

Calculus Summer Review

There are some great resources you could also preview on Khan Academy

<https://www.khanacademy.org/math/get-ready-for-ap-calc>

<https://www.khanacademy.org/math/ap-calculus-ab>

★ 1. Interval Notation

a. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each inequality. State your answer in BOTH interval notation and graphically.

b. $2x - 1 \geq 0$

c. $-4 \leq 2x - 3 < 4$

d. $\frac{x}{2} - \frac{x}{3} > 5$

★ 2. Functions

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "**f of g of x**" means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned}
 f(g(x)) &= f(x-4) \\
 &= 2(x-4)^2 + 1 \\
 &= 2(x^2 - 8x + 16) + 1 \\
 &= 2x^2 - 16x + 32 + 1 \\
 f(g(x)) &= 2x^2 - 16x + 33
 \end{aligned}$$

[Video on Sum, Difference, Product, and Quotient of Functions](#)

[Video on Function Composition](#)

a. Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

a. $f(2) =$ _____

b. $g(-3) =$ _____

c. $f(t+1) =$ _____

d. $f[g(-2)] =$ _____

e. $g[f(m+2)] =$ _____

b. Let $f(x) = \sin x$. Find each exactly.

f. $f\left(\frac{\pi}{2}\right) =$ _____

g. $f\left(\frac{2\pi}{3}\right) =$ _____

c. Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

h. $h[f(-2)] =$ _____

i. $f[g(x-1)] =$ _____

j. $g[h(x^3)] =$ _____

Find $\frac{f(x+h) - f(x)}{h}$ for the given function f .

k. $f(x) = 9x + 3$

l. $f(x) = 5 - 2x$

m. $f(x) = 2x + 1$

d. Let $g(x) = x^2 + 4$, $h(x) = 10x - 1$, and $z(x) = 5 \sin x$. Write expressions for the following:

n. $g(x) + h(x)$

o. $z(g(x))$

p. $g(x) \cdot h(x)$

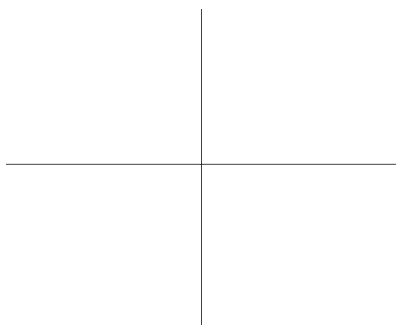
q. $g(h(x))$

★ 3. Twelve Basic Functions and a few extra

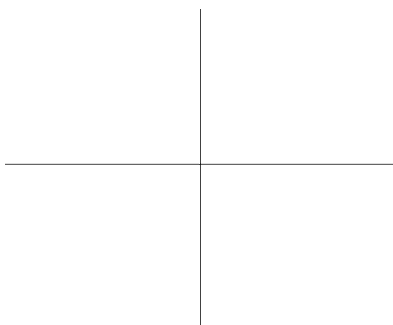
[Video on the 12 Basic Functions](#)

[Flashcards on the 12 Basic Functions](#)

