UNIT 2 *Dynamics* | 2.A Relationship Between Force and Acceleration

NAME ____________ DATE ________

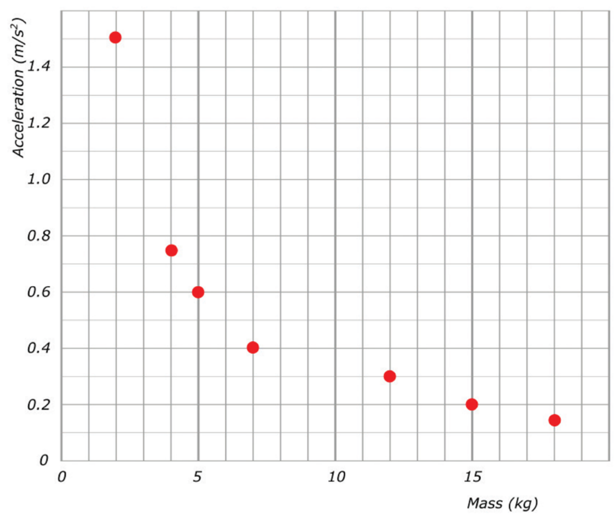
Scenario

Carlos and Dominique collect the following data from an experiment where they exerted the same force, F, to identical sized boxes with different masses and recorded the acceleration.

Trial	Mass	Acceleration
1	2 kg	1.5 m/s²
2	4 kg	0.75 m/s²
3	5 kg	0.60 m/s²
4	7 kg	0.40 m/s²
5	12 kg	0.30 m/s²
6	15 kg	0.20 m/s²
7	18 kg	0.15 m/s²

Using Representations

PART A: Plot the acceleration of the boxes versus the mass of each box.



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Relationship Between Force and Acceleration

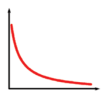
EK | 3.B.1 **SP** | 1.1, 5.1

Prepare

So far, students have seen linear and quadratic relationships. This may be the first time that they will see a relationship that is inversely proportional. If your students struggle with determining the functional relationship between variables, consider with them the equation for average speed $v_{avg} = \frac{d}{t}$, which can be rewritten $d = (v_{avg})(t)$. For a constant distance, what happens to the speed and the time? If you increase the speed, the time decreases. If you want to increase the time, you have to decrease the speed. Give them some data to graph, see that it is not linear, and decide what they could graph to make it linear (v_{avg} vs. $1/t$). The slope of that graph will be the constant distance traveled.

Average Speed (m/s)	Time to Travel Some Unknown Distance (seconds)
12	1
6	2
4	3
3	4
2.4	5
2	6
1.7	7
1.5	8

Data Analysis

Graph	Relationship
	As x increases, y increases proportionally. y is directly proportional to x.
	As x increases, y decreases. y is inversely proportional to x.
	y is proportional to the square of x.
	The square of y is proportional to x.

PART B: Based on the graph you created in Part A, identify the correct relationship between the acceleration and mass of an object. Fill in the blanks.

As mass *increases*, acceleration *decreases*. Therefore, acceleration is *inversely proportional* to mass.

PART C: Based on your analysis in Part B, what could be graphed instead of mass and acceleration that would lead to a linear relationship?

Acceleration vs. 1/mass or mass vs. 1/acceleration

PART D: What is the physical meaning of the slope of the linearized graph suggested in Part C?

Either net external force (if acceleration were graphed vs. 1/mass) or 1/force (if mass were graphed vs. 1/acceleration)

Teach

The idea of inverse relationships is powerful and later leads to both Ohm's law ($V = IR$) and $v = \lambda f$.

Now that your students know what they should graph to make the graph linear, have them create the graph. What is the magnitude of the force exerted on each box?

Can they think of a way to recreate this data themselves, so that they have a constant net force with varying accelerations based on mass? (One idea is to use a fan cart that will provide a constant force, and the students can add masses to the cart. If a motion detector is set up in front of the cart, it can collect velocity vs. time data and the slope of that line will be the acceleration of the cart.)

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

Data has been collected about the net external force on an object as well as that object's acceleration while the net external force is being exerted on it. What data should be graphed to create a linear graph? What would be the physical meaning of the slope of the graph?

What's the point?

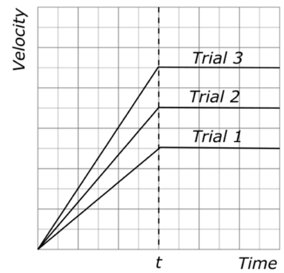
Functional relationships will be tested on the AP Physics 1 Exam. You need to be able to look at a graph and use the data presented as evidence for a claim of the relationship between the variables graphed.

UNIT 2 *Dynamics* | 2.B Force and Acceleration

NAME ______________________ DATE __________

Scenario

Angela is standing on a very low-friction skateboard while Blake pushes her away from the motion detector, which is set to record velocity as a function of time. In Trial 1, Blake pushes softly. In Trial 2, Blake pushes harder, and in Trial 3, Blake pushes Angela the hardest.



t = Blake stops pushing

Analyze Data

PART A: Use the data provided by the motion detector to find a pattern between the change in Angela's motion and the interaction with Blake. Fill in the blanks.

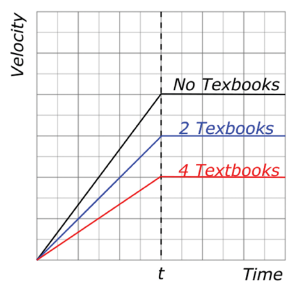
When Blake pushes harder, the slope of the velocity vs. time line increases.

Since the slope represents the acceleration, the acceleration is larger.

When Blake stops pushing, the slope of the velocity vs. time line becomes zero. Since the slope is now zero, this means that the acceleration is also zero.

Using Representations

PART B: The students then repeat the experiment, this time with Angela holding two textbooks and then four textbooks. Sketch a diagram of the velocity vs. time for Angela with two and then four textbooks. Blake pushes with the same force every time. The velocity of Angela without textbooks is already sketched.



t = Blake stops pushing

Argumentation

PART C: The following statement is written to describe what will happen after the first 5 seconds when Blake is no longer pushing. Cross out any incorrect statements and explain why they are incorrect. (Use the checklist to help you check your own writing! After you have written your answer, make sure that you can check off the statements in the list, or revise your answer!)

"After Blake stops pushing, Angela will travel at a constant speed ~~for a few seconds before she runs out of force. Then she will decelerate and stop~~."

Assuming that friction can be neglected, Angela will continue at a constant speed after Blake stops pushing. Her velocity will not change until another force is exerted on her like friction, air resistance, or a wall.

Checklist:

- ✓ I answered the question directly.
- ___ I stated a law of physics that is always true.
- ___ I connected the law or laws of physics to the specific circumstances of the situation.
- ___ I used physics vocabulary (force, mass, acceleration, velocity, constant, changing).

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Force and Acceleration

EK | 1.C.1 **SP** | 1.5, 5.1, 6.1

Prepare

This is a good demonstration for students to get a "feel" for how the speed changes under the influence of a constant force, and then what happens to the speed of the object once the force is removed. If you have access to low-friction carts and motion detectors, you could set up this experiment for students to try themselves. This worksheet can be their prediction sheet and then they can test their predictions in the lab.

In this page, we introduce students to a new tool. The "Checklist" will be provided as a scaffolding tool to help students check their own writing. By the end of the course, students should be able to ask themselves these questions without being prompted!

Teach

If you have low-friction skateboards, you can have students pull a box on the skateboard (to simulate a student) with a spring scale to see for themselves that if they want to pull with a constant force, their speed and the speed of the skateboard will increase.

Ask the students here, "What is the relationship between your speed and the speed of the skateboard?" (They should be equal.) "Why are they equal?" This will help them understand that systems that are "attached" must have the same speed at any clock reading. This may help with misconceptions later about the common speed and acceleration of systems (i.e., objects connected by strings).

Note that the sample graph provided in Part B suggests that the textbooks must be VERY massive. This is just to help students visualize that there is a relationship between net external force, mass, and acceleration. Students will explore the mathematical relationships in later scenarios.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

A box of mass m is pushed for 10 seconds with a force P across a horizontal floor with negligible friction. After 10 seconds, the person stops pushing. Sketch a velocity vs. time graph for the box. Sketch in a dotted vertical line at $t = 10$ seconds. What is different about the motion of the box before and after $t = 10$ seconds?

What's the point?

In the absence of a net force, an object in motion will continue in the same motion. All mass has a property called inertia that resists change to its motion.

UNIT 2 *Dynamics* | 2.C Force

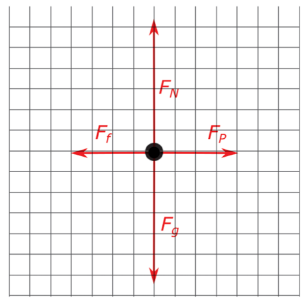
NAME ______________________ DATE __________

Scenario

Carlos pushes a block of mass, m, across a rough horizontal surface at a constant speed by applying a force, F, directly to the right.

