

Calculus Summer Review Packet Answer Key

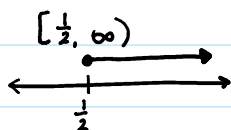
I. Interval Notation

Solution	Interval Notation	Graph
$-2 < x < 4$	$(-2, 4)$	
$-1 \leq x < 7$	$[-1, 7)$	
$-\infty < x \leq 8$	$(-\infty, 8]$	

b. $2x - 1 \geq 0$

$$2x \geq 1$$

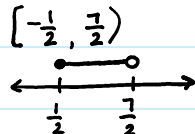
$$x \geq \frac{1}{2}$$



c. $-4 \leq 2x - 3 < 4$

$$-1 \leq 2x < 7$$

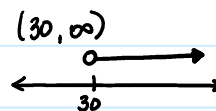
$$-\frac{1}{2} \leq x < \frac{7}{2}$$



d. $\frac{x}{2} - \frac{x}{3} > 5$

$$\frac{3x}{6} - \frac{2x}{6} > \frac{30}{6}$$

$$x > 30$$



2. Functions

a. $f(2) = 2 \cdot 2 + 1$

$$= 4 + 1$$

$$= 5$$

b. $g(-3) = 2(-3)^2 - 1$

$$= 18 - 1$$

$$= 17$$

c. $f(t+1) = 2(t+1) + 1$

$$= 2t + 2 + 1$$

$$= 2t + 3$$

d. $f(g(-2)) = f(7)$

$$= 15$$

e. $g(f(m+2)) = g(2(m+2) + 1)$

$$= g(2m + 5)$$

$$= 2(2m + 5)^2 - 1$$

$$= 2[4m^2 + 20m + 25] - 1$$

$$= 8m^2 + 40m + 49$$

f. $f(\frac{\pi}{2}) = \sin \frac{\pi}{2} = 1$



g. $f(\frac{2\pi}{3}) = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$



$$\begin{aligned} \text{h. } h(f(-2)) &= h(4) \\ &= 4^2 - 1 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{i. } f(g(x-1)) &= f(2(x-1)+5) \\ &= f(2x+3) \\ &= (2x+3)^2 \\ &= 4x^2 + 12x + 9 \end{aligned}$$

$$\begin{aligned} \text{j. } g(h(x^6)) &= g(x^6-1) \\ &= 2(x^6-1)+5 \\ &= 2x^6 + 3 \end{aligned}$$

$$\text{k. } f(x+h) = 9(x+h) + 3$$

$$f(x) = 9x + 3$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{9(x+h) + 3 - (9x + 3)}{h} \\ &= \frac{9x + 9h + 3 - 9x - 3}{h} \\ &= \frac{9h}{h} \\ &= 9 \end{aligned}$$

$$\text{l. } f(x+h) = 5 - 2(x+h)$$

$$f(x) = 5 - 2x$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{5 - 2(x+h) - (5 - 2x)}{h} \\ &= \frac{5 - 2x - 2h - 5 + 2x}{h} \\ &= \frac{-2h}{h} \\ &= -2 \end{aligned}$$

$$\text{m. } f(x+h) = 2(x+h) + 1$$

$$f(x) = 2x + 1$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{2(x+h) + 1 - (2x + 1)}{h} \\ &= \frac{2x + 2h + 1 - 2x - 1}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{n. } g(x) + h(x) &= x^2 + 4 + 10x - 1 \\ &= x^2 + 10x + 3 \end{aligned}$$

$$\text{d. } z(g(x)) = 5\sin(x^2 + 4)$$

$$\begin{aligned} \text{p. } g(x) \cdot h(x) &= (x^2 + 4)(10x - 1) \\ &= 10x^3 - x^2 + 40x - 4 \end{aligned}$$

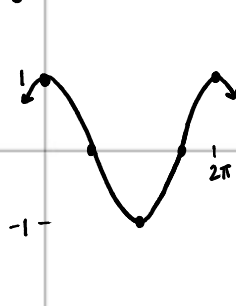
$$\begin{aligned} \text{q. } g(h(x)) &= (10x - 1)^2 + 4 \\ &= 100x^2 - 20x + 1 + 4 \\ &= 100x^2 - 20x + 5 \end{aligned}$$

