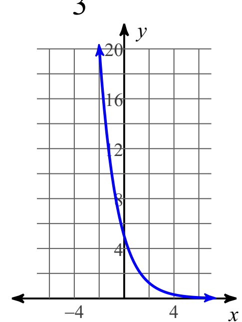
88) $y = \frac{4^x}{3}$

89) $y = 4^x + 9$

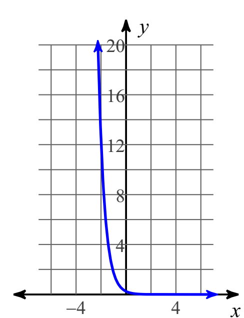
90) $y = 5^x + 5$

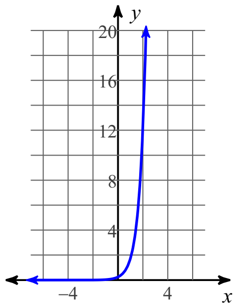
91) $y = 4^{-\frac{x}{3}}$

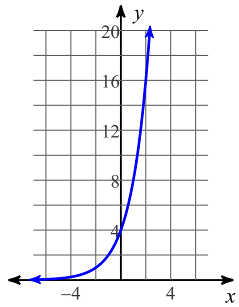
92)



93)



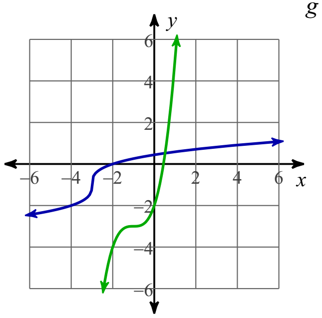
94)


95)


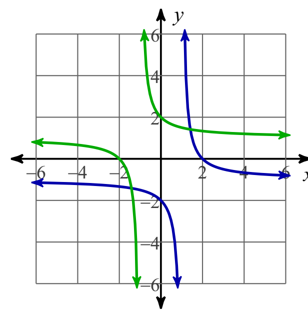
96) $y = 5 \cdot 2^x$

97) $y = 3 \cdot 2^x$

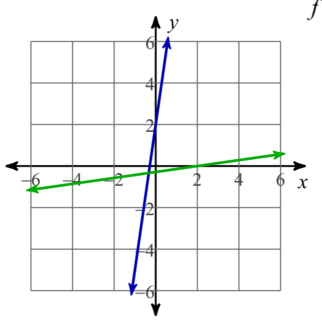
98) 1.5752
99) 0.3542
100) -0.8406
101) \$24,090.77
102) \$9,357.77
103) \$9,300.65
104) \$49,567.14
105) \$5,989.48
106) \$3,279.23
107) 11 years
108) 17 years
109) 15 years
110) 6 years
111) 20 years
112) 19 years
113) $6t$
114) $3n^2 + 3n + 10$
115) $-2x^3 - 10x - 3$
116) $4x^4 - 8x^3 - 6x^2 + 12x$
117) $8a^3 + 10a^2 + 16a + 20$
118) $-2n - 1$

119)


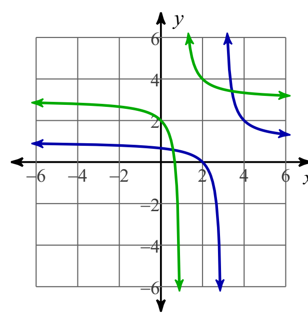
$g^{-1}(x) = (x + 1)^3 - 3$

120)


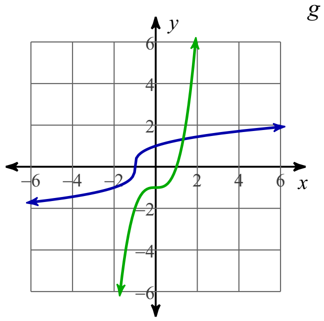
$g^{-1}(n) = -\frac{1}{-n - 1} + 1$

121)