Using Representations

PART A: The dot at right represents the block. Draw a free-body diagram showing and labeling all the forces (not components) exerted on the block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Quantitative Analysis

PART B: Blake is asked to use Newton's second law to derive an equation that relates the force of gravity and the normal force from the surface exerted on the block. Annotate his derivation by filling in the right side of the table below. For each line of the derivation, explain in words what was done mathematically. The first line is done for you as an example.

$\Sigma F_y = ma_y$	We start with Newton's second law, which says that the sum of all the forces exerted on an object is equal to the object's mass times the object's acceleration.
$F_N - F_g = ma_y$	In the vertical direction, there are two forces acting, the normal force upward and the gravitational force downward. In one dimension, direction is determined by a + or − sign. Taking up to be +, down is −.
$F_N - F_{mg} = m\cancel{a}^0$	Because the box is not accelerating in the vertical direction, the acceleration in this direction is zero.
$F_N - F_{mg} = 0$	Therefore, the normal force minus the gravitational force equals zero.
$F_N = F_{mg}$	Therefore, in this case, the magnitude of the normal force is equal to the magnitude of the gravitational force.

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Force

EK | 2.B.1, 3.A.2, 3.A.3, 3.A.4, 3.B.1, 3.B.2, 3.C.4, 4.A.2

SP | 1.1, 1.5, 2.1, 2.2, 6.1

Prepare

If you have not yet discussed how to break forces into components, you should do so before assigning this worksheet. If you start from the very beginning asking students to think about the direction of acceleration first before breaking forces into components, they will be better prepared for more difficult physical scenarios like boxes on inclines or conical pendulums.

Students should decide on the direction of the acceleration (or possible acceleration)—in this case horizontally—and then they can assign their axes to be parallel and perpendicular to that direction. (In this case, the axes should be horizontal and vertical.) Then they can analyze each force, and any force that is not parallel or perpendicular to the acceleration must be broken into components. (Part C, the force to be broken into components is F_{Pull}.)

Teach

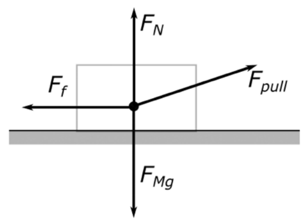
There is a *very* specific way that students will be expected to sketch a free-body diagram on the AP Physics 1 Exam. All vectors MUST start on, and point away from, the dot, and each force must be represented by its own uniquely labeled (or unambiguously labeled) vector. Unless stated otherwise, students should take special care to make sure that the lengths of the arrows represent the magnitudes of the forces and they DO NOT sketch components on the diagram. If at any point during the problem, they need a free-body diagram with components to help them analyze the physical scenario, they should feel free to sketch a second diagram somewhere else on the page that they may mark up as needed.

Make an Argument

PART C: Carlos gets tired of pushing and instead begins to pull with force F_{pull} at an angle to the horizontal. The block slides along the rough horizontal surface at a constant speed. A free-body diagram for the situation is shown below. Blake makes the following claim about the free-body diagram:

Blake: "The velocity of the block is constant, so the net force exerted on the block must be zero. Thus, the normal force F_N equals the weight F_{mg}, and the force of friction F_f equals the applied force F_{pull}."

What, if anything, is wrong with this statement? If something is wrong, identify it and explain how to correct it. If this statement is correct, explain why.

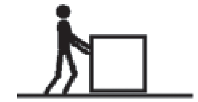


Blake is correct that the net force exerted on the block must be zero. However, he forgot that since the force of the pull is at an angle, there will be a vertical component of the pull. Therefore, it is the vertical component of the pull plus the normal force that will equal the gravitational force. So, the normal force will be less in magnitude than the gravitational force. In the horizontal direction, it is the horizontal component of the force of the pull that is equal to the friction force. Therefore, the magnitude of the force of the pull is greater than the friction force on the block.

Checklist:

- ✓ I answered the question directly.
- ____ I stated a law of physics that is always true.
- ____ I connected the law or laws of physics to the specific circumstances of the situation.
- ____ I used physics vocabulary (force, mass, acceleration, velocity, coefficient, friction).

Assess



To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

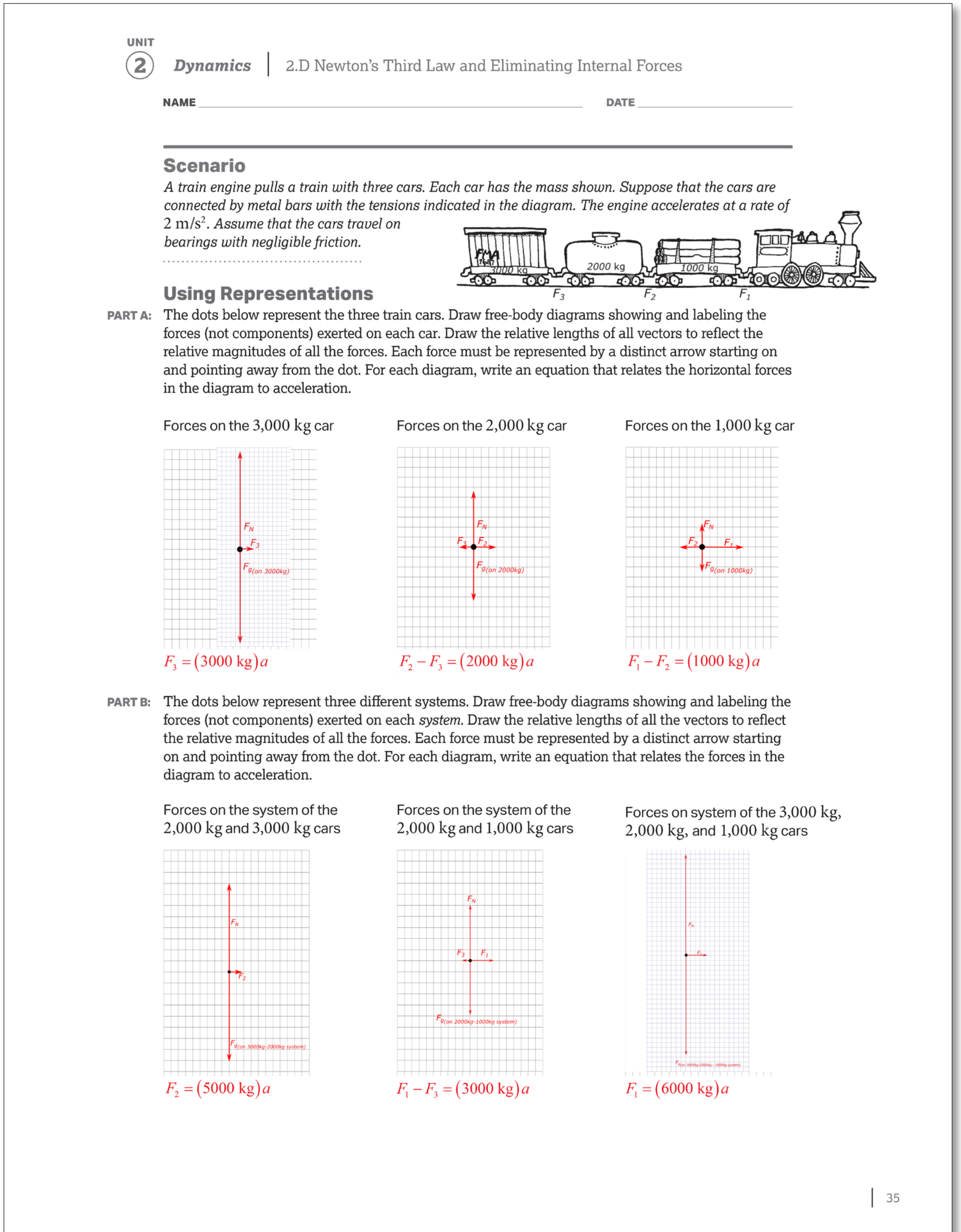
The diagram shows a block of mass m being slid at a constant speed across a horizontal concrete floor by a force parallel to the floor. Which pair of quantities could be used to determine the coefficient of kinetic friction for the block on the concrete?

A. Mass and speed of the block
B. Mass and normal force on the block
C. Friction force and speed of the block
D. Friction force and normal force on the block
E. Normal force and speed of the block

Explain how your choice of quantities could be used to determine the coefficient of kinetic friction.

What's the point?

Drawing a free-body diagram is not just busy work. A carefully sketched free-body diagram is a key to both understanding a physical scenario AND demonstrating that understanding.


UNIT 2 *Dynamics* | 2.D Newton's Third Law and Eliminating Internal Forces

NAME ______________________ DATE __________

Scenario

A train engine pulls a train with three cars. Each car has the mass shown. Suppose that the cars are connected by metal bars with the tensions indicated in the diagram. The engine accelerates at a rate of 2 m/s². Assume that the cars travel on bearings with negligible friction.

Using Representations

PART A: The dots below represent the three train cars. Draw free-body diagrams showing and labeling the forces (not components) exerted on each car. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot. For each diagram, write an equation that relates the horizontal forces in the diagram to acceleration.