3. Twelve Basic Functions and a few extra

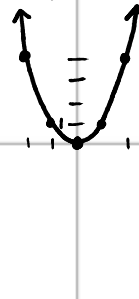
$$y = x$$



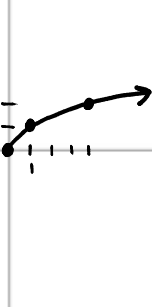
$$y = \cos x \quad (1 \text{ period})$$



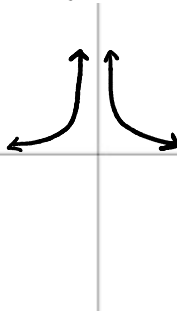
$$y = x^2$$



$$y = \sqrt{x}$$



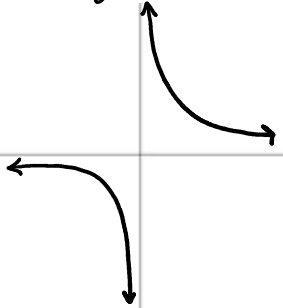
$$y = \frac{1}{x^2}$$



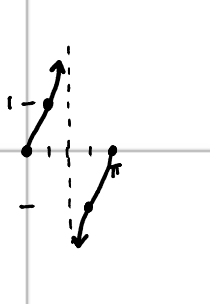
$$y = e^x$$



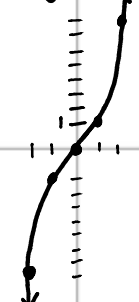
$$y = \frac{1}{x}$$

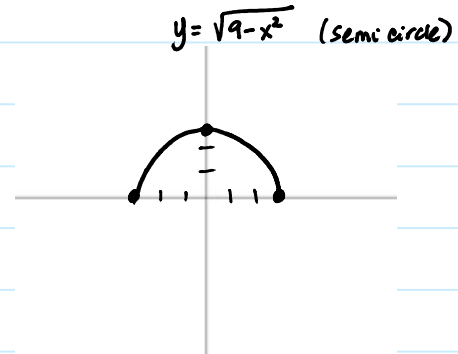
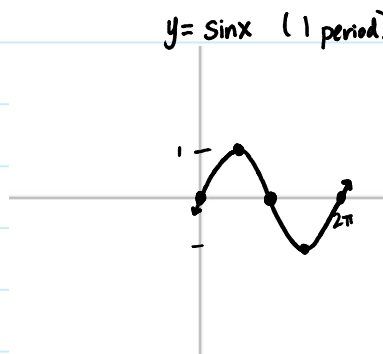
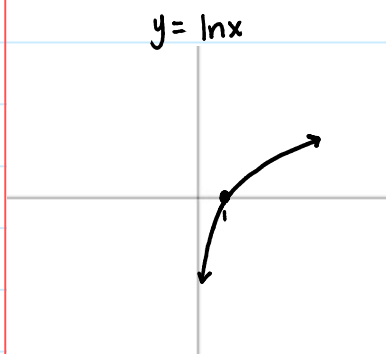
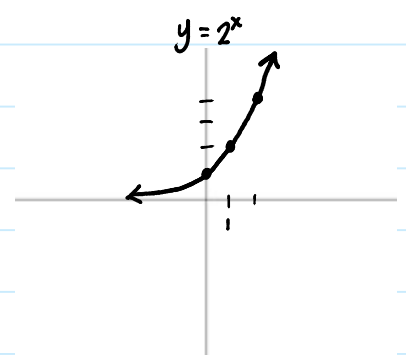
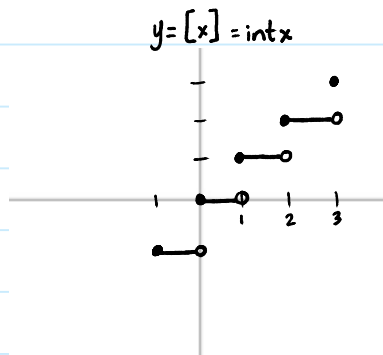
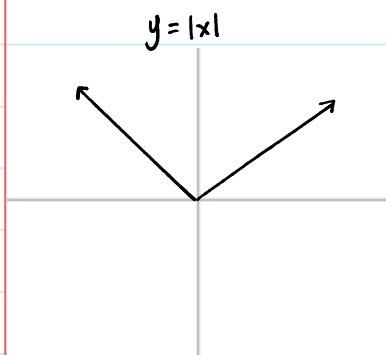


