Answer Key

Name: ______________________ **Teacher:** ________________ **Period:** ______

Geometry Summer Assignment

Solving Equations

Solve for the variable in each equation. Show your work.

1. $4f - 8 = 20$ $+8 \quad +8$ $\frac{4f}{4} = \frac{28}{4}$ $f = 7$	**2.** $-z + 7 = -8$ $-7 \quad -7$ $-z = -15$ $z = 15$	**3.** $8(-3d + 2) = 88$ $-24d + 16 = 88$ $-16 \quad -16$ $\frac{-24d}{-24} = \frac{72}{-24}$ $d = -3$
4. $x - 2(x + 10) = 12$ $x - 2x - 20 = 12$ $+20 \quad +20$ $-x = 32$ $x = -32$	**5.** $-15 = 5(3q - 10) - 5q$ $-15 = 15q - 50 - 5q$ $+50 \quad +50$ $\frac{35}{10} = \frac{10q}{10}$ $3.5 = q$	**6.** $\frac{y-8}{3} = -7$ $y - 8 = -21$ $+8 \quad +8$ $y = -13$
7. $4(3r + 2) - 3r = -10$ $12r + 8 - 3r = -10$ $-8 \quad -8$ $\frac{9r}{9} = \frac{-18}{9}$ $r = -2$	**8.** $3n + 2 = -2n - 8$ $+2n \; -2 \; +2n \; -2$ $\frac{5n}{5} = \frac{-10}{5}$ $n = -2$	**9.** $-12 + 5k = 15 - 4k$ $+12 \; +4k \; +12 \; +4k$ $\frac{9k}{9} = \frac{27}{9}$ $k = 3$

Systems of Equations

Substitution – Use when one variable is already isolated or it is easy to isolate variable.

Example: Solve the system of equations: $\begin{array}{l} 2x + y = 7 \\ 3x + 5y = 14 \end{array}$

$2x + y = 7$ $-2x \quad -2x$ $y = -2x + 7$	Solve equation for y
$3x + 5y = 14$ $3x + 5(-2x + 7) = 14$	Substitute expression into second equation
$3x - 10x + 35 = 14$ $-7x + 35 = 14$ $-7x = -21$ $x = 3$	Solve for x
$2x + y = 7$ $2(3) + y = 7$ $6 + y = 7$ $y = 1$	Substitute x into either equation to solve for y.
$(3, 1)$	Write solution as ordered pair.

Solve each system of equations using substitution.

1. $\begin{array}{l} x - 3y = 17 \\ 2x + 8y = -36 \end{array}$

$x = 3y + 17$

$2(3y + 17) + 8y = -36$

$6y + 34 + 8y = -36$

$14y + 34 = -36$

$\quad -34 \quad -34$

$\frac{14y}{14} = \frac{-70}{14}$

$y = -5$

$x = 3(-5) + 17$

$x = 2$

2. $\begin{array}{l} 6x - 5y = 3 \\ x - 9y = 25 \end{array}$

$x = 9y + 25$

$6(9y + 25) - 5y = 3$

$54y + 150 - 5y = 3$

$\quad -150 \quad -150$

$\frac{49y}{49} = \frac{-147}{49}$

$y = -3$

$x = 9(-3) + 25$

$x = -2$

Elimination – Use when equations are both in standard form (Ax + By = C)

Example: Solve the system of equations: $\begin{aligned} 2x + y &= 7 \\ 3x + 5y &= 14 \end{aligned}$

$-5(2x + y = 7)$ $3x + 5y = 14$	Multiply one (or both) equations so either the x coefficients or y coefficients will cancel out.
$-10x - 5y = -35$ $3x + 5y = 14$	Distribute to ENTIRE equation.
$-7x = -21$	Add equations together. (y-terms cancel)
$x = 3$	Solve for x.
$2x + y = 7$ $2(3) + y = 7$ $6 + y = 7$ $y = 1$	Substitute x into either equation to solve for y.
$(3, 1)$	Write solution as ordered pair.

Solve each system of equations using elimination.

3. $2(x - 3y = -3)$
$-2x + 7y = 10$

$2x - 6y = -6$
$-2x + 7y = 10$

$y = 4$

$x - 3(4) = -3$
$x - 12 = -3$
$+12 \quad +12$

$x = 9$

4. $-4x + 3y = 4$
$2(2x + 3y = 34)$

$-4x + 3y = 4$
$4x + 6y = 68$

$\frac{9y}{9} = \frac{72}{9}$

$y = 8$

$2x + 3(8) = 34$
$-24 \quad -24$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Slope and Rate of Change

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

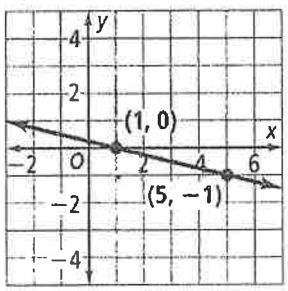
- Lines that slant upward from left to right have positive slopes.
- Lines that slant downward from left to right have negative slopes.

Two Special Cases

- Horizontal lines have slopes of 0.
- Vertical lines have slopes that are undefined.

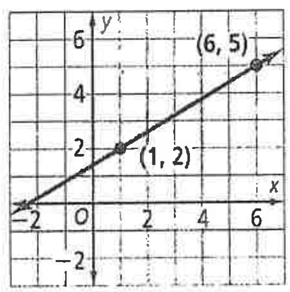
Find the slope of each line.

1.



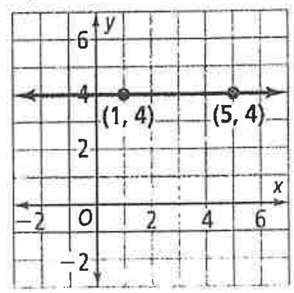
$m = \frac{-1}{4}$

2.



$m = \frac{3}{5}$

3.



