Newport-Mesa Unified School District
Office of Secondary Curriculum and Instruction
High School Course of Study

Course Title	*Enhanced Math II AB*	**Course Code**	*M0408-M0409*

Transcript Title:	*Enhanced Math II AB*	Grades Levels:	*9-12*	Board Adoption Date:	*May, 2014*
Content Area:	*Math*	GPA Scale:	*4.0*	Date Course Submitted:	*April, 2014*
Credential Req:		Graduation Subject Areas:	*Math*		
UC/CSU "A-G" Area Approvals:			School Site that wrote and submitted the course:	*R. Vettraino*	
Recommend Skill(s):	*Enhanced Math I AB*				
Next course(s):	*Enhanced Math III AB*				

COURSE DESCRIPTION *(catalog summary)***:**

The focus of Enhanced Math II is the same as Math II with an added 3 week statistics unit from Math III. It also includes added time to the probability unit to include 4 (+) standards that include: Applying the general multiplication rule in a uniform probability model, use permutations and combinations to compute probabilities of compound events and solve problems, use probabilities to make fair decisions, and analyze decisions and strategies using probability concepts. Enhanced Math II will also include added standards to circles, two added standards for complex numbers to the solving quadratic unit. Lastly, it will include additional days of added modeling and performance task projects built into some units. The goal of the enhanced pathway is so a student can bypass pre-calculus and go straight into AP Calculus.

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations

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GOALS *(expected performance outcomes for students)*:

1. Develop an understanding of the need for different number systems

Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

2. Develop an understanding of quadratic functions

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

3. Write and interpret the structure of equivalent expressions

Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

4. Understand probability concepts to evaluate outcomes and interpret data

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

5. Develop an understanding of similarity

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

6. Solve problems involving circles

In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the

coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete in Mathematics 2.

1. **Make sense of problems and persevere in solving them.**
Students persevere when attempting to understand the differences between quadratic functions and linear and exponential functions studied previously. They make diagrams of geometric problems to help them make sense of the problems.

2. **Reason abstractly and quantitatively.**
Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**
Students construct proofs of geometric theorems based on congruence criteria of triangles. They understand and explain the definition of radian measure.

4. **Model with mathematics.**
Students apply their mathematical understanding of quadratic functions to real-world problems. Students also discover mathematics through experimentation and examining patterns in data from real-world contexts.

5. **Use appropriate tools strategically.**
Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the result.

6. **Attend to precision.**
Students begin to understand that a rational number has a specific definition, and that irrational numbers exist. They make use of the definition of function when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?"

7. **Look for and make use of structure.**
Students develop formulas such as $(a\pm b)^2 = a^2 \pm 2ab + b^2$ *by applying the distributive property. Students see that the expression* $5 + (x - 2)^2$ *takes the form of "5 plus 'something' squared", and so that expression can be no smaller than 5.*

8. **Look for and express regularity in repeated reasoning.**
Students notice that consecutive numbers in the sequence of squares 1,4,9,16,25 always differ by an odd number. They use polynomials to represent this interesting finding by expressing it as $(n+1)^2 - n^2 = 2n + 1$

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<u>CALIFORNIA CONTENT STANDARDS</u> *(how the course aligns with California and/or national curriculum standards)***:**

Number and Quantity

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.
- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities equations.

Algebra

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.
- Perform arithmetic operations on polynomials.
- Create equations that describe numbers or relationships.
- Solve equations and inequalities in one variable.
- Solve systems of equations.

Functions

- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.
- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.
- Construct and compare linear, quadratic and exponential models and solve problems.
- Interpret expressions for functions in terms of the situations they model.
- Prove and apply trigonometric identities.

Geometry

- Prove geometric theorems.
- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.
- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.
- Explain volume formulas and use them to solve problems.

Statistics and Probability

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- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.
- Use probability to evaluate outcomes of decisions

EVALUATION *(how the effectiveness of the course will be monitored and assessed)***:**
Students will be assessed by district created end of unit summative assessments and SBAC formative and interim assessments.

Unit Length	**Unit Name & Sub-topics**	**Standards Addressed** **M = Major Cluster** **A/S = Additional or Supporting Cluster**	**Prior Knowledge**
1 Week	**Introductory Week** • Setting the tone • Establishing Routines		
3 Weeks	**1: Quadratic Functions** • Graph Quadratic Functions • Key Features of Graphs and Tables • Intercepts • Intervals (increasing, decreasing, positive, negative) • Max and Min • Symmetries • End Behavior • Domain and Range • Average Rate of Change	F-BF 1a, 1b, 3 F-IF 4, 5, 6, 7a, 9	• Patterns, Functions, Transformations Overview (with focus on linear and exponential)

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	• Compare properties of two functions • Write a quadratic function from a pattern, graph or table • Combine functions • The Meaning of Zeros • Focus; Directrix • Transformations – effect of k		
2 Weeks	**2: Comparing Functions (Absolute Value, Step, Piece-wise, Linear, Exponential, Quadratic)** • Key Features of Graphs and Tables • Intercepts • Intervals (increasing, decreasing, positive, negative) • Max and Min • Symmetries • End Behavior • Domain and Range • Average Rate of Change • Comparing Linear and Exponential Functions to Quadratic Functions • Observe that the exponential function will exceed linear or quadratic growth • Create an equation to represent a relationship • Graph and interpret Step and Piece-wise functions	A-CED 2 F-IF 4, 5, 6, 7b, 9 F-LE 3	
2 Weeks	**3: Operations with Polynomial Expressions** • Vocabulary: Expression, Term, Coefficient, etc. • Rewrite expressions • Add, subtract, multiply polynomials	A-SSE 1a, 2 A-APR 1	• Distributive Property • Integer Operations
3 Weeks	**4: Complex Numbers and Rational Exponents** • Definition of a rational exponent • Rewrite expressions with radicals and rational exponents using properties of exponents • Sum and Products of rational and irrational numbers • Concept of Complex Numbers • Add/Subtract/Multiply Complex Numbers • Use properties of exponents to transform an expression or an exponential function	N-RN 1, 2, 3 N-CN 1, 2, 7 A-SSE 3c	• Intervention Unit: Exponents • Exponents and Roots • Real Numbers

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4 Weeks	**5: Quadratic Expressions and Equations** • Factor quadratic expressions • Complete the square (max/min) • Factor by Grouping • Solve Physics Problems involving Quadratics • Derive the Quadratic Formula • Zeros of a Quadratic Function • Solve a Quadratic Equation • Recognize when there are complex solutions • Symmetry of the graph using zeros	A-SSE 2, 3a, 3b A-REI 4a, 4b F-LE 6 CA F-IF 8a	• Distributive Property
1 Week	**M1: Modeling with Linear, Quadratic, Exponential Functions** • Write, solve, graph from context • Include context for inverse functions	A-CED 1, 4 F-BF 1a, 1b, 4	
3 Weeks	**6: Probability** • Events and Sample Spaces • Independent Events/Dependent/Mutually Exclusive Events • Conditional Probability • Two-Way Frequency Tables • Apply the Addition Rule • Geometric Probability Models • Unions, intersections, complements	S-CP 1, 2, 3, 4, 5, 6, 7	• Basic Probability
3 Weeks	**7: Geometric Proof** • Theorems about Lines and Angles (e.g., Vertical angles; Alternate Interior) • Theorems about triangles (e.g., Interior Angles; Base Angles, Interior Angles) and parallelograms • Dilation Properties • Similarity through Transformation • CPCTC • AA Similarity • Use congruence and similarity to solve problems about triangles • Prove Pythagorean theorem by triangle similarity	G-CO 9, 10, 11 G-SRT 1, 2, 3, 4, 5 G-GMD 6	• Concept of Proof • Transformations

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3 Weeks	**8: Right Triangle Trigonometry** • $\sin^2\theta + \cos^2\theta = 1$ (weak reference) • Similarity to find trig ratios • Relationships between sine and cosine in complementary angles • Use the trig ratios and Pythagorean theorem to solve right triangles • Derive and use trig ratios for special right triangles	F-TF (might touch) G-SRT 6, 7, 8, 8.1(CA)	• Pythagorean Theorem • Triangle Similarity
4 Weeks	**9: Circles and Coordinate Geometry** • Prove all circles are similar • Relationships of angles, radii and chords • Inscribed and Circumscribed triangles and quadrilaterals • Arc length; radian measure; area of a sector • Derive the equation of a circle • Complete the square to find the radius and center of the circle • Derive the equation of a parabola • Coordinate geometry proofs; simple circle theorems, etc. • Circumference and Area	G-C 1, 2, 3, 4,5 G-GPE 1, 2 (parabola), 4, 6 G-GMD 1	• Basic Circle Vocabulary
	10: Statistics • Use Mean and Standard Deviation to Fit Data to a Normal Distribution • Estimate Area Under the Normal Curve • Understand that Statistics Allow Us to Make Inferences • Determine if a Model is Consistent with the Results • Sample Surveys, Experiments and Observational Studies: Purpose – Use Data From Margin of Error • Use Simulations to Look at Differences Between Parameters • Evaluate Reports Based on Data	SID 4 SIC 1, 2, 3, 4, 5, 6	•
4-5weeks	**11: (+) Standards** • Extend polynomial identities to the complex numbers. • Know the Fundamental Theorem of Algebra; show	N.CN.8 (+) N.CN.9 (+) S.CP.8 (+)	•

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	that it is true for quadratic polynomials. • Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. • Use permutations and combinations to compute probabilities of compound events and solve problems. • Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). • Analyze decisions and strategies using probability concepts • Construct a tangent line from a point outside a given circle to the circle	S.CP.9 (+) S.MD.6 (+) S.MD.7 (+) *G.C.4 (+)*	
1 Week	**M2: Modeling with Systems of Equations**	A-SSE 1b, A-REI 7	