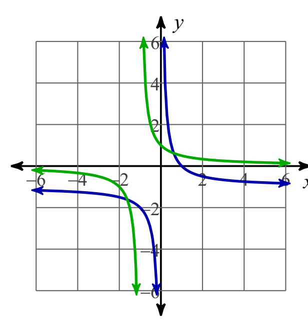
$f^{-1}(n) = \frac{1}{7}n - \frac{2}{7}$

122)


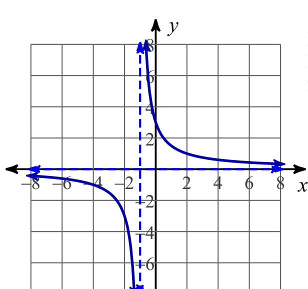
$f^{-1}(x) = \frac{1}{x - 1} + 3$

123)


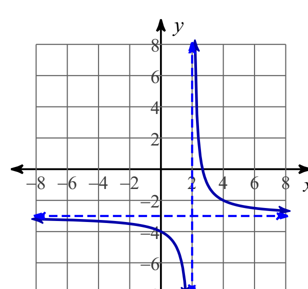
$g^{-1}(n) = n^3 - 1$

124)


$h^{-1}(x) = \frac{1}{x + 1}$

125)


Holes: None
Horz. Asym.: $y = 0$
x-intercepts: None, y-intercept: 3

126)


Holes: None
Horz. Asym.: $y = -3$
x-intercepts: $\frac{8}{3}$, y-intercept: -4

127) $-600°$

128) $390°$

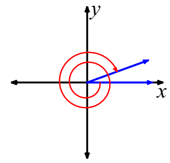
129) $745°$

130) $-780°$

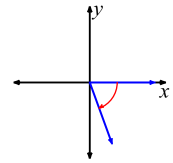
131) $-710°$

132) $805°$

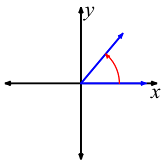
133)



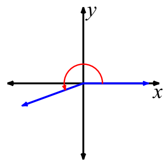
134)



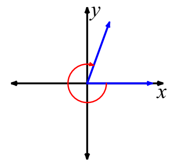
135)



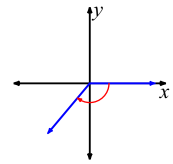
136)



137)



138)



139) $\frac{\pi}{6}$

140) $\frac{\pi}{3}$

141) $\frac{7\pi}{18}$

142) $\frac{\pi}{3}$

143) $\frac{2\pi}{9}$

144) $\frac{\pi}{6}$

145) $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

146) $\left\{\frac{\pi}{6}, \frac{11\pi}{6}\right\}$

147) $\{0, \pi\}$

148) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

149) $\left\{\frac{3\pi}{2}\right\}$

150) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

151) $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

152) $\{0\}$

153) $\{\pi\}$

154) $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

155) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

156) $\frac{25\pi}{18}$

157) -645°

158) -80°

159) $\frac{\pi}{9}$

160) $\frac{7\pi}{6}$

161) $-\frac{17\pi}{12}$

162) $\frac{5}{4}$

163) $\frac{25}{7}$

164) $\frac{13}{12}$

165) 2

166) $\frac{13}{5}$

167) $\frac{25}{7}$

168) $19.5°$

169) $23.6°$

170) $38.7°$

171) $35.7°$

172) $17.9°$

173) $69.4°$

174) $-\frac{\sqrt{2}}{2}$

175) $-\frac{1}{2}$

176) $\frac{1}{2}$

177) -2

178) 1

179) $\sqrt{3}$

180) -1

181) $\frac{\sqrt{3}}{2}$

182) 0

183) $\sqrt{2}$

184) $-\frac{1}{2}$

185) $\frac{\sqrt{3}}{3}$

186) $\sqrt{3}$

187) $\sqrt{2}$

188) $\frac{1}{2}$

189) $\sqrt{3}$

190) $\{2, -1 + i\sqrt{3}, -1 - i\sqrt{3}, -2, 1 + i\sqrt{3}, 1 - i\sqrt{3}\}$

191) $\left\{1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}, -5, \frac{5 + 5i\sqrt{3}}{2}, \frac{5 - 5i\sqrt{3}}{2}\right\}$