Forces on the 3,000 kg car | Forces on the 2,000 kg car | Forces on the 1,000 kg car

$F_3 = (3000\text{ kg})a$ | $F_2 - F_3 = (2000\text{ kg})a$ | $F_1 - F_2 = (1000\text{ kg})a$

PART B: The dots below represent three different systems. Draw free-body diagrams showing and labeling the forces (not components) exerted on each *system*. Draw the relative lengths of all the vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot. For each diagram, write an equation that relates the forces in the diagram to acceleration.

Forces on the system of the 2,000 kg and 3,000 kg cars | Forces on the system of the 2,000 kg and 1,000 kg cars | Forces on system of the 3,000 kg, 2,000 kg, and 1,000 kg cars

$F_2 = (5000\text{ kg})a$ | $F_1 - F_3 = (3000\text{ kg})a$ | $F_1 = (6000\text{ kg})a$

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Newton's Third Law and Eliminating Internal Forces

EK | 1.A.1, 1.A.5, 2.B.1, 3.A.2, 3.A.4, 3.B.1, 3.B.2, 3.C.4, 4.A.2 **SP** | 1.1, 1.2, 1.5, 2.1, 2.2, 6.1

Prepare

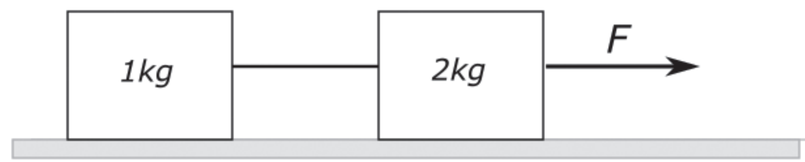
Teaching students to consider the system they are analyzing and to consciously document the system (by circling the objects) will help them be prepared to analyze systems by using energy and momentum, which also depends on the system being analyzed. If the first time they hear about the idea of a system is in Unit 4, it will be much more difficult for them to wrap their heads around the concept. A free-body diagram is used when we can approximate a system as an object (when every point on the system moves the same way) or when we are only interested in the motion of the center of mass of the system.

Teach

Have students sketch the system they are analyzing and draw a dotted box or circle around the object or objects that are part of the system. This visualization will help them to remember what interactions they can ignore as being internal to the system.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:



Quantitative Analysis

PART C: Use the equations you wrote above to find each of the three tensions: F_1, F_2, and F_3.

$F_1 = 12{,}000$ N $F_2 = 10{,}000$ N $F_3 = 6{,}000$ N

Argumentation

PART D: Without referencing any math or any numbers, explain why F_1 is the greatest tension and F_3 is the smallest tension, even though F_3 is connected to the greatest mass.

F_1 is pulling not only the 1,000-kg car, but all three. Therefore, it must accelerate the three-train-car system and so must be the largest force.

When the frictionless system shown above is accelerated by an applied force of magnitude F, if friction is negligible, the tension in the string between the blocks is:

A. F.

B. $\frac{2}{3}F$.

C. $\frac{1}{2}F$.

D. $\frac{1}{3}F$.

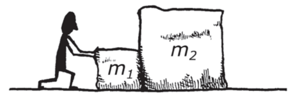
What's the point?

Newton's second law states that the SUM of all the forces exerted on an object equals the object's mass times its acceleration. It is not that ANY force can equal mass times acceleration or that EVERY force is equal to mass times acceleration, but it is the net or sum of all forces.

NAME ______________________ DATE ____________

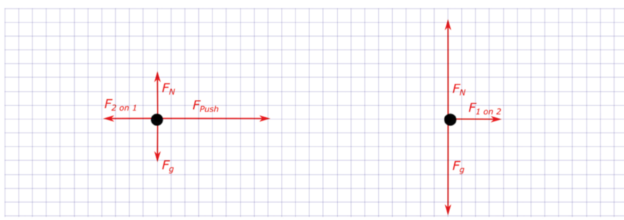
Scenario

Two blocks are being pushed across a surface with an external force F, as shown in the figure at the right. The mass m_2 of block 2 is greater than the mass m_1 of block 1. The blocks begin at rest. The surface is smooth enough that the frictional forces between the surface and the block can be neglected.



Using Representations

PART A: The dots below represent the two blocks. Draw free-body diagrams showing and labeling the forces (not components) exerted on each block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Quantitative Analysis

PART B: Derive the magnitude of the acceleration of block 2. Express your answers in terms of m_1, m_2, g, and F.

$\Sigma F_x = ma_x$	The sum of the external forces on the system will be equal to the mass of the system times the acceleration of the system.
$F_{push} = ma_x$	The net external force (in the horizontal direction) is F_{push}.
$F_{push} = (m_1 + m_2)a_x$	The mass of the system is the sum of the two masses.
$a_x = \frac{F_{push}}{(m_1 + m_2)}$	The acceleration of the system is then:
$a_2 = \frac{F_{push}}{(m_1 + m_2)}$	And since mass 2 will have the same acceleration as the system, the acceleration of mass 2 is:

Newton's Second and Third Laws

EK | 1.A.1, 1.A.5, 2.B.1, 3.A.2, 3.A.3, 3.A.4, 3.B.1, 3.B.2, 4.A.2 **SP** | 1.1, 1.5, 2.1, 2.2, 6.1

Prepare

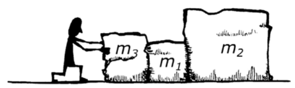
This is the first complicated derivation in the workbook. If your students struggle here, you can have them practice by assigning problems from the text where they are asked to solve for the acceleration (or tension, etc.). Replace numbers in the problems with variables and ask students to derive a symbolic solution in terms of given variables and physical constants as necessary. Have them annotate their derivations and work on using a clear sequence of thoughts that match from the "math" side to the "writing" side. (Note: We have tried to give the right number of spaces for the derivation if they do every step one at a time, but by the end of the book, this table will disappear—as it will not be given on the AP Physics 1 Exam.)

Teach

Before the students even try answering the questions—Is the system accelerating? How do you know? Sketch a dotted circle or box around the system being analyzed. What are the internal and external forces? Could the system have a net zero external force?

What is the relationship of the speed of Block 1 to the speed of Block 2? How do you know? What would it look like if the two blocks had different accelerations?

Block 3 of mass m_3 is added to the system as shown at right. The three boxes are pushed across the same surface with the same external force F.



Argumentation

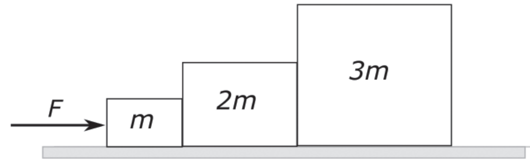
PART C: Indicate whether the magnitude of the acceleration of block 2 is now larger, smaller, or the same as in the original situation. Justify your answer.

_____ Larger __X__ Smaller _____ Same

The magnitude of the system's acceleration is now smaller since the system is more massive with the same net external (horizontal) force. The acceleration of the system is equal to the acceleration of Block 2 which is now $a_2 = \frac{F_{push}}{(m_1 + m_2 + m_3)}$

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:



Three blocks of mass m, $2m$ and $3m$ sit next to each other on a horizontal surface where friction between the blocks and the surface can be neglected. A constant force of magnitude F is applied to the right. Which of the following statements is true?

A. *Each block will have a different acceleration depending on its mass. The acceleration of each can be calculated by the equation $F = ma$, so $a = F/m$.*

B. *The acceleration of each block will be the same $a = F/m$.*

C. *The net force exerted on each block is identical and equal to F.*

D. *The magnitude of the force on block $3m$ from $2m$ is greater than the magnitude of the force back on $2m$ from $3m$.*

E. *The net force exerted on $3m$ is three times greater than the net force exerted on m.*

Explain why the answer you chose is correct and the others are incorrect.

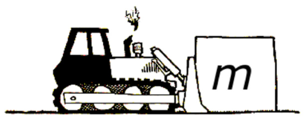
What's the point?

While you are free to choose your own system to analyze a given physical scenario, the choice of a system can greatly simplify (or greatly complicate) the analysis.

NAME ____________________ DATE ____________

Scenario

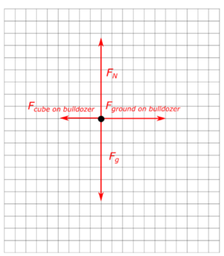
A bulldozer of mass M pushes a cube of cement of mass m across rough ground. The bulldozer and cube are speeding up.



Using Representations

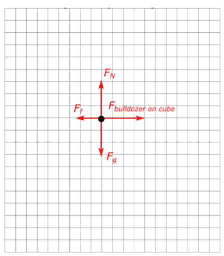
PART A: The dots below represent the bulldozer, cube, and bulldozer-cube system. Draw free-body diagrams showing and labeling the forces (not components) exerted on each system. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. For the bulldozer/cube system, draw an "external force" diagram.