$m = 0$

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

We can also find the slope from two points without graphing.

Example: Find the slope of the line passing through (5,7) and (2, 9).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-7}{2-5} = -\frac{2}{3}$$

Find the slope of the line passing through the given points.

4. (4, -2) (3, 1)

$\frac{1 + {}^{+}2}{3 - 4} = \frac{3}{-1}$

5. (5, 2), (-7, 8)

$\frac{8-2}{-7-5} = \frac{6}{-12}$

$-\frac{1}{2}$

6. (-9, 1), (0, 5)

$\frac{5-1}{0 - {}^{-}9} = \frac{4}{9}$

Linear Equations and Graphs

$y = mx + b$ is the slope-intercept form of the equation of line.

m = slope
b = y-intercept (where the line crosses the y-axis)

Write each equation in slope-intercept form and identify the slope and the y-intercept. (Solve the equation for y)

1. $-2x + y = 4$

$y = 2x + 4$

$m = 2$

$b = 4$

2. $5x - 3y = 6$

$-5x \quad -5x$

$\frac{-3y}{-3} = \frac{6}{-3} \frac{-5x}{-3}$

$y = \frac{5}{3}x - 2$

$m = \frac{5}{3}$, $b = -2$

3. $3 + y = 2x$

$-3 \quad -3$

$y = 2x - 3$

$m = 2$

$b = -3$

4. $4x + 9y = 18$

$-4x \quad -4x$

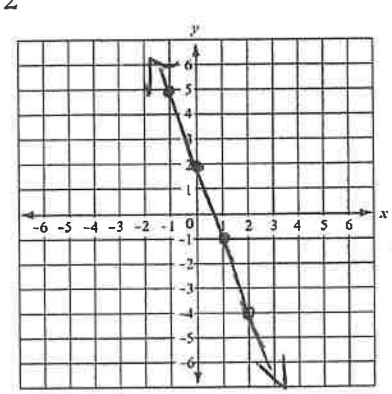
$\frac{9y}{9} = \frac{18}{9} - \frac{4x}{9}$

$y = 2 - \frac{4}{9}x$

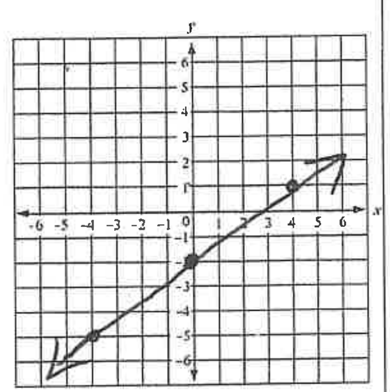
$m = -\frac{4}{9}$, $b = 2$

Graph each equation.

5. $y = -3x + 2$



6. $y = \frac{3}{4}x - 2$



Writing Equations of Lines

Example: Write the equation of a line that passes through (3,7) with a slope of 2.

$y = mx + b$	Slope-Intercept Form
$y = 2x + b$	Substitute slope for m
$7 = 2(3) + b$	Substitute (x,y) from given point
$7 = 6 + b$	Solve equation
$1 = b$	Solve for b (y-intercept)
$y = 2x + 1$	Substitute slope and y-intercept

Write the equation of a line that passes through the given point with the given slope.

7. (-1, 5), $m = 2$

$y - 5 = 2(x + {+1})$

$y - 5 = 2x + 2$

$\quad +5 \qquad +5$

$y = 2x + 7$

8. (4, -5), $m = \frac{1}{2}$

$y - {-5} = \frac{1}{2}(x - 4)$

$y + 5 = \frac{1}{2}x - 2$

$\quad -5 \qquad -5$

$y = \frac{1}{2}x - 7$

9. (2,3), $m = -1$

$y - 3 = -1(x - 2)$

$y - 3 = -x + 2$

$\quad +3 \qquad +3$

$y = -x + 5$

10. (8, -7), $m = -\frac{3}{4}$

$y + 7 = \frac{-3}{4}(x - 8)$

$y + 7 = \frac{-3}{4}x + \frac{24}{4}$

$y + 7 = \frac{-3}{4}x + 6$

$\quad -7 \qquad -7$

$y = \frac{-3}{4}x - 1$

Parallel and Perpendicular Lines

Parallel lines have the same slope and different y-intercepts.

Example: Determine if the lines are parallel or not.

$$y = \frac{2}{3}x + 5$$

$$2x - 3y = 12$$

2x – 3y = 12	Write both equations in slope-intercept form. The first is already in slope-intercept form.
-3y = -2x + 12	Subtract 2x from both sides
y = 2/3 x – 4	Divide both sides by -3
Slope = 2/3	Slopes match in both equations, therefore the lines are parallel.

Perpendicular lines have slopes that are opposite reciprocals.

$\frac{3}{4}$ and $-\frac{4}{3}$ are opposite reciprocals since they have opposite signs AND are inverses.

Example: Determine if the line AB is perpendicular to line CD using the given points.
A (3, 4) , B (7, 5), C (-3, 6), D (-2, 2)

A (3, 4) , B (7, 5) $\frac{5-4}{7-3} = \frac{1}{4}$ *slope of AB* *C (-3, 6), D (-2, 2)* $\frac{2-6}{-2-(-3)} = \frac{-4}{1}$	Find the slope of each line
Slopes are opposite reciprocals	Lines are perpendicular

Determine if the lines are parallel, perpendicular, or neither.