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Unit 1	**3 weeks**			
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Quadratic Functions • Graph Quadratic Functions • Key Features of Graphs and Tables • Intercepts • Intervals (increasing, decreasing, positive, negative) • Max and Min • Symmetries • End Behavior • Domain and Range • Average Rate of Change • Compare properties of two functions • Write a quadratic function from a pattern, graph or table • Combine functions • The Meaning of Zeros • Focus; Directrix • Transformations – effect of k	• Function • Recursive • Even • Odd • Quadratic • Linear • Exponential • Intercept • Maximum • Minimum • Domain • Range • Directrix • Increase • Decrease • Average rate of change	**F.BF.1** Write a function that describes a relationship between two quantities a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. **F.BF. 3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the	**F.BF.1** *Students in Mathematics II develop models for more complex or sophisticated situations than in previous courses, due to the expansion of the types of functions available to them.* **Example 1:** Ten bacteria are placed In a test tube and each one splits in two after one minute. After 1 minute, the resulting 10 bacteria each split in two, creating 20 bacteria. This process continues for one hour until Test tube is filled up. a. How many bacteria are in the test tube after 5 minutes? 15 minutes? *b.* Describe how you can take any current Number of bacteria to find the number of bacteria at the next minute (this is writing a NOW—Next Rule) *c.* Write an "N=…"rule that gives the number of bacteria after each minute. *d.* How many bacteria are in the test tube after one hour? *e.* For further research, Dr. Bland removes 5 bacteria after each minute from the original test tube to start the new cell structure. *f.* How does this affect your rule in parts b and c? **F.BF.3** *Identify the effect of transformations to functions in multiple modalities. Student should be fluent in representations of functions as equations, tables, graphs, and descriptions and understand the interplay among these representations.*	*Prentice Hall Algebra 1:* *9-1* *9-2* *Holt Algebra 2:* *1-6* *1-7* *1-8* *5-1* *5-8* *12-1* *12-2* *12-3* **Illustrative Math Tasks** • *Skeleton Tower* • *Summer Intern* • *Susita's Account* • *Identifying Even and Odd Functions* • *Transforming the graph of a function* • *Average Cost* • *How is the Weather?* • *Throwing Baseballs*

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value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Example 1: Fill in all the missing components to the table below.

Description of change	**Original Function Output**		**Multiply the original function output by 5**
X	F(x)	F(x) + 5	
-3	9		
-2	4		
-1	1		5
0	0		
1	1		
2	4	9	
3	9		

a. Graph and label each of the function outputs with the corresponding x-values on the same set of axis in three different colors.
b. Carefully explain the relationship between the original function and both transformed functions.

Example 2: Given the functions
$f(x)=-0.5x^2+x^2+9.5x-10$,
$g(x) = f(x-3)$,
$h(x) = f(3x)$

a. Using technology plot the above functions in different colors. Analyze g(x) and h(x) in comparison to f(x).
b. For each of the new functions, g(x) and h (x), compare the local minimum, local maximum, and zeroes to the original function f(x).
c. Complete a table of values for f(x), h(x), and g(x) for x=-5, -4, -3, -2, -1, 0, 1,2,3,4,5,

- *High School Gym*
- *Graphs of Quadratics*

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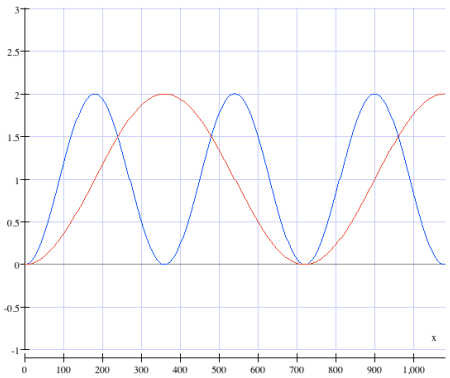
d. Explain correspondences between the function representations , graphs created in part (a), and verbal descriptions.

Example 3: Below are the graphs representing the height of two Farris Wheels at the county fair.



a. Explain how the speed is different of the blue Ferris wheel from that of the red Ferris wheel. How does this impact the positions of the local min. and max?
b. Write the rule for the red graph assuming that the blue graph is the represented by f(x).

Note: The intent of this question is NOT to develop a sophisticated trigonometric equation. It is acceptable to answer part b in terms of f(x).

Example 4:
a. Graph $f(x) = 3x^4 + x^2 + 1$ and construct a table of values from -5 to 5.
b. Graph $g(x) = 3x^3 + x + 1$ and construct a table of values from -5 to 5.
c. The graph of f(x) is said to be an even function. Using the table and graph of f(x)

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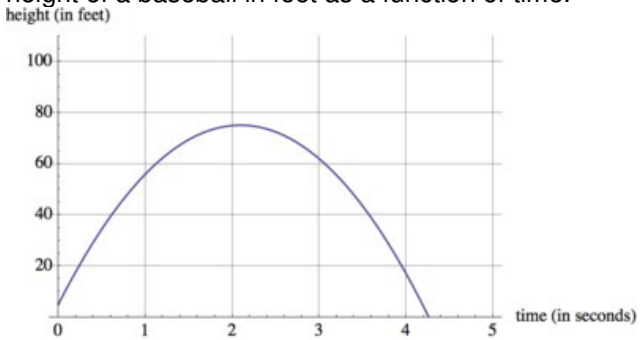
		F.IF. 4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	develop a definition of an even function. d. The graph for g(x) is said to be an odd function. Using the table and graph of f(x) develop a definition for odd function. e. In learning teams create two additional even function and two additional odd functions. Defend your created functions and definitions to your classmates. f. Are all functions either even or odd? Justify. **F.IF. 4** *This standard should be taught alongside the specific function your class is studying. Students should be able to move fluidly between graphs, tables, and words and understand the interplay between the different representations. When given a table or graph of a function that models a real---life situation, explain the meaning of the characteristics of the graph in the context of the problem.* **Example 1**: Below is a graph that represents the height of a baseball in feet as a function of time.  a. What are the x-intercepts and y-intercepts and explain their meaning in the context of the problem. b. Identify any maximums or minimums and explain their meaning in the context of the problem. c. Describe the intervals of increase and decrease	

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		F.IF. 5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes **F.IF. 6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	and explain them in the context of the problem. **F.IF. 5** *Given a function and context, determine the practical domain of the function as input values that make sense to the constraints of the problem context.* **Example 1:** Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, *n*, in attendance. If each ticket costs $30.00, find the domain and range of this function. **F.IF. 6** *Students should know that we can find the rate of change given a table of* x *and* y *values. Students should know that we can calculate the average rate of change for* any *function. Having a linear relationship is super nice, but not necessary; as long as we have initial and final values for* x *and* f(x)*, then they shouldn't have any problems.* **Example 1:** You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.	

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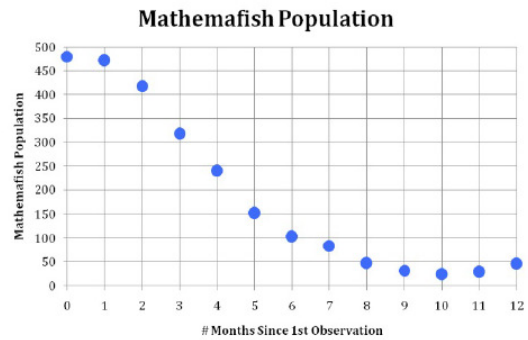
# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project. What to include in your summary report:

- Calculate the average rate of change of the mathemafish population over specific intervals. Indicate
- How and why you chose the intervals you chose.
- When was the population decreasing the fastest?
- During what month did you notice the largest effects of the EPA intervention?
- Explain the overall effects of the intervention.
- Remember to justify all your conclusions using supporting evidence.

F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases .

F.IF. 7 *This standard should be seen as related to F.IF.4 with the key difference being students can create graphs from the symbolic function in this*

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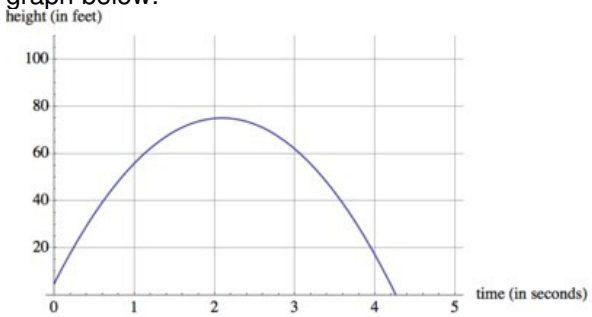
		a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **F.IF. 9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	*standard.* **Example 1:** The all star kicker kicks a field goal. The path of the ball is modeled by the $f(x) = -4.9t^2 + 20t$. Find the maximum and minimum realisitic values for the path of the ball and describe what each means in the context of this problem. **F.IF. 9** *This standard includes comparing two different functions in two different forms and comparing one function in two different forms.* **Example 1:** Suppose Brett and Andre each throw a baseball in the air. The height of Brett's ball is given by $h(t) = -16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's ball is given by the graph below:  Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.	

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			a. Who is right? b. How long is each baseball airborne? c. Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw, and explain how this can confirm your claims in parts a and b.	
Differentiation	**Support -- for students who are struggling with the content**	Content: *Unit 1* Process: Pre-unit on patterns, functions, transformation. UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 1 procedural content.		
	Extension – for high achieving students.	Content: Unit 1 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.		