Forces on Bulldozer



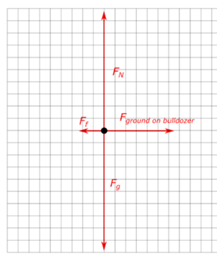
$F_{ground\ on\ bulldozer} - F_{cube\ on\ bulldozer} = Ma_x$

Forces on Cube



$F_{bulldozer\ on\ cube} - F_{friction} = ma_x$

External Forces on Bulldozer/Cube System



$F_{ground\ on\ bulldozer} - F_{friction} = (M+m)a_x$

Quantitative Analysis

PART B: In the blanks above, write an equation stating Newton's second law in the horizontal direction for the bulldozer, the cube, and the bulldozer-cube system.

PART C: Use the equation created for the external forces on the bulldozer-cube system to determine the acceleration of the bulldozer-cube system if the mass of the bulldozer is 1,000 kg, the mass of the rock is 500 kg, the force of friction on the bulldozer is 5,000 N, and the force of friction on the cube is 2,000 N.

$F_{ground\ on\ bulldozer} - F_{friction} = (M+m)a_x$

$5{,}000\text{ N} - 2{,}000\text{ N} = (1{,}000\text{ kg} + 500\text{ kg})a_x$

$3{,}000\text{ N} = (1{,}500\text{ kg})a_x$

$a_x = 2\ \frac{\text{m}}{\text{s}^2}$

Direction of Friction

EK | 1.A.1, 1.A.5, 2.B.1, 3.A.2, 3.A.3, 3.A.4, 3.B.1, 3.B.2, 3.C.4, 4.A.2

SP | 1.1, 1.3, 1.4, 2.1, 2.2

Prepare

The likelihood that your students will encounter a problem like this, where they are solving numerically for an answer on the free-response section of the AP Physics 1 Exam, is *extremely* low. However, it is possible that they will be asked to solve numerically for a solution on the multiple-choice exam. So, to that end, we have included a *few* problems in this workbook where a numerical solution is required from the students. Most of your efforts as a teacher should focus on helping your students to explain and justify results, conclusions, and ideas. While teaching your students to solve numerically for a solution may not be the focus, it can still happen—the numeric solution can't be the "end" of the analysis. Solutions should be annotated and followed up with questions about why something happens or what would happen if variables change.

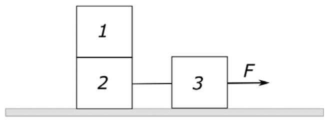
Teach

Students will likely have difficulty understanding why the force from the ground on the bulldozer is forward. Consider having a discussion with your students about how they can walk. What is the force that allows them to walk? If they tried to walk across ice which exerts negligible friction, what would happen? Which way would their foot slide? Friction prevents this motion and as you push backwards on Earth, Earth pushes forward on you. The same is true with the tread of the bulldozer.

The discussions you have about the direction of friction on the rolling treads of the bulldozer are a good preview of the ideas involved in rolling that students will see in Unit 7.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:



Three boxes of equal mass are being pulled across a smooth table top. Box 2 is connected to Box 3 by a light cord that is pulled along with a force F as shown. Block 1 is accelerated at the same rate as Block 2 because of the friction forces between the two blocks. Friction between the blocks and the table top can be neglected.

Sketch a free-body diagram of Block 1.

What's the point?

While the force of friction opposes the motion of two surfaces relative to each other, you have to think hard about what motion is being analyzed. In a single system, friction can be exerted in more than one direction.

UNIT 2 *Dynamics* | 2.G Acceleration in Two Dimensions

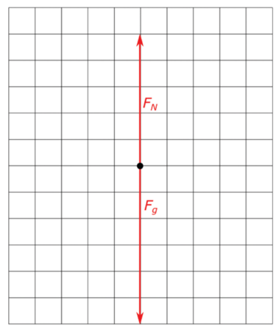
NAME ______________________ DATE __________

Scenario

A 300 kg *box rests on a platform attached to a forklift shown. Starting from rest at time* $t = 0$ *seconds, the box is lowered with a downward acceleration of* 1.5 m/s^2.

Using Representations

PART A: The dot below right represents the box. Draw a free-body diagram showing and labeling the forces (not components) exerted on the block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



PART B: In a brief sentence, support the magnitude of the normal force in comparison to the gravitational force on the box.

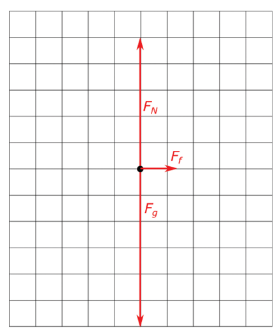
Because the box is accelerating downward, the net force must also be downward, meaning that the magnitude of the normal force must be less than the gravitational force: $F_N < F_g$.

Argumentation

PART C: Blake derives an equation for the height of the box as a function of time, makes a mistake, and comes up with $y = 9.8t$. Without deriving the correct equation, how can you tell that this equation is not plausible— in other words, why does it not make physical sense? Briefly explain your reasoning.

The box is accelerating, so it will have a changing speed, which means that position cannot depend linearly on time. Also the inclusion of 9.8 is dubious since the acceleration here is not due to gravity.

PART D: At time $t = 0$ seconds, the forklift also begins to move forward with an acceleration of 2 m/s^2 while lowering the box as described above. The box does not slip or tip over while the forklift is accelerating forward. The dot at right represents the box. Draw a free-body diagram showing and labeling the forces (not components) exerted on the block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Acceleration in Two Dimensions

EK | 2.B.1, 3.A.2, 3.B.1, 3.B.2, 3.C.4, 4.A.2 **SP** | 1.1, 1.5, 2.1, 2.2, 5.1, 6.1

Prepare

If your students have not yet stood in an elevator on a bathroom scale, that demonstration might be helpful to perform before this activity. Why does the scale read more than your "usual" weight when you first accelerate upward from rest? Why does the scale read less than your "usual" weight when you accelerate downward slowing to a stop at the top floor? Discuss the differences in weight and apparent weight with your students and which one the scale reads.

Teach

Part F can be tricky for some students. The question has given the answer and students simply need to collect evidence to support the given statement. However, some students will misunderstand the prompt and think that they are supposed to collect evidence to determine the *correctness of the given statement*. Both kinds of questions are asked on the AP Physics 1 Exam, and students need to be aware of whether they are asked to determine the correctness of a statement or support a statement that has already been determined to be true.

For an extension, you could have students complete the entire FR question from the AP Physics C 1996 #2, Part G. Derive an equation for the path of the box that expresses y (the height of the box) as a function of x (the horizontal position of the box) and not of t, assuming that at time $t = 0$, the box has a horizontal position $x = 0$ and a vertical position $y = 2$ meters above the ground with zero velocity.

PART E: Explain in a brief sentence why the force of friction points in the direction you sketched in Part D.

Because if the box was at rest, and the platform exerted no friction, the box would appear to "slide backward" as the platform moved forward. Because there is friction, that friction force keeps the box stationary relative to the platform, while the platform moves forward. Therefore the friction force is forward.

Checklist:
- ✓ I answered the question directly.
- I stated a law of physics that is always true.
- I connected the law or laws of physics to the specific circumstances of the situation.
- I used physics vocabulary (force, mass, acceleration, velocity, constant, changing).

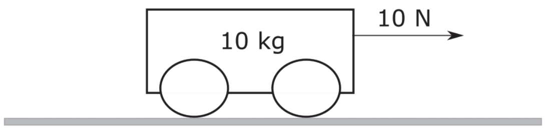
Quantitative Analysis

PART F: When the box is only being accelerated forward, a_{max} has one value. (a_{max} is the maximum acceleration the forklift can have before the box begins to slide.) When the box is both accelerating forward and down, a_{max} is less. Explain in a clear, coherent paragraph-length response why this is true.

Because the friction force depends on the normal force, when the forklift is only accelerating forward, the maximum acceleration that it can have without the box sliding is related to the coefficient of friction and the normal force (which in this case is equal to the weight of the box). When the box is also being accelerated downward, the normal force is less, so there is a smaller maximum static friction force, and hence, a smaller maximum acceleration the forklift can supply without causing the box to slip.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:



Multiple Correct

The cart of mass 10 kilograms shown above moves along a smooth surface on a horizontal table. A 10-newton force pulls on the cart horizontally to the right. Which of the following describes a manner in which this cart could be moving? Select two answers.

A. *Moving left and speeding up*

B. *Moving left and slowing down*

C. *Moving right and speeding up*

D. *Moving right and slowing down*

What's the point?

Although we don't often deal with objects accelerating in two directions, they certainly can. The forces are then modeled using Newton's laws both in the horizontal and vertical direction.

Forces on Inclined Planes

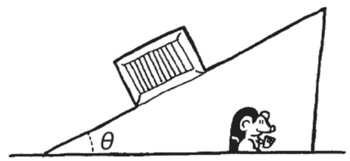
EK | 2.B.1, 3.A.2, 3.B.1, 3.B.2, 3.C.4, 4.A.2 **SP** | 1.1, 1.5, 2.1, 2.2, 5.1, 6.1

UNIT 2 *Dynamics* | 2.H Forces on Inclined Planes

NAME ______________________ DATE __________

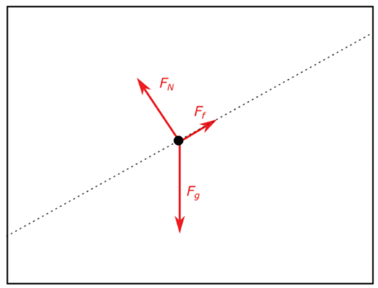
Scenario

Angela and Carlos are asked to determine the relationship between the normal force on a box of mass m and the angle of incline of the box θ as the box sits at rest on the incline.



