AP Physics Summer Work 2023-2024

for AP Physics 1 and AP Physics C: Mechanics

Introduction

Welcome to AP Physics! This summer work is designed to prepare students taking *either* AP Physics 1 *or* AP Physics C: Mechanics. The physics content is identical for the two courses and consists of these topics:

- Kinematics
- Linear Motion
- Circular Motion
- Gravitation
- Momentum
- Energy
- Oscillations
- Rotational Motion

The difference in the courses is primarily the level of math and the way that math is used to explore the physical universe. In AP Physics 1 we will use algebra, geometry, and some triangle trig. In AP Physics C: Mechanics, the "C" stands for calculus.

This summer work is designed to let you hit the new school year running with the basics skills needed in both classes. This work is designed to let you practice the critical skills needed to tackle the physics concepts. These basic skills include:

- Significant figures
- Conversions: metric and other units
- Solving algebraic equations
- Right triangle trigonometry
- Graphing
- Solving word problems

Please read the following and use these readings as a reference for solving the practice problems:

- Chapter 1: Collecting and Reporting Data, pp 1-11
    - Memorize the info in Table 1.2
- Chapter 2: Data Analysis, pp 19-27, 33
- Chapter 8: Quantitative Skills and Advanced Calculus Topics in AP Physics C: Mechanics, pp 128-140 [AP Physics C: Mechanics only]

You may watch the recommended videos in the readings or watch these six Introductory Concepts videos on YouTube from Flipping Physics:

- https://www.youtube.com/watch?v=mxbsgqFnNVY&list=PLPyapQSxH6maR-JEosZJ9rxW2cw3ACczG

If you have any questions please email me at shulbert@stpaulsmd.org.

Significant Figures

In the space provided, write the number of significant figures in each number.

Number	# sig figs	Number	# sig figs
246.32		14.600	
107.854		0.0001	
100.3		700,000	
0.678		350.670	
1.008		1.0000	
0.00340		320,001	

Convert the following numbers into scientific notation and indicate how many significant figures there are in each.

Number	Number in scientific notation	# sig figs
55,690		
1,200,000		
832		
0.00459		
0.0000116		
3,200,000,000		
0.123		
103,000,000		
4.05		

Conversions

Convert the following measurements into the base units for the SI system using the factor-labor method (aka the "multiply by 1" approach). Write your answer in scientific notation. Show your work for how you did the conversion. Do not just move the decimal place by counting.

65 km

126 cm

1,000 cm

0.05 km

0.10 g

550 mg

1,000,000 g

89 ms

50 µs

315,569,520 s

Convert the following values into the requested units using the factor-labor method (aka the "multiply by 1" approach). Write your answer in scientific notation. Show your work for how you did the conversion.

$1\frac{km}{s}\ to\ \frac{m}{s}$

$1\ year\ to\ s$

$1\ \frac{g}{cm^3}\ to\ \frac{kg}{m^3}$

$1\ km^2\ to\ m^2$

Algebra

Solve the following algebraic equations for the requested variable. Write you answer with the requested variable on the left side of the equal sign. Show all work.

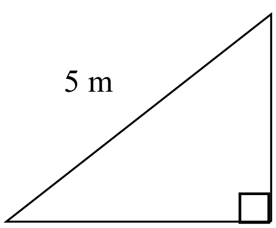
Solve for v, given $\frac{1}{2}mv^2 = mgh$	Solve for θ_1, given $n_1 sin(\theta_1) = n_2 sin(\theta_2)$
Solve for x, given $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$	Solve for v, given $\frac{GMm}{r^2} = \frac{mv^2}{r}$
Solve for a, given $v_f = v_0 + a \cdot \Delta t$	Solve for g, given $T = 2\pi\sqrt{\frac{l}{g}}$

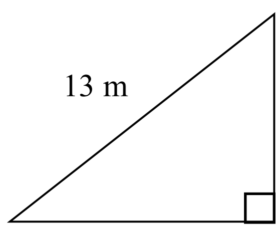
Solve for F_A in terms of m, g, and θ, given

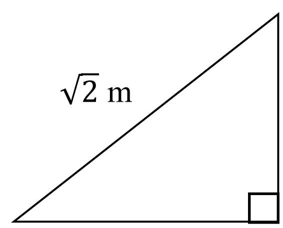
$$T sin(\theta) = F_A$$
$$T cos(\theta) = mg$$

Right Triangle Trigonometry

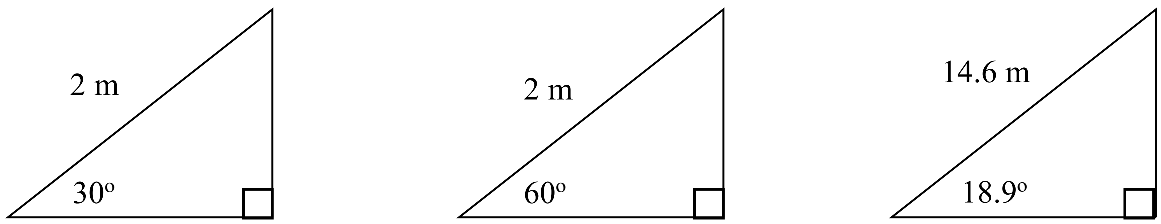
Label the integer lengths of the sides of the "special" right triangles show below, given the length of the hypotenuse.







Calculate the decimal lengths of the sides of the right triangles show below, given the length of the hypotenuse and the measure of one other angle.



Graphing

Graph the data in the following table on a separate piece of graph paper, provided.

Table 1: A ball rolling across the table

Time (s)	Position (m)
0	2.0
1	4.1
2	5.8
3	7.9
4	10.1

- Draw a line of best fit through the data points.
- Determine the slope of the line of best fit.
- Show you work below.

Problem Solving

An intrepid SPSG physicist measured the dimensions of a block of pink rigid foam and found that block is 2.5 m long, 0.5 m wide, and 0.1 m thick. Another SPSG physicist measured the mass of the block to be 15 kg. Calculate the density of the block of foam in $\frac{kg}{m^3}$. Use the Find/Given technique described in "How to Solve Some Problems in Physics". Recall that equation for density is: $\rho = \frac{m}{V}$.

CHAPTER 1

Collecting and Reporting Data

Types of Data

Data can be classified as qualitative or quantitative:

- Qualitative data
  - Are observed rather than measured
  - Include written descriptions, videos, photographs, or live observations
  - Examples include observations of appearance, behaviors, smell, taste, etc.
- Quantitative data
  - Are measured and recorded in numerical form
  - Examples include absorbance, size, time, height, and mass

Qualitative data and quantitative data are both important and not always used completely separate from each other. Qualitative data can be coded or organized in a quantitative way for the purpose of interpretation or analysis. For instance, in the AP Biology Enzyme Catalysis lab, a color palette (figure 1.1) is used to qualitatively determine the amount of oxygen produced when hydrogen peroxide is degraded by the turnip peroxidase enzyme. By numbering the colors 1–10, the qualitative data obtained from the experiment can be converted to quantitative data. Alternately, quantitative data can be obtained from this experiment by using a spectrophotometer to measure the absorbance or percent transmittance of the samples. The purpose or anticipated outcome of an experiment will determine which type of data you choose to collect and how to organize it.

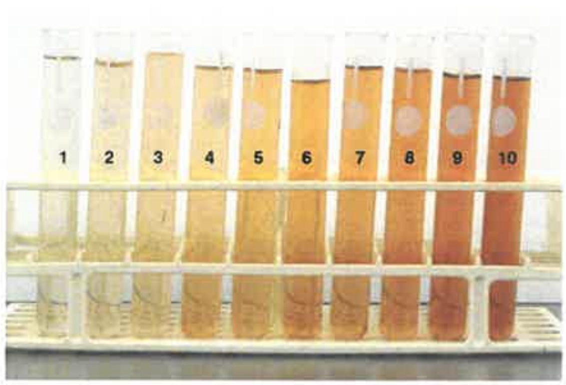


Figure 1.1: Turnip Peroxidase Color Chart

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Measurement

Units: Use of Metric Prefixes

There are two commonly used systems of measurement in the world, which differ in the units they use for length, mass, and time. The first is the United States Customary System (USCS, formerly called the English system) of feet, pounds, and seconds. The second is the metric system of meters, kilograms, and seconds. In 1960, the metric system was adopted by an international committee in Paris as the worldwide standard for science and is now referred to as the Système International or SI. A subset of the metric system is the centimeter-gram-second (cgs) system that is commonly used in atomic physics and chemistry. The meter-kilogram-second (mks) system is another subset commonly used in physics (specifically mechanics). In science, medicine, and government in the United States, the SI system is often used alongside the USCS system. In this guide, we will use the SI system of measurement, which is the preferred measurement system of science.

Fundamental Units

Most physical quantities, such as velocity, acceleration, force, momentum, and energy can ultimately be expressed in terms of three basic units of length, mass, and time. These three units are referred to as **fundamental units** because they can be used to define all other elements in a particular system of measurement.

Table 1.1 summarizes the fundamental units for the metric and USCS systems of measurement.

Table 1.1: Fundamental Units of Measurement

System	Length	Mass	Time
SI (MKS)	meter	kilogram	second
SI (CGS)	centimeter	gram	second
USCS	feet	slug	second

The units of measure in the SI system are often preceded by prefixes to indicate the appropriate size of a measurement. Each prefix represents a power of 10 and has a symbol that is added to the measurement for reporting. For example, the prefix *milli-* indicates 1/1000, which means that there are 1000 milligrams in a gram. So when describing the mass of an object or substance that is very small, it is reported as 3.42 milligrams rather than 0.00342 grams. Table 1.2 lists the prefixes that you will most commonly use in your AP science courses.

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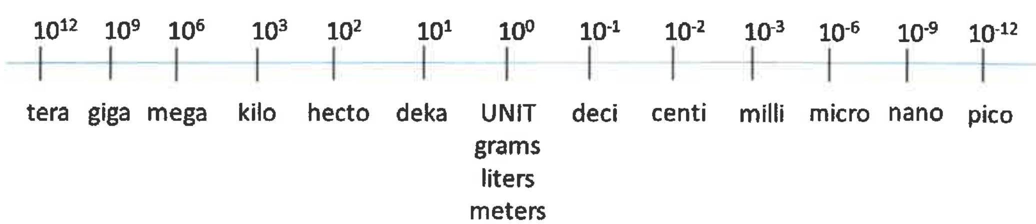
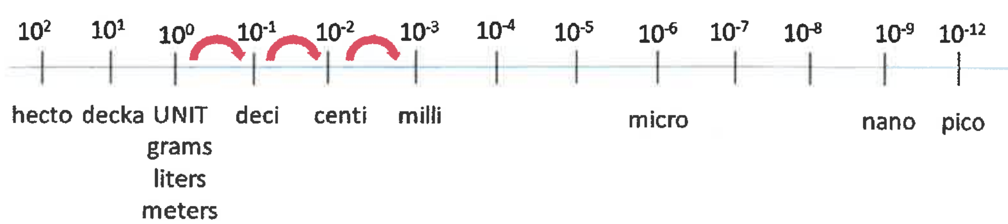
Table 1.2: Common SI Prefixes

Prefix	Symbol	Multiplier	Number
tera	T	10^{12}	1 trillion
giga	G	10^{9}	1 billion
mega	M	10^{6}	1 million
kilo	k	10^{3}	1,000
hecto	h	10^{2}	100
deka	da	10^{1}	10
UNIT (grams/ liters/meters)		10^{0}	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	µ	10^{-6}	1 millionth
nano	n	10^{-9}	1 billionth
pico	p	10^{-12}	1 trillionth

Another way of thinking about this is to use a place-value representation:



One way to convert from one unit to another is to use the above representation to count how many decimal places should be used for the adjustment. So, using the example above, if you wanted to convert 0.00342 grams to milligrams, you would start at the base unit then count until you get to the *milli-* prefix, as shown below.



Since we moved three places to the right, we will move the decimal point in our number—0.00342—to the right also. By doing this, we find that 0.00342 grams is equal to 3.42 milligrams. If we move the decimal point two places to the right instead of three, we find that

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0.00342 grams is equal to 0.342 centigrams, and moving the decimal point one place to the right shows that 0.00342 grams is equal to 0.0342 decigrams.

If you need further review of the metric system, try this tutorial:

Khan Academy: U.S. customary and metric units

Dimensional Analysis: Unit Conversions

In AP science courses you will frequently have to analyze relationships between physical quantities. This may require you to convert between units to describe equivalent amounts of the data you are reporting. In doing this, the amounts of data you are describing remain the same. You are only changing the way you report these amounts. Converting units is a type of **dimensional analysis** for which the **factor-label method** is helpful. For example, let's say we want to convert 650 mL to liters.

We know that there are 1000 mL in one liter:

$$1L = 1000mL$$

We first convert this equation to conversion factors:

$$\frac{1L}{1000mL} \qquad \frac{1000mL}{1L}$$

Multiplying a quantity by these conversion factors changes the units, but leaves the quantity unchanged. We next choose a conversion factor that will convert our quantity, 650 mL, from units of mL to units of liters:

$$650\cancel{mL} \times \frac{1L}{1000\cancel{mL}} = \frac{650L}{1000} = 0.650L$$

The conversion factor was chosen so that when units are cancelled out (the diagonal lines in the accompanying examples), the desired unit remains. In choosing the conversion factor, we put the mL in the denominator so that it cancels out, and we are left with L. By cancelling out mL, we converted from mL to L.