1. $2x+5y=10$
$10x-4y=8$

$-10x \quad -10x$

$\frac{-4y}{-4}=\frac{8}{-4}-\frac{10x}{-4}$

$y=-2+\frac{5}{2}x$

$2x+5y=10$

$-2x \quad -2x$

$\frac{5y}{5}=\frac{10}{5}-\frac{2x}{5}$

$y=2-\frac{2}{5}x$

perpendicular

2. $y=\frac{1}{3}x-7$
$2x-6y=12$

$-2x \quad -2x$

$\frac{-6y}{-6}=\frac{12}{-6}-\frac{2x}{-6}$

$y=-2+\frac{1}{3}x$

parallel

3. Line AB and CD
A(-7,2), B(-5, 8), C (4, 3), D(7, 4)

$\frac{8-2}{-5+{}^{+}7}=\frac{6}{2}=3$

$\frac{4-3}{7-4}=\frac{1}{3}$

Neither

4. Line AB and CD
A(-2, 1), B(4, 10), C (5, 2), D(8, 0)

$\frac{10-1}{4-{}^{-}2}=\frac{9}{6}=\frac{3}{2}$

$\frac{0-2}{8-5}=\frac{-2}{3}$

perpendicular

Write an equation of the line described.

5. Through the point (-4, -1) parallel to $y=-\frac{1}{2}x-1$

$y-{}^{-}1=\frac{-1}{2}(x+{}^{+}4)$

$y+1=\frac{-1}{2}x-2$

$-1 \quad -1$

$y=\frac{-1}{2}x-3$

6. Through the point (2,-2) perpendicular to $y=\frac{1}{3}x+4$

$m=\frac{1}{3}$

$m_{\perp}=-3$

$y-{}^{-}2=-3(x-2)$

$y+2=-3x+6$

$-2 \quad -2$

$y=-3x+4$

Midpoint & Distance in the Coordinate Plane

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

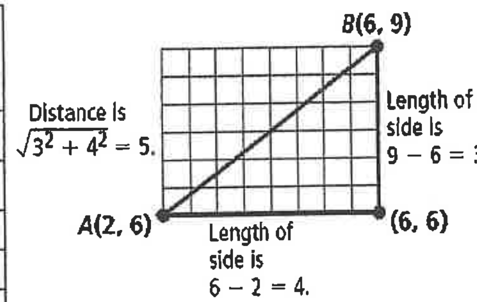
Find the midpoint of the given coordinates.

1. A(4, 3) B(8, 11) $\left(\frac{4+8}{2}, \frac{3+11}{2}\right)$ $\left(\frac{12}{2}, \frac{14}{2}\right)$ $(6, 7)$	**2.** A(-4, 5) B(2, 9) $\left(\frac{-4+2}{2}, \frac{5+9}{2}\right)$ $\left(\frac{-2}{2}, \frac{14}{2}\right)$ $(-1, 7)$
3. A(7, -3) B(-8, 10) $\left(\frac{7+{}^-8}{2}, \frac{-3+10}{2}\right)$ $\left(-\frac{1}{2}, \frac{7}{2}\right)$	**4.** A(-7, 1) B(-5, -9) $\left(\frac{-7+{}^-5}{2}, \frac{1+{}^-9}{2}\right)$ $\left(\frac{-12}{2}, \frac{-8}{2}\right)$ $(-6, -4)$

Distance Formula

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Example: Find the distance between (6,9) and (2,6)

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Distance Formula
$d=\sqrt{(2-6)^2+(6-9)^2}$	Substitute coordinates into formula
$d=\sqrt{(-4)^2+(-3)^2}$	Subtract
$d=\sqrt{16+9}$	Square each number
$d=\sqrt{25}$	Simplify
$d=5$	Take square root



Find the distance between each pair of points.

5. (2, 3), (5, 8)

$d=\sqrt{(5-2)^2+(8-3)^2}$

$d=\sqrt{(3)^2+(5)^2}=\sqrt{9+25}$

$d=\sqrt{34}\approx 5.831$

6. (3, 5), (-2, 10)

$d=\sqrt{(-2-3)^2+(10-5)^2}$

$d=\sqrt{(-5)^2+(5)^2}=\sqrt{25+25}$

$d=\sqrt{50}=\sqrt{25\cdot 2}=5\sqrt{2}$

$d\approx 7.071$

7. (-4, 11), (-3, -9)

$d=\sqrt{(-3+{}^{+}4)^2+(9-11)^2}$

$d=\sqrt{(1)^2+(-2)^2}=\sqrt{1+4}$

$d=\sqrt{5}\approx 2.236$

8. (-2, -8), (8, -3)

$d=\sqrt{(8+{}^{+}2)^2+(-3+{}^{+}8)^2}$

$d=\sqrt{(10)^2+(5)^2}$

$d=\sqrt{100+25}=\sqrt{125}=$

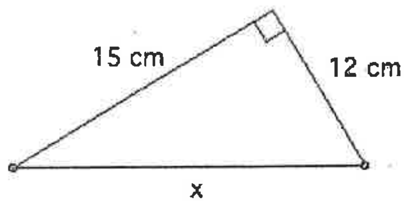
$d=\sqrt{25\cdot 5}=5\sqrt{5}\approx 11.180$

The Pythagorean Theorem

$a^2 + b^2 = c^2$ where a and b are the legs of the triangle and c is the hypotenuse (longest side – opposite the right angle)

Solve for the missing side length in each right triangle.

1.



$a^2 + b^2 = c^2$

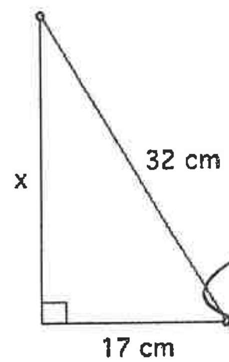
$15^2 + 12^2 = c^2$

$225 + 144 = c^2$

$\sqrt{369} = \sqrt{c^2}$

$c = 19.209$

2.



$x^2 + 17^2 = 32^2$

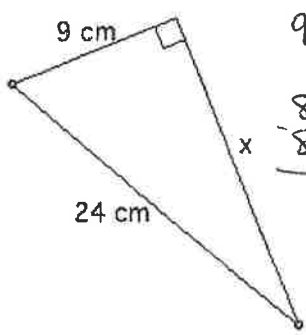
$x^2 + 289 = 1024$

$-289 \quad -289$

$\sqrt{x^2} = \sqrt{735}$

$x = 27.111$

3.