Using Representations

PART A: The dot at right represents the block on the incline. Draw a free-body diagram showing and labeling the forces (not components) exerted on the block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot. The dotted line represents the incline.



Quantitative Analysis

PART B: Start with Newton's second law to derive an equation that relates the normal force with the angle of incline. For each line of the derivation, explain in words what you did mathematically. The first line is done for you as an example. Express your answer in terms of m, θ, and physical constants as appropriate.

$\Sigma F_y = ma_y$	Newton's second law states that the sum of the forces in the "y" direction will be equal to the mass of the box times the acceleration of the box in the y-direction, therefore:
$F_N - mg\cos\theta = 0$	Since the box would accelerate down the incline (if it were to accelerate) we choose our axis to be down the incline and perpendicular to the incline, so we will find the components of the gravitational force (since it is the only force not parallel or perpendicular to the incline). Then in the "vertical" (perpendicular to the incline) direction, the two forces are the normal force and one of the components of the gravitational force, and since the box is not accelerating in this direction, these two forces sum to zero.
$F_N = mg\cos\theta$	Therefore, the normal force is equal to one component of the gravitational force.

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Prepare

Remember to have students determine the direction of acceleration first when they encounter a problem where the forces are at angles. In this case, the box is being held at rest by the friction force and will not accelerate. However, if we made this a ramp with negligible friction, which way would the box accelerate? Down the ramp! So, make the "down the ramp" direction the "x" direction and make perpendicular to the ramp the "y" direction. Which forces are then neither parallel nor perpendicular to the ramp? The gravitational force—so students should find the components of the gravitational force.

Teach

This scenario can easily be turned into an experiment that the students can execute in class. Have them write up the procedure, either individually or in groups. Once they have collected the data, have them analyze it according to what they wrote in Part C. This is also a great experiment for a discussion of errors. Students should know the equation for, and be able to calculate, both percent error and percent difference.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

Derive an equation for the friction force necessary to hold the block on the incline as a function of the angle of the ramp. Does the derived quantity make physical sense? Check $\theta = 0$ and $\theta = 90$ degrees. What would the value of the friction force be in each of these extremes? Does that make sense?

(For extra linearization practice, have them linearize and determine the coefficient of static friction from their graph.)

What's the point?

Practicing the skill of linearization is important and can be done quickly every time the students do a derivation. "What would we graph, if we had data, to make this graph linear?"

Analyze Data

Angela and Carlos then perform an experiment to test the equation they derived in Part B. The following data are collected.

Normal Force (N)	**Angle** (degrees)
97	10
95	15
85	30
80	35
75	40
63	50
49	60

PART C: Based on the equation you created in Part B, what data should be plotted to create a linearized graph for this experiment?

Students should graph the normal force vs. the cosine of the incline angle.

PART D: What is the physical meaning of the slope?

The physical meaning of the slope will be the weight of the box.

Stopping Distance

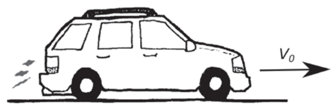
EK | 2.B.1, 3.A.2, 3.B.1, 3.B.2, 3.C.4, 4.A.1, 4.A.2 **SP** | 1.1, 1.5, 2.1, 2.2, 5.1, 6.1

UNIT 2 *Dynamics* | 2.I Stopping Distance

NAME ______________________ DATE __________

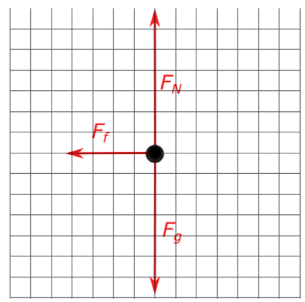
Scenario

Consider a car of mass m moving with initial speed v_0 on a straight, flat road. At time $t = 0$, the driver fully applies the brakes to avoid colliding with debris in the road in front of the car. The car's wheels lock, causing the car to slide on the roadway until the car stops, before running over the debris. The distance that the car slides is D. The coefficient of kinetic friction between the car's tires and the roadway is a constant value μ_k.



Using Representations

PART A: The dot at right represents the car. Draw a free-body diagram showing and labeling the forces (not components) exerted on the car, while the car slides to a stop. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot. Note that the car is moving to the right.



Argumentation

PART B: The stopping distance D depends on the value of v_0 and μ_k.

i. Does the value of D increase or decrease with increasing initial speed v_0? Give a physical explanation why this is the relationship.

__X__ Increase ____ Decrease ____ Remains the same

The maximum acceleration depends on the coefficient of friction and does not change with changing speed. So, since the maximum acceleration is constant, if the initial value of the velocity increases, it will take a longer distance to come to a stop with the same acceleration: $v_f^2 = v_0^2 + 2a(\Delta x)$.

ii. Does the value of D increase or decrease with increasing coefficient of friction μ_k? Give a physical explanation why this is the case.

____ Increase __X__ Decrease ____ Remains the same

The maximum acceleration depends directly on the coefficient of friction, if the coefficient of friction increases, the maximum acceleration will also increase. If the acceleration increases, with the same initial velocity, the distance to come to a stop will decrease.

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Prepare

The translation between the written argument in Part B and the Quantitative Analysis in Part C is a critical skill for success on the AP Physics 1 exam. Again, as with other skills, if your students struggle with this, you can assign classic textbook questions to them for practice with slight modifications. First, remove the numbers given in favor of symbols and ask them to predict the result when one of the variables changes. Then have them derive a symbolic solution (with annotations) and show how their derived expression supports their prediction made from principles of physics. For example, consider a classic Atwood machine (a pulley of negligible mass over which two objects are suspended on a string of negligible mass) if the suspended objects have masses m and M, where $M>m$, what happens to the acceleration of the system as M is reduced? Have them predict, using principles of physics to support their answer. They may reference equations but should not derive anything. Then have them derive an expression for the magnitude of the acceleration of the system and use that expression to justify their claim.

Teach

This can be a difficult concept for students to understand if they don't have personal experience riding in cars. While nothing can replace experience, there are ways to simulate this experience in the classroom, from having the students sit on skateboards and drag their feet to riding wheeled toys or bicycles, if available.

Students should sketch two velocity vs. time graphs both with the same initial velocity: one with a gentle stop and one with an emergency stop (so that the graphs have small or large accelerations). Have students compare the times to stop as well as the displacement before coming to rest (the areas under the curves). What would the graphs look like if we factor in reaction time?

PART C: In the spaces below, derive two equations, one in the "y" direction and one in the "x" direction, expressing Newton's second law using the symbols m, g, a, μ and physical constants as appropriate. For each line of the derivation, explain mathematically what was done (i.e., annotate your derivation). The first line is done for you as an example.

$\Sigma F_y = ma_y$	Newton's second law states that the sum of the forces in the "y" direction will be equal to the mass of the car times the acceleration of the car in the "y" direction, therefore:
$F_N - F_g = ma_y$	Since the normal force and the force of gravity are both purely in the y direction, a sign is enough to specify their directions. Thus, the normal force minus the force of gravity equals ma.
$F_N - F_g = 0$	But since the car is not accelerating in the vertical direction, $a_y = 0$.
$F_N = F_g$	Therefore, in this case, the normal force has the same magnitude as the gravitational force.
$\Sigma F_x = ma_x$	Newton's second law states that the sum of the forces in the x direction will be equal to the mass of the car times the acceleration of the car in the "x" direction, therefore:
$F_f = ma_x$	The friction force is the only horizontal force, so the friction force is equal to the mass of the car times the car's acceleration.
$\mu F_N = ma_x$	The friction force is equal to the normal force times the coefficient of kinetic friction.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

What data should be collected if the students wanted to experimentally determine the coefficient of kinetic friction between the tires and the road? What would the students graph to determine the coefficient of kinetic friction?

What's the point?

Only forces belong on free-body diagrams. While it is often helpful to mark the direction of the initial velocity on the sketch of the physical situation, they should never appear on a free-body diagram.

$\mu mg = ma_x$	The normal force has the same magnitude as the force of gravity.
$\mu g = a_x$	Therefore, the acceleration of the car is equal to the coefficient of friction times the acceleration due to gravity.

Quantitative Analysis

PART D: Use your equations from Part C along with an appropriate kinematic equation to do the following:

i. Write an equation for D in terms of v_0, g, and μ.

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}(\Delta x)$$

$$0 = \vec{v}_0^2 + 2(-\mu g)(D - 0)$$

$$D = \frac{v_0^2}{2\mu g}$$

ii. Explain how your equation in Part D (i) supports your reasoning about the relationships among D, v_0, and μ outlined in Part B.

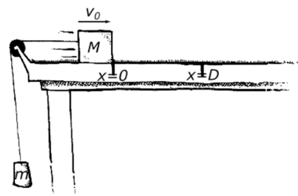
μ is on the bottom of the equation, so when μ increases, D decreases as stated in Part B. Also, v_0^2 is on the top of the equation, so when v_0^2 increases, so does D, as stated in Part B.