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Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*

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Unit 2	**2 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Comparing Functions (Absolute Value, Step, Piece-wise, Linear, Exponential, Quadratic) • Key Features of Graphs and Tables • Intercepts • Intervals (increasing, decreasing, positive, negative) • Max and Min • Symmetries • End Behavior • Domain and Range • Average Rate of Change • Comparing Linear and Exponential Functions to Quadratic Functions • Observe that the exponential function will exceed linear or quadratic	• Exponential • Quadratic • Linear • Variable • Domain • Range • Average Rate of Change • Exponential growth • Exponential decay • Linear growth • Step function • Piece-wise • Absolute value	**A.CED. 2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F.IF. 4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	**A.CED.2** *Given a contextual situation, write equations in two variables that represent the relationship that exists between the quantities. Also graph the equation with appropriate labels and scales. Make sure students are exposed to a variety of equations arising from the functions they have studied.* **Example 1:** The intensity of light radiating from a point source varies inversely as the square of the distance from the source. Write an equation to model the relationship between these quantities given a fixed energy output. **F.IF. 4** *This standard should be taught alongside the specific function your class is studying. Students should be able to move fluidly between graphs, tables, and words and understand the interplay between the different representations. When given a table or graph of a function that models a real---life situation, explain the meaning of the characteristics of the graph in the context of the problem.* **Example 1**: Below is a graph that represents the height of a baseball in feet as a function of time.	*Prentice Hall Algebra 1:* *4-2* *4-3* *Holt Algebra 2:* *1-9* *7-8* *9-1* *9-2* *9-3* *9-6* *IMP Activities* **<u>Illustrative Math Tasks</u>** • *Influenza Epidemic* • *Warning and Cooling* • *The Canoe Trip* • *Identifying graphs of functions* • *Throwing Baseballs* • *Throwing a*

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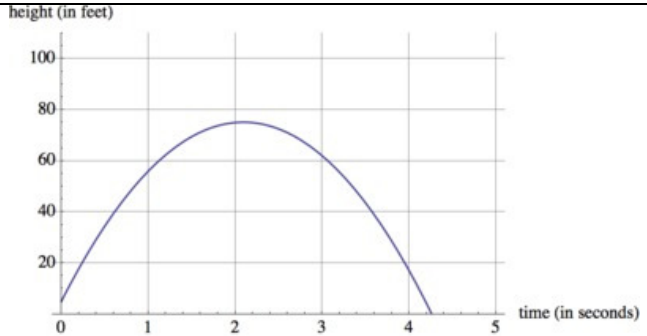
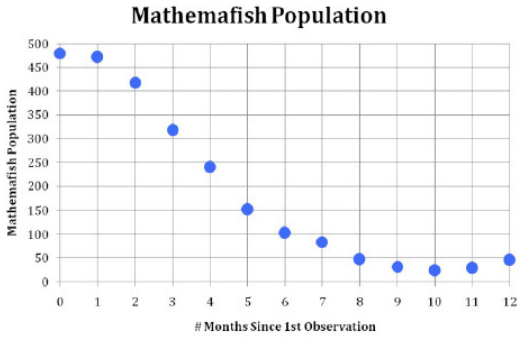
growth • Create an equation to represent a relationship • Graph and interpret Step and Piece-wise functions			 a. What are the x-intercepts and y-intercepts and explain their meaning in the context of the problem. b. Identify any maximums or minimums and explain their meaning in the context of the problem. c. Describe the intervals of increase and decrease and explain them in the context of the problem.	*Ball*
		F.IF. 5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes **F.IF. 6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a	**F.IF. 5** *Given a function and context, determine the practical domain of the function as input values that make sense to the constraints of the problem context.* **Example 1:** Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, *n*, in attendance. If each ticket costs $30.00, find the domain and range of this function. **F.IF. 6** *Students should know that we can find the rate of change given a table of* x *and* y *values. Students should know that we can calculate the average rate of change for* any *function. Having a linear relationship is super nice, but not necessary; as long as we have initial and final values*	

		specified interval. Estimate the rate of change from a graph.	*for* x *and* f(x)*, then they shouldn't have any problems.* **Example 1:** You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.	

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project. What to include in your summary report:

- Calculate the average rate of change of the mathemafish population over specific intervals. Indicate
- How and why you chose the intervals you chose.
- When was the population decreasing the fastest?
- During what month did you notice the largest

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		F.IF. 7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases b. Use the properties of exponents to interpret expressions for exponential functions. **F.IF. 9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which	effects of the EPA intervention? • Explain the overall effects of the intervention. • Remember to justify all your conclusions using supporting evidence. **F.IF. 7** *This standard should be seen as related to F.IF.4 with the key difference being students can create graphs from the symbolic function in this standard.* **Example 1:** The all star kicker kicks a field goal. The path of the ball is modeled by the $f(x) = -4.9t^2 + 20t$. Find the maximum and minimum realisitic values for the path of the ball and describe what each means in the context of this problem. **F.IF. 9** *This standard includes comparing two different functions in two different forms and comparing one function in two different forms.* **Example 1:** Suppose Brett and Andre each throw a baseball in the air. The height of Brett's ball is given by $h(t) = -16t^2 + 79t + 6$, where h is in feet and t is in seconds. The height of Andre's ball is given by the graph below:	

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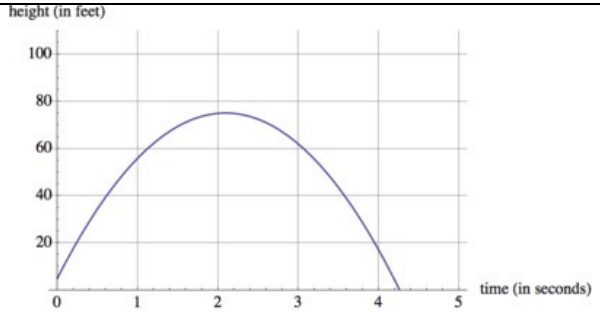
		has the larger maximum. **F.LE. 3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	 Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher. a. Who is right? b. How long is each baseball airborne? Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw, and explain how this can confirm your claims in parts a and b. **F.LE. 3** *In Mathematics II, students continue their investigation of exponential functions by 281 comparing them with linear and quadratic functions, observing that they will always eventually grow larger than any polynomial function.* **Example 1**: Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves: i. Two dollars for *each* bag of leaves filled, ii. She will be paid for the number of bags of leaves she rakes as follows:	

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			two cents for filling one bag, four cents for filling two bags, eight cents for filling three bags, and so on, with the amount doubling for each additional bag filled. a) If Celia rakes enough to five bags of leaves, should she opt for payment method 1 or 2? What if she fills ten bags of leaves? b) How many bags of leaves would Celia have to fill before method 2 pays more than method 1?	
Differentiation	**Support -- for students who are struggling with the content**	Content: *Unit 2* Process: UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 2 procedural content.		
	Extension – for high achieving students.	Content: Unit 2 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.		
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*			

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	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*

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Unit 3	**2 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Operations with Polynomial Expressions • Vocabulary: Expression, Term, Coefficient, etc. • Rewrite expressions • Add, subtract, multiply polynomials	• Expression • Term • Co-efficient • Polynomial • Expression • Factor • Sum • Difference • Equivalent •	**A.SSE. 1** Interpret expressions that represent a quantity in terms of its context a. Interpret parts of an expression, such as terms, factors, and coefficients. **A.SSE. 2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. **A.APR. 1** Understand that polynomials form a system analogous	**A.SSE. 1** *Students manipulate the terms, factors, and coefficients in difficult expressions to explain the meaning of the individual parts of the expression. Use them to make sense of the multiple factors and terms of the expression.* **Example 1**: The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically in the air. Interpret the terms and coefficients of the expression in the context of this situation. **A.SSE. 2** *Students rewrite algebraic expressions by combining like terms or factoring to reveal equivalent forms of the same expression.* **Example 1**: The expression $4000p - 250p^2$ represents the income at a concert, where p is the price per ticket. Rewrite this expression in another form to reveal the expression that represents the number of people in attendance based on the price charged. **A.APR. 1** *The Closure Property*	*Prentice Hall Algebra 1:* *8-1* *8-2* *8-3* *8-4* *8-5* *8-6* *8-7* *8-8* *Holt Algebra 2:* *6-1* *6-2* *6-4*

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		to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	*means that when adding, subtracting or multiplying polynomials, the sum, difference, or product is also a polynomial. Polynomials are not closed under division because in some cases the result is a rational expression rather than a polynomial.* **Example 1**: Simplify. a. $4b\,(cb - zd)$ b. $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$ c. $(x + 4)(x - 2)(3x + 5)$ **Example 2**: A small online company estimates that the cost, in dollars, of producing *x* units of one of their products is given by the expression $0.001x^2 + 2x + 550$. The revenue from selling *x* units is given by the expression $8.50x$. Write a polynomial expression to represent the profit generated by selling *x* units of the product. What would be the profit from sales of 2000 units?	
Differentiation	**Support -- for students who are struggling with the content**	Content: *Unit 3* Process: UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 3 procedural content.		
	Extension – for high achieving students.	Content: Unit 3 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.		