$9^2 + x^2 = 24^2$

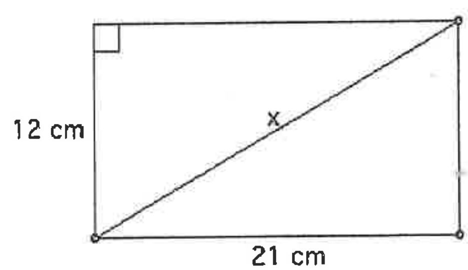
$81 + x^2 = 576$

$-81 \quad -81$

$\sqrt{x^2} = \sqrt{495}$

$x = 22.249$

4.



12cm

$12^2 + 21^2 = x^2$

$144 + 441 = x^2$

$\sqrt{585} = \sqrt{x^2}$

$x = 24.187$

Order of Operations and Radical Expressions

ORDER OF OPERATIONS

REMEMBER: PLEASE EXCUSE MY DEAR AUNT SALLY

P—Perform all operations that occur within grouping symbols such as ( ), { }, or [ ].

E— Evaluate exponents (powers and roots)

M & D—Perform multiplication and division operations from left to right

A & S—Perform addition and subtraction operations from left to right.

Simplify the following expressions.

1] $(-4+2)(-2+5)^2$
$(-2)(3)^2$
$(-2)(9) = -18$

2] $5 - 12 \div 3 - 7$
$5 - 4 - 7$
-6

3] $8 \div (6-2) + 5$
$8 \div 4 + 5$
$2 + 5$
7

4] $11 \times 622 - 3$
$6842 - 3$
6839

RADICAL EXPRESSIONS

Simplify.

1. $\sqrt{56}$ Solution: $\sqrt{56} = \sqrt{4 \cdot 14} = 2\sqrt{14}$

2. $\dfrac{\sqrt{7}}{\sqrt{3}}$ Solution: $\sqrt{\dfrac{7}{3}} = \dfrac{\sqrt{7}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{21}}{\sqrt{9}} = \dfrac{\sqrt{21}}{3}$

3. $(3\sqrt{7})^2$ Solution: $(3\sqrt{7})^2 = (3\sqrt{7} \cdot 3\sqrt{7}) = 3(3)(\sqrt{7} \cdot \sqrt{7}) = 9(7) = 63$

Simplify the following.

1. $\sqrt{36} = \pm 6$
2. $\sqrt{81} = \pm 9$
3. $\sqrt{24} = \sqrt{4 \cdot 6}$
$2\sqrt{6}$
4. $\sqrt{98} = \sqrt{49 \cdot 2}$
$7\sqrt{2}$
5. $\sqrt{300} = \sqrt{100 \cdot 3}$
$10\sqrt{3}$
6. $\sqrt{\dfrac{1}{4}} = \dfrac{\sqrt{1}}{\sqrt{4}} = \dfrac{1}{2}$
7. $\sqrt{\dfrac{80}{25}} = \dfrac{\sqrt{80}}{\sqrt{25}}$
$\dfrac{4\sqrt{5}}{5}$
8. $\dfrac{\sqrt{5}}{\sqrt{3}}$ $\dfrac{\sqrt{3}}{\sqrt{3}}$
$\dfrac{\sqrt{15}}{3}$
9. $\dfrac{2\sqrt{13}}{\sqrt{12}}$ $\dfrac{\sqrt{12}}{\sqrt{12}}$
$\dfrac{2\sqrt{4 \cdot 3 \cdot 13}}{12} = \dfrac{4\sqrt{39}}{12} = \dfrac{\sqrt{39}}{3}$
10. $\sqrt{\dfrac{205}{48}}$ $\dfrac{\sqrt{205}}{\sqrt{48}}$
$\dfrac{\sqrt{205}}{4\sqrt{3}} \cdot \dfrac{4\sqrt{3}}{4\sqrt{3}}$
$\dfrac{4\sqrt{615}}{36} = \dfrac{\sqrt{615}}{9}$
11. $\sqrt{13^2} = 13$
12. $(\sqrt{17})^2 = 17$

Solve for x.

1. $2^2 + x^2 = 4^2$
Solution: $4 + x^2 = 16$
$x^2 = 12$
$x = \sqrt{12}$
$x = 2\sqrt{3}$

2. $x^2 + (3\sqrt{2})^2 = 9^2$
Solution: $x^2 + (9)(2) = 81$
$x^2 + 18 = 81$
$x^2 = 63$
$x = \sqrt{63}$
$x = \sqrt{9 \cdot 7}$
$x = 3\sqrt{7}$

Solve for x. Assume x represents a positive number.

1. $3^2 + 4^2 = x^2$
$\sqrt{9+16} = \sqrt{x^2}$
$x = 5$

2. $x^2 + 4^2 = 5^2$
$-16 \quad -16$
$\sqrt{x^2} = \sqrt{9}$
$x = 3$

3. $5^2 + x^2 = 13^2$
$-25 \quad -25$
$\sqrt{x^2} = \sqrt{144}$
$x = 12$

4. $x^2 + 3^2 = 4^2$
$x^2 + 9 = 16$
$-9 \quad -9$
$\sqrt{x^2} = \sqrt{7}$
$x = \sqrt{7}$

5. $4^2 + 7^2 = x^2$
$16 + 49 = x^2$
$\sqrt{65} = \sqrt{x^2}$
$\sqrt{65} = x$

6. $x^2 + 5^2 = 10^2$
$-25 \quad -25$
$\sqrt{x^2} = \sqrt{75}$
$x = \sqrt{75}$

Quadratics

FACTORING

Examples: Factor.