$$y = \tan x \quad (1 \text{ period})$$



$$y = x^3$$





4. Intercepts

a. $y = 2x - 5$

x-int: $(\frac{5}{2}, 0)$

y-int: $(0, -5)$

b. $y = x^2 + x - 2$

x-int: $(-2, 0), (1, 0)$

y-int: $(0, -2)$

c. $y = x\sqrt{16-x^2}$

x-int: $(0, 0), (4, 0), (-4, 0)$

y-int: $(0, 0)$

d. $y^2 = x^3 - 4x$

x-int: $(0, 0), (2, 0), (-2, 0)$

y-int: $(0, 0)$

5. Domain and Range

a. $f(x) = x^2 - 5$

D: $(-\infty, \infty)$

R: $[-5, \infty)$

b. $f(x) = -\sqrt{x+3}$

D: $[-3, \infty)$

R: $(-\infty, 0]$

c. $f(x) = 3\sin x$

D: $(-\infty, \infty)$

R: $[-3, 3]$

d. $f(x) = \frac{2}{x-1}$

D: $(-\infty, 1) \cup (1, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

6. Equation of a Line

a. $y = 3x + 5$

b. $x = 5$

c. $y = 2$

d. $y - 5 = \frac{2}{3}(x - 0)$

e. normal $\Rightarrow$ perpendicular

$$y - 8 = -\frac{6}{5}(x - 2)$$

f. $y = 7$

g. Slope = $\frac{2-6}{1-3} = \frac{-4}{-2} = 2$

$$y - 2 = 2(x - 1) \text{ or } y - 6 = -1(x + 3)$$

h. Slope = $\frac{3-0}{0-2} = -\frac{3}{2}$

Ans: $y = -\frac{3}{2}x + 3$

i. $2x - 3y = 15$

$$-3y = -2x + 15$$

$$y = \frac{2}{3}x - 5$$

↑
slope

Ans:

$$y + 1 = -\frac{3}{2}(x - 5)$$

j. $2(3x - y) = -3$

$$6x - 2y = -3$$

ANS: $y - 0 = -\frac{1}{3}(x + \frac{1}{2})$

$$-2y = -6x - 3$$

$$y = 3x + \frac{3}{2}$$

↑
slope

x-int: $(-\frac{1}{2}, 0)$

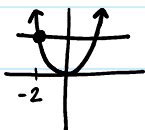
k. $g(-2) = -1$

$$g(3) = -6$$

$$\text{slope} = \frac{-6 - (-1)}{3 - (-2)} = -1$$

$$y + 1 = -1(x + 2) \quad \text{or} \quad y + 6 = -1(x - 3)$$

l.



$$y - 4 = \frac{-1}{3}(x + 2)$$

1. Systems

a. $x + y = 8$

$$\underline{4x - y = 7}$$

$$5x = 15$$

$$x = 3$$

$$y = 5$$

$$(3, 5)$$

b. $x^2 + y = 6$

$$x + y = 4$$

$$y = 4 - x$$

$$x^2 + 4 - x = 6$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2, -1$$

$$(2, 2), (-1, 5)$$

8. Factor completely

a. $4x^2 - 9 = (2x - 3)(2x + 3)$

b. $x^2 - 4x - 32 = (x - 8)(x + 4)$

c. $2x^2 + 13x - 7 = (2x - 1)(x + 7)$

d. $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

e. $2\sin^2 x - 3\sin x + 1 = (2\sin x - 1)(\sin x - 1)$

f. $x^3 + 3x^2 - 4x - 12$

$$= x^2(x + 3) - 4(x + 3)$$

$$= (x + 3)(x^2 - 4)$$

$$= (x + 3)(x - 2)(x + 2)$$

g. $x^3 + 3x^2 - 10x = x(x^2 + 3x - 10)$

$$= x(x + 5)(x - 2)$$

h. $12x^3 - 26x^2 - 10x = 2x(6x^2 - 13x - 5)$

$$= 2x(2x - 5)(3x + 1)$$

i. $3x^3 - 48x = 3x(x^2 - 16)$

$$= 3x(x - 4)(x + 4)$$

9. Solve for x.

a. $4x^2 + 12x + 3 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 4 \cdot 3}}{2 \cdot 4}$$