192) $\{-4, 2 + 2i\sqrt{3}, 2 - 2i\sqrt{3}\}$

193) $\{i\sqrt{2}, -i\sqrt{2}, 2i, -2i\}$

194) $\{i\sqrt{6}, -i\sqrt{6}, 3i, -3i\}$

195) $\{1 \text{ mult. } 2, -1 \text{ mult. } 2, i, -i\}$

196) $\{2i, -2i, \sqrt{3}, -\sqrt{3}\}$

197) $\{\sqrt{5}, -\sqrt{5}, i\sqrt{6}, -i\sqrt{6}\}$

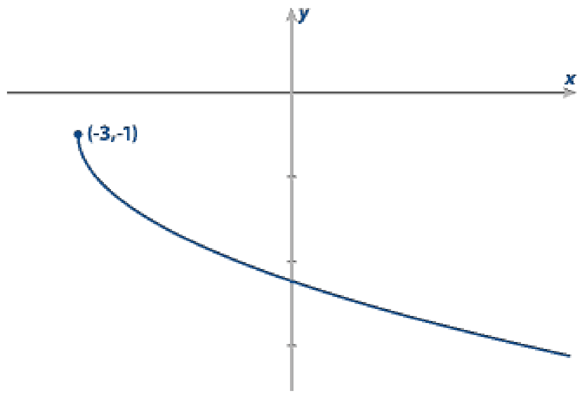
198) $\{0, \sqrt{6}, -\sqrt{6}, 2\sqrt{2}, -2\sqrt{2}\}$

199) $\left\{1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}, -1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}\right\}$

200) $\{3, -3, i\sqrt{6}, -i\sqrt{6}\}$

SL2 Summer work *[66 marks]*

1a. The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. *[3 marks]*



Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.

1b. A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$. *[1 mark]*

State the range of f.

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1c. Find an expression for $f^{-1}(x)$, stating its domain. *[5 marks]*

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1d. Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. *[5 marks]*

2a. The temperature $T\ ^{\circ}\mathrm{C}$ of water t minutes after being poured into a cup can be modelled by $T = T_0 \mathrm{e}^{-kt}$ where $t \geq 0$ and $T_0,\ k$ are positive constants. *[1 mark]*

The water is initially boiling at $100\ ^{\circ}\mathrm{C}$. When $t = 10$, the temperature of the water is $70\ ^{\circ}\mathrm{C}$.

Show that $T_0 = 100$.

2b. Show that $k = \frac{1}{10}\ln\frac{10}{7}$. *[3 marks]*

2c. Find the temperature of the water when $t = 15$. *[2 marks]*

2d. Sketch the graph of T versus t, clearly indicating any asymptotes with their equations and stating the coordinates of any points of intersection with the axes. *[4 marks]*

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2e. Find the time taken for the water to have a temperature of $50\ ^{\circ}\mathrm{C}$. Give your answer correct to the nearest second. *[4 marks]*

2f. The model for the temperature of the water can also be expressed in the form $T = T_0 a^{\frac{t}{10}}$ for $t \geq 0$ and a is a positive constant. *[3 marks]*

Find the exact value of a.

3a. The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f\left(x\right) = a(x - h)^2 + k$. *[3 marks]*

3b. The quadratic function g is defined by $g(x) = px^2 + (t-1)x - p$ where $x \in \mathbb{R}$ and $p,\ t \in \mathbb{R},\ p \neq 0$. *[4 marks]*

In the case where $g(-3) = g(1) = 4$,

find the value of p and the value of t.