We can also multiply by a series of conversion factors. For example, consider converting from three miles to meters, given the conversion from miles to feet (there are 5280 feet in a mile) and the conversion from feet to meters (there are 3.28 feet in a meter).

There are 5280 feet in a mile, and 3.28 feet in a meter:

$$1\text{ mile} = 5280\text{ feet}$$

$$3.28\text{ feet} = 1\text{ meter}$$

This gives us four conversion factors:

$$\frac{1mile}{5280\,feet} \qquad \frac{5280\,feet}{1mile} \qquad \frac{3.28\,feet}{1meter} \qquad \frac{1meter}{3.28\,feet}$$

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So the conversion would look like this:

$$2\cancel{miles} \times \frac{5280\,\cancel{ft}}{1\,\cancel{mile}} \times \frac{1m}{3.28\,\cancel{ft}} = 3219m$$

You will often use this method to determine how to make solutions. For example, how many grams of sodium hydroxide (NaOH, molar mass = 40 g/mol) would we need if we wanted to make 500 mL of a 0.40 M (moles per liter) solution? In this case, the molar mass of NaOH (1 mol NaOH = 40 g NaOH) leads to the following conversion factors:

$$\frac{1\,mol\,NaOH}{40\,g\,NaOH} \qquad \frac{40\,g\,NaOH}{1\,mol\,NaOH}$$

Similarly, our target concentration of 0.40 M gives us the following ratios:

$$\frac{0.40\,mol\,NaOH}{1\,L} \qquad \frac{1\,L}{0.40\,mol\,NaOH}$$

To determine the mass of NaOH needed to make 500 mL of solution we start with 500 mL and multiply as follows:

$$500\cancel{mL} \times \frac{1\cancel{L}}{1000\cancel{mL}} \times \frac{0.4\cancel{mol}}{1\cancel{L}} \times \frac{40g}{1\cancel{mol}} = 8\,g\,NaOH\,needed$$

These tutorials will help you refresh your memory on how to do unit conversions:

Khan Academy: Unit conversion within the metric system

Khan Academy: Converting within the metric system

Significant Digits

To ensure that you are reporting your data to the correct degree of precision, the data you record during an experiment should include only **significant digits** (also called **significant figures**). These are:

- The digits that are meaningful in a measurement or a calculation.
- Determined by the measurement device used during the experiment.
    - If you use a digital device, record the measurement value exactly as it is shown on the screen.
    - If you read the result from a ruled scale (such as a ruler or graduated cylinder), the value that you record should include each digit that is certain and one uncertain digit.

For example, figure 1.2 shows the same measurement made with two different scales, which vary in their precision of measurement. On the left, the digits 8 and 4 are certain because they are shown by markings on the scale and it is clear that the measurement is at least 8.4.

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The digit 2 is an estimate of how far the measurement is beyond 8.4, so that is the uncertain digit. This measurement (8.42 cm) has three significant digits. The scale on the right has markings at 8 and 9. The 8 is certain, but you must estimate how far the measurement is beyond 8, so 4 is the uncertain digit. This measurement is 8.4 cm. Even though the measurement on the right is the same as the measurement on the left, it has only two significant digits because the markings are farther apart, and thus there is less precision to the measurement being made.

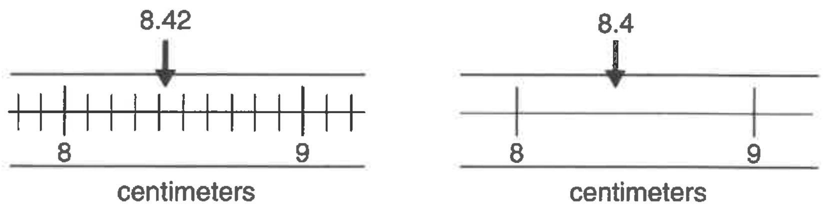

Figure 1.2: Different Significant Digits from Different Scales

Uncertainties in measurements should always be rounded to one significant digit. When measurements are made with devices that have a ruled scale, the uncertainty is half the value of the precision of the scale. The markings on the device will show the precision. Looking at the example shown in figure 1.2 above, the scale on the left has markings every 0.1 cm, so the uncertainty is half this, which is 0.05 cm. The correct way to report this measurement is 8.43 ± 0.05 cm. The scale on the right has markings every 1 cm, so the uncertainty is 0.5 cm. The correct way to report this measurement is 8.4 ± 0.5 cm.

Table 1.3 presents the rules you should follow in determining which digits in a number that represents a measured value are meaningful (in the sense described above) and therefore significant.

Table 1.3: Rules for Significant Figures

Rule	Examples
Non-zero digits are always significant.	4,735 km has four significant digits. 573.274 in. has six significant digits.
Zeros before other digits are not significant.	0.38 m has two significant digits. 0.002 in. has one significant digit.
Zeros between other digits are significant.	42.907 km has five significant digits. 0.00706 in. has three significant digits. 8,005 km has four significant digits.
Zeros to the right of all other digits are significant if they are to the right of the decimal point.	975.3810 cm has seven significant digits. 471.0 m has four significant digits.
It is impossible to determine whether zeros to the right of all other digits are significant if the number has no decimal point.	8,700 km has at least two significant digits, but the exact number is unknown. 20 in. has at least one significant digit, but the exact number of significant digits is unknown.

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Table 1.3: Rules for Significant Figures *(continued)*

Rule	Examples
If a number is written with a decimal point, zeros to the right of all other numbers are significant.	620.0 km has four significant digits. 5,100.4 m has five significant digits. 670. in. has three significant digits.
All digits in the coefficient of a number written in scientific notation are significant.	6.02×10^4 cm has three significant digits

Note that it is good scientific practice to use **scientific notation** (see chapter 2). If you use scientific notation, then all digits shown are always significant.

If you need additional review on significant figures, this tutorial can help:

Khan Academy: Intro to significant figures

Data Tables

Data tables allow you to gather your data in one place so that it can be organized, compared, or analyzed in a meaningful way for interpretation. When constructing a data table you need to be sure to include both the independent and dependent variables.

- **Independent variable**
  - Also called the explanatory or controlled variable
  - The variable that the researcher controls or manipulates
  - Not changed by the other variable(s) measured in the experiment
  - Examples: time, distance, velocity, acceleration, concentration, light intensity
- **Dependent variable**
  - Also called the response or experimental variable.
  - The response to the independent variable—what is measured.
  - Example: population growth: The number of individuals in a population will change with time, so the growth of the population is a dependent variable since it is dependent on time (the independent variable).
  - Example: If you were interested in the velocity of an object as a function of time, then velocity could be a dependent variable, while time would be the independent variable. On the other hand, velocity could be an independent variable if you investigated acceleration as a function of velocity.

Elements of Effective Data Tables

You may often use computer software to create data tables to communicate the results of an investigation. However, whether you are using software or drawing by hand, you should keep in mind the following elements of effective data tables, shown in figure 1.3:

1. A meaningful title: This is a title that informs the reader about the experiment and exactly what is being measured.

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2. Independent and dependent variables: These are typically with the independent variable on the left side of the data table and the dependent variables on the right.
3. Units: Be sure that units are clearly indicated for each variable.
4. Data: There should be data for each repeated trial.

Meaningful title

Effect of Glucose Concentration on Plant Growth by Mass

Independent variable with units

Dependent variable with units

Glucose Solution (M)	Mass of Plants (g)						
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Average
0.0							
0.2							
0.4							
0.6							
0.8							
1.0							

Data for each repeated trial

Figure 1.3: Elements of Effective Data Tables

Graphs

One of the best ways to communicate the results of a scientific investigation is by creating a graph of the data that have been counted, measured, or calculated. Graphs can help you to easily see patterns more easily through a visual display of data and can also help you clearly see how two measured variables affect one another.

Elements of Effective Graphs

Just as with data tables you may use computer software to create your graphs. However, whether you are using software or graphing by hand, you should keep in mind the following elements required of nearly all effective graphs (illustrated in figure 1.4):

1. A meaningful title: This is a title that informs the reader about the experiment and exactly what is being measured.
2. Labeled axes with units:
   - The **x-axis** is the horizontal axis, and it usually denotes the independent variable.
   - The **y-axis** is the vertical axis, and it usually denotes the dependent variable.
   - Note that the axes do not always need to denote dependent versus independent variables. In physics, we often choose the axes for straight-line fitting so that the slope or y-intercept provides physical information. For example, for various satellites orbiting the Earth, we might choose to graph period squared (T^2) on the y-axis, and radius cubed (R^3) on the x-axis in order to see if the orbits obey Kepler's third law.
3. Uniform intervals: For example, if one interval on the x-axis corresponds to five minutes, each interval must be the same and not change to 10 minutes or one minute. If there is a break in the graph, such as a time course over which little happens for an extended period, it should be noted with a break in the axis and a corresponding break in the data line. It is not necessary to label each interval.
4. Identifiable lines or bars: Using different colors or patterns and including a legend will help the reader distinguish one line or bar from the others. You can also label each line or bar.

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5. Origin: The graph should clarify whether the data and any **trend lines** start at the origin (0,0) or not. A trend line should not be extended to the origin if the data do not start there. In addition, the line should not be extended beyond the last data point (extrapolation) unless a dashed line clearly indicates that this is a prediction about what may (or could) happen if additional data were to be obtained.
6. Error bars: For some of the labs you perform in class, you should consider the variability (or confidence) of your data in your analysis and use error bars on your graphical displays when appropriate (see the discussion of standard deviation and standard error later in this chapter).

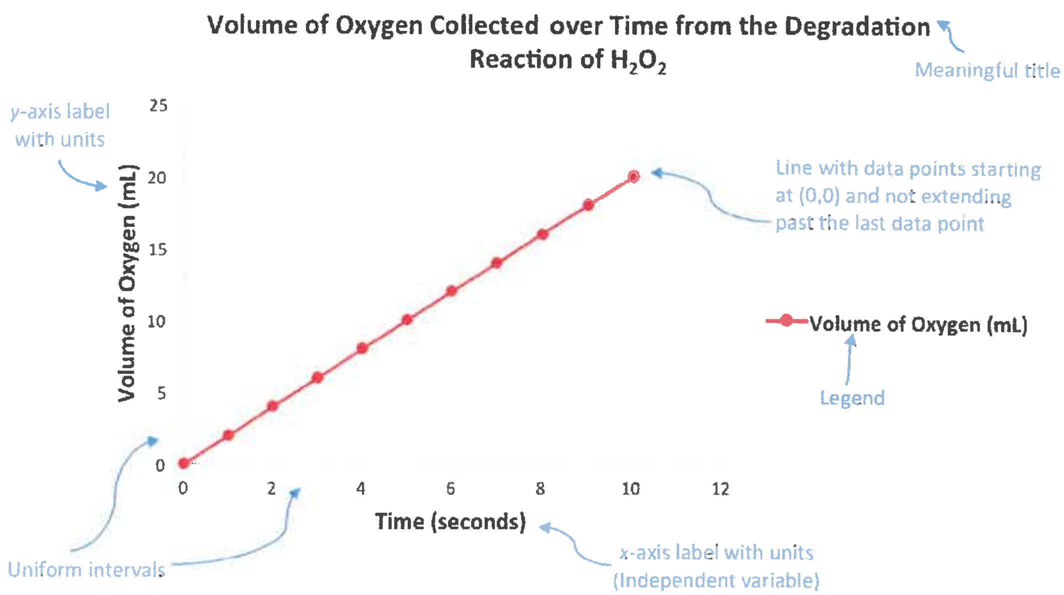

Figure 1.4: Example of an Effective Graph

Types of Graphs

Line Graphs

Line graphs are plotted on *x*-*y* axes and offer a good visual representation of the relationship between two variables; in other words, how one variable is affected by the other as it increases or decreases. Line graphs can contain one line or multiple lines that represent the data. Clear trends in the data can be seen by the direction of the line(s) on a graph. Line graphs are advantageous because they can sometimes allow you to predict the results of data that have not yet been collected, since the line implies a continuous response of the dependent variable.

Figure 1.5 shows an example of a line graph. It is a type of rate graph called a progress curve, because it shows an amount of a substance on the *y*-axis and time on the *x*-axis. There are several different curves plotted on the graph and each one is labeled with a different temperature.