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Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*

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Unit 4	**3 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Complex Numbers and Rational Exponents • Definition of a rational exponent • Rewrite expressions with radicals and rational exponents using properties of exponents • Sum and Products of rational and irrational numbers • Concept of Complex Numbers • Add/Subtract/Multiply Complex Numbers • Use properties of exponents to transform an expression or an exponential function	• Complex number • Radical • Rational exponent • Rational • Irrational • Exponent • Expression • Quadratic • Solutions • Sum • Product • Coefficient	**N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**N.RN.1** *Students make meaning of the representation of radicals with rational exponents.* **Example 1:** Simplify $(\sqrt{2}+\sqrt{3})^2$ **Example 2:** Write in order from least to greatest. $\sqrt[5]{7}, \sqrt[3]{7}, \sqrt{7}, \sqrt[4]{7}, \sqrt[6]{7}$ **N.RN.2** *Students should be able to rewrite expressions involving rational exponents as expressions involving radicals and simplify those expressions. Students should be able to rewrite expressions involving radicals as expressions using rational exponents and use the properties of exponents to simplify those expressions.* **Example 1:** The expression $8^{\frac{2}{3}}$ can be written as $\left(8^{\frac{1}{3}}\right)^2$ *or* $(8^2)^{\frac{1}{3}}$. Write each expression in a form that involves radicals. Simplify your answers to confirm they are equivalent. **Example 2:** Given $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3$, which form would be easiest for you to calculate without a calculator? Why?	*Prentice Hall Algebra 1:* *10-1* *10-3* *Holt Algebra 2:* *1-5* *5-4* *5-5* *5-6* *5-9* *8-6* **<u>Illustrative Math Tasks</u>** • Extending the definition of exponents • Rational or Irrational? • Checking a calculation of decimal exponent • Operations with Rational and Irrational Numbers

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		N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**N.RN.3** *Students explain that the sum or product of two rational numbers is rational, by say, arguing that the sum of two fractions with integer numerator and denominator is also a fraction of the same type, showing that the rational numbers are closed under the operations of addition and multiplication.* **Example 1:** Experiment with sums and products of two numbers from the following list to answer the questions that follow: $5, \frac{1}{2}, 0, \sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}}, \pi$ Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true? a. The sum of a rational number and a rational number is rational. b. The sum of a rational number and an irrational number is irrational. c. The sum of an irrational number and an irrational number is irrational. d. The product of a rational number and a rational number is rational. e. The product of a rational number and an irrational number is irrational. f. The product of an irrational number	• Complex Number Patterns • Powers of a complex number • Completing the square

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		N.CN.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. **N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.	and an irrational number is irrational. **N.CN.1** *Students begin to work with complex numbers, first by finding simple square roots of negative numbers.* **Example 1:** Write each of the following in the form a + bi. • $i^2 + i + 1$ • $i^3 + i^2 + i + 1$ • $i^4 + i^3 + i^2 + i + 1$ • $i^5 + i^4 + i^3 + i^2 + i + 1$ • $i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$ • $i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$ • $i^8 + i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$ **N.CN.2** **Example 1:** Let $z = 1 + i$, where $i^2 = -1$. Find z^3, z^4, z^5, and z^{100}. **N.CN.7** *Students work with examples of quadratic functions and solve quadratic equations, where they encounter situations in which a resulting equation does not have a solution that is a real number.* **Example 1:** Below shows how Renee solves the equation $x^2+x+1=0$	

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		A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions	*First I will rewrite this as a square plus some number.* $x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}.$ *Now I can subtract* $\frac{3}{4}$ *from both sides of the equation* $\left(x+\frac{1}{2}\right)^2=-\frac{3}{4}.$ *But I can't take the square root of a negative number so I can't solve this equation.* a. Show how Renee might have continued to find the complex solutions. **A.SSE.3** *Use properties of exponents to write an equivalent form of an exponential function to reveal and explain specific information about the rate of growth or decay.* **Example 1:** The equation $y= 14000(0.8)^x$ represents the value of an automobile *x* years after purchase. Find the yearly and the monthly rate of depreciation of the car.	

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Differentiation	**Support -- for students who are struggling with the content**	Content: *Unit 4* Process: Pre-Unit on Exponents, UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 4 procedural content.
	Extension – for high achieving students.	Content: Unit 4 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*	
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*	

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Unit 5	**4 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Quadratic Expressions and Equations • Factor quadratic expressions • Complete the square (max/min) • Factor by Grouping • Solve Physics Problems involving Quadratics • Derive the Quadratic Formula • Zeros of a Quadratic Function • Solve a Quadratic Equation • Recognize when there are complex solutions • Symmetry of the graph using zeros	• Quadratic • Expression • Maximum • Minimum • Zeroes • Discriminant • Equivalent • Solution • Complex • Symmetry • Function • Factor • Completing the square	**A.SSE. 2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. **A.SSE. 3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the	**A.SSE. 2** *Students rewrite algebraic expressions by combining like terms or factoring to reveal equivalent forms of the same expression.* **Example 1**: The expression $4000p - 250p^2$ represents the income at a concert, where *p* is the price per ticket. Rewrite this expression in another form to reveal the expression that represents the number of people in attendance based on the price charged. **A.SSE. 3ab** *Students will write expressions in equivalent forms to solve problems involving quadratics and exponentials.* **Example 1:** The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If *p* is the price of the item, then three equivalent forms for the profit are: Standard form: $-2p^2 + 24p - 54$ Factored form: $-2(p-3)(p-9)$ Vertex form: $-2(p-6)^2 + 18$. Which form is most useful for finding: a. The prices that give a profit of zero dollars?	*Prentice Hall Algebra 1:* *9-4* *9-5* *9-6* *9-7* *9-8* *Holt Algebra 2:* *5-3* *5-4* *5-6* **<u>Illustrative Math Tasks</u>** • *Computations with Complex Numbers* • *Animal Populations* • *Completing the Square* • *Profit of a Company* • *Springboard Dive* • *Braking Distance* • *Which function?*

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		maximum or minimum value of the function it defines. **A.REI.4** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.	b. The profit when the price is zero? c. The price that gives the maximum profit **A.REI.4ab** *Solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square. Use the quadratic formula to solve any quadratic equation, recognizing the formula always produces solutions.* **Example 1:** Find the solutions to the following quadratic equations. a. $x^2 - 7x - 18 = 0$ b. $x^2 = 81$ c. $x^2 - 10x + 5 = 0$ **F.LE 6** *Students experiment with quadratic functions and discover their uses in representing real-world*	

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		Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **F.LE 6** Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity **F.IF 8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *a.* Use the process of factoring and completing the square in a quadratic function to show zeros.	*phenomena, such as projectile motion.* **Example 1**: A simple activity that involves tossing a ball and recording its height with video as it rises and falls can reveal the height as a function of time to be approximately quadratic. Afterwards, students can derive a quadratic expression that determines the height of the ball at time *t*38T using a graphing calculator or other software, and they can compare the values of the function with their data. **F.IF 8a** *Students should take a quadratic function and manipulate it in a different form (standard to factored) so that they can show and explain special properties of the function such as; zeros, extreme values, and symmetry.* **Example 1:** Coyote was chasing road runner, seeing no easy escape, Road Runner jumped off a cliff towering above the roaring river below. Molly Mathematician was observing the chase and obtained a digital picture of this fall. Using her mathematical knowledge, Molly modeled the Road Runner's fall with the following quadratic functions: $-16t^2 + 32t + 48$ $-16(t + 1)(t – 3)$ a. How can Molly have two equations?	

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			b. Which of the rules would be most helpful in answering each of these questions? Explain. c. What is the maximum height the Road Runner reaches and when will it occur? d. When would the Road Runner splash into the river? e. At what height was the Road Runner when he jumped off the cliff?	
Differentiation	**Support -- for students who are struggling with the content**	Content: Unit 5 Process: Pre-Unit on Exponents, UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 5 procedural content.		
	Extension – for high achieving students.	Content: Unit 5 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.		
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*			
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*			

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Unit 6	**3 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Probability • Events and Sample Spaces • Independent Events/Dependent/Mutually Exclusive Events • Conditional Probability • Two-Way Frequency Tables • Apply the Addition Rule • Geometric Probability Models • Unions, intersections, complements	• *Event* • *Sample Space* • *Independent* • *Dependent* • *Mutually exclusive* • *Disjoint* • *Probability* • *Union* • *Intersection* • *Complement* • *Outcome* • *Subset* • *Two-way frequency table* • *Conditional* • *Model* • *Addition Rule* • *Interpret* • *Data* • *Random sample*	**S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	**S.CP.1** *Students will define the sample space for a given situation. Students will describe an event in terms of categories or characteristics of the outcome of the sample space. Students will describe an event as a union, intersection, or complement of other events.* **Example 1:** What is the sample space for rolling a dice? **Example 2:** Describe different subsets of outcomes for rolling using a single category or characteristic. **Example 3:** Describe the following subset of outcomes for choosing one card from a standard deck of cards as the intersection of two events. **S.CP.2** *Student will understand that two events A and B are independent if and only if P(A and B) = P(A)P(B). Students will need to determine if two events of independent.* **Example 1:** For the situation of drawing a card from a standard	*Holt Algebra 2:* *11-2* *11-3* *11-4* **<u>Illustrative Math Tasks</u>** • *The Titanic 1* • *Cards and Independence* • *Rain and Lightning* • *The Titanic 2* • *Return to Fred's Fun Factory* • *Lucky Envelopes* • *The Titanic 3* • *How do you get to school?* • *Breakfast before school* • *Coffee at Mom's Diner*

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			deck of cards, consider the two events of “draw a diamond” and “draw an ace.” Determine if these two events are independent. **Example 2:** Create and prove two events are independent from drawing a card from a standard deck.	
		S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	**S.CP.4** **Example 1:** Collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. **S.CP. 5** *Given an everyday situation describing two events, use the context to construct an argument as to whether the events are independent or dependent.* **Example 1**: Felix is a good chess player and a good math student. Do you think that the events “being good at playing chess” and “being a good math student” are independent or dependent? Justify your answer.	