3c. find the range of g. *[3 marks]*

3d. The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R},\ p \neq 0$. *[6 marks]*

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t.

4. The population of a town t years after 1 January 2014 can be modelled by the function *[7 marks]*

$$P(t) = 15\ 000\mathrm{e}^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.

5a. The following table shows values of $f(x)$ and $g(x)$ for different values of x. *[1 mark]*

Both f and g are one-to-one functions.

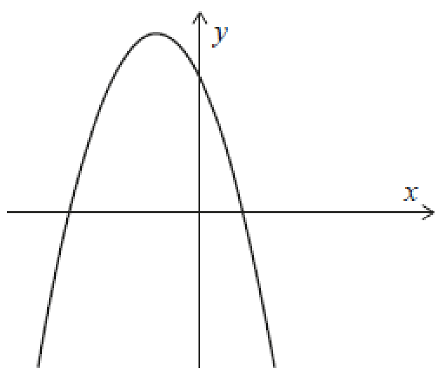
x	−2	0	3	4
$f(x)$	8	4	0	−3
$g(x)$	−5	−2	4	0

Find $g(0)$.

5b. Find $(f \circ g)(0)$. *[2 marks]*

5c. Find the value of x such that $f(x) = 0$. *[2 marks]*

6a. Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f. *[2 marks]*



For the graph of f

find the x-coordinates of the x-intercepts.

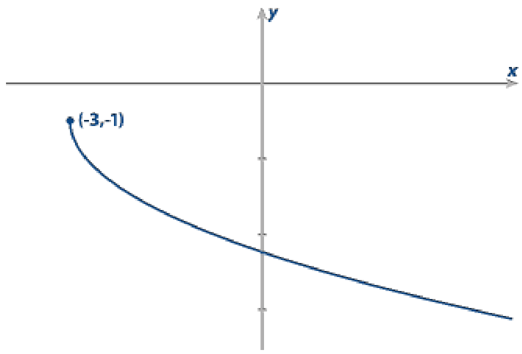
6b. find the coordinates of the vertex. *[3 marks]*

6c. The function f can be written in the form $f(x) = -2(x-h)^2 + k$. *[2 marks]*

Write down the value of h and the value of k.

SL2 Summer work *[66 marks]*

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



1a. Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. *[3 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

for example,

a reflection in the x-axis (in the line $y = 0$) **A1**

a horizontal translation (shift) 3 units to the left **A1**

a vertical translation (shift) down by 1 unit **A1**

Note: Award **A1** for each correct transformation applied in a correct position in the sequence. Do not accept use of the “move” for a translation.

Note: Award **A1A1A1** for a correct alternative sequence of transformations. For example,

a vertical translation (shift) down by 1 unit, followed by a horizontal translation (shift) 3 units to the left and then a reflection in the line $y = -1$.

[3 marks]

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

1b. State the range of f. *[1 mark]*

Markscheme

range is $f(x) \leq -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$, $(-\infty, -1]$ or $y \leq -1$.

[1 mark]

1c. Find an expression for $f^{-1}(x)$, stating its domain. *[5 marks]*

Markscheme

$-1 - \sqrt{y+3} = x$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$\sqrt{y+3} = -x - 1 (= -(x+1))$ **A1**

$y + 3 = (x+1)^2$ **A1**

so $f^{-1}(x) = (x+1)^2 - 3 \left(f^{-1}(x) = x^2 + 2x - 2\right)$ **A1**

domain is $x \leq -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$ or $(-\infty, -1]$.

