



*Princeton High School*

*Mathematics Department*

*Calc AB Summer Assignment (for Calc BC)*

## **Packet is Optional**

**Summer assignment vision and purpose:** The Mathematics Department of Princeton Public Schools aims to build confidence and competence in students as they strengthen their mathematical understanding. As such, students can complete the summer assignment in preparation for the coming school year. Success in mathematics is dependent on comprehending critical concepts. Such concepts will be extended and applied in more challenging contexts in successive years. For this reason, the department is supporting and providing summer assignments for students. These assignments will serve as a reinforcement of previously learned skills.

**Directions:** Please complete all of the following questions. ***Be sure to show all of your work and attach all completed work on additional pages. All graphs should be completed on graph paper.*** Questions are divided into subgroups based on skill and concept. Some procedures and examples have been provided to help reinforce or remind you of previously covered material. Also, please be sure to complete the following information:

AP Calculus BC Summer Work

1.1

Find each derivative.

(a)  $h(t) = 2t \cos t + t^2 \sin t$

(b)  $f(x) = 2x^2 \cot x$

(c)  $f(x) = \frac{\tan x}{\sin x + 1}$

5. Find the equation(s) of the tangent line(s) to the graph of  $y = \frac{x+1}{x-1}$  that are parallel to the line  $2y + x = 6$ .

1.2

5. Find the linear approximation of the function  $f(x) = \sqrt{1-x}$  at  $a = 0$  and use it to approximate the numbers  $\sqrt{0.9}$  and  $\sqrt{0.99}$ . Illustrate by sketching a graph of  $f$  and the tangent line.

## 1.3

6. If  $g(5) = -3$ ,  $g'(5) = 6$ ,  $h(5) = 3$ , and  $h'(5) = -2$ , find  $f'(5)$  (if possible) for each of the following. If it is not possible, state what additional information is required.

(a)  $f(x) = \frac{g(x)}{h(x)}$

(b)  $f(x) = g(h(x))$

(c)  $f(x) = g(x)h(x)$

\_\_\_\_ 15. If  $g(x) = (1-x)^3(4x+1)$ , then  $g'(x) =$

(A)  $-12(1-x)^2$

(B)  $(1-x)^2(1+8x)$

(C)  $(1-x)^2(1-16x)$

(D)  $3(1-x)^2(4x+1)$

(E)  $(1-x)^2(16x+7)$

## 1.4

2. Find  $\frac{dy}{dx}$  at the indicated point, then find the equation of the indicated line at the point.

(a)  $y^2 = \frac{x^2 - 4}{x^2 + 4}$  at  $(2, 0)$ , tangent line

(b)  $(x+y)^3 = x^3 + y^3$  at  $(-1, 1)$ , normal line

\_\_\_\_\_ 11. Find the equation of the tangent line to the graph of  $y^2 - xy - 12 = 0$  at the point  $(1, 4)$ .

- (A)  $3y = 2x + 10$
- (B)  $3y + 2x = 10$
- (C)  $y = 4x$
- (D)  $7y = 4x + 24$
- (E)  $7y + 4x = 24$

1.5

5. If  $f(x) = x^5 + 2x^3 + x - 1$  and  $f(g(x)) = x = g(f(x))$  find

(a)  $f(1)$

(b)  $g'(3)$ .

1.6

Find the derivative.

1.  $y = e^{2x^2+2x}$

2.  $y = 6^{2x}$

3.  $y = \sin^2 x + 2^{\sin x}$

4.  $y = xe^2 - e^{x^2}$

2.1

5. Find the absolute extrema of  $f$  on the given interval.

(a)  $f(x) = 2x^3 - 3x^2 - 12x + 1, [-2, 3]$

(b)  $f(x) = (x^2 - 1)^3, [-1, 2]$

2.2

- \_\_\_\_\_ 3. Determine if the function  $f(x) = x^3 - x - 1$  satisfies the hypothesis of the MVT on  $[-1, 2]$ . If it does, find all possible values of  $c$  satisfying the conclusion of the MVT.
- (A)  $-\frac{1}{2}$   
(B)  $-1, 1$   
(C)  $0$   
(D)  $1$   
(E) hypothesis not satisfied