UNIT 2 *Dynamics* | 2.J Modified Atwood Machines

NAME ______________________ DATE __________

Scenario

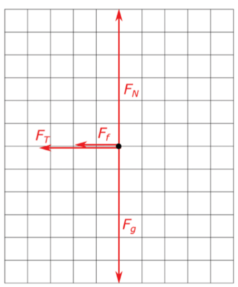
In the diagram shown to the right a block of mass M has taken a quick hit from a bat. After the strike, its front end is at position $x = 0$ at time $t = 0$ and it is moving to the right with initial speed v_0. The block slides on a rough surface and is also connected to a hanging mass object of mass m by a string that passes over an ideal pulley. The front end of the block reaches position $x = D$ at time $t = t_1$, the instant that the block comes to rest. The block then returns to position $x = 0$ at time $t = t_2$, having a leftward speed v_2 at that time.



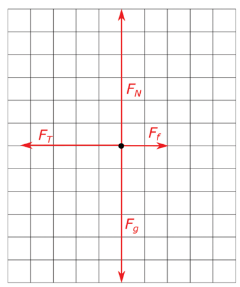
Using Representations

PART A: The dots below represent the block on the table during the interval $0 < t < t_1$ and $t_1 < t < t_2$. Draw free-body diagrams showing and labeling the forces (not components) exerted on the block during each of those intervals. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force should be a single arrow that originates on the dot.

Forces during $0 < t < t_1$



Forces during $t_1 < t < t_2$



Analyze Data

PART B: Is the magnitude of the block's acceleration greater before the block reaches $x = D$ or after? Explain your reasoning in terms of the forces that you drew in the above diagrams.

The acceleration of the block is greater before the block reaches $x = D$ since between time $t = 0$ and $t = t_1$, both the friction force and the force of tension are pointing to the left. After $t = t_1$, the friction force points to the right, while the force of tension points to the left, making the net force (horizontally) less and therefore making the acceleration less.

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Modified Atwood Machines

EK | 1.A.1, 2.B.1, 3.A.2, 3.A.3, 3.A.4, 3.B.1, 3.B.2, 3.C.4, 4.A.1, 4.A.2 **SP** | 1.1, 1.5, 6.1, 7.1

Prepare

An ideal pulley means that the pulley's mass is negligible and any friction in the pulley may also be ignored. Later in the course (Unit 7: Torque and Rotation), we will introduce the idea of pulleys that have mass and analyze what that means for the system, but for now, all pulleys will be ideal. You can challenge your students to think about what would happen to the acceleration of the system if the pulley is real and not ideal.

Teach

Sketching a graph onto a blank grid can be difficult for students. Rather than expecting them (or the students expecting themselves) to sketch the correct shape on the first try, have them focus on plotting the points that they KNOW to be true. For example, on this graph, v_0 is positive, and at t_1, the velocity is zero, etc.

For Part B, whenever students are asked to compare two scenarios, they must talk about the physical quantities of each situation and compare the similarities and differences.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

Is the force of tension in the string the same in both cases? Is t_1 equal to t_2?

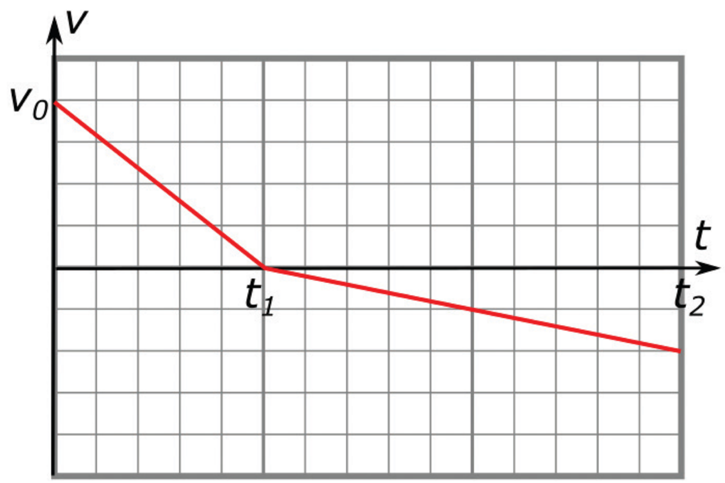
Quick Quiz

A massive chain (as opposed to a string of negligible mass) is hung over the edge of a table, where $\frac{1}{10}$ the length of the chain is over the edge and $\frac{9}{10}$ of the length rests on the table. If friction between the chains and the table is negligible, will the chain stay at rest or start to slide off the edge of the table? Support your answer. Will the acceleration of the chain be constant or changing? If the acceleration will change, will it increase or decrease? Support your answer by referencing the net force on the chain vs. the mass of the system.

What's the point?

Remembering that friction can change direction depending on the direction of motion is important. Equally important is remembering that the direction of forces does not depend on the direction of motion.

PART C: On the grid below, sketch a graph of the block's velocity as a function of time, taking right to be positive. Label the values v_0, t_1, and t_2 on the axes. Make sure that your graph is sketched to show that the block travels the same distance forward and backward.



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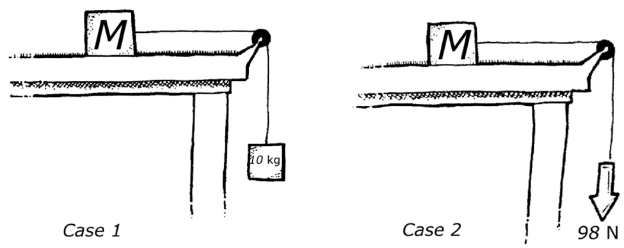


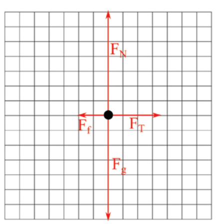
Scenario

In both cases shown above, a block of mass M is set on a rough table. The block is connected to a string that passes over an ideal pulley. In Case 1, the free end of the string is connected to a hanging object of mass $m = 10$ kg. In Case 2, the hanging object is removed and a person grabs the free end of the string and pulls with a constant force equal to 98 N, the weight of the hanging object in Case 1. In both cases, the block is released from rest the same distance from the right edge of the table.

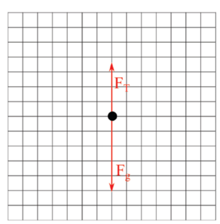
Using Representations

PART A: The dots below represent each object in Case 1. Draw the forces acting on those objects after the system is released. Use the grids to draw longer arrows to represent stronger forces. Assume that $m < M$. Recall that the system is accelerating.

Case 1: Block M



Case 1: Hanging object m



Acceleration of Systems

EK | 1.A.1, 2.B.1, 3.A.2, 3.A.3, 3.A.4, 3.B.1, 3.B.2, 3.C.4, 4.A.1

SP | 1.1, 1.5, 6.1

Prepare

Students often find this concept difficult to grasp. You can demonstrate this in class or have them replicate it themselves with low-friction carts and motion sensors to experience it for themselves.

Teach

Depending on your students, they may find that Part B (i) or (ii) is easier and more intuitive. It is important that students be pushed to find explanations beyond their comfort zone and be able to support claims with multiple lines of evidence.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

Ask students to sketch acceleration vs. time and velocity vs. time graphs for each situation.

Argumentation

PART B: Angela and Dominique are observing this demonstration and note that the block accelerates in both cases. However, the block reaches the right edge of the table in less time in Case 2 even though the force on the string in this case is the same as the weight of the hanging object in Case 1.

i. This occurs because there is a different amount of tension in the two cases. Explain why the block reaches the end in less time in Case 2 in terms of the different tension force in each case.

In Case 2, the acceleration is larger, so it reaches the edge faster than in Case 1. This happens because the force of tension, in Case 2, is now 98 N, whereas in Case 1, the tension is less than 98 N. Analyzing the hanging object in Case 1, we can see that if the tension were equal to 98 N, the hanging object would not accelerate downward—so for the hanging object to accelerate downward, the force of tension, in Case 1, must be less than 98 N.

ii. This can also be explained by considering systems. Let the system in Case 1 consist of both the hanging object and the block on the table. Let the system in Case 2 consist only of the block on the table. Explain how Newton's second law, when applied to these systems, predicts that the block in Case 2 reaches the end of the table in less time.

The system in Case 1 consists of two objects M and m. The net external force that causes the acceleration $F_g - F_f$ has to accelerate both objects. In Case 2, the net external force $F_g - F_f$ only has to accelerate one block, M, so the acceleration must be larger making the time to travel the same distance smaller.

NAME ____________________ DATE ____________

Scenario

Consider a spring and a rubber band. Both have elastic properties, which means that as their lengths increase, they exert increasing amounts of force. Let "stretch length" represent the difference between the spring or rubber band's length while it exerts force and its length while it exerts no force.

Carlos suggests that both the spring and rubber band exert a force that is directly proportional to their stretch length.