[5 marks]

1d. Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. *[5 marks]*

Markscheme

the point of intersection lies on the line $y = x$

EITHER

$(x + 1)^2 - 3 = x$ **M1**

attempts to solve their quadratic equation **M1**

for example, $(x + 2)(x - 1) = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \frac{-1 \pm 3}{2}\right)$

OR

$-1 - \sqrt{x + 3} = x$ **M1**

$\left(-1 - \sqrt{x + 3}\right)^2 = x^2 \Rightarrow 2\sqrt{x + 3} + x + 4 = x^2$

substitutes $2\sqrt{x + 3} = -2(x + 1)$ to obtain $-2(x + 1) + x + 4 = x^2$

attempts to solve their quadratic equation **M1**

for example, $(x + 2)(x - 1) = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \frac{-1 \pm 3}{2}\right)$

THEN

$x = -2, 1$ **A1**

as $x \leq -1$, the only solution is $x = -2$ **R1**

so the coordinates of the point of intersection are $(-2,\ -2)$ **A1**

Note: Award **R0A1** if $(-2,\ -2)$ is stated without a valid reason given for rejecting $(1,\ 1)$.

[5 marks]

The temperature T °C of water t minutes after being poured into a cup can be modelled by $T = T_0 e^{-kt}$ where $t \geq 0$ and $T_0,\ k$ are positive constants.

The water is initially boiling at 100 °C. When $t = 10$, the temperature of the water is 70 °C.

2a. Show that $T_0 = 100$. *[1 mark]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

when $t = 0, T = 100 \Rightarrow 100 = T_0 \mathrm{e}^0$ **A1**

so $T_0 = 100$ **AG**

[1 mark]

2b. Show that $k = \frac{1}{10} \ln \frac{10}{7}$. *[3 marks]*

Markscheme

correct substitution of $t = 10, T = 70$ **M1**

$70 = 100\mathrm{e}^{-10k}$ or $\mathrm{e}^{-10k} = \frac{7}{10}$

EITHER

$-10k = \ln \frac{7}{10}$ **A1**

$\ln \frac{7}{10} = -\ln \frac{10}{7}$ or $-\ln \frac{7}{10} = \ln \frac{10}{7}$ **A1**

OR

$\mathrm{e}^{10k} = \frac{10}{7}$ **A1**

$10k = \ln \frac{10}{7}$ **A1**

THEN

$k = \frac{1}{10} \ln \frac{10}{7}$ **AG**

[3 marks]

2c. Find the temperature of the water when $t = 15$. *[2 marks]*

Markscheme

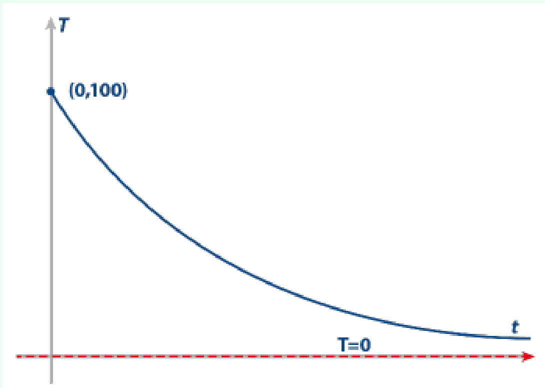
substitutes $t = 15$ into T **(M1)**

$T = 58.6\,(^{\circ}\mathrm{C})$ **A1**

[2 marks]

2d. Sketch the graph of T versus t, clearly indicating any asymptotes with their equations and stating the coordinates of any points of intersection with the axes. *[4 marks]*

Markscheme



a decreasing exponential **A1**

starting at $(0,\ 100)$ labelled on the graph or stated **A1**

$T \to 0$ as $t \to \infty$ **A1**

horizontal asymptote $T = 0$ labelled on the graph or stated **A1**

Note: Award **A0** for stating $y = 0$ as the horizontal asymptote.