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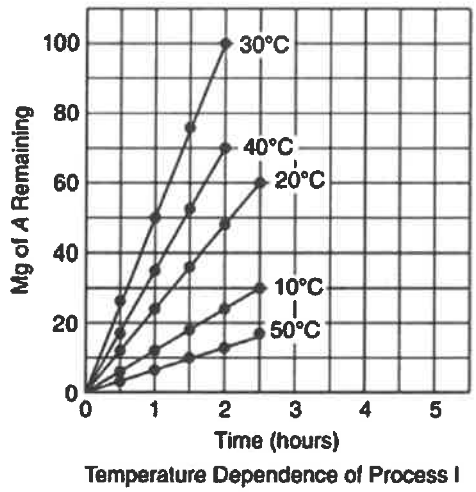

Figure 1.5: Example of a Line Graph with Several Sets of Data

If you need additional review on line graphs, try this tutorial:

Khan Academy: Introduction to line plots

Scatter Plots

Scatter plots are plotted on x-y axes and are also used to compare two variables. However, in scatter plots, data are presented as an assortment of points that may or may not show one or more of the linear relationships between the two variables that are commonly presented in line graphs. In order to determine whether there is a linear relationship between the two variables, a linear regression (see the Curve Fitting section later in this chapter) can be calculated and plotted to help make the pattern clearer. Keep in mind that the data shown in scatter plots do not have to have a linear relationship.

Figure 1.6 is an example of a scatter plot with a linear regression line. Linear regression lines can indicate a pattern in the data that may not be apparent by looking at the dots alone. We see from the graph that there is a relationship between heart rate and temperature.

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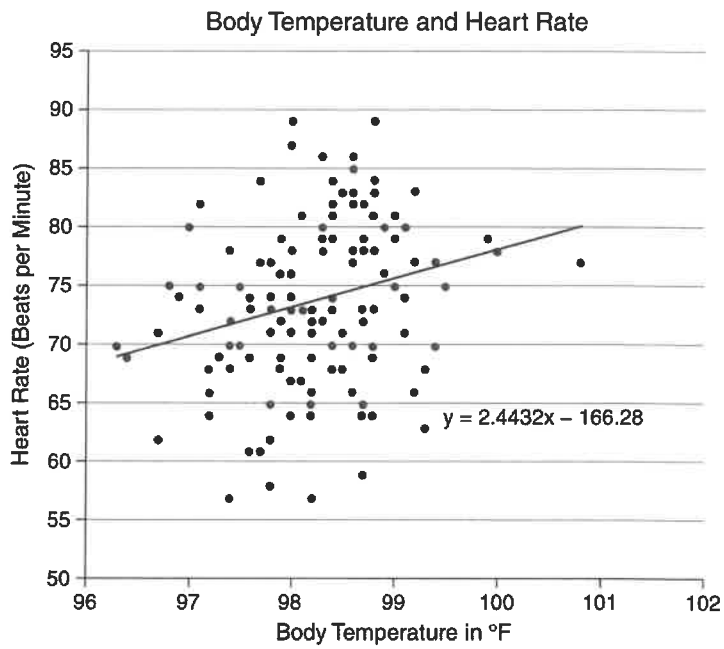

Figure 1.6: A Scatterplot with a Linear Regression Line

To learn more about scatter plots, review these tutorials:

Khan Academy: Constructing a scatter plot

Khan Academy: Regression line example

Histograms

Histograms are plotted on *x*-*y* axes and show the distribution of numerical data. Creating this kind of graph requires setting up grouped intervals called bins for the range of values tested. The range is divided into equal intervals, then the number of measurements that fit into each bin are counted and graphed, which results in a frequency diagram. In figure 1.7, the bins (i.e., the grouped intervals) are the five-day age ranges plotted on the *x*-axis: the number of flies in each five-day interval is the measurement represented by the height of each bar as read from the *y*-axis.

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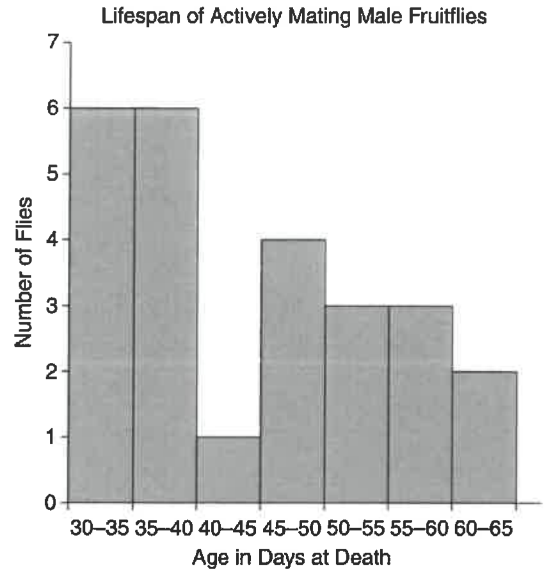

Figure 1.7: A Histogram Showing Nonparametric Data

You can use the tutorials below to review creating and interpreting histograms:

Khan Academy: Creating a histogram

Khan Academy: Interpreting a histogram

Bar Graphs

Bar graphs are plotted on *x*-*y* axes and are used to compare data: typically they represent the summary statistics of a data set, such as the mean or median. Drawn either vertically or horizontally, each bar can represent a category or variable. The relationship between the variable and each group can be determined based on the height or length of the bars, respectively. In figure 1.8, the *x*-axis shows the categories of leaf habitat (sunny and shady) that are being compared, and the *y*-axis shows the mean width of leaves measured in each environmental condition. Note that when the independent variable is numerical, you should use a histogram (see the previous section) to represent your data instead of a bar graph.

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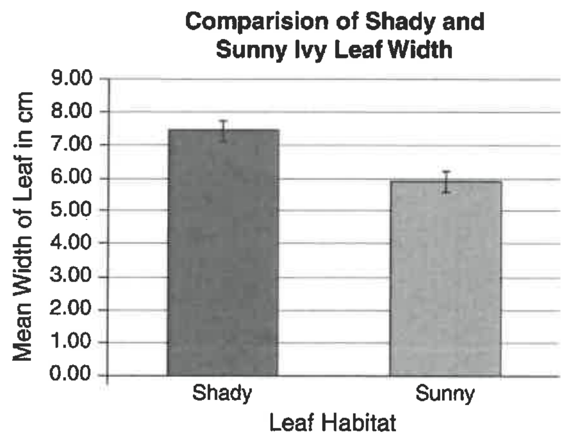

Figure 1.8: Example of a Bar Graph

The following tutorials can help you learn more about creating and reading bar graphs:

Khan Academy: Creating a bar graph

Khan Academy: Reading bar graphs

Box and Whisker Plots

Box and whisker plots are plotted on *x*-*y* axes and are used to look at the range of data that has been measured. The advantages of box and whisker plots are that they give you a quick idea of the spread (variability) of your data, the skewness (the amount of skew, or asymmetric distribution), and how different it is from the other data. Box and whisker plots also provide a quick estimate for comparing data sets. A limitation is that you really need at least 10 data points per sample in order to construct an effective box and whisker plot.

To construct a box and whisker plot, gather a five-number summary (or five statistical summary) of data: the (1) minimum, (2) maximum, (3) median, (4) first quartile, and (5) third quartile. You start by numerically ordering your data, and then determine the **median**, which is the number in the middle (or the average of the two middle numbers if the data set contains an even number of observations) of the ordered data set. Once this has been done, the data can be divided into **quartiles** then plotted on a graph.

For example, let's look at the following data set, where researchers measured the percentage of leaf decay in bags containing three different types of leaves.

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Bag	% Decay		
Number	Ash	Sycamore	Beech
1	51	40	34
2	63	33	15
3	44		26
4		52	21
5	48	48	
6	32	35	11
7	70	44	19
8	48	63	32
9	57	40	

First we numerically order each set of data from least to greatest. Then we find the median, which is the number in the middle of this ordered list:

	% Decay			
	Ash	Sycamore	Beech	
minimum	32	33		
	44	35	11	
	48	40	15	
	48	40	19	
	51	44	21	median
	57	48	32	
	63	52	34	
maximum	70	63		

In each case, since there is an even number of data points, the median is the average of the two middle points: 49.5 for the ash, 42 for the sycamore, and 20 for the beech. The **minimum** in each data set is the smallest number and the **maximum** is the largest number.

Once you have determined the median, you then determine the quartiles by dividing each half of the data in half:

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% Decay		
Ash	**Sycamore**	**Beech**
32	33	
44	35	11
48	40	15
48	40	19
51	44	21
57	48	32
63	52	34
70	63	

1st quartile
3rd quartile
minimum
median
maximum

So, the five-number summary for this data is as follows:

% Decay: Five-Number Summary

	Ash	Sycamore	Beech
Minimum	32	33	11
Quartile 1	46	37.5	13
Median	49.5	42	20
Quartile 3	60	50	33
Maximum	70	63	34

These five values are used to create a box and whisker plot representing the spread of values in a data set. The box and whisker plot for this data is shown in figure 1.9.

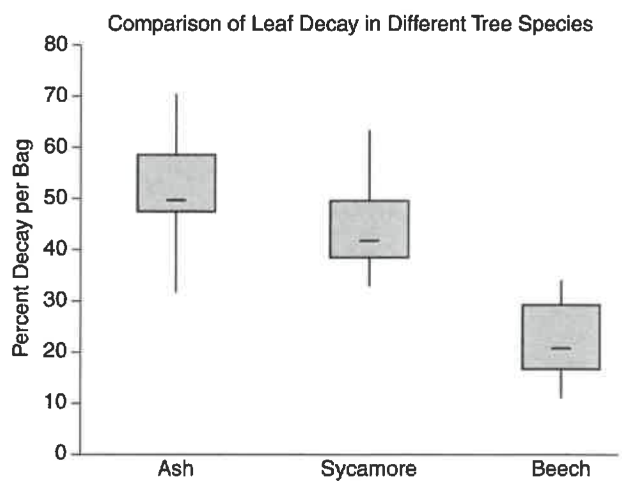

Figure 1.9: Example of a Box and Whisker Plot

In the graph, the tops and bottoms of the vertical lines typically show the range of the data set. The top of each box shows the upper (third) quartile, the bottom of each box shows the lower (first) quartile, and the horizontal line inside the box represents the median.

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The following tutorials can help you learn more about creating and reading box and whisker plots:

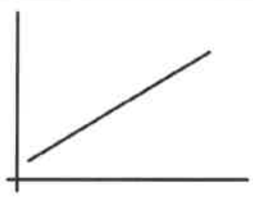
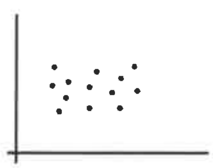
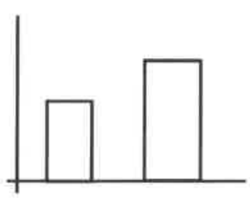
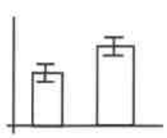
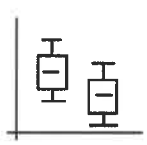
Khan Academy: Constructing a box plot

Khan Academy: Reading box plots

Summary: Types of Graphs

We have covered several different types of graphs in this chapter. Table 1.4 summarizes all of them and the instances in which you would use each one.

Table 1.4: Types of Graphs and When to Use Them

Type of Graph	Example	Examples of When to Use
Line Graph		▪ To track changes over time/ concentration, etc. ▪ To compare changes over the same time period for multiple groups/ treatments
Scatterplot		▪ To compare two variables that may or may not have a linear relationship
Histogram		▪ To show how values in a data set are distributed across evenly spaced (or equal) intervals ▪ To explore the relationship between two or more (in a three-dimensional plot) variables
Bar Graph		▪ To compare multiple groups/ treatments to each other
Box and Whisker Plots		▪ To show the variability in your sample

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For more detailed information on using Microsoft Excel or Google Sheets for graphing, watch the following tutorials:

Create a chart from start to finish

Making a graph in Google Spreadsheet and inserting it into a Google Doc

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CHAPTER 2

Data Analysis

When you complete a laboratory investigation, it is important to make sense of your data by summarizing it, describing the distributions, and clarifying "messy" data. Analyzing your data will allow you to do this.

Working with Data

Data analysis may involve calculations, such as dividing mass by volume to determine density or subtracting the mass of a container from the total mass to determine the mass of the contents. Using the correct rules for significant digits during these calculations is important to avoid misleading or incorrect results.

When adding or subtracting quantities, the result should have the same number of decimal places (digits to the right of the decimal) as the fewest number of decimal places in any of the numbers that you are adding or subtracting.

Table 2.1 presents examples and explains how the proper results should be written.

Table 2.1: Writing Your Results When Adding or Subtracting

Example	Explanation
3.7 cm + 4.6083 cm = 8.3 cm	The result is written with one decimal place because the number 3.7 has only one significant digit to the right of the decimal.
48.3506 m – 6.28 m = 42.10 m	The result is written with two decimal places because the number 6.28 has only two significant digits to the right of the decimal.
(8 km – 4.2 km) + 1.94 km = 6 km	The result is written with zero decimal places because the number 8 has zero significant digits to the right of the decimal.

Notice that the result of adding and subtracting has the correct number of significant digits if you consider significant digits to the right of the decimal.

When multiplying and dividing a set of numbers, look for the number with the fewest significant digits. Your result should have that number of significant digits. Table 2.2 explains how to apply this concept.

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Table 2.2: Writing Your Results When Multiplying or Dividing

Example	Explanation
5.246 in. × 2.30 in. = 12.1 in.	The result is written with three significant digits because 2.30 has three significant digits.
0.038 cm ÷ 5.273 cm = 0.0072 cm	The result is written with two significant digits because 0.038 has two significant digits.
76.34 m × 2.8 m = 2.1×10^2 m	The result is written with two significant digits because 2.8 has two significant digits. [Note that scientific notation had to be used because writing the result as 210 would have an unclear number of significant digits.]