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		S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **S.CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	**Example 2**: Juanita flipped a coin 10 times and got the following results: T, H, T, T, H, H, H, H, H, H. Her math partner Harold thinks that the next flip is going to result in tails because there have been so many heads in a row. Do you agree? Explain why or why not. **S.CP. 6** *Students will understand that when finding the conditional probability of A given B, the sample space is reduced to the possible outcomes for event B. Therefore, the probability of event A happening is the fraction of event B's outcomes that also belong to A. Students will understand that drawing without replacement produces situations involving conditional probability. Students will calculate conditional probabilities using the definition. Students will interpret the probability in context.* **Example 1**: Peter has a bag of marbles. In the bag are 4 white marbles, 2 blue marbles, and 6 green marbles. Peter randomly draws one marble, sets it aside, and then randomly draws another marble. What is the probability of Peter drawing out two green marbles? **S.CP. 7** *Students will understand*	

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			that two events A and B are mutually exclusive if and only if $P(A \text{ and } B)=0$. In other words, mutually exclusive events cannot occur at the same time. Students will determine whether two events are disjoint. Given events A and B, calculate $P(A \text{ or } B)$ using the Addition Rule. **Example 1**: Given the situation of rolling a six---sided die, determine whether the following pairs of events are disjoint: a. rolling an odd number; rolling a five b. rolling a six; rolling a prime number c. rolling an even number; rolling a three d. rolling a number less than 4; rolling a two **Example 2**: Given the situation of drawing a card from a standard deck of cards, calculate the probability of the following: a. drawing a red card or a king b. drawing a ten or a spade c. drawing a four or a queen d. drawing a black jack or a club e. drawing a red queen or a spade	
Differentiation	**Support -- for students who are struggling with the content**	Content: Unit 6 Process: UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 6 procedural content.		

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	Extension – for high achieving students.	Content: Unit 6 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*	
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*	

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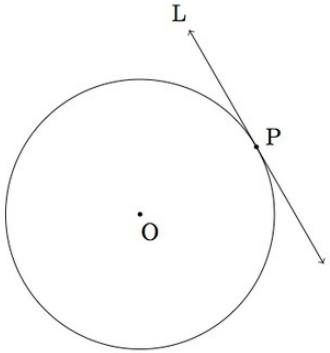
Unit 7	**3 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Geometric Proof • Theorems about Lines and Angles (e.g., Vertical angles; Alternate Interior) • Theorems about triangles (e.g., Interior Angles; Base Angles, Interior Angles) and parallelograms • Dilation Properties • Similarity through Transformation • CPCTC • AA Similarity • Use congruence and similarity to solve problems	• Theorem • Vertical angle • Transversal • Parallel • Alternate interior • Corresponding angles • Perpendicular bisector • Equidistant • Segment • Endpoint • Congruent • Interior angles • Isosceles triangle • Dilation • Midpoint • Median • Parallelogram • Scale factor • Similarity • Transformation • Ratio	**G.CO. 9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints **G.CO. 10** Prove theorems about triangles. Theorems	***G.CO. 9*** *Students will prove theorems pertaining to lines and angles. Students will prove vertical angles are congruent. Students will prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent. Students will prove points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **Example 1:** Consider a circle with center *O* and let *P* be a point on the circle. Suppose *L* is a tangent line to the circle at *P*, that is *L* meets the circle only at *P*. Show that OP is perpendicular to L.  **G.CO. 10** *Students will prove theorems pertaining to triangles. Students will prove the measures of interior angles of a triangle have a sum of 180º. Students will prove base angles of isosceles triangles are congruent. Students will prove the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.*	*Glencoe Geometry:* *3-1* *3-2* *3-5* *3-6* *2-8* *4-4* *4-5* *4-6* *4-7* *5-1* *5-2* *5-3* *5-4* *5-5* *6-2* *6-3* *7-3* *7-4* *7-5* *6-7* *9-5* **<u>Illustrative Math Tasks</u>** • *Tangent Lines and Radius of a Circle* • *Points equidistant from two points in a plane* • *7 Circles 1* • *Classifying*

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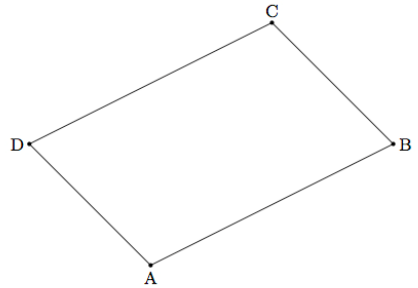
about triangles • Prove Pythagorean theorem by triangle similarity		include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. **G.CO. 11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	*Students will prove the medians of a triangle meet at a point.* **Example 1**: Using any method you choose, construct the medians of a triangle. Each median is divided up by the centroid. Investigate the relationships of the distances of these segments. Can you create a deductive argument to justify why these relationships are true? Can you prove why the medians all meet at one point for all triangles? **G.CO. 11** *Students will prove theorems pertaining to parallelograms. Students will prove opposite sides are congruent. Students will prove opposite angles are congruent. Students will prove the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **Example 1:** In quadrilateral *ABCD* pictured below, *AB* is congruent to *CD* and *BC* is congruent to *AD*. Can we deduce that ABCD is a parallelogram? Explain.  	*Triangles* • *Is this a parallelogram?* • *Parallelogram and Translations* • *Dilating a Line* • *Are they similar?* • *Similar triangles* • *Pythagorean Theorem* • *Defining Trigonometric Ratios* • *Tangent line to 2 circles* • *Bank Shot*

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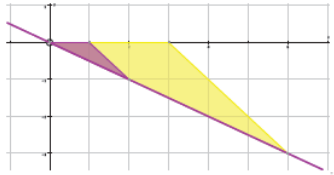
		G.SRT.1 Verify experimentally the properties of dilation given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using	**G.SRT.1** *When given a center of dilation and a scale factor, students will verify using coordinates that when dilating a figure in a coordinate plane, a segment of the pre---image that does not pass through the center of the dilation, is parallel to its image when the dilation is preformed. However, a segment that passes through the center does not change.* **Example 1:**  **Rule for the transformation is (x,y)→(⅓x,⅓y).** Comment on the slopes and the lengths of the segments that make up the triangles. **G.SRT.2** *Students will use the idea of dilation transformations to develop the definition of similarity. When given two figures, students will determine whether they are similar and explain their similarity based on the equality of corresponding angles and the proportionality of corresponding sides.* **Example 1:** In the picture given below, line segments *AD* and *BC* intersect at *X*. Line segments *AB* and *CD* are drawn, forming two triangles *AXB* and *CXD*.	

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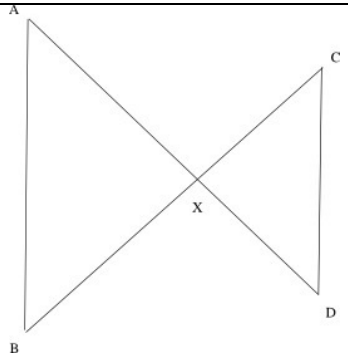
		similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3** Use the properties of similarity transformations to establish the	 In each part (a)-(d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not. a. The lengths *AX* and *XD* satisfy the equation $2AX=3XD$. b. Lines *AB* and *CD* are parallel. c. Angle *XAB* is congruent to angle *XCD*. **G.SRT.3** *Students will use the properties of similarity transformations to develop the criteria for proving similar triangles; AA.* **Example 1:** In the two triangles below, the measure of angle A is equal to the measure of angle D. The measure of angle B is equal to the measure of angle E.	

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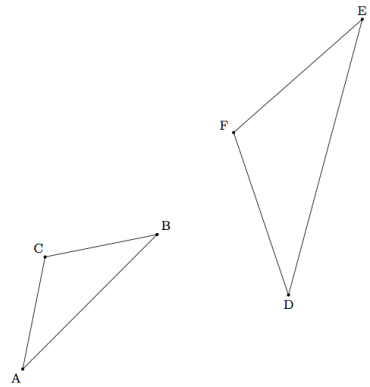
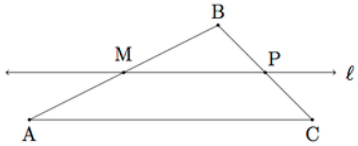
		Angle-Angle (AA) criterion for two triangles to be similar. **G.SRT.4** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle	 Using a sequence of rotations, reflections, translations, and/or dilations show that triangle ABC is similar to triangle DEF. **G.SRT.4** *Students will use AA, SAS, SSS similarity theorems to prove triangles are similar. Students will use triangle similarity to prove other theorems about triangles. Students will prove a line parallel to one side of a triangle divides the other.* **Example 1:** Suppose ABC is a triangle. Let M be the midpoint of segment AB and the line *l* is parallel to line AC through M.  Show that angle CAB is congruent to angle PMB and that angle BPM is congruent to angle BCA. Conclude that triangle MBP is similar to triangle ABC.	