$$= \frac{-12 \pm \sqrt{96}}{8}$$

$$= \frac{-12 \pm 4\sqrt{6}}{8}$$

$$= \frac{-3 \pm \sqrt{6}}{2}$$

b. $\frac{2x+1}{1} = \frac{5}{x+2}$

$$2x^2 + 5x + 2 = 5$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$x = \frac{1}{2}, -3$$

c. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

$$\frac{x+1}{x} = \frac{x}{x+1}$$

$$x^2 = x^2 + 2x + 1$$

$$0 = 2x + 1$$

$$x = -\frac{1}{2}$$

10. Solve for the indicated variable

a. $A = P + xrP$; P

$$A = P(1 + xr)$$

$$P = \frac{A}{1 + xr}$$

b. $2x - 2yd = y + xd$; d

$$2x - y = 2yd + xd$$

$$2x - y = d(2y + x)$$

$$d = \frac{2x - y}{2y + x}$$

11. Vertical Asymptotes

a. $f(x) = \frac{1}{x^2}$

$$x = 0$$

b. $f(x) = \frac{x^2}{x^2 - 4} = \frac{x^2}{(x-2)(x+2)}$

$$x = 2, -2$$

c. $f(x) = \frac{2+x}{x^2(1-x)}$

$$x = 0, 1$$

12. Horizontal Asymptotes

a. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

$$y = 0$$

b. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

$$y = -\frac{5}{3}$$

c. $f(x) = \frac{4x^5}{x^2 - 7}$

No H.A.

13. a. $\frac{x^2 - 9}{x^2 + 8x + 15} = \frac{(x-3)(x+3)}{(x+5)(x+3)} = \frac{x-3}{x+5}$

b. $\frac{3x+4}{3x} = \frac{3x}{3x} + \frac{4}{3x} = 1 + \frac{4}{3x}$

c. $\frac{x^3 - 125}{x-5} = \frac{(x-5)(x^2 + 5x + 25)}{x-5} = x^2 + 5x + 25$

d. $\frac{a^2 - ab}{b^2 - ab} = \frac{a(a-b)}{b(b-a)} = \frac{a(a-b)}{b \cdot -1(a-b)} = -\frac{a}{b}$

e. $\frac{3x(x+1) - 2x(x+4)}{(x+1)^2} = \frac{3x^2 + 3x - 2x^2 - 8x}{(x+1)^2} = \frac{x^2 - 5x}{(x+1)^2}$

f. $\frac{\frac{25}{a} - a}{5+a} = \frac{\frac{25}{a} - \frac{a^2}{a}}{5+a} \cdot \frac{a}{a} = \frac{25 - a^2}{a(5+a)} = \frac{(5-a)(5+a)}{a(5+a)} = \frac{5-a}{a}$

$$g. \frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}} \cdot \frac{x+2}{x+2} = \frac{2x+4-4}{5x+10+10} = \frac{2x}{5x+20}$$

$$h. \frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}} \cdot \frac{2x-3}{2x-3} = \frac{8x-12-12}{10x-15+15} = \frac{8x-24}{10x} = \frac{4x-12}{5x}$$

$$i. \frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}} \cdot \frac{x(x+1)}{x(x+1)} = \frac{x^2 - x - 1}{x^2 + x + 1}$$

$$j. \frac{1 - \frac{2x}{3x-4}}{x + \frac{32}{3x-4}} \cdot \frac{3x-4}{3x-4} = \frac{3x-4-2x}{3x^2-4x+32} = \frac{x-4}{3x^2-4x+32}$$