When calculations involve a combination of operations, you must retain one or two extra digits at each step to avoid any round-off errors; at the end of the calculation, you must round to the correct number of significant digits.

An exception to these rules occurs when a calculation involves count data, such as the number of times a ball bounces, or the number of waves that pass a point during a time interval. As shown in the following example, do not consider exact numbers when determining significant digits in a calculation.

Example

While performing the Millikan oil-drop experiment, you find that a drop of oil has an excess of three electrons. What is the total charge of the drop?

$$\text{Charge} = (\text{number of electrons})(\text{charge per electron})$$
$$q = ne$$
$$= (3\ \text{electrons})(1.6\times10^{-19}\ \text{C/electron})$$
$$= 4.8\times10^{-19}\ \text{C}$$

When determining the number of significant digits in the answer we ignore the number of electrons because it is an exact number.

Scientific Notation

When manipulating data, there will be many times when the numbers that you calculate will be either too large or too small to be conveniently expressed as decimals. To make it easier to work with these very large or very small numbers, scientists use scientific notation. In scientific notation, a number is written as a coefficient multiplied by the base 10 raised to some exponent. Let's look at Avogadro's number to better understand the components:

$$\underbrace{6.022}_{\text{coefficient}} \times \underbrace{10}_{\text{base}}{}^{\overset{\text{exponent}}{23}}$$

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The coefficient must be between 1 and 10, and the exponent must be an integer. Very large numbers will have a positive exponent, while very small numbers will have a negative exponent; for example:

$$10000 = 1 \times 10^4$$

$$1000 = 1 \times 10^3$$

$$100 = 1 \times 10^2$$

$$10 = 1 \times 10^1$$

$$1 = 10^0$$

$$1/10 = 0.1 = 1 \times 10^{-1}$$

$$1/100 = 0.01 = 1 \times 10^{-2}$$

$$1/1000 = 0.001 = 1 \times 10^{-3}$$

$$1/10000 = 0.0001 = 1 \times 10^{-4}$$

So, a number such as 0.00000000000757 would be written in scientific notation as 7.57×10^{-12}, while a number like 218,000,000 would be written as 2.18×10^8.

Another way of thinking about this is to use the following representation for the place values:

You can rewrite a number in scientific notation by simply using this representation to count how many decimal places to move the decimal point. If the number you are converting is greater than 10, then the decimal point is moved to the left on the line, while if it is less than 1, the decimal point is moved to the right. For instance, if you wanted to convert 0.000436 into scientific notation, you would start at the base unit—the 1—then count the decimal places you would have to move until the coefficient is between 1 and 10, as shown below.

This tells us that we need to move the decimal point four places to the right:

So, 0.000436 would be written in scientific notation as 4.36×10^{-4}.

For more detailed information on using scientific notation, watch the following tutorial:

Khan Academy: Introduction to scientific notation

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Calculations Using Percentages

Percent Change

When working with data, sometimes we need to compare unequal quantities or scales; in order to do this we normalize the data. One way to do this is to compare the percent change over time. We use the following formula to calculate percent change:

$$\% \; change = \frac{final\,value - initial\,value}{initial\,value} \times 100$$

For example, in the AP Biology Diffusion and Osmosis lab investigation, dialysis bags are first filled with a sucrose solution and then placed in water for 30 minutes. We measure the mass of each bag before and after it sits in the water for 30 minutes, and report this as a percent change in mass. If the mass of a dialysis bag at the beginning of the experiment was 12.2 g and at the end of the experiment it was 16.7 g, the percent change is

$$\frac{16.7-12.2}{12.2} \times 100 = 36.9\%$$

Percent change can also be negative. What if in the previous example the mass at the beginning of the experiment was 16.7 g and the mass at the end of the experiment was 12.2 g? Let's look at this new calculation:

$$\frac{12.2-16.7}{16.7} \times 100 = -26.9\%$$

In the first calculation the positive result indicates that the dialysis bags gained mass. However, in the second calculation the negative result indicates that the dialysis bag lost mass.

Percent Difference

There are times when you may need to calculate the percent difference between two experimental results to see how they compare to each other. To calculate percent difference, we use the following formula:

$$\% \; difference = \left| \frac{x_1 - x_2}{(x_1 + x_2)/2} \right| \times 100$$

where x_1 is the first data point and x_2 is the second data point. The numerator is the difference between the measurements, and the denominator is the average of the measurements. The two vertical lines on either side of the fraction indicate that we are using the absolute value of the calculation.

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Example

There are two cars traveling at different speeds: one at 25 mph and the other at 33 mph. We want to know the percent difference between the speeds of the two cars. The calculation would be

$$\begin{aligned}
\%\text{ difference} &= \left|\frac{x_1 - x_2}{(x_1 + x_2)/2}\right| \times 100 \\
&= \left|\frac{25-33}{(25+33)/2}\right| \times 100 \\
&= \left|\frac{-8}{29}\right| \times 100 \\
&= 0.2759 \times 100 = 27.6\%
\end{aligned}$$

This means that there is a 28% difference between the speeds of the two cars.

Percent Error

Percent error is a calculation that is done when you want to compare your results to a known or predicted theoretical value. We use the following formula to calculate percent error:

$$\%\,error = \frac{|experimental\ value - theoretical\ value|}{theoretical\ value} \times 100$$

Notice that we are using the absolute value of the difference between the experimental value and the theoretical value.

Example

Calculate the percent error of a titration of 3.0% hydrogen peroxide (H_2O_2) with potassium permanganate ($KMnO_4$), as in AP Chemistry Investigation 8: Oxidation-Reduction Titration. If we performed this investigation and calculated the concentration of H_2O_2 in our sample to be 2.74%, our calculation would be:

$$\begin{aligned}
\%\,error &= \frac{|2.74 - 3.0|}{3.0} \times 100 \\
&= \frac{0.26}{3.0} \times 100 \\
&= 8.67\%
\end{aligned}$$

This means that our titration yielded data that was in error by 8.67% relative to what was expected.

Rate Calculations

You may occasionally have to determine a rate of change when you are processing data from an experiment. Examples include a rate of reaction, growth rate, speed, and acceleration. Each of these describes how a quantity changes over time. The change over time can be expressed as

$$\frac{\Delta Y}{\Delta t} = \frac{\text{the change in the dependent variable}}{\text{the change in time}}$$

where ΔY represents the change on the y-axis and Δt represents the change on the x-axis (time).

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Example

Suppose you were doing an AP Physics lab and wanted to calculate the magnitude of the average velocity (speed) of an object. You would do this by calculating the displacement traveled during a particular period of time. So, if you pushed a toy car across the floor and it traveled in a straight line from 1.0 meter to 4.0 meters in 8 seconds, you would calculate the speed as follows:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{(4.0-1.0)\text{meters}}{(8-0)\text{seconds}} = \frac{3.0\,\text{meters}}{8\,\text{seconds}} = 0.375\,\text{meters/sec}$$

Example

Suppose you are doing an AP Chemistry lab and needed to calculate the rate of the decompostion of NO_2 from 60 to 120 seconds:

$$NO_2(g) \rightarrow 2NO(g) + O_2(g)$$

Time (seconds)	$[NO_2]$	$[NO]$	$[O_2]$
0	0.0150	0	0
60	0.0085	0.0027	0.0018
120	0.0071	0.0041	0.0024

$$Rate = \frac{\Delta A}{\Delta t} = \frac{\Delta[NO_2]}{\Delta t}$$

$$Rate = \frac{0.0071M - 0.0085M}{120\,\text{sec} - 60\,\text{sec}}$$

$$Rate = \frac{-0.0014M}{60\,\text{sec}} = -2.33\times10^{-5}\,M/\text{sec}$$

Note that the negative sign indicates that the NO_2 is being consumed in the reaction. We could also calculate the rate using the one of the products:

$$Rate = \frac{\Delta A}{\Delta t} = \frac{\Delta[NO]}{\Delta t}$$

$$Rate = \frac{0.0041M - 0.0027M}{120\,\text{sec} - 60\,\text{sec}}$$

$$Rate = \frac{0.0014M}{60\,\text{sec}} = 2.33\times10^{-5}\,M/\text{sec}$$

Notice that the rate of product formation is the same as the rate of consumption of the reactant. The rate is positive in magnitude because product is being formed.

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The following tutorials can help you review how to do rate calculations:

Khan Academy: Intro to rates

Khan Academy: Introduction to average rate of change

Linear Relationships and Curve Fitting

Graphing Data as a Straight Line

When you plot data on *x*-*y* axes, a straight line is the simplest relationship that data might have. Graphing data points as a straight line is useful because you can easily see where data points belong on the line.

You can represent data as a straight line on a graph as long as you can identify its slope (m) and its y-intercept (b) in a linear equation: $y = mx + b$. The slope is a measure of how y varies with changes in x, $m = \Delta y / \Delta x$. The y-intercept is where the line crosses the y-axis (where $x = 0$).

Linearizing Data

Even if the data you measure do not have an apparent linear relationship, you may be able to represent the data as a straight line by revising the form of the variables in your graph. One method is to transform the equation to represent the relationship so that it has the linear form of $y = mx + b$ by substitution. For powers of x, the data would be in the form $y = Ax^c + b$. To linearize this data, substitute x^c for the x in the linear equation. Then you can plot $y\ vs.\ x^c$ as a linear graph. For example, graphing kinetic energy, *KE*, and velocity, v, for the function $KE = \frac{1}{2}mv^2$, yields a parabola, as shown in figure 2.1a. But if we set the horizontal axis variable equal to v^2 instead, the graph is linear, as shown in figure 2.1b, and the slope is equal to 1/2m [Note that "1/2" should be a built-up fraction, with m setting next to it.].

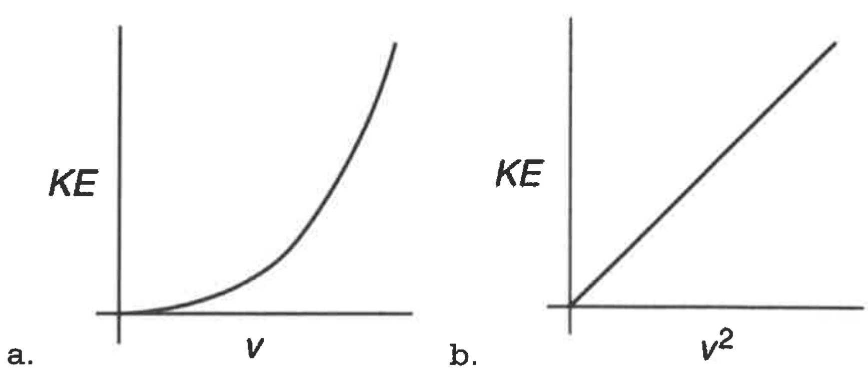

Figure 2.1: Changing the Variable on the *x*-axis to Produce a Linear Graph

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If the data is exponential, as in $y = Ae^{bx}$, or is a power of x, as in, $y = ax^n$, taking the log of both sides of the equation will linearize them. For exponential data, the equation you obtain is $ln(y) = ln(A) + bx$. The data will approximate a line with y-intercept ln(A) and slope b.

Similarly, for an equation with a power of x, taking the log of both sides of $y = ax^n$ results in $log(y) = log(a) + nlog(x)$. If you plot log($y$) versus log($x$), the data will approximate a line with y-intercept log(a) and slope n, as shown in figures 2.2a and 2.2b.

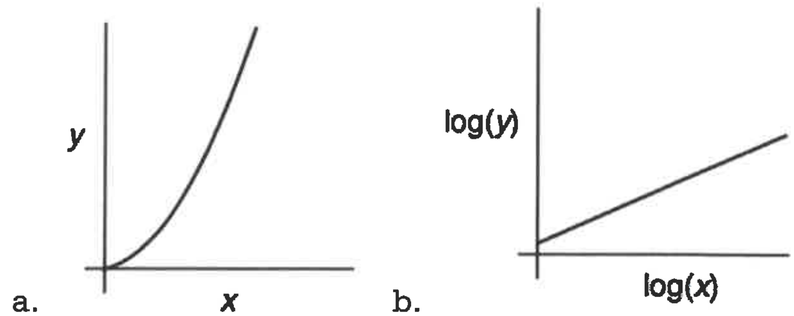

Figure 2.2: Linear Graphs of Equations with a Power of x

Curve Fitting

A useful way to analyze data is to determine whether it corresponds to a certain mathematical model. A mathematical relationship or function will allow you to make a prediction if you know the function and an initial condition. The first step is to plot the points and see if they follow a recognizable trend, such as a linear, quadratic, or exponential function. Figure 2.3 shows examples of each of these types.