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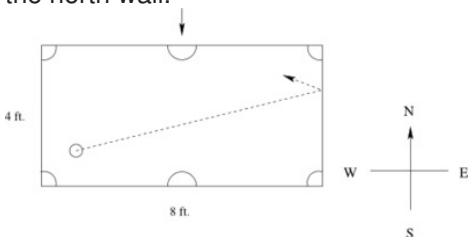
		similarity. **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute	**G.SRT.5** *Students will prove that two triangles are congruent. using similarity theorems. Students will prove geometric figures, other than triangles, are similar and/or congruent.* **Example 1:** Pablo is practicing bank shots on a standard 4 ft.-by-8 ft. pool table that has a wall on each side, a pocket in each corner, and a pocket at the midpoint of each eight-foot side. Pablo places the cue ball one foot away from the south wall of the table and one foot away from the west wall, as shown in the diagram below. He wants to bank the cue ball off of the east wall and into the pocket at the midpoint of the north wall.  At what point should the cue ball hit the east wall? **G.SRT.6** *Students will show that the relationships of the side ratios are the same, which leads to the definition of trigonometric ratios for acute angles, Using a corresponding angle of similar right triangles.* **Example 1:**	

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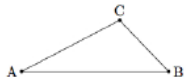
		angles.	 a. Draw a triangle DEF which is similar (but not congruent) to $\triangle ABC$. b. How do $\frac{\lvert DE\rvert}{\lvert DF\rvert}$ and $\frac{\lvert AB\rvert}{\lvert AC\rvert}$ compare? Explain. c. When $\angle B$ is a right angle, the ratio $\lvert AB\rvert : \lvert AC\rvert$ is called the cosine of $\angle A$ while the ratio $\lvert BC\rvert : \lvert AC\rvert$ is called the sine of $\angle A$. Why do these ratios depend only on $\angle A$? d. The ratios in part (c) make sense whether or not $\angle B$ is a right angle but they are only given names (sine and cosine) in this special case. What is special about the case where B is a right angle?	
Differentiation	**Support -- for students who are struggling with the content**	Content: Unit 7 Process: UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies. Product: Proficiency in Unit 7 procedural content.		
	Extension – for high achieving students.	Content: Unit 7 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.		
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*			
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*			

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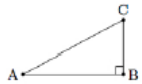
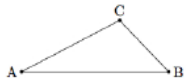
Unit 8	**3 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Right Triangle Trigonometry • $\sin^2\theta + \cos^2\theta = 1$ (weak reference) • Similarity to find trig ratios • Relationships between sine and cosine in complementary angles • Use the trig ratios and Pythagorean theorem to solve right triangles • Derive and use trig ratios for special right triangles	• Trigonometric fuctions • Similarity • Right triangle • Sine • Cosine • Tangnet • Quadrant • Acute • Compelementary • Right triangle •	**F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant. **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** Explain	**F.TF.8**  a. In the triangle pictured above show that $\left(\frac{\lvert AB\rvert}{\lvert AC\rvert}\right)^2 + \left(\frac{\lvert BC\rvert}{\lvert AC\rvert}\right)^2 = 1$ b. Deduce that $\sin^2\theta + \cos^2\theta = 1$ for any acute angle θ. c. If θ is in the second quadrant and $\sin\theta = \frac{8}{17}$ what can you say about $\cos\theta$? Draw a picture and explain. **G.SRT.6** *Students will show that the relationships of the side ratios are the same, which leads to the definition of trigonometric ratios for acute angles, Using a corresponding angle of similar right triangles.* **Example 1:**  a. Draw a triangle DEF which is similar (but not congruent) to $\triangle ABC$. b. How do $\frac{\lvert DE\rvert}{\lvert DF\rvert}$ and $\frac{\lvert AB\rvert}{\lvert AC\rvert}$ compare? Explain. c. When $\angle B$ is a right angle, the ratio $\lvert AB\rvert : \lvert AC\rvert$ is called the cosine of $\angle A$ while the ratio $\lvert BC\rvert : \lvert AC\rvert$ is called the sine of $\angle A$. Why do these ratios depend only on $\angle A$? d. The ratios in part (c) make sense whether or not $\angle B$ is a right angle but they are only given names (sine and cosine) in this special case. What is special about the case where B is a right angle?	*Glencoe Geometry:* *8-1* *8-2* *8-3* *8-4* *8-5* *Holt Algebra 2:* *13-1* *13-2* **<u>Illustrative Math Tasks</u>** • *Defining Trigonometric Ratios* • *Sine and Cosine of Complementary Angles* • *Setting up Sprinklers* • *7 Circles 3* • *Ask the pilot* • *Coins in a circular pattern* • *Trigonometric Ratios and the Pythagorean Theorem*

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		and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **G.SRT.8.1** Derive and use the trigonometric ratios for special right triangles (30°, 60°, 90°and 45°, 45°, 90°).	**G.SRT.7** *Students will explore the sine of an acute angle and the cosine of its complement and determine their relationship.* **Example 1:** Suppose $0<a<90$ is the measure of an acute angle. Draw a picture and explain why $\sin a = \cos(90-a)$ Are there any angle measures $0<a<90$ for which $\sin a = \cos a$. Explain. **G.SRT.8** *Students will apply both trigonometric ratios and Pythagorean Theorem to solve application problems involving right triangles.* **Example 1:** An onlooker stands at the top of a cliff 119 meters above the water's surface. With a clinometer, she spots two ships due west. The angle of depression to each of the sailboats is 11° and 16°. Calculate the distance between the two sailboats. What is the distance from the onlooker's eyes to each of the sailboats? What is the difference of those distances? Explain why or why not this difference is not the same as the distance between the two sailboats. **G.SRT.8.1 :** Selina has a 9 meter by 12 meter lawn that she wants to water. She has two sprinklers, each of which can water grass within an 8.4-meter radius. Selina wants to set up the two sprinklers so that they are on corners of the lawn. She would like for the sprinklers to water as much of the lawn as possible, because she will have to manually water the part of the lawn that is not covered by the sprinklers. *http://www.illustrativemathematics.org/illustrations/607*	
Differentiation	**Support -- for students who are**	Content: Unit 8 Process: UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk		

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	struggling with the content	strategies. Product: Proficiency in Unit 8 procedural content.
	Extension – for high achieving students.	Content: Unit 8 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*	
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*	

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Unit 9	**4 weeks**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Circles and Coordinate Geometry • Prove all circles are similar • Relationships of angles, radii and chords • Inscribed and Circumscribed triangles and quadrilaterals • Arc length; radian measure; area of a sector • Derive the equation of a circle • Complete the square to find the radius and center of the circle • Derive the equation of a parabola • Coordinate geometry proofs; simple circle theorems, etc. • Circumference and Area	• Circles • Similar • Angle • Diameter • Radius • Chord • Inscribed • Circumscribed • Right angle • Perpendicular • Tangent • Intersect • Arc • Radian • Sector • Area • Quadrilateral • Proportional • Circumference • Area • Parabola • Focus • Directrix • Line segment • Ratio	**G.C. 1** Prove that all circles are similar. **G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular	**G.C. 1** *Students will prove that all circles are similar, using the fact that the ratio of diameter to circumference is the same for circles.* **Example 1:** For this problem, (a,b) is a point in the x-y coordinate plane and r is a positive number. a. Using a translation and a dilation, show how to transform the circle with radius r centered at (a,b) into the circle of radius 1 centered at (0,0) . b. Explain how to use your work in part (a) to show that any two circles are similar. **G.C. 2** *Students will identify and describe relationships among inscribed angles, radii, and chords, using definitions, properties, and theorems. Students will understand that inscribed angles on a diameter are right angles. Students will understand that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	*Glencoe Geometry:* *10-1* *10-2* *10-3* *10-4* *10-5* *10-6* *10-7* *10-8* **Illustrative Math Tasks** • *Similar Circles* • *Right triangles inscribed in circles 1* • *Right triangles inscribed in circles 2* • *Locating Warehouse* • *Placing Fire Hydrant* • *Tangent to a circle from a point* • *Mutually tangent circles*

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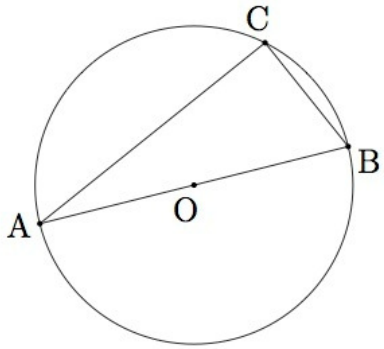
		to the tangent where the radius intersects the circle. **G.C. 3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	**Example 1:** Suppose segment AB is a diameter of a circle and C is point on the circle different from A and B.  a. Show that triangles COB and COA are both isosceles triangles. b. Use part a and the fact that the sum of the angles in triangle ABC is 180 degrees to show that angle C is a right angle. **G.C. 3** *Students will construct inscribed circles of a triangle and construct circumscribed circles of a triangle. Students will prove properties of angles for a quadrilateral inscribed in a circle, using definitions, properties, and theorems.* **Example 1:** The *circumcenter* of a triangle *ABC* is the point in the plane which is equidistant from the three vertices *A*, *B*, and *C*. a. If *O* is the circumcenter	• *Slopes and Circles* • *Midpoint Miracle* • *Finding Triangle Coordinates*

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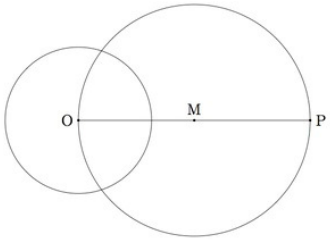
		G.C. 4 Construct a tangent line from a point outside a given circle to the circle.	of $\triangle ABC$, show that $\triangle ABC$ can be inscribed in a circle with center O. b. Show that $\triangle ABC$ cannot have more than one circumcenter. **G.C. 4** *Students will construct a tangent line from a point outside a given circle to the circle.* **Example 1:** Suppose C is a circle with center O and P is a point outside of C. Let M be the midpoint of segment OP and let D be the circle with center M passing through O.  Let A and B be the two points of intersection of C and D, pictured below along with several line segments of interest:	

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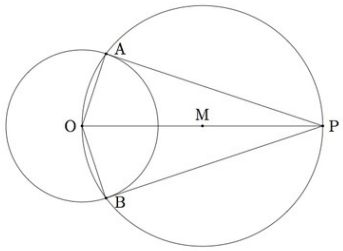
		G.C. 5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians. **G.GPE. 1** Derive the equation of a circle of given center and radius	 a. Show that angles *OAP* and *OBP* are right angles. b. Show that *PA* and *PB* are tangent lines from *P* to the circle *C*. **G.C. 5** *Students will use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius, identifying the constant of proportionality as the radian measure of the angle. Students will find the arc length of a circle. Using similarity, students derive the formula for the area of a sector. Students will find the area of a sector in a circle.* **Example 1:** Three circles, each having radius 2, are mutually tangent as pictured below.	