14. Unit Circle Trig

$$a. \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} \quad b. \cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2} \quad c. \tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}} \quad d. \sin 6\pi = 0$$

$$e. \csc \frac{5\pi}{4} = -\sqrt{2} \quad f. \sec \frac{4\pi}{3} = -2 \quad g. \cot \frac{\pi}{3} = \frac{1}{\sqrt{3}} \quad h. \tan \frac{3\pi}{2} = \text{und.}$$

$$i. \sec \frac{3\pi}{4} = -\sqrt{2} \quad j. \sin \frac{\pi}{2} = 1 \quad k. \cos \frac{8\pi}{3} = -\frac{1}{2} \quad l. \tan 3\pi = 0$$

$$m. \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3} \quad n. \arccos(-1) = \pi \quad o. \arctan(-1) = -\frac{\pi}{4}$$

$$p. \sec^{-1} 1 = 0 \quad q. \sec^{-1}\left(\frac{1}{2}\right) = \text{und} \quad r. \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$s. \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \quad t. \cot^{-1}(-1) = \frac{3\pi}{4} \quad u. \csc^{-1} \sqrt{2} = \frac{\pi}{4}$$

15. Trig Equations

a. $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

b. $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

c. $\sin^2 x = \frac{1}{2}$

$\sin x = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

d. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

e. $\sin^2 x = \sin x$

$\sin^2 x - \sin x = 0$

$\sin x (\sin x - 1) = 0$

$\sin x = 0, \sin x - 1 = 0$

$x = 0, \frac{\pi}{2}$

f. $2\cos^2 x - 1 - \cos x = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2} \quad \cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$

16. Laws of Exponents

a. $\frac{(2a)^3}{b} = \frac{8a^3}{b}$

b. $\sqrt{9ab^3} = 3ba^{1/2}b^{1/2}$

c. $\frac{a^{-2}b^4c^0}{a^{-1}b^5c^5} = \frac{a^5b^3}{c^5}$

d. $\frac{a^{\frac{1}{3}}a^{\frac{2}{5}}}{a^{\frac{3}{4}}} = \frac{a^{\frac{20}{60}}a^{\frac{24}{60}}}{a^{\frac{45}{60}}} = \frac{1}{a^{\frac{1}{60}}}$

17. Laws of Logs

a. $\log_2 8 = 3$

b. $4\ln e = 4$

c. $\ln 1 = 0$

d. $\log_{10} \left(\frac{1}{10^x} \right) = -x$

e. $7^{2\log_7 8}$
 $= 7^{\log_7 64}$
 $= 64$

$$\begin{array}{lll} \text{f. } \log_2 5 + \log_2 10 & \text{g. } 2\log_2 9 - \log_2 3 & \text{h. } e^{\ln 5} = 5 \\ = \log_2 50 & = \log_2 81 - \log_2 3 & \\ & = \log_2 27 & \end{array}$$

$$\text{i. } \log \sqrt{10} = \log_{10} 10^{1/2} = \frac{1}{2} \quad \text{j. } \ln(e^{-x}) = -x$$

18. Solving Exponential and Log Equations

$$\begin{array}{ll} \text{a. } 5^{x+1} = 25 & \text{b. } \frac{1}{3} = 3^{2x+2} \\ 5^{x+1} = 5^2 & 3^{-1} = 3^{2x+2} \\ x+1 = 2 & -1 = 2x+2 \\ x = 1 & -3 = 2x \\ & x = -\frac{3}{2} \end{array}$$

$$\begin{array}{ll} \text{c. } \ln(5x+4) = \ln(3x+18) & \text{d. } \ln(x+7) = 4 \\ 5x+4 = 3x+18 & e^4 = x+7 \\ 2x = 14 & x = e^4 - 7 \\ x = 7 & \end{array}$$

$$\begin{array}{ll} \text{e. } \log_2 x^2 = 3 & \text{f. } \log_3 x^2 = 2\log_3 4 - 4\log_3 5 \\ 2^3 = x^2 & \log_3 x^2 = \log_3 \frac{16}{625} \\ x = \pm 2\sqrt{2} & x^2 = \frac{16}{625} \\ & x = \pm \frac{4}{25} \end{array}$$

19. Calculator

a. B

b. 41

c. $y_1(B) = 0.6817$

d. $(0.9022, 1.2658)$

e. $x = -0.5042, 0.7576$