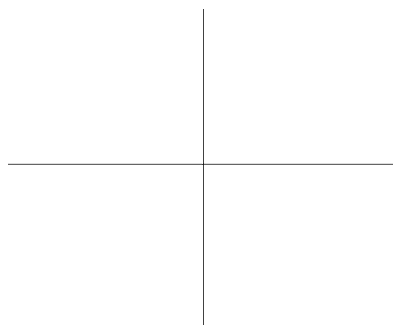
$$y = x$$



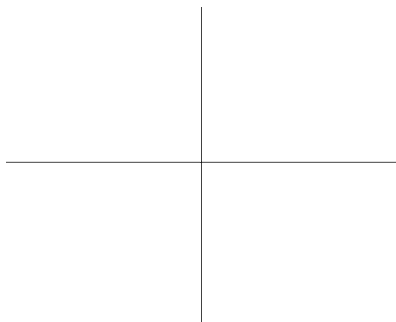
$$y = \cos x$$



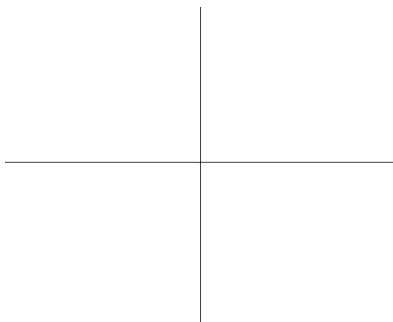
$$y = x^2$$



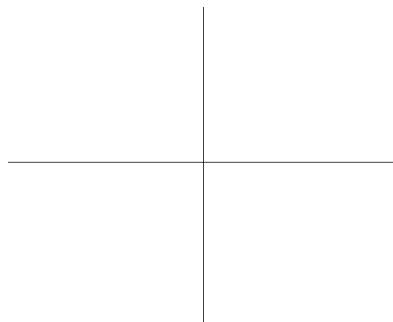
$$y = \sqrt{x}$$



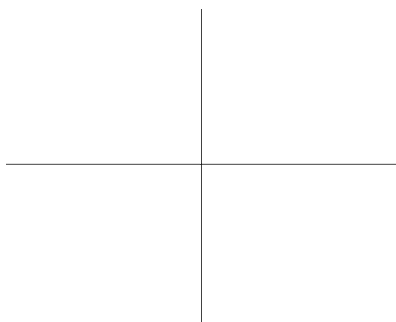
$$y = \frac{1}{x^2}$$



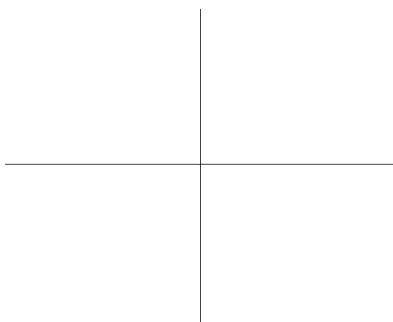
$$y = e^x$$



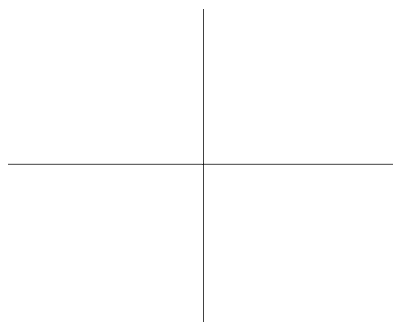
$$y = \frac{1}{x}$$



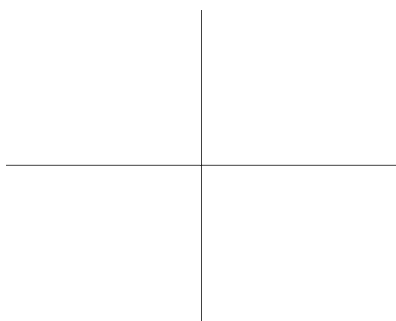
$$y = \tan x$$



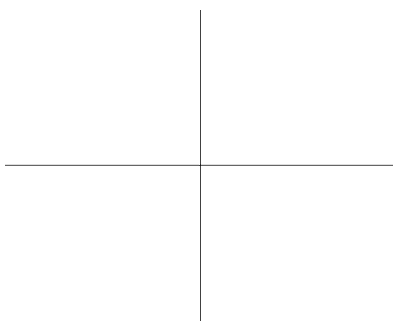
$$y = x^3$$



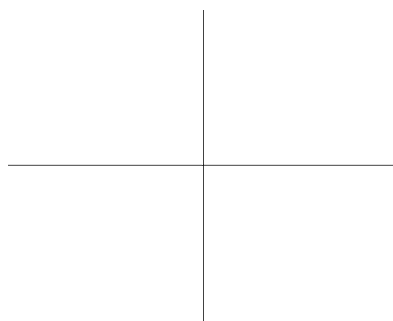
$$y = |x|$$



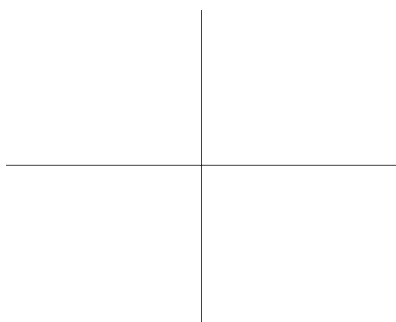
$$y = [x] \text{ or } y = \text{int } x$$



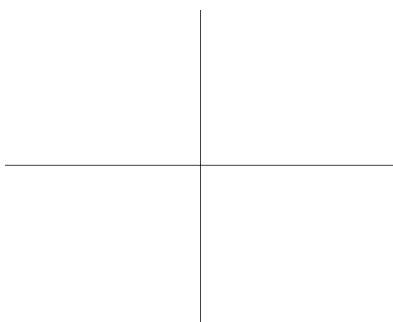
$$y = 2^x$$



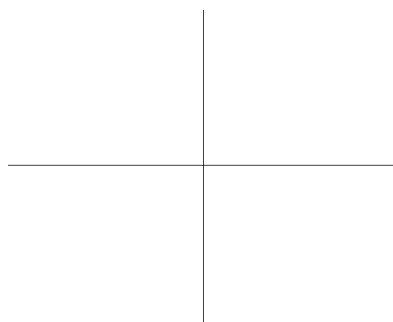
$$y = \ln x$$



$$y = \sin x$$



$$y = \sqrt{9 - x^2}$$



★ 4. Intercepts

Find the x and y intercepts for each.

a. $y = 2x - 5$

b. $y = x^2 + x - 2$

c. $y = x\sqrt{16 - x^2}$

d. $y^2 = x^3 - 4x$

★ 5. Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

a. $y = 2x - 5$

b. $f(x) = -\sqrt{x + 3}$

c. $f(x) = 3 \sin x$

d. $f(x) = \frac{2}{x - 1}$

★ 6. Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

[Video on Writing Linear Equations](#)

[Video on Slope Intercept Form](#)

[Video on Point Slope Form](#)

[Video on Writing Equations in All Forms](#)

- Use slope-intercept form to find the equation of the line having a slope of 3 and a y -intercept of 5.
- Determine the equation of a line passing through the point (5, -3) with an undefined slope.
- Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
- Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $2/3$.

- e. Find the equation of a line passing through the point (2, 8) and normal to the line $y = \frac{5}{6}x - 1$.
- f. Find the equation of a line perpendicular to the y-axis passing through the point (4, 7).
- g. Find the equation of a line passing through the points (-3, 6) and (1, 2).
- h. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).
- i. Find the equation of the line perpendicular to $2x - 3y = 15$ and through (5, -1).
- j. Find the equation of the line through the x-intercept of $2(3x - y) = -3$ and is perpendicular to this line.
- k. Given $g(x) = -x^2 + 3$, write the equation of the line through $g(-2)$ and $g(3)$.
- l. Write the equation of the line with slope $\frac{-1}{3}$ and through the intersection point in quadrant 2 between $y = 4$ and $y = x^2$.

★ 7. Systems

Use substitution or elimination method to solve the system of equations.