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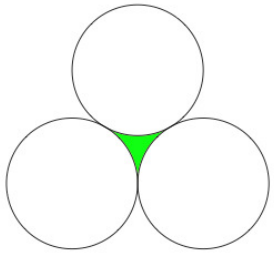
		using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G.GPE. 2** Derive the equation of a parabola given a focus and directrix. **G.GPE. 4** Use coordinates to prove simple geometric theorems algebraically.	 What is the total area of the circles together with the shaded region? **G.GPE. 1** *Students will see the Pythagorean Theorem to derive the equation of a circle, given the center and the radius. When given an equation of a circle, students will complete the square to find the center and radius of a circle.* **Example 1:** Given a coordinate and a distance from that coordinate develop a rule that shows the locus of points that is that given distance from the given point (based on the Pythagorean theorem). **G.GPE.2** *When given a focus and directrix, students will derive the equation of a parabola. When given a parabola, students will identify the vertex, focus, directrix, and axis of symmetry, noting that every point on the parabola is the same distance from the focus and the directrix.*	

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		G.GPE. 6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**G.GPE. 4** *Students will use coordinate geometry to prove geometric theorems algebraically; such as prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **Example 1:** Draw a quadrilateral *ABCD*. Try to draw your quadrilateral so that no two sides are congruent, no two angles are congruent, and no two sides are parallel. a. Let *P*, *Q*, *R*, and *S* be the midpoints of sides *AB*, *BC*, *CD*, and *DA*, respectively. Use a ruler to locate these points as precisely as you can, and join them to form a new quadrilateral *PQRS*. What do you notice about the quadrilateral *PQRS*? b. Suppose your quadrilateral *ABCD* lies in the coordinate plane. Let ($x1$,$y1$) be the coordinates of vertex *A*,($x2$,$y2$) the coordinates	

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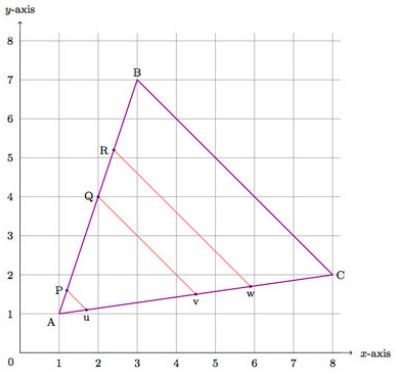
			of *B*, (*x*3,*y*3) the coordinates of *C*, and (*x*4,*y*4) the coordinates of *D*. Use coordinates to prove the observation you made in part (a). **G.GPE. 6** *When given two points, students will find the point on the line segment between the two points that divides the segment into a given ratio.* **Example 1:** Below is a picture of a triangle *ABC* on the coordinate grid. The red lines are parallel to *BC.*  *S*uppose *P*=(1.2,1.6), *Q*=(2,4), and *R*=(2.4,5.2). Find the coordinates of the points *u*, *v*, and *w*.	
Differentiation	**Support -- for students who are struggling with the content**	Content: Unit 9 Process: UCI Intervention Units, manipulatives, flexible grouping, visual organizers and student talk strategies.		

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		Product: Proficiency in Unit 9 procedural content.
	Extension – for high achieving students.	Content: Unit 9 Process: Robust and differentiated problem solving involving a Leveled Problem of the Month. Product: Analyze content and use it to solve real-world and mathematical problems.
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess students understanding of math targeted content and the application of the 8 Common Core Mathematical Practices as well as to and determine whether to continue with the lesson or if re-teaching is required. Strategies may include, but are not limited to check for understanding, quizzes, performance tasks, and homework review, with emphasis of the 8 Mathematical Practices.*	
	Summative Assessments *(unit final evaluation):* *Typically used to assess students' achievement at the end of units or end of course. Strategies may include, but are not limited to, district wide unit tests, Smarter Balanced Interim Assessments, Smarter Balanced Assessments and final exams.*	

Unit 10	**Length of Unit … 720 minutes…16 traditional periods…8 block periods**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
Statistics **Use mean and standard**	Mean Standard deviation	S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate	S.ID.4 Understand the characteristics of the Standard Normal Distribution including symmetry, the Empirical Rule and the fact that mean, median, and mode are all equal.	*IMP*

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deviation to fit data to a Normal distribution **Estimate area under the Normal curve** **Understand the statistics allows us to make inferences** **Determine if a model is consistent with results** **Sample surveys, experiments, and observational studies: purpose – use data from margin of error** **Use simulations to look at differences between parameters** **Evaluate**	z-score normal curve	population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. S.IC.1 Understand statistics as a process for making inferences to be made about population	Ex. Describe the characteristics of the Standard Normal Distribution. S.ID.4 Find the number of standard deviations a value is from the mean by calculating the z-score. Ex. IQ scores are approximately normally distributed with a mean of 100 and a standard deviation of 15. How many standard deviations below the mean is an IQ score of 75? S.ID.4 Use the Empirical Rule to estimate population percentages for a set of data that is approximately normally distributed. Ex. IQ scores are approximately normally distributed with a mean of 100 and a standard deviation of 15. What percent of the population has IQ scores above 115? What percent have IQ scores between 85 and 130? S.ID.4 Use a table of z-scores, spreadsheets, and calculators to find areas under the curve to estimate population percentages. Interpret these percentages in context. Ex. The heights of adult males are approximately normally distributed with a mean of 70 in. and a standard deviation of 3 in. What percentage of adult males are between 5.5 and 6.5 feet tall? What percentage of males are 7 feet tall or taller? S.IC.1 Explain the difference between a population parameter and a sample statistic. S.IC.1 Understand that random samples tend to be representative of the population they are drawn from and therefore we can draw conclusions about the population based	*Lessons:* *Skew the data* *What can I conclude?* *Who won the election?* *Opening a store?*

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reports based upon data		parameters based on a random sample from that population. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	on the sample. If conclusions cannot be drawn from the random sample, discuss why and propose a better way to select a random sample. Ex. From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection. I. Select the first three names on the class roll. II. Select the first three students who volunteer. III. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix. IV. Select the first three students who show up for class tomorrow. Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class? Explain the weaknesses of the three you did not select as the best. S.IC.2 Ex. A random sample of 100 students from a specific high school resulted in 45% of them favoring a plan to implement block scheduling. Is it plausible that a majority of the students in the school actually favor the block schedule? Simulation can help answer the questions. The accompanying plot shows a simulated distribution of sample proportions for samples of size 100 from a population in which 50% of the students favor the plan, and another distribution from a population in which 60% of the students favor the plan. (Each simulation contains 200 runs.)	

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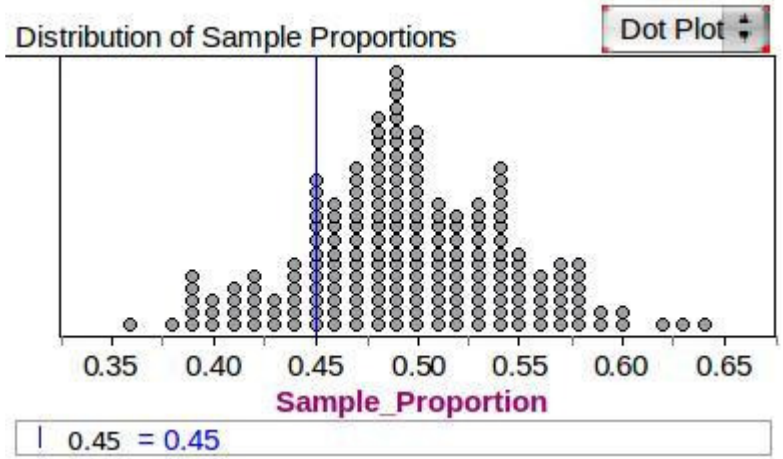
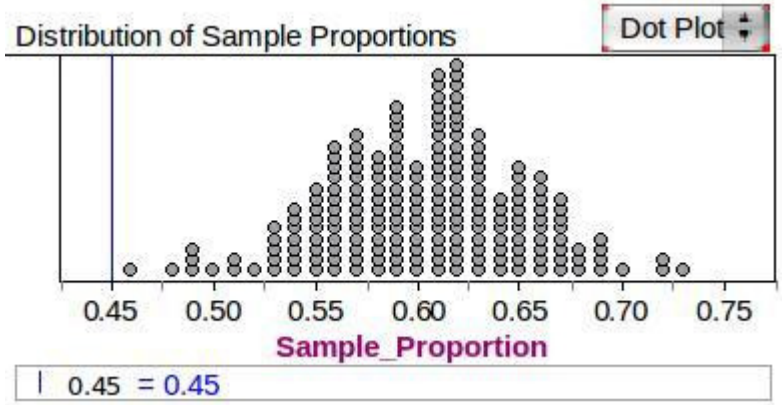
			1. What do you conclude about the plausibility of a population proportion of 0.50 when the sample proportion is only 0.45? 2. What about the plausibility of 0.60 for the population proportion? Population proportion 0.5; sample size 100   Population proportion .6; Sample size 100	

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		S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models	S.IC 3 Ex. Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as “strict”. They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students. 1. Describe the parameter of interest and a statistic the students could use to estimate the parameter. 2. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning. 3. The students quickly realized that, as there is no definition of “strict”, they could not simply ask a student, “Are your parents or guardians strict?” Write three questions that could provide objective data related to strictness. 4. Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above. S.IC.4 Ex. Sometimes hotels, malls, banks, and other businesses will present a display of a large, clear container holding a large number of items and ask customers to estimate some aspect of the items in the container as a contest. In some cases, contestants are allowed to sample items from the jar; but usually, contestants simply have to estimate based on visual inspection of the jar. A local bank is running such a contest, but one of the bank employees is concerned.	