2.3

- \_\_\_\_\_ 4. The derivative of a function  $f$  is given for all  $x$  by  $f'(x) = (2x^2 + 4x - 16)(1 + g^2(x))$  where  $g$  is some unspecified function. At which value(s) of  $x$  will  $f$  have a local maximum?
- Note:  $g^2(x) = (g(x))^2$
- (A)  $x = -4$   
(B)  $x = 4$   
(C)  $x = -2$   
(D)  $x = 2$   
(E)  $x = -4, 2$

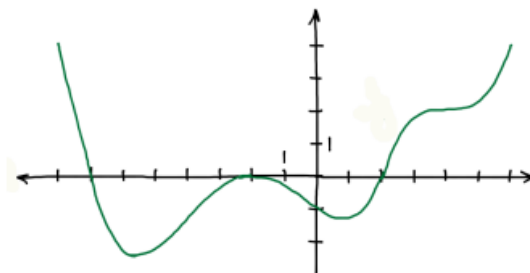
2.4

- \_\_\_\_\_ 3. The  $x$ -coordinates of the points of inflection of the graph of  $y = x^5 - 5x^4 + 3x + 7$  are
- (A) 0 only    (B) 1 only    (C) 3 only    (D) 0 and 3    (E) 0 and 1

- \_\_\_\_\_ 4. Which of the following conditions would enable you to conclude that the graph of  $f$  has a point of inflection at  $x = c$ ?
- (A) There is a local max of  $f'$  at  $x = c$     (B)  $f''(c) = 0$     (C)  $f''(c)$  does not exist  
 (D) The sign of  $f'$  changes at  $x = c$     (E)  $f$  is a cubic polynomial and  $c = 0$

- \_\_\_\_\_ 13. Let  $f$  be the function defined by  $f(x) = 2x^3 - 3x^2 - 12x + 18$ . On which of the following intervals is the graph of  $f$  both increasing and concave down?
- (A)  $(-\infty, -1)$     (B)  $(-1, \frac{1}{2})$     (C)  $(-1, 2)$     (D)  $(\frac{1}{2}, 2)$     (E)  $(2, \infty)$

- \_\_\_\_\_ 14.

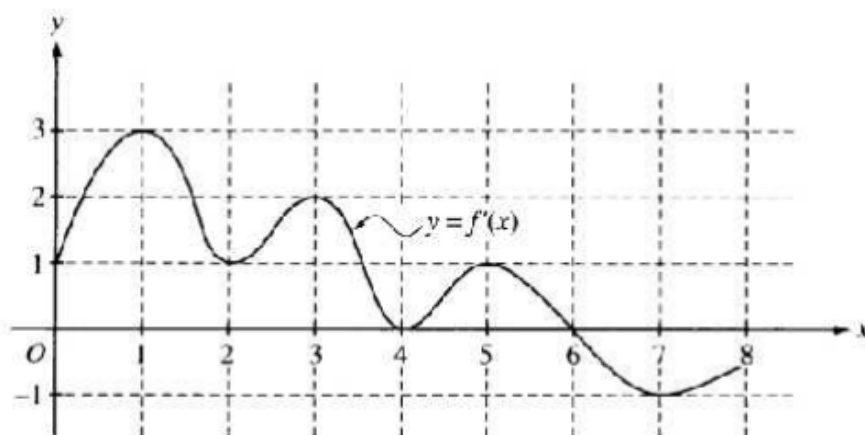


The graph of  $f'(x)$

The figure above shows the graph of  $f'$ , the derivative of function  $f$ , for  $-8 < x < 6$ . Of the following, which best describes the graph of  $f$  on the same interval?