Example:

$$x^2 + y - 16x + 39 = 0$$

$$x^2 - y^2 - 9 = 0$$

Elimination Method

$$2x^2 - 16x + 30 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ and } x = 5$$

Plug $x = 3$ and $x = 5$ into one original

$$3^2 - y^2 - 9 = 0 \quad 5^2 - y^2 - 9 = 0$$

$$-y^2 = 0$$

$$16 = y^2$$

$$y = 0$$

$$y = \pm 4$$

Points of Intersection (5, 4), (5, -4) and (3, 0)

Substitution Method

Solve one equation for one variable.

$$y^2 = -x^2 + 16x - 39$$

(1st equation solved for y)

$$x^2 - (-x^2 + 16x - 39) - 9 = 0$$

Plug what y^2 is equal to into second equation.

$$2x^2 - 16x + 30 = 0$$

(The rest is the same as previous example)

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ or } x = 5$$

Find the point(s) of intersection of the graphs for the given equations.

$$x + y = 8$$

$$x^2 + y = 6$$

a. $4x - y = 7$

b. $x + y = 4$

8. Factor Completely

[Video on Factoring Methods](#)

[Video on Factoring a Quadratic Expression with Lead Coefficient of 1](#)

[Video on Factoring a Quadratic Expression with Lead Coefficient not 1](#)

[Video on Factoring by Grouping](#)

[Video on Factoring a Difference of Two Squares](#)

[Video on Factoring Sum or Difference of Cubes](#)

a. $4x^2 - 9$

b. $x^2 - 4x - 32$

c. $2x^2 + 13x - 7$

d. $x^3 - 8$

e. $2\sin^2 x - 3\sin x + 1$

f. $x^3 + 3x^2 - 4x - 12$

g. $x^3 + 3x^2 - 10x$

h. $12x^3 - 26x^2 - 10x$

i. $3x^3 - 48x$

★ 9. Solve for x

[Video on Solving Quadratics](#)

a. $4x^2 + 12x + 3 = 0$

b. $2x + 1 = \frac{5}{x + 2}$

c. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

★ 10. Solve for the indicated variable

[Video on Solving Literal Equations](#)

[Website on Solving Literal Equations](#)

a. $A = P + xrP$, for P

b. $2x - 2yd = y + xd$, for d

★ 11. Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

a. $f(x) = \frac{1}{x^2}$

b. $f(x) = \frac{x^2}{x^2 - 4}$

c. $f(x) = \frac{2 + x}{x^2(1 - x)}$

★ 12. Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

a. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

b. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

c. $f(x) = \frac{4x^5}{x^2 - 7}$

★ 13. Simplify a Rational Expression, if possible

Video on Simplifying Rational Expressions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x - 7 - 6}{5} = \frac{-7x - 13}{5}$$

$$\frac{-\frac{2}{x} + \frac{3x}{x-4}}{5 - \frac{1}{x-4}} = \frac{-\frac{2}{x} + \frac{3x}{x-4}}{5 - \frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4) + 3x(x)}{5(x)(x-4) - 1(x)} = \frac{-2x + 8 + 3x^2}{5x^2 - 20x - x} = \frac{3x^2 - 2x + 8}{5x^2 - 21x}$$

Simplify each of the following.

a. $\frac{x^2 - 9}{x^2 + 8x + 15}$

b. $\frac{3x + 4}{3x}$

c. $\frac{x^3 - 125}{x - 5}$

d. $\frac{a^2 - ab}{b^2 - ab}$

e. $\frac{3x(x+1) - 2x(x+4)}{(x+1)^2}$

f. $\frac{\frac{25}{a} - a}{5 + a}$

g. $\frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}}$

h. $\frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}$

i. $\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$

j. $\frac{1 - \frac{2x}{3x-4}}{x + \frac{32}{3x-4}}$

★ 14. Unit Circle Trigonometry

Evaluate, leaving answers in radians if necessary.