		for random sampling.	The bank has placed 1,500 marbles in a very large, clear jar near the customer entrance. Since the bank's logo's colors are blue and white, some of the 1,500 marbles are blue and the rest are white. In order to enter the contest, a customer must fill in an entry form with his/her estimate for the percentage of blue marbles in the jar and then place the entry form in a ballot box. A random drawing will be held and the first entry drawn that correctly estimates the percentage of blue marbles in the whole jar will receive a $100 gift certificate. The entry form says the following: Name: _____ Phone: _____ I think that 1 out of every _____ marbles in this jar is blue. (Fill in the blank with a "2", "3", "4", "5", or "6".) Note that for the ease of the contestants, the estimate is to be stated as "1 out of every 2" instead of "50%," "1 out of every 3" instead of "33.3%," and so on. Now the concerned employee is fairly confident that the true proportion of blue marbles is 25% (1 out of every 4), but he has heard other employees (some of whom are responsible for the contest) state a true proportion value that is different. The employee is worried enough that he wants to investigate but he certainly does not want to empty the jar and inspect all 1,500 marbles! He decides to select a random sample of marbles from the jar and calculate the percentage of blue marbles in his sample. The percentage of blue marbles in the random sample will be his estimate for the actual percentage of blue marbles in the jar. He selects a random sample of 5 marbles, and only 1 of the marbles is blue. Based on this sample which gives him an estimate of 20% (1 out of 5) blue marbles, the employee is	

concerned, but he decides to stick with his original claim of 25% blue marbles in the jar. However, he is now inspired to take even larger samples. He records his results in the table below (additional spaces will be filled in eventually).

Sample Number	**Sample Size**	**Total Number of Blue Marbles in Sample**	**Percentage of Blue Marbles in Sample**
1	5	1	20 %
2	12	2	
3	20	5	
4		32	
5			
6			
7			
8			
9			

1. His second random sample consists of 12 marbles. Only 2 of the marbles are blue. (This is recorded in the table above.) Compute the sample percentage of blue marbles for this sample and record it in the table. Based on this sample, do you think the employee should stick with his original claim of 25% blue marbles in the jar, or should he come up with a different estimate? Explain why you think this.
2. He then takes a random sample of 20 marbles (Sample 3). Five of the 20 marbles are blue. Compute the sample percentage of blue marbles for this sample and record it in the table. Based on this sample, do you think the employee should stick with his original claim of 25% blue marbles in the jar, or should he come up with a different estimate? Explain why you think this.
3. He then takes a random sample of 32 marbles (Sample 4). Eight of the marbles are blue. Enter this information

			on the table, and compute the sample percentage of blue marbles for this sample. Based on this sample, do you think the employee should stick with his original claim of 25% blue marbles in the jar, or should he come up with a different estimate? Explain why you think this. At this point, the employee feels compelled to talk to the bank manager who is responsible for the contest. The bank manager is a little surprised by the results, but she is not overly concerned. She is quite confident that the true proportion of blue marbles is 33.3%, or 1 in every 3 (i.e., 5,000 blue, 10,000 white marbles), and she asks the concerned employee to go back and look at an even larger random sample of marbles. 4. He then takes a random sample of 40 marbles (Sample 5) and 13 of the marbles are blue. Add this information to the table. Based on this sample, and mindful that the correct, true proportion of blue marbles in the jar is 1 in 2, or 1 in 3, or 1 in 4, etc., do you think the employee should challenge the bank manager's claim that 1 in every 3 marbles is blue? Explain why you think this. 5. Here are the results of some additional random samples. Record each of these in the table and compute the blue marble percentage for each sample. Sample 6, sample size = 55, 17 blue. Sample 7, sample size = 65, 21 blue. Sample 8, sample size = 75, 24 blue. Sample 9, sample size = 85, 27 blue. 6. Based on the random sample of 85 marbles, and mindful that the correct, true proportion of blue marbles in the jar is 1 in 2, or 1 in 3, or 1 in 4, etc., do you think that the employee should challenge the bank manager's claim that 1 in every 3 marbles is blue? Explain why you think this. 7. Keeping in mind that the samples were random samples,	

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			and assuming that the bank manager's claim is correct that the true proportion of blue marbles is 33.3% (1 in every 3), did the employee get more accurate estimates from the small samples or from the large samples.	
		S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	S.IC.5 Determine whether one treatment is more effective than another treatment in a randomized experiment. Ex. You conduct a survey at your school to determine who is most likely to be voted as Prom Queen. You randomly survey 20 students and receive 8 votes for Gabby, 5 votes for Anna, 4 votes for Sarah, and 3 votes for Lola. How likely is it that Gabby will be Prom Queen? Create an argument justifying your conclusion. S.IC.4 Use simulation to collect multiple samples. Calculate the sample mean or proportion for each and use this information to determine a reasonable margin of error for the population estimate. Ex. For the situation above, use technology to generate random numbers to simulate multiple samples of 20 votes. Calculate the proportion of votes for Gabby in each sample and use this information to determine a reasonable margin of error for the proportion of students who are going to vote for Gabby for Prom Queen.	
		S.IC.6 Evaluate reports based upon data.	S.IC.6 Evaluate reports based on data on multiple aspects (e.g. experimental design, controlling lurking variables, representativeness of samples, choice of summary statistics, etc) Ex. Find a statistical report based upon an experiment and evaluate the experimental design.	
Differentiation		Content: *Unit 10*		

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	Support -- for students who are struggling with the content	Process: *UCI intervention units, flexible grouping, organizers,* Product: *Proficiency in Unit 1 content*
	Extension – for high achieving students.	Content: *Unit 10* Process: *Robust and differentiated problem solving* Product: *Solutions to real world and mathematical problems*
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess student understanding of Math 3 common core standards in unit 1 and the application of the mathematical practices to determine whether to move forward with lessons or to reteach. Strategies may include, but are not limited to informal checking for understanding, quizzes, performance tasks, and homework review.*	
	Summative Assessments *(unit final evaluation):* ***Typically used to assess student achievement at the end of the unit. Strategies include but are not limited to, district wide unit tests, Smarter Balanced Assessments, a final exam, and a modeling performance task.***	

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Unit 11 Added Plus (+) Standards to Enhanced Math II	**Length of Unit … 720 minutes…16 traditional periods…8 block periods**			
	Key Vocabulary	**Standards (referenced)**	**Model Tasks**	**Tools / Texts**
	degree of a polynomial *Fundamental Theorem of Algebra* *roots* *solutions* *x-intercepts* *zeros* conditional probability dependent events independent events intersecting sets subset two way table	N.CN.8 (+) Extend polynomial identities to the complex numbers. N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	**N.CN.8:** **Example**: Rewrite x + 4 as (x+ 2i)(x-2i) **N.CN.9:** **Example:** How many solutions does a polynomial function posses? How does the Fundamental Theorem of Algebra apply to the solutions of a polynomial function? How do you show that a quadratic polynomial has at most two solutions? **S.CP.8:** **Example:** How can the probability of a series of events, or multiple events occurring simultaneously, be computed? Apply the Multiplication Rule with intersecting sets A and B (e.g., a Venn Diagram or two-way table to apply Multiplication Rule) and interpret the Multiplication rule with intersecting sets A and B. **S.CP.9:** **Example:** What is the difference between permutations and combinations? How can they be used to calculate probabilities? Determining the number of outcomes of events can be done using the formulas for permutations and combinations. Determine when a problem requires permutations and when it requires combinations.	*Houghton Mifflin Calculus with Pre-Calculus 2nd Edition* *Pearson/ Prentice Hall Sullivan Pre-Calculus*

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	Venn Diagram	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Solve problems involving permutations and combinations. Solve probability problems involving permutations and combinations. Derive the formulas for the number of combinations and permutations of n objects taken r at a time. **S.MD.6:** **Example:** How can probability be used to make fair decisions? Determine the fairness of a decision based on the available data. Determine the fairness of a decision by comparing theoretical and experimental probability. Use counting principles to determine the fairness of a decision. **S.MD.7:** **Example:** Suppose a town has about 25,000 adults in its population. If the unemployment rate in the town is 5%, and you sampled 200 adults from the town at random, what is your expected count of unemployed? **Example 2:** Consider the following experiment. Two 6-sided dice are thrown 1000 times and the sum of the numbers on the dice are recorded. A sum of 7 occurred 175 times. Assume that the dice are fair. a. What is the expected number of 7s? b. What is the difference between the observed and the expected number of 7s? c. What is the relative frequency of 7s? d. What is the difference between the relative frequency of 7s and the probability of 7s?	

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		G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle	**Example 3:** http://www.insidemathematics.org/problems-of-the-month/pom-gotyournumber.pdf **G.C.4:** Example 1: How can one construct a tangent line from a point outside a given circle to the circle? **Example 2**: http://www.illustrativemathematics.org/illustrations/963	
Differentiation	**Support -- for students who are struggling with the content**	Content: *Unit 11* Process: *UCI intervention units, flexible grouping, organizers,* Product: *Proficiency in Unit 1 content*		
	Extension – for high achieving students.	Content: *Unit 11* Process: *Robust and differentiated problem solving* Product: *Solutions to real world and mathematical problems*		
Evaluation	***Formative Assessments*** *(ongoing & mid-lesson):* *Used to assess student understanding of Math 3 common core standards in unit 1 and the application of the mathematical practices to determine whether to move forward with lessons or to reteach. Strategies may include, but are not limited to informal checking for understanding, quizzes, performance tasks, and homework review.*			
	Summative Assessments *(unit final evaluation):* ***Typically used to assess student achievement at the end of the unit. Strategies include but are not limited to, district wide unit tests, Smarter Balanced Assessments, a final exam, and a modeling performance task.***			

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