APPLIED STATISTICS Graded Course of Study

Conceptual Category: **Statistics and Probability**	
Domain: Interpreting Categorical and Quantitative Data **S-ID**	
Clusters	• Summarize, represent, and interpret data on a single count or measurement variable.
	• Summarize, represent, and interpret data on two categorical and quantitative variables.
	• Interpret linear models.

Conceptual Category: **Statistics and Probability**	
Domain: Making Inferences and Justifying Conclusions **S-IC**	
Clusters	• Understand and evaluate random processes underlying statistical experiments.
	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Conceptual Category: **Statistics and Probability**	
Domain: Conditional Probability and the Rules of Probability **S-CP**	
Clusters	• Understand independence and conditional probability and use them to interpret data.
	• Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Conceptual Category: **Statistics and Probability**	
Domain: Using Probability to Make Decisions **S-MD**	
Clusters	• Calculate expected values and use them to solve problems.
	• Use probability to evaluate outcomes of decisions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions.
First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Unit #1: Descriptive Statistics

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

- Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each
- Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments
- Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- Compute basic statistics and understand the distinction between a statistic and a parameter
- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools
- Display and discuss bivariate data where at least one variable is categorical
- Recognize how linear transformations of univariate data affect shape, center, and spread

Statistics and Probability - Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

Unit #2: Probability

Develop and evaluate inferences and predictions that are based on data.

- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference
- Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions
- Understand how basic statistical techniques are used to monitor process characteristics in the workplace

Statistics and Probability – Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

(+) **S.MD.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.★

(+) **S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★

Use probability to evaluate outcomes of decisions.

(+) **S.MD.7** Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★

(+) **S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★

(+) **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★

Unit #3: Inferential Statistics

Understand and apply concepts of probability.

- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
- Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases
- Use simulations to construct empirical probability distributions
- Compute and interpret the expected value of random variables in simple cases
- Understand the concepts of conditional probability and independent events
- Understand how to compute the probability of a compound event

Statistics and Probability - Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

- **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
- **b.** Informally assess the fit of a function by discussing residuals.
- **c.** Fit a linear function for a scatterplot that suggests a linear association.

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S.ID.9 Distinguish between correlation and causation.★

Statistics and Probability – Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

(+) **S.MD.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.★

(+) **S.MD.2** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.★

(+) **S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers*

obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.★
(+) **S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★

Use Probability to evaluate outcomes of decisions.

(+) **S.MD.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★

- **a.** Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
- **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Statistics and Probability – Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

S.IC.6 Evaluate reports based on data.★