1. $24x^3 - 32x^2$ *Hint:* $8x^2$ is the greatest common factor between the two terms.
   $= 8x^2(3x - 4)$
2. $x^2 - 12x - 28$
   $(x - 14)(x + 2)$

Factor the trinomial. If the trinomial cannot be factored, say so.

1. $x^2 + 5x + 4$
   $(x+4)(x+1)$
2. $x^2 - 8x + 12$
   $(x-6)(x-2)$
3. $5x^2 + 5x - 10$
   $5(x^2 + x - 2)$
   $5(x+2)(x-1)$
4. $3x^2 + 54x + 243$
   $3(x^2 + 18x + 81)$
   $3(x+9)(x+9) = 3(x+9)^2$
5. $-x^2 + 2x - 1$
   $-(x^2 - 2x + 1)$
   $-(x-1)(x-1) = -(x-1)^2$

QUADRATIC EQUATIONS

Example: $3x^2 + 14x + 8 = 0$ *Solution:* $(3x+2)(x+4) = 0$
$3x + 2 = 0$ or $x + 4 = 0$

1) $x^2 + 5x - 6 = 0$
   $(x+6)(x-1) = 0$ $x = -6$, $x = 1$
2) $x^2 - 7x - 18 = 0$
   $(x-9)(x+2) = 0$ $x = 9$, $x = -2$
3) $x^2 = 20x - 36$
   $x^2 - 20x + 36 = 0$
   $(x-2)(x-18) = 0$ $x = 2$, $x = 18$
4) $x^2 + 8x = 20$
   $x^2 + 8x - 20 = 0$
   $(x-2)(x+10) = 0$
   $x = 2, \quad x = -10$
5) $4x^2 + 15 = 17x$
   $-17x \quad -17x$
   $4x^2 - 17x + 15 = 0$
   $(4x-5)(x-3) = 0$ $x = 3$, $x = \frac{5}{4}$
6) $3x^2 - 13x - 10 = 0$
   $(3x+2)(x-5) = 0$ $x = 5$, $x = \frac{-2}{3}$

The Quadratic Formula: For quadratic equations: $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve each equation using the Quadratic Formula.

1. $4x^2 + 11x - 20 = 0$

$$\frac{-11 \pm \sqrt{11^2 - 4(4)(-20)}}{2(4)}$$

$$\frac{-11 \pm \sqrt{121 + 320}}{8} = \frac{-11 \pm \sqrt{441}}{8}$$

$$\frac{-11 + 21}{8} = \frac{10}{8} = \frac{5}{4} \qquad \frac{-11 - 21}{8} = \frac{-32}{8} = -4$$

2. $x^2 - 5x - 24 = 0$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-24)}}{2(1)}$$

$$\frac{5 \pm \sqrt{25 + 96}}{2} = \frac{5 \pm \sqrt{121}}{2}$$

$$\frac{5 + 11}{2} = 8 \qquad \frac{5 - 11}{2} = -3$$

3. $x^2 = 3x + 3$

$x^2 - 3x - 3 = 0$

$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$$

$$\frac{3 \pm \sqrt{9 + 12}}{2} = \frac{3 + \sqrt{21}}{2} \qquad \frac{3 - \sqrt{21}}{2}$$

4. $x^2 + 5 = -5x$

$x^2 + 5x + 5 = 0$

$$\frac{-5 \pm \sqrt{(5)^2 - 4(1)(5)}}{2(1)}$$

$$\frac{-5 \pm \sqrt{25 - 20}}{2} = \frac{-5 + \sqrt{5}}{2} \qquad \frac{-5 - \sqrt{5}}{2}$$

Multiplying Binomials and adding and subtracting Polynomials

Multiply binomials and the FOIL pattern

Find the product $(9x^2 - x + 6)(5x - 2)$.

Solution

$(9x^2 - x + 6)(5x - 2)$	Write product.
$= 9x^2(5x - 2) - x(5x - 2) + 6(5x - 2)$	Distributive property
$= 45x^3 - 18x^2 - 5x^2 + 2x\ 130x - 12$	Distributive property
$= 45x^3 - 23x^2 + 32x - 12$	Combine like terms.

Find the product $(2x - 1)(7x + 6)$.

Solution

$(2x - 1)(7x + 6)$	Write product.
$=(2x)(7x) + (2x)(6) + (-1)(7x) + (-1)(6)$	Write product of terms.
$=14x^2 + 12x + (-7x) + (-6)$	Multiply.
$=14x^2 + 5x - 6$	Combine like terms.

Find the product.

1. $(m^2 + 6m + 4)(3m - 1)$

   $3m^3 + 18m^2 + 12m - m^2 - 6m - 4$

   $3m^3 + 17m^2 + 6m - 4$

2. $(2n + 7)(3n + 4)$

   $6n^2 + 8n + 21n + 28$

   $6n^2 + 29n + 28$

3. $(2p^2 - p + 6)(p + 7$

   $2p^3 - p^2 + 6p + 14p^2 - 7p + 42$

   $2p^3 + 13p^2 - p + 42$

4. $(6q^2 - 5q - 4)(2q - 3)$

   $12q^3 - 10q^2 - 8q - 18q^2 + 15q + 12$

   $\boxed{12q^3 - 28q^2 + 7q + 12}$

5. $(5t + 9)(3t - 8)$

   $15t^2 - 40t + 27t - 72$

   $\boxed{15t^2 - 13t - 72}$

6. $(8s - 7)(9s - 7)$

   $72s^2 - 56s - 63s + 49$

   $72s^2 - 119s + 49$

Adding or subtracting polynomials

Examples: Find the sum.

a. $(3x^4 - 2x^3 + 5x) + (7x^2 + 9x^3 - 2x)$

b. $(11x^2 + 6x - 1) - (2x^2 - 7x + 5)$

Solution

a. Vertical format: Align like terms in vertical columns.

$$\begin{array}{r} 3x^4 - 2x^3 + 5x^2 \qquad\quad \\ +\qquad 9x^3 + 7x^2 - 2x \\ \hline 3x^4 + 7x^3 + 12x^2 - 2x \end{array}$$

b **Horizontal format:** Group like terms and simplify.

$$\begin{aligned}(11x^2 - 6x - 1) - (2x^2 - 7x + 5) &= 11x^2 + 6x - 1 - 2x^2 + 7x - 5 \\ &= (11x^2 - 2x) + (6x + 7x) + (-1 - 5) \\ &= 9x^2 + 13x - 6\end{aligned}$$

Find the sum or difference.