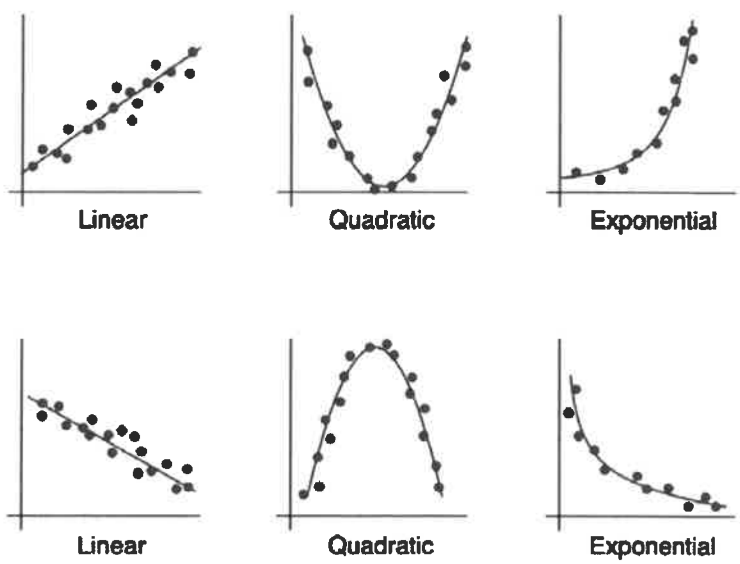

Figure 2.3: Common Mathematical Models

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The general equation of a **linear function** is $y = mx + b$, as noted above, in which m is slope and b is the y-intercept. For example, a linear function in physics is the time dependence of the velocity of an object undergoing constant acceleration, $v = v_0 + at$, where the acceleration, a, is the slope and the initial velocity, v_0, is the y-intercept. An example of a linear function in biology is the amount of oxygen consumption by an endotherm over time at a constant temperature. In chemistry, an example of a linear function is the relationship between the concentration of a solution and the amount of light that is transmitted through the solution.

The general equation of a **quadratic function** is $y = ax^2 + bx + c$, where a, b, and c are constants. An example of a quadratic function in physics is the potential energy of a spring, $U = \frac{1}{2}kx^2$, where x is the distance the spring is stretched from equilibrium, k is the spring constant, and in this case the constants b and c are zero. Another example of a quadratic function is the position as a function of time for a constantly accelerating object, $x = \frac{1}{2}at^2 + v_0t + x_0$, where a is acceleration, v_0 is initial velocity, and x_0 is initial position.

The general equation of an **exponential function** is $y = Ae^{bx}$, where A and b are arbitrary constants. An example of the exponential function in physics is the number of radioactive particles left after a certain time of radioactive decay: $N = N_0e^{-\lambda t}$, where N_0 is the original number of particles, and λ is the decay rate. Population growth is an example of an exponential function in biology and environmental science (see the section on population growth later in this chapter).

If the pattern of the data is clearly linear, or if you can plot the data using linearization, you can use a straightedge to draw a **best-fit line** that has approximately the same number of data points above and below the line. You can then determine an equation of the line by identifying the slope and y-intercept of the best-fit line.

If a more exact equation is desired, or if the data do not clearly follow a linear pattern, you can use a graphing calculator or a computer to fit the data to a mathematical model. In this case, you input the data and choose the model that you think will best fit the data. This is called **regression analysis**. Regression analysis is a common curve-fitting procedure. An analysis using this procedure provides parameters for the equation you have chosen for the fit, as well as parameters that describe how well the data fit the model. Figure 2.4 shows the same data using a linear model and a quadratic model. The value r^2 is the **coefficient of determination**. It indicates how well the model fits the data. A value closer to 1 indicates a better fit. In the examples in figure 2.4, both models are a good fit for the data, but the r^2 values show that the quadratic model is better because 0.9826 is closer to 1 than 0.95492 is.

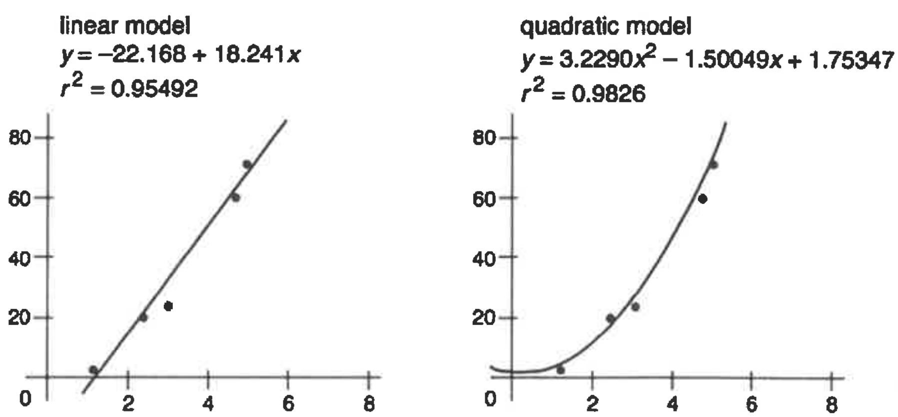

Figure 2.4: Different Models Have Different r^2 Values

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For more detailed information on linear functions, watch the following tutorial:

Khan Academy: Comparing linear functions word problem

Descriptive Statistics

Mean, Standard Deviation, and Standard Error

You can describe the uncertainty in data by calculating the mean and the standard deviation. The **mean** of a set of data is the sum of all the measurement values divided by the number of measurements. If your data is a sample of a population, then the mean you calculate is an estimate of the mean of a population. The mean, $\overline{X}$, is determined using the formula

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + x_3 + \dots}{n}$$

where x_1, x_2, etc. are the measurement values and n is the number of measurements.

Standard deviation is a measure of how spread out the data values are. If your measurements have similar values, then the standard deviation is small: each value is close to the mean. If your measurements have a wide range of values, then the standard deviation is high: some values may be close to the mean, but others are far from it. In general, if you make a large number of measurements, then the majority of them are within one standard deviation above or below the mean. (See the section on confidence intervals for a graph of the standard deviation ranges later in this chapter.)

Since standard deviations are a measure of uncertainty, they should be standard using only one significant digit. Standard deviation is commonly represented by the letter s. You calculate sample standard deviation using this formula:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

When you make multiple measurements of a quantity, the **standard error** (SE) of the data set is an estimate of the precision to which you know the mean of the quantity. The standard error is related to how spread out the data is, but it also includes the fact that when you measure a quantity many times and take the average of your measurements, you get a more precise value than if you only measure the quantity a few times. You calculate standard error using this formula:

$$\mathrm{SE} = \frac{s}{\sqrt{n}}$$

Because the number of measurements, n, is in the denominator, the more measurements you take, the smaller the standard error.

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Example

Suppose you measure the following values for the temperature of a substance:

Trial	1	2	3	4
Temperature (°C)	20.5	22.0	19.3	23.0

The mean of the data is

$$\bar{x} = \sum_{i=1}^{4} \frac{x_i}{4} = \frac{20.5+22.0+19.3+23.0}{4} = 21.2^{\circ}\text{C}$$

The standard deviation of the data is

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{4}(x_i - \bar{x})^2}{4-1}}$$

$$= \sqrt{\frac{(20.5-21.2)^2+(22.0-21.2)^2+(19.3-21.2)^2+(23.0-21.2)^2}{3}}$$

$$= 1.63 = 2 \text{ (rounded to one significant digit)}$$

The SE is

$$\text{SE} = \frac{s}{\sqrt{n}} = \frac{1.63}{\sqrt{4}} = 0.8 \text{ (rounded to one significant digit)}$$

Using one standard deviation, we would report the temperature of the substance as 21.2 ± 2°C, meaning the typical temperature is in a range that is 2° above or 2° below the mean temperature. Since we only have a few data values, a standard deviation of 2°C shows that most of the data values were close to the mean. However, if we had taken a large number of measurements the standard deviation would show that the majority (specifically, 68%; see the Confidence Intervals section later in this chapter) of the data values were between 19.2°C and 23.2°C. Alternatively, the data could be reported using the standard error as 21.2 ± 0.8°C. This tells us how our data compare to the true population mean with 95% confidence. In other words, because we took four measurements we have 95% confidence that the average temperature is within 0.8°C of 21.2°C. The standard error tells us how confident we are in our determination of the mean, while the standard deviation tells us how far we expect any individual measurement to be from the mean.

A graph of your data showing the statistics can clearly summarize the data in a way that is easy to understand and interpret.

Example

Suppose you conducted an investigation to determine if English ivy leaves in a shady area have a greater width than English ivy leaves in a sunny area. Table 2.3 shows the raw data from your experiment.

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Table 2.3: Leaf Measurement Data

Shady Leaves (in cm)	Sunny Leaves (in cm)
3.7	3.2
5.2	3.5
5.4	4.1
5.7	4.3
5.8	4.4
5.8	4.6
6.0	5.0
6.1	5.0
6.5	5.2
6.5	5.2
6.6	5.3
6.8	5.4
7.0	5.6
7.3	5.7
7.3	5.7
7.4	5.8
7.7	6.0
7.9	6.0
8.0	6.4
8.1	6.5
8.1	6.7
8.2	6.7
8.3	7.1
8.9	7.1
9.0	7.1
9.4	7.3
9.9	7.5

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Table 2.3: Leaf Measurement Data *(continued)*

Shady Leaves (in cm)	Sunny Leaves (in cm)
9.9	7.9
9.9	8.0
10.4	8.2

The statistics from each experimental group can be calculated and shown in a table such as table 2.4.

Table 2.4: Descriptive Statistics

	Shady Leaves	Sunny Leaves
Mean	7.43	5.88
Standard Deviation	1.63	1.32
N	30	30
Standard Error	0.30	0.24

Using this information, you can graph your data to visually compare the means of the two groups of leaves:

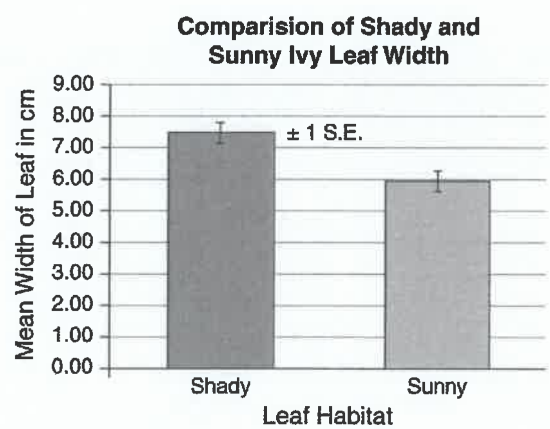

Figure 2.5: Comparison of Shady and Sunny Ivy Leaf Width

We see that the mean width of leaves gown in a shady environment is greater than the mean width of the leaves grown in a sunny environment. We also see that the error bars (± 1 SE) for the two means do not overlap. This supports our claim that the two populations are different, in other words, that English ivy leaves grown in a shady environment have a greater width than English ivy leaves grown in a sunny environment.

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Confidence Intervals

A **confidence interval** is a range of values that the true value of the population has a probability of being within. If you measure a single quantity such as the mass of a certain isotope multiple times, you would expect a small standard deviation compared to the mean: the confidence intervals would be narrow. A wide confidence interval in this case would indicate the possibility of random errors in your measurements.

Confidence intervals can be presented in different ways. Figure 2.6 illustrates a commonly used method.

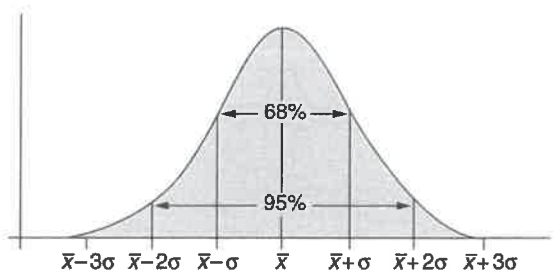

Figure 2.6: Confidence Intervals for a Normal Distribution

This method applies only to data that has a normal (bell-shaped) distribution. The mean lies at the peak of the distribution. Confidence intervals on either side of the peak describe multiples of the standard deviation from the mean. The percentage associated with each confidence interval (68%, 95%) has been determined by calculating the area under the curve.

A wide variety of data types in various subjects follow a **normal distribution** (i.e., a bell curve). In science, normal distributions apply to repeated measurements of a single value, such as multiple measurements of fluorescence decay time. A normal distribution is not appropriate when more than one central value is expected, or when only a few measurements are made.

If you need more information, the following tutorials can help to further explain these concepts:

Khan Academy: Measures of spread: range, variance & standard deviation

Khan Academy: Standard error of the mean

Khan Academy: Confidence interval 1

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Accuracy, Precision, and Experimental Error

Communication of data is an important aspect of every experiment. You should strive to analyze and present data that is as accurate as possible. Keep in mind that in the laboratory neither the measuring instrument nor the measuring procedure is ever perfect. Every experiment is subject to experimental error. Data reports should describe the experimental error for all measured values.

Experimental error affects the accuracy and precision of data.

- **Accuracy:** how close a measurement is to a known or accepted value. For example, suppose the mass of a sample is known to be 5.85 g. A measurement of 5.81 g would be more accurate than a measurement of 6.05 g because 5.81 g is closer to actual value of the measurement.
- **Precision:** how close several measurements are to each other. The closer measured values are to each other, the higher their precision.