- (A) 1 local minimum, 1 local maximum, and 3 inflection points  
 (B) 1 local minimum, 1 local maximum, and 4 inflection points  
 (C) 2 local minima, 1 local maximums, and 2 inflection points  
 (D) 2 local minima, 1 local maximum, and 4 inflection points  
 (E) 2 local minima, 2 local maxima, and 3 inflection points

2.5

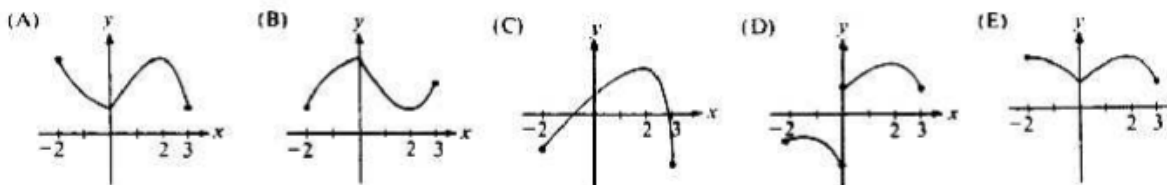


The graph above shows  $f'(x)$  for some function  $f(x)$  on  $[0, 8]$ .

\_\_\_\_ 2. How many points of inflection does the graph of  $f$  have on  $[0, 8]$ ?

- (A) Two
- (B) Three
- (C) Four
- (D) Five
- (E) Six

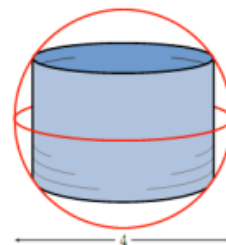
\_\_\_\_ 7. Let  $f$  be a function that is continuous on the closed interval  $[-2, 3]$  such that  $f'(0)$  does not exist,  $f'(2) = 0$ , and  $f''(x) < 0$  for all  $x$  except  $x = 0$ . Which of the following could be the graph of  $f$ .



2.6

3. A right circular cylinder is inscribed in a sphere with diameter 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.

- (A)  $A = 8 \text{ cm}^2$       (B)  $A = 16 \text{ cm}^2$       (C)  $A = 16\pi \text{ cm}^2$   
 (D)  $A = 2 \text{ cm}^2$       (E)  $A = 8\pi \text{ cm}^2$       (F)  $A = 4\pi \text{ cm}^2$   
 (G)  $A = 4 \text{ cm}^2$       (H)  $A = 2\pi \text{ cm}^2$       (I)  $A = 2 \text{ cm}^2$



4. The point on the curve  $y = \sqrt{2x}$  that is nearest the point  $(1, 4)$  occurs at the point

- (A)  $(1, \sqrt{2})$       (B)  $(2, 2)$       (C)  $(4, 2\sqrt{2})$       (D)  $(0, 0)$       (E)  $(8, 4)$

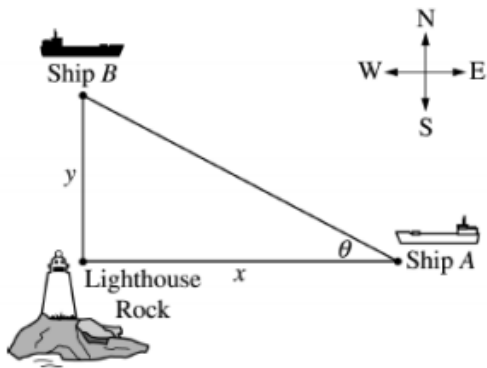
2.7

4. A point is moving on the graph of  $5x^3 + 6y^3 = xy$ . When the point is at  $\left(\frac{1}{11}, \frac{1}{11}\right)$ , its  $y$ -coordinate is increasing at a speed of 5 units per second. What is the speed of the  $x$ -coordinate at that time and in which direction is the  $x$ -coordinate moving?

- (A) 8 units/sec, increasing  $x$       (B)  $\frac{17}{2}$  units/sec, decreasing  $x$       (C)  $\frac{17}{2}$  units/sec, increasing  $x$   
 (D)  $\frac{33}{4}$  units/sec, decreasing  $x$       (E) 8 units/sec, decreasing  $x$       (F)  $\frac{35}{4}$  units/sec, decreasing  $x$



6. (2002B AB6) Ship  $A$  is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship  $B$  is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let  $x$  be the distance between Ship  $A$  and Lighthouse Rock at time  $t$ , and let  $y$  be the distance between Ship  $B$  and Lighthouse Rock at time  $t$ , as shown in the figure at right.