1. $(2a^2 + 7) + (7a^2 + 4a - 3)$

   $9a^2 + 4a + 4$

2. $(9b^2 - b + 8) + (4b^2 - b - 3)$

   $13b^2 - 2b + 5$

3. $(7c^3 - 6c + 4) + (9c^3 + 5c^2 + c)$

   $-2c^3 + 5c^2 - 5c + 4$

4. $(d^2 - 15d + 10) + (+12d^2 + 8d + 1)$

   $13d^2 - 23d + 11$

Proportions and Solving Linear Inequalities

PROPORTIONS

Example:

1. $\frac{3}{2} = \frac{y}{22}$
   $3(22) = 2y$
   $66 = 2y$
   $33 = y$

2. $\frac{x+4}{5} = \frac{x-2}{3}$
   $3(x+4) = 5(x-2)$
   $3x + 12 = 5x - 10$
   $22 = 2x$
   $11 = x$

Solve the following proportions using the format used in the examples.

1. $\frac{7}{2} = \frac{y}{3}$ $=$ $\frac{2y}{2} = \frac{21}{2}$

   $y = 10.5$

2. $\frac{7}{3} = \frac{21}{x}$ $\frac{7x}{7} = \frac{63}{7}$

   $x = 9$

3. $\frac{25}{15} = \frac{10}{x}$ $\frac{25x}{25} = \frac{150}{25}$

   $x = 6$

4. $\frac{10}{6x+7} = \frac{6}{2x+9}$

   $20x + 90 = 36x + 42$
   $-20x \ -42 \quad -20x \ -42$

   $\frac{48}{16} = \frac{16x}{16}$

   $x = 3$

5. $\frac{4}{x-3} = \frac{6}{x+3}$

   $4x + 12 = 6x - 18$
   $-4x + 18 \quad -4x + 18$

   $30 = 2x$

   $x = 15$

6. $\frac{3x-5}{2} = \frac{x-15}{4}$

   $12x - 20 = 2x - 30$
   $-2x + 20 \quad -2x + 20$

   $\frac{10x}{10} = \frac{-10}{10}$

   $x = -1$

Solve a linear inequality

Example Solve the inequality.

a. $-\frac{1}{3}(x + 12) < 5$

b. $9x + 2 < 5x - 18$

Solution

a. $-\frac{1}{3}(x + 12) < 5$ Write original inequality.

$-\frac{x}{3} - 4 < 5$ Distributive property

$-\frac{x}{3} < 9$ Add 4 to each side.

$x > -27$ Multiply each side by –3. Reverse the inequality symbol.

The solutions are all real numbers greater than –27.

b. $9x + 2 < 5x - 18$ Write original inequality

$9x < 5x - 20$ Subtract 2 from each side.

$4x < -20$ Subtract $5x$ from each side

$x < -5$ Divide each side by 4.

The solutions are all real numbers less than –5.

Solve the inequality.

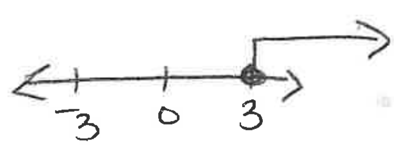
1. $3(2x - 7) > 15$

   $6x - 21 > 15$
   $+21 \quad +21$

   $\frac{6x}{6} > \frac{36}{6}$ $x > 6$

2. $10 - 3x \leq 5x - 14$
   $-10 \ -5x \ -5x \ -10$

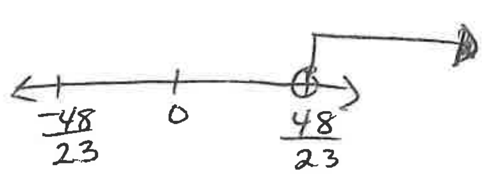
   $\frac{-8x}{-8} \leq \frac{-24}{-8}$ $x \geq 3$



3. $\frac{1}{2}(8x + 6) < 3(9x - 15)$

   $4x + 3 < 27x - 45$
   $-27x \ -3 \quad -27x \ -3$

   $\frac{-23x}{-23} < \frac{-48}{-23}$ $x > \frac{48}{23}$



Classify Polygons

Vocabulary

A **polygon** is a closed plane figure formed by three or more line segments called **sides**. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear. Each endpoint of a side is a **vertex** of the polygon.

A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon.

A polygon that is not convex is called nonconvex or **concave**.

The term ***n*-gon**, where n is the number of a polygon's sides, can be used to name a polygon.

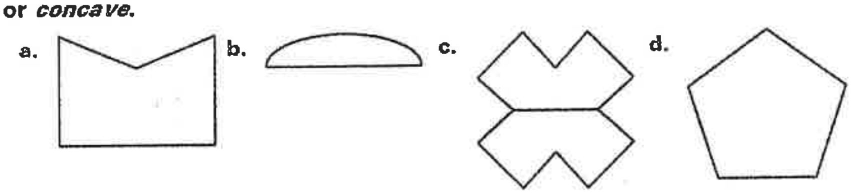
In an **equilateral** polygon, all sides are congruent.