Measurements can be precise even if they are not accurate. Consider again a sample with a known mass of 5.85 g. Suppose several students each measure the sample's mass, and all of the measurements are close to 8.5 g. The measurements are precise because they are close to each other, but none of the measurements are accurate because they are all far from the known mass of the sample.

Systematic errors are errors that occur every time you make a certain measurement.

- They result in measurements that can be inaccurate or incorrect by making measurements that are consistently either higher or lower than they would be if there were no systematic errors.
- Examples include errors due to the calibration of instruments and errors due to faulty procedures or assumptions; for example, using a balance that is not correctly calibrated.

Random errors are errors that cannot be predicted.

- This includes errors of judgment in reading a meter or a scale and errors due to fluctuating experimental conditions.
- If the random errors in an experiment are small, the experiment is said to be *precise.*
- For example, when having numerous groups of students making temperature measurements of a classroom at the same time, they will have random variations due to local variation and instrument fluctuation.

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CHAPTER 8

Quantitative Skills and Advanced Calculus Topics in AP Physics C: Mechanics

This chapter focuses on some of the quantitative skills that are important in your AP Physics C: Mechanics course. These are not all of the skills that you will learn, practice, and apply during the year, but these are the skills you will most likely encounter as part of your laboratory investigations or classroom experiences, and potentially on the AP Physics C Exam.

Vector Products

Dot Product

The **dot product** is also called the scalar product. It is the product that takes two vectors and performs an operation where the product's value is a scalar value. Here is the definition of the dot product:

$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

The angle theta is defined as the angle between the two vectors when the two vectors are drawn from a common origin.

Figure 8.1 shows the definition of the dot product:

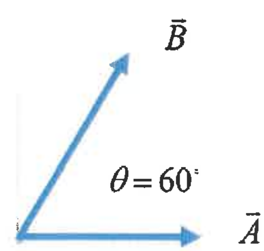

Figure 8.1: Two Vectors (A and B) Drawn in *x*-*y* Coordinate Frame

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Example

Defining the two vectors in figure 8.1:

$$|\vec{A}| = 10\,units$$
$$|\vec{B}| = 12\,units$$

and also defining the vectors in unit vector notation:

$$\vec{A} = 10\hat{i}$$
$$\vec{B} = 6\hat{i} + 10.4\hat{j}$$

Using the definition of the dot product stated above, the product of vector A and vector B is

$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}|\cos\theta = (10)(12)\cos 60^\circ = 60$$

This value of 60 is a scalar value (without direction!) and has the units' defined physical quantity.

There is a second way to compute the dot product. This process is defined by using the components of the vectors. Here is this definition:

$$\vec{A} \bullet \vec{B} = A_x \bullet B_x + A_y \bullet B_y + A_z \bullet B_z$$

Apply this rule to the example given, and we should get the same product result:

$$\vec{A} \bullet \vec{B} = (10)(6) + (0)(10.4) + (0)(0)$$
$$\vec{A} \bullet \vec{B} = 60$$

Cross Product

The cross product is sometimes called the vector product. In this vector operation two vectors are taken through a process to produce a third vector. Here is one of the definitions of the cross product:

$$\vec{C} = \vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta$$

The operation looks very similar to the dot product. The angle definition is also the same definition as the angle definition defined in the dot product. However, there is an important difference besides the obvious (the sine function contribution verses the cosine contribution). This difference is that there is a direction to the cross product. That direction is determined by the right-hand screw rule. The vector C is defined to be in the direction that is perpendicular to the plane created by the two vectors A and B. When rotating A into B with the right hand, the thumb will point in the direction of the vector created by the cross product. A good visual example of this is to take the cross product of the unit vector $\hat{i}$ with the unit vector $\hat{j}$. So imagine rotating the x-axis into the y-axis with the right hand, and the thumb is in the z-direction. Therefore, the cross product vector is the unit vector $\hat{k}$. This is because the z-axis direction is perpendicular to the plane created by the x-y axes. This is essentially what creates our right-handed coordinate systems.

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Example

Compute the cross product of the previously defined vectors (A and B).

$$\vec{C}=\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\sin\theta=(10)(12)\sin 60=104$$

The vector C would have a magnitude of 104 units and a direction in the $+z$ direction. The direction is determined by the right-hand rule, so the direction that is perpendicular to the plane contained by vectors in the x-y plane is the $+z$-direction.

If you need more information, the following tutorials can help to further explain these concepts:

Khan Academy: Vector dot product and vector length

Calculus for AP Physics C

Derivatives

The derivative allows you to calculate the slope of a function or the rate of change in one quantity with respect to another quantity. For example, the slope of a position versus time graph would give us the velocity, so by taking the derivative of a position function, we can determine the velocity function.

Power Rule

The power rule allows you to determine the derivative of a function of the form $x(t)=t^n$ where n is a rational number. The power rule states:

$$x(t)=t^n$$
$$\frac{dx}{dt}=n\cdot t^{n-1}$$

If the position of a ball falling from rest from a height y_0 is equal to $y=y_0-\frac{g}{2}t^2$, the derivative of this function will give the velocity function.

$$\frac{dy}{dt}=v_y(t)=0-2\cdot\frac{g}{2}t^{(2-1)}=gt$$
$$v_y(t)=gt$$

Example

Determine an expression for the velocity as a function of time for a ball with a position function equal to $x(t)=x_0+v_0t+\frac{1}{2}at^2$.

$$\frac{dx}{dt}=v_x(t)=0+v_0+\left(2\cdot\frac{1}{2}at^1\right)$$
$$v_x(t)=v_0+at$$

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The following tutorial can help you learn more about the power rule:

Khan Academy: Power rule

Chain Rule

For composite functions, the chain rule can help to find the derivative. For a function, $y=f(g(x))$, where $y=f(h)$ and $h=g(x)$, the derivative of y with respect to x is

$$\frac{dy}{dx}=\frac{dy}{dh}\frac{dh}{dx}$$ This could also be written as: $$\frac{dy}{dx}\left[f(g(x))\right]=f'(g(x))\cdot g'(x)$$

A quick and easy way of remembering the chain rule is to think about the function $y=f(g(x))$ as having an "inside function," $h=g(x)$, and an "outside function," $y=f(h)$. The derivative is then the derivative of the outside function (with the inside function left alone), times the derivative of the inside function.

Example

Find the derivative of the function, $y(x)=(5x-8)^{\frac{1}{2}}$.

In this case, the "outside function" is the ½ power (or square root) and the inside function is $5x-8$. So the derivative $\frac{dy}{dx}$ will be equal to:

$$\frac{dy}{dx}=\frac{1}{2}(5x-8)^{\left(\frac{1}{2}-1\right)}\cdot(5)$$

$$\frac{dy}{dx}=\frac{5}{2}(5x-8)^{\frac{-1}{2}}$$

The following tutorial can help you learn more about the chain rule:

Khan Academy: Chain rule

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Antiderivatives and Definite Integrals

Evaluating a definite integral allows you to determine the area of the region between the graph of a function and the x-axis. That area gives you information about the net change in the quantity whose derivative is graphed.

Given a function, $f(x)$, the value of the definite integral would equal the difference between the values of the *anti*derivative at each of the two endpoints. For example, if $f(x)=x^n$ then

$$\int_{x_1}^{x_2} f(x)dx=\int_{x_1}^{x_2} x^n dx=\left.\frac{x^{(n+1)}}{(n+1)}\right|_{x_1}^{x_2}=\frac{x_2^{(n+1)}}{(n+1)}-\frac{x_1^{(n+1)}}{(n+1)}$$

Note that the antiderivative for x^n inverts the operations in the power rule for derivatives.

Given an expression for velocity as a function of time for a car, $v(t)$, the net change in position for the car over an interval of time can be calculated by evaluating a definite integral of this function from $t=0$ to any time, t: $\int_0^t v(t)dt$. Given an initial position, x_0, the position, x as a function of time will be given by $x(t)=x_0+\int_0^t v(t)dt$. For example, if $v(t)=v_0+at$:

$$x(t)=x_0+\int_{t_0=0}^{t} v(t)dt$$

$$x(t)=x_0+\int_{t_0=0}^{t} (v_0+at)dt$$

$$x(t)=x_0+\left.\left(v_0 t+\frac{at^2}{2}\right)\right|_{t_0=0}^{t}$$

$$x(t)=x_0+v_0 t+\frac{at^2}{2}, \text{ provided } t_0=0.$$

The following tutorial can help you learn more about anti-derivatives and integrals:

Khan Academy: Anti-derivatives and indefinite integrals

First Order Differential Equations

A first order linear differential equation is a function that includes both a variable and the first derivative of that variable. For example, the function $-x=4\frac{dx}{dt}$ is an example of a first order differential equation because it contains both x and the rate of change of x with respect to time $\frac{dx}{dt}$.

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Steps to solving a simple first order differential equation:

1. Rearrange the equation so like variables are together (separate variables).
2. Integrate both sides, being sure to account for initial conditions in limits of integration.
3. Solve for an expression for the variable whose derivative is represented in the differential equation (in this case x).

Example

A box, given an initial speed v_0, slows to rest under the influence of air resistance, which applies a force $F = -kv$, where k is a constant and v is the velocity of the box as a function of time. Derive an expression for the velocity of the box as a function of time.

Since the force from the air is the only force slowing the box, the force from the air will be equal to the net force on the box.

$$\underbrace{\sum F:}_{-kv}$$

$$-kv = ma$$

$$-kv = m\frac{dv}{dt}$$

Now we can see that this is a differential equation since it contains both v and dv/dt. First rearrange the equation with like terms together:

$$\frac{-k}{m}dt = \frac{dv}{v}$$

Note that both k and m are constants, so k/m is a constant.

Now integrate both sides and add limits to account for initial conditions:

$$\int_{t=0}^{t} \frac{-k}{m}dt = \int_{v_0}^{v} \frac{dv}{v}$$

$$\left.\frac{-kt}{m}\right|_{t=0}^{t} = \ln v\Big|_{v_0}^{v}$$

$$\frac{-kt}{m} = \ln v - \ln v_0 = \ln\left(\frac{v}{v_0}\right)$$

$$e^{\frac{-kt}{m}} = e^{\ln\left(\frac{v}{v_0}\right)}$$

$$e^{\frac{-kt}{m}} = \frac{v}{v_0}$$

Finally, solve for v:

$$v_0 e^{\frac{-kt}{m}} = v(t)$$

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The following tutorial can help you learn more about solving first order differential equations:

Khan Academy: Separable equations introduction

Definition of Work (Calculus)

The complete definition of **work** involves two advanced mathematical ideas: the dot product and the integral. Here is the precise definition of work:

$$W = \int_{A}^{B} \vec{F} \cdot d\vec{r}$$

This would be defined as the integral of the dot product of the force and the displacement vector as the object is moved from position A to position B; "dr" is used to define a displacement linearly in any direction (x, y, z, or radial direction or combination). If the force being used in the definition is a constant force, then the definition can be written more simply (AP Physics 1 and 2 style) as

$$W = |\vec{F}||\vec{r}|\cos\theta = F_{\parallel} \cdot d$$

where $F_{//}$ represents the parallel component of the force in the direction of the displaced distance *d*.

Example

Let's try a complete example using vectors and calculus. Assume a force has an expression of the following relationship:

$$F = 2x^2\hat{i}$$

and the object has a displaced from a position in the x-direction of $x = 0$ to $z = 5$.

The displacement vector is the position vector in the x-y plane or $d\vec{r} = dx\hat{i} + dy\hat{j}$. This is what the computation would look like:

$$W = \int_{x=0}^{x=5} (2x^2\hat{i}) \cdot (dx\hat{i} + dy\hat{j})$$

$$W = \int_{x=0}^{x=5} (2x^2\,dx) = \left[\frac{2}{3}x^3\right]_{x=0}^{x=5} = \frac{250}{3} = 83.3\,J$$

Moment of Inertia

Rotation is a major concept in the AP Physics C: Mechanics course. One of the fundamental physical quantities in this area of mechanics is the idea of **rotational inertia**. This idea is qualitatively defined as the measurement of inertia of an extended body (system) in response to a torque acting on the system; in other words, how easy or difficult it is to mechanically rotate an extended body about some given rotational axis. The formal name for this type of inertia is the moment of inertia and the symbol for this quantity is I. The moment of inertia is mathematically defined in the following way for a discrete mass system:

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$$I = \sum_i m_i r_i^2$$

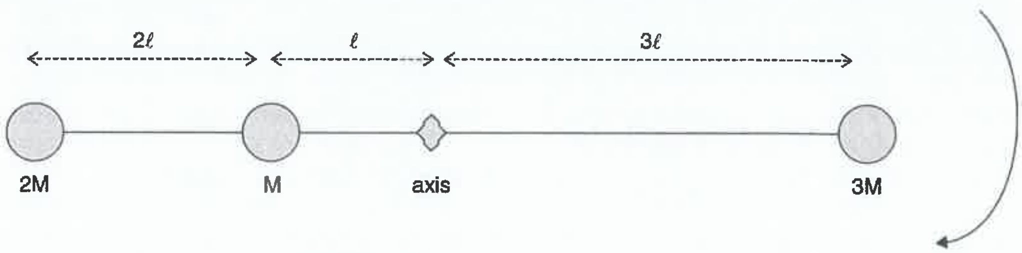
where I is inertia, m is the mass of the object, and r is the radius of rotation. The units for the moment of inertia measurement are $kg \cdot m^2$.