Experimental Design

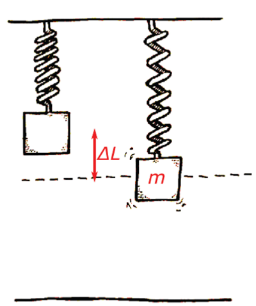
PART A: Describe a procedure that Carlos could perform to make measurements that would allow him to show evidence for his claim. Assume that Carlos has access to a spring, a rubber band, and equipment typically found in a school physics laboratory. Describe the measurements to be made and with what equipment. Include enough detail that another student could follow the procedure. Draw a diagram of the experimental setup.

What Needs to Be Measured and Algebraic Symbols

Stretch length (ΔL)

Applied force (F)

Labeled Diagram of the Setup



Procedure: Hang the spring and rubber band each from a hook on a stand so that the spring and rubber band are hanging vertically. Measure the original length of the rubber band and spring. For each, add an amount of known mass (different for each trial) and measure the length of stretch (final length - original length). Do 10 trials for the rubber band and 10 for the spring. The applied force is the weight of the known mass ($F = mg$)

Analyze Data

PART B: How would the measurements be analyzed in order to test Carlos's claim about the behavior of a spring and a rubber band?

Make a graph of force vs. stretch length. (Two graphs—one for the spring and one for the rubber band.) If force is directly proportional to stretch length, the graph(s) will be a straight line through the origin.

Hooke's Law Springs

EK | 2.B.1, 3.C.4

SP | 4.1, 4.2. 5.1, 5.3, 6.1, 6.4, 7.1

Prepare

This can and should be done in class. If this page is sent home as pre-lab homework, students can meet in small groups at the beginning of the next class to agree on a procedure before doing the activity themselves in class.

Since springs and rubber bands change shape when a force is exerted on them, they cannot be modeled as objects. Remember, in AP Physics 1, "object" is reserved for something which can be modeled as having no internal structure. Since a rubber band or a spring can stretch, it has internal structure that cannot be ignored.

Teach

This is the first workbook page where students are being asked to write their own procedure from scratch. While we believe the scaffolding provided to be helpful for students to see visually where they should be putting their information, most students will need more help in scaffolding experimental design. Consider introducing your students to the "SQUARED" method of writing procedures:

S—Setup

Q—Quantities (variables) to be measured

U—Units

A—Apparatus (tools)

R—Repetition (multiple trials)

E—Error reduction (more data points, or more trials of each point)

D—Diagram (labeled)

If you test this experiment with your students be sure to note that systematic errors can lead to graphs that do not pass directly through the origin. This is acceptable, but students should be taught not to force their line through (0, 0).

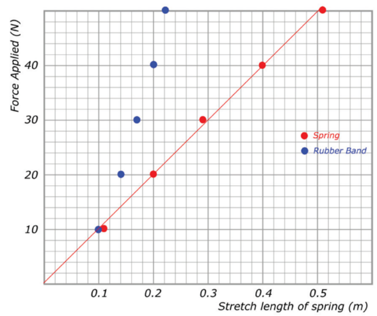
PART C: Carlos collects the measurements shown below. Graph the data on the axis below.

Force Applied [N]	10	20	30	40	50
Stretch Length of Spring [m]	0.11	0.20	0.29	0.40	0.51
Stretch Length of Rubber Band [m]	0.10	0.14	0.17	0.20	0.22

Argumentation

PART D: Does the graph show evidence that supports Carlos's claim? Be sure to address both the spring's behavior and the rubber band's behavior.

If the claim is correct, the ratio F/stretch will be the same on each trial. So, a graph of F vs. stretch should be a straight line. The student's claim is correct for the spring but not for the rubber band.



Checklist:

- ✓ I answered the question directly.
- ____ I stated a law of physics that is always true.
- ____ I connected the law or laws of physics to the specific circumstances of the situation.
- ____ I compared the situations (stated what was the same in both cases)
- ____ I contrasted the situations (stated what was different in both cases.)
- ____ I used physics vocabulary (force, mass, stretch).

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

A Hookean spring obeys Hooke's law ($F = -kx$). How could you design a test for a new kind of plastic spring to see if it can be labeled "Hookean"?

What's the point?

On lab design questions, there is almost always a point for reducing error by doing multiple trials! Don't forget that quick and easy point!

Limiting Cases

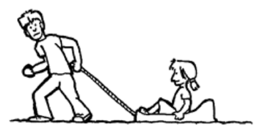
EK | 2.B.1, 3.A.2, 3.B.1, 3.B.2, 3.C.4, 4.A.2 **SP** | 1.1, 1.5, 2.1, 2.2, 6.1

UNIT 2 *Dynamics* | 2.M Limiting Cases

NAME ______________________ DATE ____________

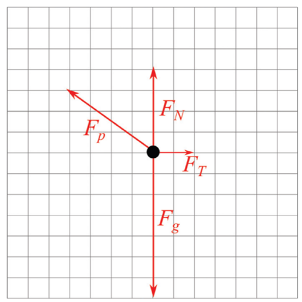
Scenario

Blake accelerates a sled carrying his little sister from rest. The sled and sister have a total mass m, the coefficient of kinetic friction between the sled and the ground is μ_k, and Blake causes the rope connected to the sled to have a constant tension force F_p that makes an angle θ with the horizontal.



Using Representations

PART A: The dot at right represents the sister and sled system. Draw a free-body diagram showing and labeling the forces (not components) exerted on the system. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Argumentation

PART B: Suppose that Blake increases the angle θ slightly but keeps the angle less than 90°. Angela and Carlos debate how this will change the magnitude of the sled's acceleration.

i. Angela suggests that increasing the angle will <u>decrease</u> the acceleration of the sled. Explain why this is possible in terms of the free-body diagram you drew above.	ii. Carlos suggests that increasing the angle will <u>increase</u> the acceleration of the sled. Explain why this is possible in terms of the free-body diagram you drew above.
As the angle increases (θ goes to 90°), the cosine of θ goes to zero, so as the angle increases, the component of the pull that can contribute to the acceleration decreases which decreases the acceleration of the sled.	As the angle increases (θ goes to 90°), the sine of θ goes to one, so as the angle increases, the normal force decreases, reducing the friction force, which increases the acceleration of the sled.

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Prepare

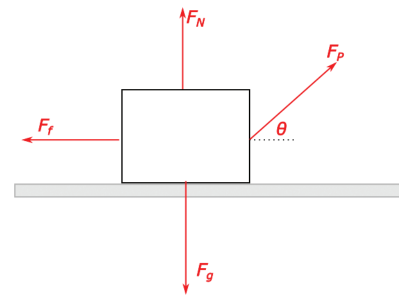
This problem asks students to think about limiting cases. If you have yet to discuss this form of analysis with your class, they may find this piece of the question difficult. Keep circling back to this kind of thinking and ask students to practice limiting-case analysis often.

Teach

Have students verbalize how they decided which force to break into components. Although it is straightforward here (break the pulling force into components because the sled-sister system is accelerating to the left), the more they practice verbalizing this, the easier it will be later.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:



A student pulls a wooden box across a rough horizontal floor at a constant speed by means of a force F_p as shown above. Which of the following must be true?

A. $F_p > F_f$ and $F_N < F_g$
B. $F_p > F_f$ and $F_N = F_g$
C. $F_p = F_f$ and $F_N > F_g$
D. $F_p = F_f$ and $F_N = F_g$

What's the point?

Being able to discuss limiting cases is an important skill necessary to analyze mathematical representations on the AP Physics 1 Exam.

Quantitative Analysis

PART C: Derive expressions for the following in terms of m, μ, F_p, and θ:

i. The force that the ground exerts on the system.	ii. The force of friction the ground exerts on the system.	iii. The acceleration of the system
$\Sigma F_y = ma$ $F_N + F_P \sin\theta - mg = ma$ $F_N = mg - F_P \sin\theta$	$F_f = \mu F_N$ $F_f = \mu(mg - F_P \sin\theta)$	$\Sigma F_x = ma$ $F_P \cos\theta - F_f = ma$ $F_P \cos\theta - \mu(mg - F_P \sin\theta) = ma$ $a = \dfrac{F_P \cos\theta - \mu(mg - F_P \sin\theta)}{m}$

UNIT 2 *Dynamics* | 2.N Experimental Procedure Design

NAME ______________________ DATE ____________

Scenario

Dominique reads that race cars have wide tires because the increased area of contact between the tire and the road results in a stronger force of friction. She hypothesizes that the force of kinetic friction on an object is directly proportional to the area of the object in contact with the surface and wants to test this hypothesis.

Dominique and Blake take a long wooden plank and cut the plank into pieces that have different lengths but the same width and height. The students also have access to other equipment commonly available in a school physics laboratory.

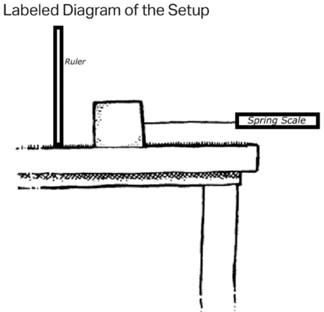
Experimental Design

PART A: Explain how Dominique and Blake could determine whether the force of friction on an object is directly proportional to the area of the object in contact with the surface.