In an **equiangular** polygon, all angles in the interior of the polygon are congruent.

A polygon is **regular** if all sides and all angles are congruent.

EXAMPLE 1 **Identify polygons**

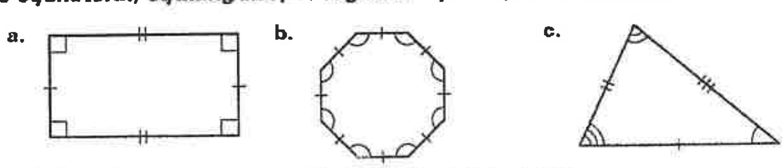
Tell whether the figure is a polygon and whether it is *convex* or *concave*.



a. The figure is a concave polygon.
b. Part of the figure is not a segment, so it is not a polygon.
c. Some segments intersect more than two segments, so it is not a polygon.
d. The figure is a convex polygon.

EXAMPLE 2 **Classify polygons**

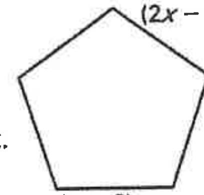
Classify the polygon by the number of sides. Tell whether the polygon is *equilateral*, *equiangular*, or *regular*. *Explain* your reasoning.



a. The polygon has 4 sides, so it is a quadrilateral. The angles in the interior of the polygon are congruent, so it is equiangular. Not all of the sides are congruent, so it is not equilateral. So, the polygon is not regular.
b. The polygon has 8 sides. It is equilateral and equiangular, so it is a regular octagon.
c. The polygon has 3 sides, so it is a triangle. It is not equilateral or equiangular, so it is not regular.

EXAMPLE 3 **Find side lengths**

The figure shown at the right is a regular pentagon. Find the length of a side.



(2x − 1) cm

(x + 3) cm

First, write and solve an equation to find the value of x.
Use the fact that the sides of a regular pentagon are congruent.

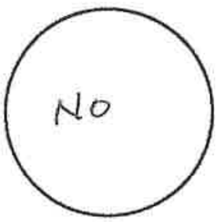
$2x - 1 = x + 3$	Write equation.
$x - 1 = 3$	Subtract x from each side.
$x = 4$	Add 1 to each side.

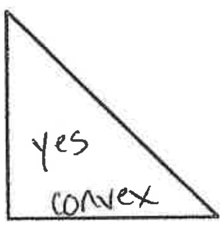
Then find a side length. Evaluate one of the expressions when $x = 4$.

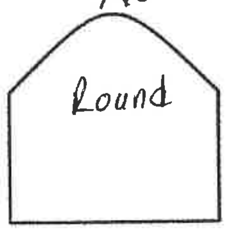
$2x - 1 = 2(4) - 1 = 7$

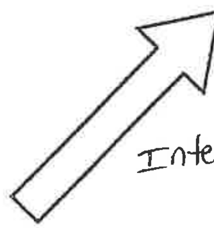
The length of a side of the pentagon is 7 centimeters.

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

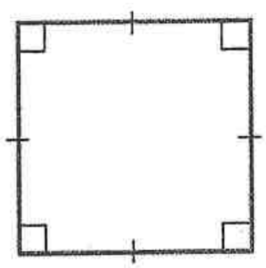
1. 
No

2. 
yes
convex

3. No 
Round

4. 
NO
Intersects

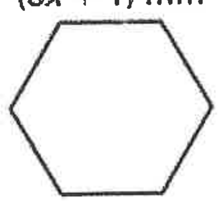
5. Classify the polygon by the number of sides. Tell whether the polygon is *equilateral*, *equiangular*, or *regular*. *Explain* your reasoning.



Square – Quad.
Equiangular
Equilateral
Regula

6. The figure shown at the right is a regular hexagon. Find the length of a side.

(3x + 1) mm



(2x + 6) mm

$3x+1 = 2x+6$

$-2x\ -1 \quad -2x\ -1$

$x=5$

$2(5)+6$

$10+6 = 16$ mm

Perimeter and Area

Vocabulary

Formulas for the perimeter P, area A, and circumference C of some common plane figures are given below.

Square

side length s

$P = 4s$

$A = s^2$

Rectangle

length ℓ and width w

$P = 2\ell + 2w$

$A = \ell w$

Triangle

side lengths a, b, and c,

base b, and height h

$P = a + b + c$

$A = \frac{1}{2}bh$

Circle

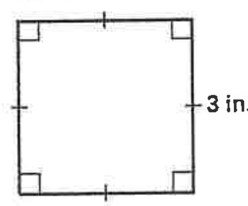
diameter d and radius r

$C = \pi d = 2\pi r$

$A = \pi r^2$

EXAMPLE 1 **Find the perimeter and area of a square**

Find the perimeter and area of the square shown at the right.



Solution

Perimeter

$P = 4s$

$= 4(3)$

$= 12$

Area

$A = s^2$

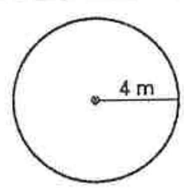
$= (3)^2$

$= 9$

The perimeter is 12 inches and the area is 9 square inches.

EXAMPLE 2 **Find the circumference and area of a circle**

Find the circumference and area of the circle shown at the right.



Solution

Use 3.14 to approximate the value of π.

Circumference

$C = 2\pi r$

$\approx 2(3.14)(4)$

$= 25.12$

Area

$A = \pi r^2$

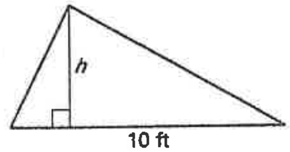
$\approx 3.14(4)^2$

$= 50.24$

The circumference is about 25.1 meters and the area is about 50.2 square meters.

EXAMPLE 3 **Find unknown length**

The base of a triangle is 10 feet. Its area is 30 square feet. Find the height of the triangle.