For a uniform shape or object (such as disks, spheres, rings, rods, etc.) there are known relationships for the moments of inertia. All of these relationships can be derived (although some of the derivations are beyond the scope of the AP Physics C course) from the calculus definition of the moment of inertia:

$$I = \int r^2\, dm$$

Example (noncalculus definition)

A very light (negligible mass) rigid rod is shown with three masses attached along the rod. Each mass has a value shown in the figure. The distances are also shown in terms of ℓ. The system of masses will be rotated about the pivot point (center of rod) shown in the plane of the paper. The figure shows the rotating mass system viewed from above and the system is rotating in the plane of the paper. Compute the moment of inertia of this system.



The $3M$ mass is located 3ℓ from the pivot, the M mass is located a distance of ℓ from the pivot, and the $2M$ mass is located a distance of 3ℓ from the pivot.

$$I = \sum m_i r_i^2$$

$$I = \left[\left(3M \cdot (3\ell)^2\right) + \left(M \cdot (\ell)^2\right) + \left(2M \cdot (3\ell)^2\right)\right]$$

$$I = 46M\ell^2$$

Example (calculus definition)

A rigid rod has a total mass of M and a length of L. The rod has a nonuniform linear mass density. The linear mass density is defined as

$$\lambda(x) = \lambda_o \frac{x}{L}$$

This density varies with the position of x along the rod, where $\lambda_o = \frac{2M}{L}$ is a constant measured in $\frac{kg}{m}$. This means at greater distances of the x-position the density value will increase in direct proportion to the x-value: essentially the rod is weighted more on the far end. This is more of an example to show the mathematical nature of the moment of inertia definition and is not intended to be a realistic physical rod. Here is a diagram of our rod and frame of reference:

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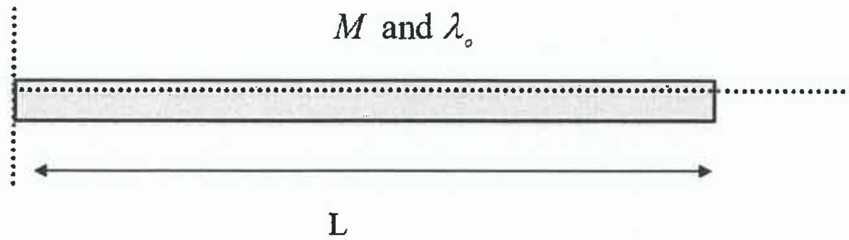
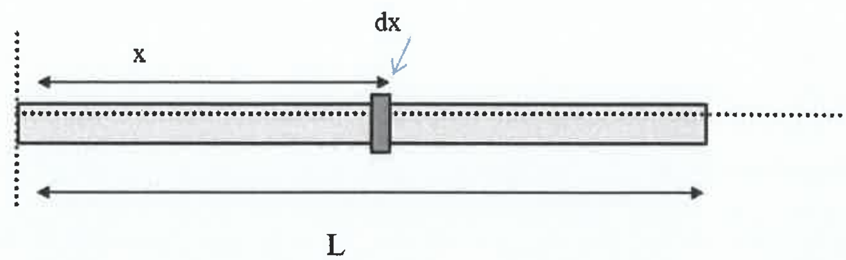


An infinitesimal slice mass along the rod needs to be defined as ***dm***. The ***dm*** slice has an infinitesimal length in the x-direction that is defined as length of ***dx***.



The calculus definition can now be applied to the rod. We will integrate the *dm* slices from a length of $x = 0$ to a length of $x = L$.

$$dm = \lambda(x)dx = \lambda_0 \frac{x}{L} dx$$

$$r = x$$

$$I = \int dm x^2 = \int_0^L \lambda_0 \frac{x}{L} dx \cdot x^2 = \int_0^L \lambda_0 \frac{x^3}{L} dx = \left[\lambda_0 \frac{x^4}{4L}\right]_0^L$$

$$I = \lambda_0 \frac{L^3}{4}$$

Let's give a value to λ_o of $\frac{2M}{L}$ to further simplify the above expression for the moment of inertia of this particular rod, where M is the mass of the rod and L is the length of the rod. This reduces the expression of moment of inertia to a more familiar expression:

$$I = \lambda_o \frac{L^3}{4} = \frac{2M}{L} \cdot \frac{L^3}{4} = \frac{1}{2} ML^2$$

If you need more information, the following tutorial can help to further explain these concepts:

Khan Academy: More on moment of inertia

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Total Mass Computation and Center of Mass Computation

Total Mass Computation

Using the rod example from the last section, we can use calculus to determine that the total mass of the rod is *M*. This is more for a verification exercise, but it certainly gives more practice with integrating expressions in physics. To verify that the total mass of the rod has a value of *M*, we simply integrate the *dm* slices from zero to L.

$$dm = \lambda(x)dx = \lambda_o \frac{x}{L}dx$$

$$M_{total} = \int_0^L dm = \int_0^L \lambda_o \frac{x}{L}dx$$

$$M_{total} = \frac{\lambda_o}{L}\int_0^L x\,dx = \frac{\lambda_o}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{\lambda_o}{L}\cdot\frac{L^2}{2}$$

$$M_{total} = \lambda_o \frac{L}{2} = \frac{2M}{L}\cdot\frac{L}{2} = M$$

Center of Mass Computation

Using this same rod and variable linear mass density, we can use the calculus definition of the center of mass (CM) to determine the center of mass of this rod. The definition for the center of mass is

$$x_{CM} = \frac{\int x\,dm}{M_T}$$

Example: center of mass using the calculus definition

We have already computed the total mass above of a long non uniform rod, (value is *M*) and we simply do a different integral that is defined in the numerator of the CM expression.

$$\int_0^L x\,dm = \int_0^L x\cdot\lambda_o\frac{x}{L}dx = \frac{\lambda_o}{L}\int_0^L x^2\,dx = \frac{\lambda_o}{L}\cdot\left[\frac{x^3}{3}\right]_0^L = \frac{\lambda_o L^2}{3}$$

Then we substitute the value for λ_o into the expression:

$$\int_0^L x\,dm = \frac{\lambda_o L^2}{3} = \frac{2M}{L}\cdot\frac{L^2}{3} = \frac{2}{3}ML$$

Lastly we divide this (numerator) by the total mass *M*, (the denominator in the CM expression) which gives:

$$X_{cm} = \frac{2}{3}L$$

So the CM of this nonuniformly massed rod is at a distance of two thirds along the rod and not the expected value of the CM being in the center of a uniformly weighted rod.

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Circular Motion and Rotation

Definition of Torque in AP Physics C

Torque is used and defined in AP Physics 1. However, the definition that is used in AP Physics C is the definition without any modification or simplification. Torque is defined as

$$\vec{\tau}=\vec{r}\times\vec{F}$$

This can also be expressed in the same way as in AP Physics 1:

$$\tau=\left|\vec{r}\right|\left|\vec{F}\right|\cdot\sin\vartheta$$

where the magnitude of $\left|\vec{r}\right|\sin\vartheta$ is defined as the moment arm.

Torque is a vector that comes from the vector cross product of two other vectors. The direction of torque is always a vector that is perpendicular to the plane that contains both of the vectors $\vec{r}$ and $\vec{F}$. If $\vec{r}$ and $\vec{F}$ are also perpendicular to each other, then all three $(\vec{r},\vec{F},\vec{\tau})$ vectors are mutually perpendicular to each other. Torque is measured in $N\cdot m$. The units are stated in this way to avoid confusion with the units of energy (joules).

If you need more information, the following tutorials can help to further explain these concepts:

Khan Academy: Introduction to torque

Definition of Angular Momentum of a Linearly Moving Point Object

Because of the nature of the interaction with systems of point masses and extended bodies, it is sometimes necessary in physics to define a linearly moving object as having angular momentum about some conveniently (but mathematically arbitrary) taken frame of reference. This is commonly phrased as the angular momentum of an object about some point O.

The fact that a linearly moving body has an angular momentum does sound physically weird, but it is a necessary definition in mechanics. It is totally possible to define a point at which a linear moving particle has an angular momentum value about that point. For example, this becomes necessary when a baseball is thrown at an open door and has a collision. The momentum of the ball is transferred into angular momentum of the door as it rotates about the hinges. In order for our conservation laws to hold in all cases, we need a way to account for an angular momentum of the linearly moving ball. The definition for the angular momentum of a linear moving particle is

$$\vec{L}=\vec{r}\times\vec{p}$$

where $\vec{r}$ is defined as the position vector. This vector is defined as being measured from the origin of the frame of reference defining the angular momentum to the position of the linearly moving object. The symbol $\vec{p}$ is the linear momentum of the particle. The nature of this cross product gives rise to what is referred to as a moment arm in the world of rotation. The moment

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arm is defined as the perpendicular distance between the line of the velocity vector and the line containing the axis of rotation. In other words, this distance (moment arm) is always the $|\vec{r}|\sin\theta$ value from the cross product. Here is what this would look like in an example:

Example: dart striking a wheel

Figure 8.2 shows a dart striking a wheel mounted on an axle free to rotate without any resistive forces. The dart clearly has linear velocity and momentum. The dart strikes the wheel and imbeds itself into the rubber wheel. The entire wheel/dart system begins to rotate about the central axle of the wheel with some constant angular velocity.

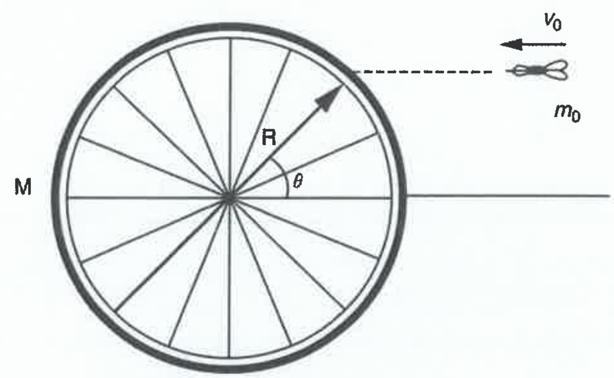

Figure 8.2: A Dart Striking a Wheel

In terms of M, m_0, v_0, R, θ and fundamental constants as appropriate, determine the angular velocity of the wheel after the collision with the dart.

To determine the angular velocity of the wheel, the conservation of angular momentum must be used. There are no external torques on the system of the wheel and the dart. The only torque on the wheel or dart is exerted as the dart collides with the wheel, which would be an internal torque. This means that the total angular momentum of the system before the collision must be equal to the total angular momentum after the collision. Now we see why we need our definition! Since the wheel is at rest prior to the collision, it initially has no angular momentum. The only angular momentum is defined as the dart's angular momentum about the center of the wheel. It is the wheel and the final angular momentum that defines our point of reference for the entire approach to the problem.

$$\vec{L}_{dart} = \vec{r} \times \vec{p} = (R)(m_o)v_o \sin\theta = mv_o R \sin\theta$$

After the collision the wheel and dart system is rotating and has an angular momentum equivalent to

$$L = I\omega$$

Note that the total moment of inertia of the system is the sum of the two inertias about the center of the wheel: the inertia of the wheel and the inertia of the dart stuck in the wheel a distance R away from the center of the wheel. Using the definition of moment of inertia of a ring (wheel) and the definition of a point mass moment of inertia, the total moment of inertia can be found.

$$I_{system} = I_{wheel} + I_{dart}$$
$$I_{system} = MR^2 + m_0 R^2$$

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This gives an angular momentum of the wheel after the collision as

$$L_{wheel/system} = I_{system}\omega = (M+m_0)R^2\omega$$

Now equate the angular momentum of the dart before the collision equal to the angular momentum of the wheel after the collision:

$$m_0 v_0 R \sin\theta = \left(M+m_0\right)R^2\omega$$

$$\omega = \frac{m_0}{M+m_0}\cdot\frac{v_0}{R}\sin\theta$$

This example shows why the definition of angular momentum for a linearly moving object is a necessary definition in mechanics.

If you need more information, the following tutorial can help to further explain this concept:

Khan Academy: Angular momentum

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How to Graph in Physics v6.0

Collecting Data in a Table

- The purpose of the data table is to provide a clear record of all data collected.
- Data tables should be uncluttered and easy to read.
- All straight lines drawn to organize the data must be drawn using a ruler.
- Data tables have rows and columns.
- Columns have headers. The headers contain the variable label and units. The units for the data recorded in the columns are included in parentheses. The appropriate abbreviation for the units is used.
- The independent variable is recorded in the first (left-hand) column.
- Add a useful title to help clarify things and to tell a story. Don't just repeat the column headers.