[4 marks]

2e. Find the time taken for the water to have a temperature of $50\ ^{\circ}\mathrm{C}$. Give your answer correct to the nearest second. *[4 marks]*

Markscheme

$100\mathrm{e}^{-kt} = 50$ where $k = \frac{1}{10}\ln\frac{10}{7}$ **A1**

EITHER

uses an appropriate graph to attempt to solve for t **(M1)**

OR

manipulates logs to attempt to solve for t e.g. $\ln\frac{1}{2} = \left(-\frac{1}{10}\ln\frac{10}{7}\right)t$ **(M1)**

$t = \frac{\ln 2}{\frac{1}{10}\ln\frac{10}{7}} = 19.433\ldots$ **A1**

THEN

temperature will be $50\ ^\circ\mathrm{C}$ after 19 minutes and 26 seconds **A1**

[4 marks]

2f. The model for the temperature of the water can also be expressed in the form $T = T_0 a^{\frac{t}{10}}$ for $t \geq 0$ and a is a positive constant. *[3 marks]*

Find the exact value of a.

Markscheme

METHOD 1

substitutes $T_0 = 100$, $t = 10$ and $T = 70$ into $T = T_0 a^{\frac{t}{10}}$ **(M1)**

$70 = 100a^{\frac{10}{10}}$ **A1**

$a = \frac{7}{10}$ **A1**

METHOD 2

$100a^{\frac{t}{10}} = 100\mathrm{e}^{-kt}$ where $k = \frac{1}{10}\ln\frac{10}{7}$

EITHER

$\mathrm{e}^{-k} = a^{\frac{1}{10}} \Rightarrow a = \mathrm{e}^{-10k}$ **(M1)**

OR

$a = \left(\mathrm{e}^{\left(-\frac{1}{10}\ln\frac{10}{7}\right)t}\right)^{\frac{10}{t}}$ **(M1)**

THEN

$a = \mathrm{e}^{-\ln\frac{10}{7}}\left(= \mathrm{e}^{\ln\frac{7}{10}}\right)$ **A1**

$a = \frac{7}{10}$ **A1**

[3 marks]

3a. The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f\left(x\right) = a(x-h)^2 + k$. *[3 marks]*

Markscheme

correct substitution of $h = 3$ and $k = 2$ into $f(x)$ ***(A1)***

$f(x) = a(x-3)^2 + 2$

correct substitution of $(5,\ 0)$ ***(A1)***

$0 = a(5-3)^2 + 2\ \ \left(a = -\frac{1}{2}\right)$

Note: The first two A marks are independent.

$f(x) = -\frac{1}{2}(x-3)^2 + 2$ ***A1***

[3 marks]

The quadratic function g is defined by $g(x) = px^2 + (t-1)x - p$ where $x \in \mathbb{R}$ and $p,\ t \in \mathbb{R},\ p \neq 0$.

In the case where $g(-3) = g(1) = 4$,

3b. find the value of p and the value of t. *[4 marks]*

Markscheme

METHOD 1

correct substitution of $(1,\ 4)$ ***(A1)***

$p+(t-1)-p=4$

$t=5$ ***A1***

substituting their value of t into $9p-3(t-1)-p=4$ ***(M1)***

$8p-12=4$

$p=2$ ***A1***

METHOD 2

correct substitution of ONE of the coordinates $(-3,\ 4)$ or $(1,\ 4)$ ***(A1)***

$9p-3(t-1)-p=4$ OR $p+(t-1)-p=4$

valid attempt to solve their two equations ***(M1)***

$p=2,\ t=5$ ***A1A1***

$\left(g(x)=2x^2+4x-2\right)$

[4 marks]

3c. find the range of g. *[3 marks]*

Markscheme

attempt to find the x-coordinate of the vertex ***(M1)***

$x=\frac{-3+1}{2}(=-1)$ OR $\frac{-4}{2\times 2}$ OR $4x+4=0$ OR $2(x+1)^2-4$

y-coordinate of the vertex $=-4$ ***(A1)***

correct range ***A1***

$[-4,\ +\infty[$ OR $y\geq -4$ OR $g\geq -4$ OR $[-4,\ \infty)$

[3 marks]

3d. The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R},\ p \neq 0$. *[6 marks]*

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t.