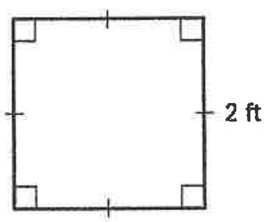
Solution

$A = \frac{1}{2}bh$	Write formula for the area of a triangle.
$30 = \frac{1}{2}(10)h$	Substitute 30 for A and 10 for b.
$6 = h$	Solve for h.

The height is 6 feet.

Find the perimeter and area of the figure. If necessary, round to the nearest tenth.

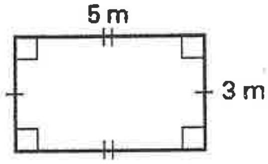
1.



$P = 4s$
$P = 4(2)$
$P = 8ft$

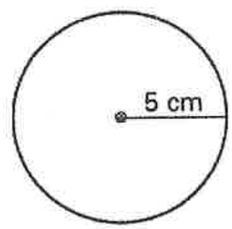
$A = s^2$
$A = (2)^2 = 4ft^2$

2.



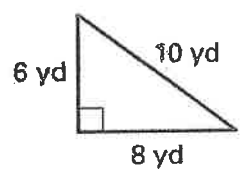
$P = 2l + 2w$
$P = 2(5) + 2(3)$
$P = 10 + 6 = 16m$
$A = l \cdot w = 5(3) = 15m^2$

3. Find the circumference and area of the circle. If necessary, round to the nearest tenth.



$C = 2\pi r$
$C = 2\pi(5)$
$C = 10\pi = 31.5cm$

4. Find the perimeter and area of the triangle.



$P = 6 + 8 + 10$
$P = 24$ yd

5. The base of a triangle is 12 meters. Its area is 42 square meters. Find the height of the triangle.

$A = \frac{1}{2}bh$

$\frac{42}{6} = \frac{\frac{1}{2}(12)h}{6}$

$7 = h$

Measure and Classify Angles

Vocabulary

An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

An **acute angle** has measure greater than 0° and less than 90°.

A **right angle** has measure equal to 90°.

An **obtuse angle** has measure greater than 90° and less than 180°.

A **straight angle** has measure equal to 180°.

Two angles are **congruent angles** if they have the same measure.

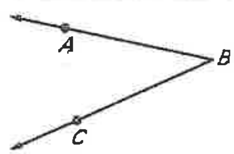
An **angle bisector** is a ray that divides an angle into two angles that are congruent.

Postulate 3 Protractor Postulate: Consider $\overleftrightarrow{OB}$ and a point A on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180. The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Postulate 4 Angle Addition Postulate: If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$.

EXAMPLE 1 **Name angles**

Write three names for the angle and name the vertex and sides of the angle.

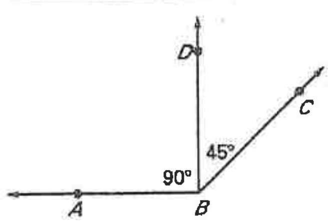


Solution

Three names for the angle are $\angle ABC$, $\angle CBA$, or $\angle B$. The vertex of the angle is point B. The sides of the angle are $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

EXAMPLE 2 **Measure and classify angles**

In the diagram, $m\angle ABD = 90°$ and $m\angle DBC = 45°$. Find $m\angle ABC$. Then classify each angle as *acute, right, obtuse,* or *straight.*



Solution

Use the Angle Addition Postulate to find $m\angle ABC$.

$m\angle ABC = m\angle ABD + m\angle DBC$	Angle Addition Postulate
$m\angle ABC = 90° + 45°$	Substitute angle measures.
$m\angle ABC = 135°$	Add.

So, $m\angle ABC = 135°$.

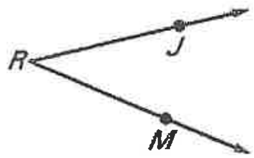
Because $m\angle ABD = 90°$, $\angle ABD$ is a right angle.

Because $m\angle DBC = 45°$, $\angle DBC$ is an acute angle.

Because $m\angle ABC = 135°$, $\angle ABC$ is an obtuse angle.

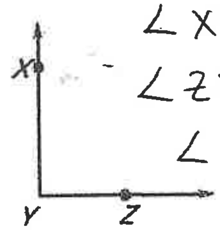
Write three names for the angle and name the vertex and sides of the angle. (Straight line)

1.



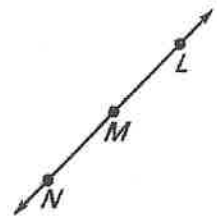
$\angle JRM$
$\angle MRJ$
$\angle R$

2.



$\angle XYZ$
$\angle ZYX$
$\angle Y$

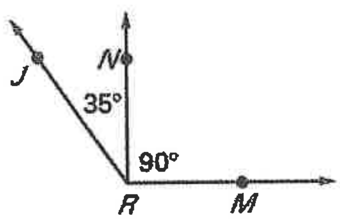
3.



$\angle LMN$
$\angle NML$
$\angle L$

Find the indicated angle measure. Then classify each angle in the diagram as *acute, right, obtuse,* or *straight.*

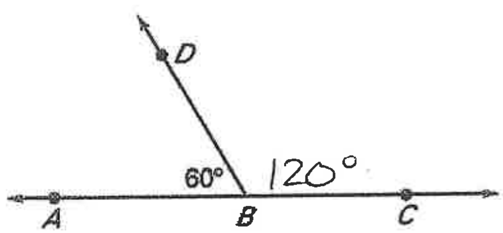
4. $m\angle JRN = 35°$ and $m\angle NRM = 90°$.
Find $m\angle JRM$.



$35 + 90 = 125$

$\angle JRM = 125°$

5. $m\angle ABD = 60°$ and $m\angle ABC = 180°$.
Find $m\angle DBC$.



$180 - 60 = 120$

$\angle DBC = 120°$