Graphing Physics Data

- The purpose of your graph is to tell the story of your experiment.
- Hand-drawn graphs should be done in pencil.
- Use a ruler to draw all straight lines.
- Use the grid on the graph paper. Do not try to "extend" the grid.
- Draw the axes using a ruler. Do not add arrow heads to the end of your axes. Allow space for labels between each axis and the edge of the paper. **If you have negative data values don't draw your axes until you determine your scales!**
- Use as much of the page as possible. See "Setting the Scale" below.
- Tie the scale to the lines of the grid not to the spaces between the lines.
- Include and label the origin (0, 0).
- Don't compress the scale, i.e., don't use that little squiggle.
- Label both axes. Include the units in parentheses. Use the standard abbreviation for the units. For example: Length (cm)
- Label both scales. Use integers, decimals, or scientific notation. No fractions. Don't label every grid line. Skip some to make your graph more readable.
- Plot data points with dots. (i.e., Scatter plot)
- Plot all of your data.
- Add a useful title to help clarify things and to tell a story. Don't just restate the axis labels in the title. In other words, don't title your graph "y vs x". The title may be the same as the title of your data table.
- Don't "connect the dots". Look for a pattern and add a "line of best fit" as appropriate.
- Strive to make your graph legible/readable. Avoid clutter and unnecessary adornments. (No "chart junk")
- Ensure that your graph tells the story of your experiment.

How to Graph in Physics v6.0

How to calculate the scale for your graph (assuming positive-valued data)

1. NOTE: Each axis scale is effecitvely a ruler for you data. Your goal is to create a scale that will make it easy for you to estimate where to plot the data points and easy for the person using your graph to read and understand your plot.
2. Start with your horizontal axis. Count the number of squares along the horizontal axis. Write this number down, somewhere.
3. Find the maximum value for the data you are plotting on the horizontal axis. We're assuming here that your minimum possible value is 0.
4. Divide the max value by the number of squares.
5. **Round up** to a "nice" number such as 1,2,5,10,25. You can use 4 in a pinch.
   E.g. 160 -->200 or 7.9 --> 10 or 3.3 --> 5 or 0.008 --> 0.01. Always round up. (Why?)
   "Nice" means "easy to interpolate". (Ask if you unsure what interpolate means.)
6. Label the first line, 0. This is the left hand side of the first square.
7. Don't label every line. Label every few to make your graph more readable.
   E.g. If your scale is 2 units per box, label the first line 0, then count five boxes and label the line 10. If your scale is 5 units per box, label the first line 0 then count two boxes and label the line 10, then two more and label the line 20, etc.
8. Repeat this process for the vertical axis.

How to draw a Line of Best Fit (LoBF)

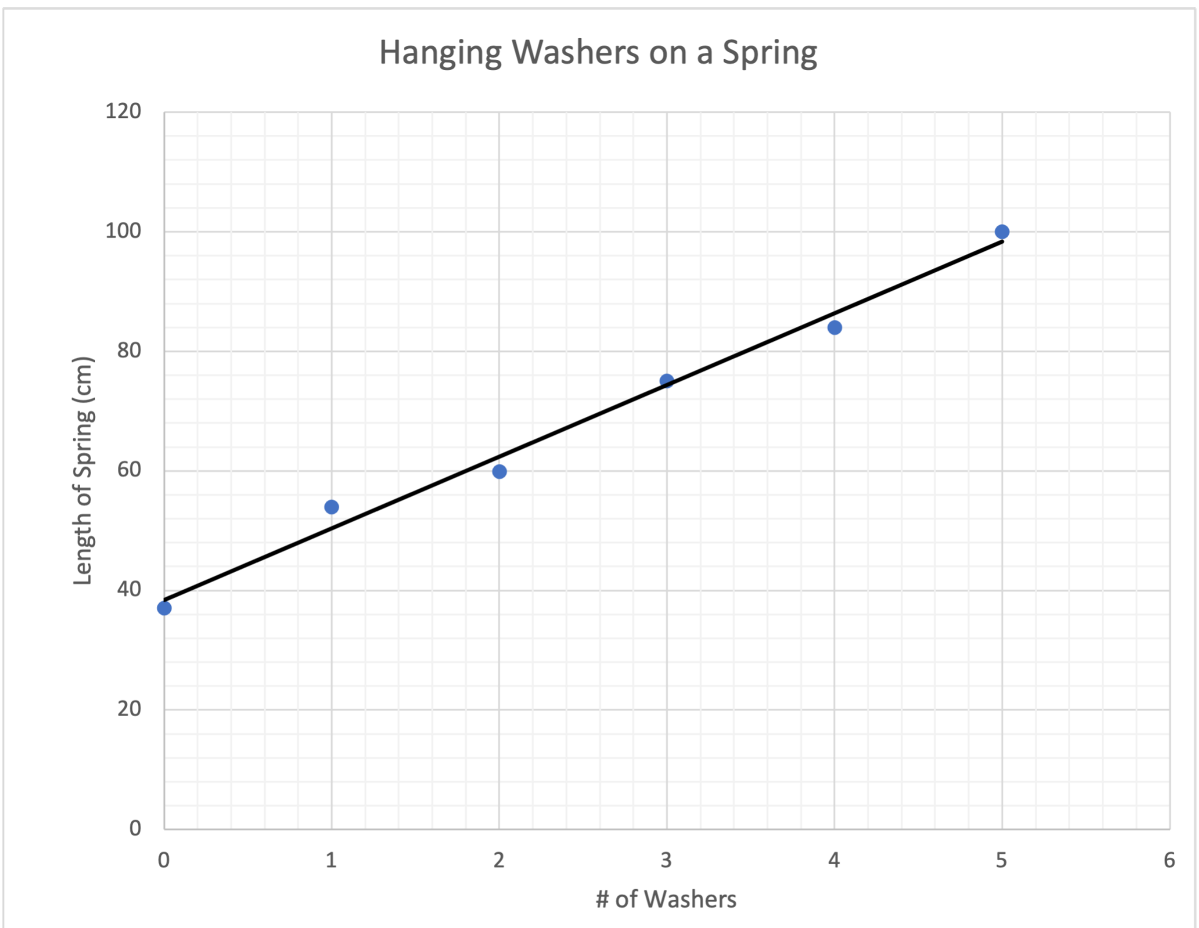
- A line of best fit is a straight line drawn through the data on a scatter plot balancing about an equal number of points above and below the line.
- The LoBF doesn't need to "go through" any of the data points.
- Use a clear plastic ruler to help draw your LoBF**.**
- The LoBF is used to represent or model the data.

How to Graph in Physics v6.0

Example Data Table and Graph

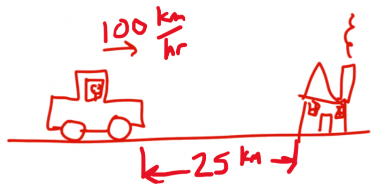
Hanging Washers on a Spring

# Washers	Length of Spring (cm)
0	37.1
1	54.0
2	60.0
3	75.0
4	84.0
5	100.0



How to Solve Some Problems in Physics

Example Problem #1: Jane Doe, a physicist at SPSG, decides to take a trip to her friend's house. Jane jumps in her Jeep and drives the 25 km trip averaging 100 $\frac{km}{hr}$. How many seconds did Jane's trip last?

Step	Directions	Example
1	Draw a picture of what is described in the problem.	
2	Write down the word "Find" followed by the physics variable name that describes what you're looking for. OPTIONAL: Add notes to help you.	Find: Δt [time in seconds]
3	Write down the word "Given" followed by the physics variable names and values of any information given in the problem. Include all units. Use accepted abbreviations for units. OPTIONAL: Add notes to help you. Draw a line under your last given.	Given: $\Delta\vec{x} = 25\ km$ [change in pos] $\vec{v}$ = 100 $\frac{km}{hr}$ [avg v] ____________________

How to Solve Some Problems in Physics

4	Write down the physics equation that you think is relevant to solving the problem.	$\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$
5	Rework the equation to solve for the variable you wish to find. Do this algebraically. Write the equation so that the variable you are solving for is on the left.	$\vec{v} \cdot \Delta t = \Delta\vec{x}$ $\Delta t = \frac{\Delta\vec{x}}{\vec{v}}$
6	Substitute the values you know into the equation. Include all units.	$\Delta t = \frac{(25\ km)}{100\ \frac{km}{hr}}$
7	Do the calculations. Report your answer as a decimal. Include all units.	$\Delta t = 0.25\ \frac{km}{\frac{km}{hr}}$ $\Delta t = 0.25\ hr$
8	Convert units *as needed* by multiplying by "1". Include all units	$t = 0.25\ hr \cdot \frac{60\ min}{1\ hr} \cdot \frac{60\ s}{1\ min}$ $t = 900\ s$
9	Draw a box around your final answer. The thing in the box should be what you were trying to find, with units.	$\boxed{t = 900\ s}$

How to Solve Some Problems in Physics

On the next page, check out what your answer to the example problem above would look like.

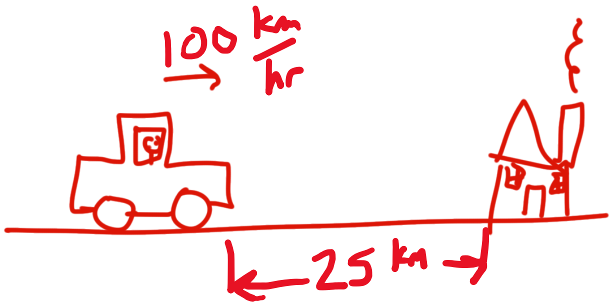
Notes

- Solutions should be written in a column.
- If you run out of room in the current column, then start a new column. Draw an arrow from the bottom of the first to the top of the new column.
- **Include comments about what you are doing to clarify the flow of the solution. I recommend using "#" to show the start of a comment. See example, below.**

How to Solve Some Problems in Physics

Example Problem #1: Jane Doe, a physicist at SPSG, decides to take a trip to her friend's house. Jane jumps in her Jeep and drives the 25 km trip averaging 100 $\frac{km}{hr}$. How many seconds did Jane's trip last?

Example Problem #1 Answer



Find: t [time in seconds]

Given:

$\Delta\vec{x} = \ 25\ km$ [change in pos]

$\vec{\vec{v}}$ = 100 $\frac{km}{hr}$ [avg v]

$$\vec{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t}$$

$$\vec{\vec{v}} \cdot \Delta t = \ \Delta\vec{x}$$

$$\Delta t = \frac{\Delta\vec{x}}{\vec{\vec{v}}}$$

$$\Delta t = \frac{(25\ km)}{100\ \frac{km}{hr}}$$

$$\Delta t = \ 0.25\ \frac{km}{\frac{km}{hr}}$$

$$\Delta t = \ 0.25\ hr$$

Need to convert hr to s

$$\Delta t = 0.25\ \cancel{hr}\ \cdot\ \frac{60\ \cancel{min}}{\cancel{hr}} \cdot \frac{60\ s}{\cancel{min}}$$

$$\boxed{\Delta t = 900\ s}$$

How to Solve Some Problems in Physics

Problem Solving Template: Fill in the blanks. The Step numbers are just to help you work through the procedure. These Step numbers have no special meaning, and you shouldn't write them when solving problems.

Step	Directions	Example
1	Draw a picture of what is described in the problem.	
2	Write down the word "Find" followed by the physics variable name that describes what you're looking for. OPTIONAL: Add notes to help you.	Find:
3	Write down the word "Given" followed by the physics variable names and values of any information given in the problem. Include all units. Use accepted abbreviations for units. OPTIONAL: Add notes to help you.	Given:

How to Solve Some Problems in Physics

4	Write down the physics equation that you think is relevant to solving the problem.	
5	Rework the equation to solve for the variable you wish to find. Do this algebraically. Write the equation so that the variable you are solving for is on the left.	
6	Substitute the values you know into the equation. Include all units.	
7	Do the calculations. Report your answer as a decimal. Include all units.	
8	Convert units *as needed* by multiplying by "1". Include all units.	
9	Draw a box around your final answer. The thing in the box should be what you were trying to find, with units.	

How to Solve Some Problems in Physics

Multiplying by 1

If we have measurements where we want to change units or if we have a calculation where we're combining units we have to be sure that we don't change the physical meaning of anything. The way we do this is multiplying by 1. You've learned in math that 1 is the muliplicative identity, in other words, anything multiplied by 1 remains the same. That's exactly what we're looking for.

Example 1

Convert 1.7 m to cm.

We know that $1\ m\ =\ 100\ cm$, which means, $\frac{1\ m}{100\ cm} = 1$ and $\frac{100cm}{1\ m} = 1$ (why?)

So to convert 1.7 m to cm we multiply 1.7 meters by 1, but we carefully choose how we represent 1.

In this case we need to end up with cm so we pick $\frac{100cm}{1\ m} = 1$

and then we get

$$1.7\ \cancel{m} \cdot \frac{100cm}{1\ \cancel{m}} = 170\ cm$$

Example 2

Calculate the number of minutes in a year.

We know, $1\ year\ =\ 365\ days$, $1\ day\ =\ 24\ hours$,
and $1\ hour\ =\ 60\ minutes$

So, we get

$$1\ \cancel{year} \cdot \frac{365\ \cancel{days}}{1\ \cancel{year}} \cdot \frac{24\ \cancel{hours}}{1\ \cancel{day}} \cdot \frac{60\ minutes}{1\ \cancel{hour}} = 525600 \text{ minutes}$$

(Sound familiar?)