[Videos on Evaluating Trig Expressions](#)

[Video on Evaluating Inverse Trig](#)

a. $\sin \frac{2\pi}{3}$

b. $\cos \frac{7\pi}{4}$

c. $\tan \frac{5\pi}{6}$

d. $\sin 6\pi$

e. $\csc \frac{5\pi}{4}$

f. $\sec \frac{4\pi}{3}$

g. $\cot \frac{\pi}{3}$

h. $\tan \frac{3\pi}{2}$

i. $\sec \frac{3\pi}{4}$

j. $\sin \frac{\pi}{2}$

k. $\cos \frac{8\pi}{3}$

l. $\tan 3\pi$

m. $\arcsin \frac{-\sqrt{3}}{2}$

n. $\arccos(-1)$

o. $\arctan(-1)$

p. $\sec^{-1} 1$

q. $\sec^{-1} \left(\frac{1}{2} \right)$

r. $\sin^{-1} \left(\frac{\sqrt{2}}{2} \right)$

s. $\cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$

t. $\cot^{-1}(-1)$

u. $\csc^{-1} \sqrt{2}$

★ 15. Trigonometric Equations

Solve each of the equations in the interval $0 \leq x < 2\pi$.

a. $\sin x = -\frac{1}{2}$

b. $2 \cos x = \sqrt{3}$

c. $\sin^2 x = \frac{1}{2}$

d. $4 \cos^2 x - 3 = 0$

e. $\sin^2 x = \sin x$

f. $2 \cos^2 x - 1 - \cos x = 0$

Trig Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

★ 16. Laws of Exponents

Example:

$$x^a x^b = x^{a+b}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^y)^z = x^{yz}$$

[Website on Exponents](#)

[Video on Exponential Growth and Decay](#)

Write each of the following expressions with positive exponents.

a. $\frac{(2a)^3}{b}$

b. $\sqrt{9ab^3}$

c. $\frac{a^{-2}b^4c^0}{a^{-7}bc^5}$

d. $\frac{a^{\frac{1}{3}}a^{\frac{2}{5}}}{a^{\frac{3}{4}}}$

★ 17. Laws of Logarithms

Example:

$$\log_a x + \log_a y = \log_a (x \cdot y)$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y} \right)$$

$$\log_a x^n = n \log_a x$$

$$\log_e x = \ln x$$

$$e^{\ln m} = m$$

$$\ln e^m = m$$

[Videos on Logarithms](#)

[Website on Log Properties](#)

Simplify each of the following:

a. $\log_2 8$

b. $4 \ln e$

c. $\ln 1$

d. $\log_{10} \left(\frac{1}{10^x} \right)$

e. $7^{2 \log_7 8}$

f. $\log_2 5 + \log_2 10$

g. $2 \log_2 9 - \log_2 3$

h. $e^{\ln 5}$

i. $\log \sqrt{10}$

j. $\ln(e^{-x})$

★ 18. Solving Exponential and Logarithmic Equations

Solve for x .

a. $5^{x+1} = 25$

b. $\frac{1}{3} = 3^{2x+2}$

c. $\ln(5x+4) = \ln(3x+18)$

d. $\ln(x+7) = 4$

e. $\log_2 x^2 = 3$

f. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

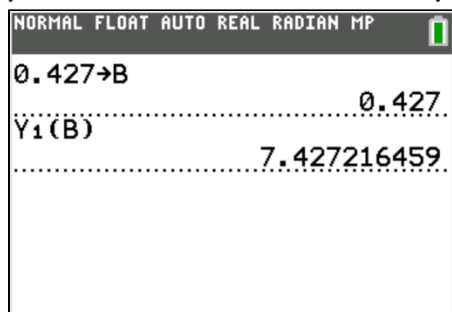
★ 19. Use the Graphing Calculator

a. Store the value 0.427 as variable B. (0.427 STO B)

b. Put $f(x) = \ln x + e^x$ into Y1.

c. Compute $f(0.427)$ on your homescreen using the following steps

(Vars YVars Function Y1 Y1(B))



d. Graph $y = 2 - x^3$ and $y = \tan x$. Find the intersection point in quadrant 1 and round to 4 decimal places.

e. In radian mode, find the zeroes of $y = -2\sin x + 8x^2 - 3$ and round answer(s) to 4 decimal places.