Markscheme

equating the two functions or equations ***(M1)***

$g(x) = j(x)$ OR $px^2 + (t-1)x - p = -x + 3p$

$px^2 + tx - 4p = 0$ ***(A1)***

attempt to find discriminant (do not accept only in quadratic formula) ***(M1)***

$\Delta = t^2 + 16p^2$ ***A1***

$\Delta = t^2 + 16p^2 > 0$, because $t^2 \geq 0$ <u>and</u> $p^2 > 0$, therefore the sum will be positive ***R1R1***

Note: Award ***R1*** for recognising that Δ is positive and ***R1*** for the reason.

There are two distinct points of intersection between the graphs of g and j. ***AG***

[6 marks]

4. The population of a town t years after 1 January 2014 can be modelled by the function *[7 marks]*

$P(t) = 15\ 000\mathrm{e}^{kt}$, where $k < 0$ and $t \geq 0$.

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.

Markscheme

recognition that initial population is 15000 (seen anywhere) ***(A1)***

$P(0) = 15000$ OR 0.11×15000 OR 0.89×15000

population after 11% decrease is $15000 \times 0.89 (= 13350)$ ***(A1)***

recognizing that $t = 8$ on 1 January 2022 (seen anywhere) ***(A1)***

substitution of their value of t for 1 January 2022 and their value of $P(8)$ into the model ***(M1)***

$15000 \times 0.89 = 15000\mathrm{e}^{8k}$ OR $13350 = 15000\mathrm{e}^{8k}$

$k = \frac{\ln 0.89}{8} (-0.014566)$ ***(A1)***

substitution of $t = 2041 - 2014 (= 27)$ and their value for k into the model ***(M1)***

$P(27) = 15000e^{-0.0145\ldots \times 27}$

$10122.3\ldots$

$P(27) = 10100\ (10122)$ ***A1***

[7 marks]

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

5a. Find $g(0)$. *[1 mark]*

Markscheme

$g(0) = -2$ ***A1***

[1 mark]

5b. Find $(f \circ g)(0)$. *[2 marks]*

Markscheme

evidence of using composite function ***(M1)***

$f(g(0))$ OR $f(-2)$

$(f \circ g)(0) = 8$ ***A1***

[2 marks]

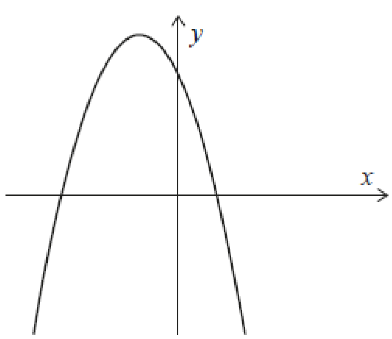
5c. Find the value of x such that $f(x) = 0$. *[2 marks]*

Markscheme

$x = 3$ ***A2***

[2 marks]

Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



For the graph of f

6a. find the x-coordinates of the x-intercepts. *[2 marks]*

Markscheme

setting $f(x) = 0$ ***(M1)***

$x = 1,\ x = -3$ (accept $(1, 0)$,$(-3, 0)$) ***A1***

[2 marks]

6b. find the coordinates of the vertex. *[3 marks]*

Markscheme

METHOD 1

$x = -1$ ***A1***

substituting their x-coordinate into f ***(M1)***

$y = 8$ ***A1***

$(-1, 8)$

METHOD 2

attempt to complete the square ***(M1)***

$-2\left((x+1)^2 - 4\right)$ ***(M1)***

$x = -1,\ y = 8$ ***A1A1***

$(-1, 8)$

[3 marks]

6c. The function f can be written in the form $f(x) = -2(x-h)^2 + k$. *[2 marks]*

Write down the value of h and the value of k.

Markscheme

$h = -1$ ***A1***

$k = 8$ ***A1***

[2 marks]