- (a) Find the distance, in kilometers, between Ship  $A$  and Ship  $B$  when  $x = 4$  km and  $y = 3$  km.
- (b) Find the rate of change, in km/hr, of the distance between the two ships when  $x = 4$  km and  $y = 3$  km.
- (c) Let  $\theta$  be the angle shown in the figure. Find the rate of change of  $\theta$ , in radians per hour, when  $x = 4$  km and  $y = 3$  km.

2.8

6. Evaluate the following:

(a)  $\int (\sqrt{x^3} + 2x + 1) dx$

(b)  $\int \left( \frac{x^3 + 2x - 3}{x^4} \right) dx$

(c)  $\int (2t^2 - 1)^2 dt$

(d)  $\int (\theta^2 + \sec^2 \theta - \csc \theta \cot \theta) d\theta$

(e)  $\int \left( \frac{\cos x}{1 - \cos^2 x} \right) dx$

(f)  $\int (\cos x + 3^x) dx$

2.9

$x$	0	1	2	3	4	5	6
$f(x)$	9.3	9.0	8.3	6.5	2.3	-7.6	-10.5

6. The table above gives the values of a function obtained from an experiment. Use them to estimate

$$\int_0^6 f(x) dx \text{ using **three equal subintervals** using}$$

(a) right endpoints (REP)

(b) left endpoints (LEP)

(c) midpoints (MDPT)

(d) the trapezoidal rule (TRAP)

## 3.1

1. (Calculator Permitted) What is the average value of  $f(x) = \cos x$  on the interval  $[1, 5]$ ?  
(A)  $-0.990$  (B)  $-0.450$  (C)  $-0.128$  (D)  $0.412$  (E)  $0.998$

2. If the average value of the function  $f$  on the interval  $[a, b]$  is 10, then  $\int_a^b f(x) dx =$   
(A)  $\frac{10}{b-a}$  (B)  $\frac{f(a)+f(b)}{10}$  (C)  $10b-10a$  (D)  $\frac{b-a}{10}$  (E)  $\frac{f(a)+f(b)}{20}$

3. (Calculator Permitted) Let  $f'(x) = \ln(2 + \sin x)$ . If  $f(3) = 4$ , then  $f(5) =$   
(A)  $0.040$  (B)  $0.272$  (C)  $0.961$  (D)  $4.555$  (E)  $6.667$

5. What is the linearization of  $f(x) = \int_{\pi}^x \cos^3 t dt$  at  $x = \pi$ ?  
(A)  $y = -1$  (B)  $y = -x$  (C)  $y = \pi$  (D)  $y = x - \pi$  (E)  $y = \pi - x$

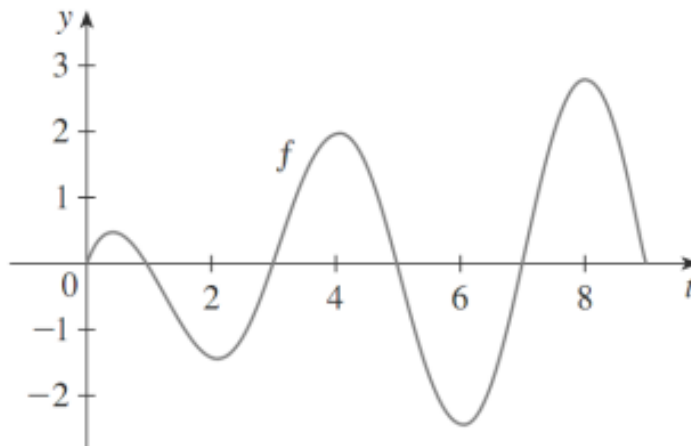
6. (Calculator Permitted) The area of the region enclosed between the graph of  $y = \sqrt{1-x^4}$  and the  $x$ -axis is  
(A)  $0.886$  (B)  $1.253$  (C)  $1.414$  (D)  $1.571$  (E)  $1.748$

9. Find  $\frac{dy}{dx}$

(a)  $y = \int_{-\pi}^x \frac{2 - \sin t}{3 + \cos t} dt$

(b)  $y = \int_x^7 \sqrt{2m^4 + m + 1} dm$

17. Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is the function whose graph is given below.



(a) At what values of  $x$  do the local maximum and local minimum of  $g$  occur? Justify.

(b) Where does  $g$  attain its absolute maximum value?