What Needs to Be Measured and Algebraic Symbols

Friction force (F_f)
Length of block (l)
Width of block (w)
Height of block (h)

Labeled Diagram of the Setup



Procedure:

Pull each piece of wood across the table with a spring scale at a constant speed. Measure the friction force as the force registered on the scale as the piece moves. Measure the length, width, and height of each block to calculate the area and the volume of each block.

Experimental Procedure Design

EK | 3.C.4 **SP** | 4.1, 4.4, 5.1, 6.1

Prepare

This lab question has a built-in mistake (that the lab group doesn't take into account the mass of the blocks is also changing). Most students will not notice until the end when it is specifically called out, but if your students notice, be prepared to discuss that this question is designed to test if lab results support the hypothesis (regardless of the methods) AND their understanding of good lab procedure. Remind students that it is not their job to argue with the question. For example, they should not use Part A to discuss why this lab won't give the desired results. Go with what is given. Do not argue with the question.

Teach

Have students determine a way to control for mass and repeat the lab. (Use the SQUARED method of procedure writing for scaffolding!) Do the results provide evidence for the reasoning that race cars have wide tires because the increased area results in a stronger force of friction? Now may be a good point in the class to discuss that the friction force that allows a driver to control a car is not simple kinetic or static friction as has been discussed in the course; it is more complicated.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

Sketch a graph of friction force vs. area and friction force vs. mass. Explain in a few short sentences why these graphs have the shapes they have.

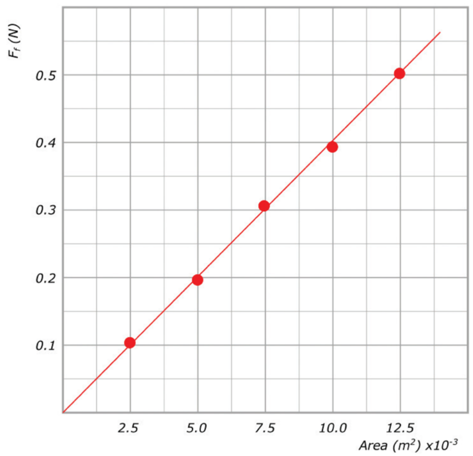
What's the point?

There are often variables that are linked (like area and mass), and a change in one results in a change in the other. If students are not careful and conscious about how variables might be linked, they can invalidate their results by not controlling their variables.

Block	A	B	C	D	E
Area [m²]	0.0025	0.0050	0.0075	0.0100	0.0125
Volume [m³]	0.00005	0.00010	0.00015	0.00020	0.00025
Kinetic Friction Force [N]	0.11	0.19	0.31	0.39	0.50

Analyze Data

PART B: On the grid, plot a graph of the data that could be used to test Dominique's SPECIFIC hypothesis. Label both axes with quantities, units, and an appropriate scale. Draw a best-fit line to the data.



PART C: Does the graph itself support Dominique's hypothesis? Why or why not?

Yes, because the graph shows that the force is directly proportional to the area (a line through the origin).

Argumentation

PART D: There was a flaw in the procedure that renders the conclusion invalid. Briefly explain what this flaw was.

Each piece of wood has a different volume, so they all have different masses. The friction force depends on the normal force, which depends on the mass. Since they didn't hold the mass constant, the conclusions above are invalid.

NAME ______________________ DATE __________

Scenario

Angela is given a spring, meterstick, stopwatch, and set of objects of known mass, that can be connected to the spring. She also has a stand with a clamp from which the spring can be connected to and hang vertically.

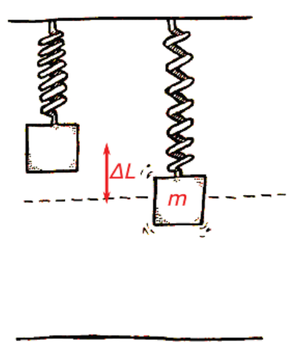
Experimental Design

PART A: Briefly explain how Angela can obtain a value for the spring constant of the spring using this equipment.

What Needs to Be Measured and Algebraic Symbols

Weight of objects (either massed or recorded if mass is known) (W) Spring stretch Δx

Labeled Diagram of the Setup



Procedure: Hang the spring vertically. Hang the known mass m from the spring and measure the stretch length (final length of the spring - original length of the spring) with the meterstick. Repeat for at least five values of mass. At equilibrium, the force upward from the spring is equal to the weight of the object, so

$mg = kx$

$W = k(\Delta x)$

The students should graph the weight of the objects (hung from the spring) vs. the spring stretch, and the slope of the graph will be the spring constant k

Angela is then shown to an elevator that is currently on the ground floor. She needs to use this equipment to analyze the motion of the elevator between the time that the doors close and the time that the elevator reaches its maximum upward speed. Specifically, Angela is to determine the elevator's maximum upward speed v_{max} and the upward acceleration a that the elevator has as its speed increases to maximum.

The elevator has no windows, so there is no way for her to make measurements relative to anything outside the elevator.

Spring Force and Acceleration

EK | 2.B.1, 3.B.2, 3.C.4 **SP** | 4.1, 4.2, 5.1, 6.1

Prepare

If possible, have students perform experiments in an elevator. Whether they bring springs and known masses, spring scales, force sensors, or just simple analog bathroom scales, seeing and feeling the changes in the normal force (or apparent weight) is powerful for student understanding.

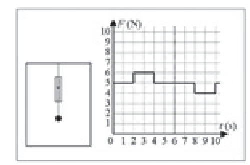
Teach

Remind students that even if they are not specifically asked to sketch a free-body diagram, drawing one can help them to solidify their ideas about the magnitudes and the directions of the forces. If they do draw a free-body diagram and want to reference it in their response, make sure that they specifically call attention to the free-body diagram. Remember that anything written or drawn outside the answer area will not be graded unless the reader is specifically told to do so.

Assess

To further assess student understanding of the concepts addressed in this scenario, you may want to ask students the questions below:

See Problem #1 on AP Physics B Exam from 1993 for more elevator problems. It is not necessary for students to even complete the whole question from 1993. Ask them to describe the motion for one or two of the segments of the motion. For example, ask, "What is happening to the position, velocity, and acceleration of the elevator during the segment?"



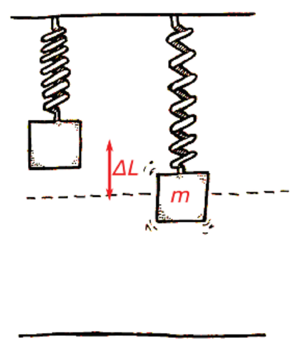
An object of known mass hangs from a force sensor inside of an elevator. As the elevator moves from the bottom floor to the top floor, the upward force exerted on the object as a function of time is recorded in the following graph. Which of the following questions could NOT be answered by the data in the graph and the known mass?

A. *What is the acceleration of the elevator as it leaves the ground floor?*
B. *What is the maximum speed attained by the elevator?*
C. *What is the approximate height of the building?*
D. *All of these questions could be answered by these data.*

What's the point?

In AP Physics 1, ideas never disappear. Just because you learned something in the last chapter, doesn't mean that it won't pop up again!

PART B: Outline a brief procedure that explains how measurements are to be made that can be used to calculate v_{max} and a.

<table>
<tr>
<td>What Needs to Be Measured and Algebraic Symbols
Time that the elevator accelerates (t) additional stretch (ΔL)</td>
<td rowspan="2">Procedure: Hang the known mass on the spring before the elevator begins to accelerate. Start the stopwatch when the elevator begins to accelerate and stop the stopwatch when the elevator stops accelerating. WHILE the elevator is accelerating, measure the stretch of the spring (final stretch − initial stretch).</td>
</tr>
<tr>
<td>Labeled Diagram of the Setup

</td>
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</table>

Analyze Data

PART C: Explain how the measurements made in the procedure outlined in Part B can be used to determine the values of v_{max} and a.

$$ma = k(\Delta L)$$

$$a = \frac{k(\Delta L)}{m}$$

$$v_{max} = at$$

$$v_{max} = \frac{k(\Delta L)t}{m}$$

Argumentation

PART D: An object connected to the spring causes the spring to be 10-cm long before the elevator begins to move. When the elevator has reached half its maximum speed, the spring is 12 cm long.

i. How long is the spring when the elevator has finished accelerating and reached its maximum speed? Explain your reasoning.

10 cm because the forces on the object are the same for rest and for constant velocity—in both cases the acceleration is zero.

ii. How long is the spring when the elevator is moving upward but slowing down with the same magnitude acceleration a that it had while speeding up?

______ Longer than 10 cm ______ Equal to 10 cm __X__ Shorter than 10 cm

Explain your reasoning.

When the elevator is moving upward but slowing down, it means that the acceleration of the elevator is downward, so the net force on the object should be downward, and the gravitational force (downward) has to be greater than the spring force (upward). When the elevator isn't moving, the spring force is equal to the gravitational force and the spring is 10 cm long. This means that when the elevator is accelerating downward, the spring stretch should be less than 10 cm, providing a force smaller than the gravitational force.