(c) On what approximate intervals is  $g$  concave downward?

(d) Sketch the graph of  $g$ .

## 3.2

1. Find the most general function  $f$  such that  $f''(x) = 9 \cos 3x$

- (A)  $f(x) = -3 \sin x + Cx^2 + D$    (B)  $f(x) = -\cos 3x + Cx + D$    (C)  $f(x) = -3 \cos 3x + Cx^2 + D$   
(D)  $f(x) = \sin x + Cx + D$    (E)  $f(x) = 3 \sin 3x + Cx + D$

4. Evaluate the definite integral  $\int_0^1 (4 - 2x)e^{8x - 2x^2} dx$

- (A)  $\frac{1}{2}(e^6 - 1)$    (B)  $\frac{1}{2}(e^{-6} - 1)$    (C)  $e^{-6} + 1$    (D)  $\frac{1}{2}(e^6 + 1)$    (E)  $e^6 - 1$

5. Evaluate  $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x} + 5}{\cos^2 x} dx$

- (A)  $2e + 3$    (B)  $2e$    (C)  $2e - 3$    (D)  $e$    (E)  $e + 5$

8. Evaluate  $\int_0^1 \frac{6x}{1+x^2} dx$

- (A)  $\frac{3}{2}$    (B)  $3$    (C)  $6$    (D)  $3 \ln 2$    (E)  $\frac{3}{2} \ln 2$

## 3.3

2. (Calculator Permitted) Let  $R$  be the region in the first quadrant bounded by the graph of  $y = 8 - x^{3/2}$ , the  $x$ -axis, and the  $y$ -axis. Which of the following gives the best approximation of the volume of the solid generated when  $R$  is revolved about the  $x$ -axis?  
 (A) 60.3    (B) 115.2    (C) 225.4    (D) 319.7    (E) 361.9
3. Let  $R$  be the region enclosed by the graph of  $y = x^2$ , the line  $x = 4$ , and the  $x$ -axis. Which of the following gives the best approximation of the volume of the solid generated when  $R$  is revolved about the  $y$ -axis.  
 (A)  $64\pi$     (B)  $128\pi$     (C)  $256\pi$     (D) 360    (E) 512
4. Let  $R$  be the region enclosed by the graphs of  $y = e^{-x}$ ,  $y = e^x$ , and  $x = 1$ . Which of the following gives the volume of the solid generated when  $R$  is revolved about the  $x$ -axis?  
 (A)  $\int_0^1 (e^x - e^{-x}) dx$     (B)  $\int_0^1 (e^{2x} - e^{-2x}) dx$     (C)  $\int_0^1 (e^x - e^{-x})^2 dx$   
 (D)  $\pi \int_0^1 (e^{2x} - e^{-2x}) dx$     (E)  $\pi \int_0^1 (e^x - e^{-x})^2 dx$
7. (Calculator Permitted) Let  $R$  be the region bounded by the graphs of  $y = \sqrt{x}$ ,  $y = e^{-x}$ , and the  $y$ -axis.  
 (a) Find the area of  $R$ .
- (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a semicircle whose diameter runs from the graph of  $y = \sqrt{x}$  to the graph of  $y = e^{-x}$ . Find the volume of this solid.

3.4

5. ('93-AB42) (**Calculator permitted**) A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during the first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?
- (A) 4.2 pounds    (B) 4.6 pounds    (C) 4.8 pounds    (D) 5.6 pounds    (E) 6.5 pounds

For problems 9 – 18, find the general solution to the following differential equations, then find the particular solution using the initial condition. **You may have to factor and/or rewrite the expression in order to separate your x-factors and y-factors.**

9.  $\frac{dy}{dx} = \frac{x}{y}, y(1) = -2$

10.  $\frac{dy}{dx} = -\frac{x}{y}, y(4) = 3$

11.  $\frac{dy}{dx} = \frac{y}{x}, y(2) = 2$

12.  $\frac{dy}{dx} = 2xy, y(0) = -3$

13.  $\frac{dy}{dx} = xy + 5x + 2y + 10, y(0) = -1$

14.  $\frac{dy}{dx} = \cos^2 y, y(0) = 0$