Summer Review
For Students Entering
<u>Geometry Honors</u>

Dear Parents and Students,

This packet is designed to provide a review of Algebra I Honors skills that were taught during this school year. Students entering Geometry Honors will still need understanding of these skills in Geometry and Algebra II later on. This packet is designed to be a refresher of those skills so students can be well prepared as they enter Geometry Honors.

DIRECTIONS:
Student will need to use pencil and show work on lined paper or in the packet. All work must accompany the completed packet in order to receive full credit.

RESOURCES:
In addition to the instruction and examples included in the packet, here are some websites that provide additional assistance:

- www.ixl.com
- www.khanacademy.org

GRADING:
Students are expected to bring the completed packet with all work shown to school on the first day. Teachers will collect the work and award a completion grade for homework. Teachers will return the work and provide correct answers as well as go over any questions students may have in class.

FRACTIONS

In Honors Geometry we will be requiring more exact answers (fractions) and less approximate answers (decimals).

Adding and Subtracting

Example $\frac{3}{4} + \frac{4}{5}$

1. Find the common denominator; 20

2. Convert your fractions so that the common denominator is the denominator for both.

   $\frac{15}{20} + \frac{16}{20}$

3. Add the numerators. Keep the common denominator

   $\frac{15+16}{20} = \frac{31}{20}$

4. Simplify if possible.

Multiplying and Dividing.

Dividing fractions is equivalent to multiplying by the reciprocal. When multiplying fractions, multiply the numerators together and the denominators together. Then simplify.

Example $\frac{5}{7} \cdot \frac{10}{12} = \frac{50}{84} = \frac{25}{42}$

Example $\frac{-2}{6} \div \frac{5}{9}$

$$= \frac{-2}{6} \cdot \frac{9}{5} = \frac{-18}{30} = \frac{-3}{5}$$

Practice. Perform the indicated operation. Leave all answers in simplest form. Do not use a calculator.

1. $\frac{7}{10}+\frac{3}{5}$

2. $\frac{3}{4}\div\frac{1}{2}=$

3. $\frac{5}{8}+\frac{3}{4}-\frac{1}{2}=$

4. $\frac{5}{4}\cdot\frac{-4}{7}=$

5. $\frac{4}{5}\cdot 12=$

6. $4\frac{1}{2}+\frac{-3}{4}=$

7. $\left(\frac{3}{16}-\frac{3}{32}\right)\cdot\frac{1}{8}\div\frac{1}{16}=$

8. $\frac{7}{15}\cdot\left(4-\frac{7}{10}\right)+5\cdot 12\frac{2}{5}=$

ORDER OF OPERATIONS

The order of operations is used to provide consistency when solving mathematical expressions. The most common phrase associated with order of operations is "Please Excuse My Dear Aunt Sally," which stands for the operations listed below. Expressions should always be simplified using this order.

Parentheses (all grouping symbols—parentheses, brackets, braces, fraction bar, radical sign, absolute value)

Exponents

Multiplication/Division (whichever comes first from left to right)

Addition/Subtraction (whichever comes first from left to right)

Example: $3 - \frac{2^3}{4}$

$3 - \frac{8}{4}$

$3 - 2$

1

Practice:

You should be able to work all of these without a calculator.

9. $-7 - 4^2 \bullet 3$

10. $(10 + \sqrt{10 + 6}) + \frac{6}{-3}$

11. $\frac{8^2}{(3 \cdot 2 + 2)}$

12. $[\,9 - (2 + 3)\,]^3$

13. $[\,7 \bullet (2 + 3)\,]^2$

14. $[\,7 \bullet 2 + 3]^2$

EQUATIONS

Solve equations using inverse operations. When solving equations the order of operations is reversed.

Ex 1: $-3x - 7 = 8$

$$-3x - 7 = 8$$

$$+7 \quad +7$$

$$-3x = 15$$

$$\frac{-3x}{-3} = \frac{15}{-3}$$

$$x = -5$$

Remember $\frac{x}{2}$ is the same as $\frac{1}{2}x$ and $-x$ is the same as $-1x$.

Practice: Do not use a calculator.

15. $9x + 4 = -41$

16. $11 - 2x = 11$

17. $15 = 8x - 1$

18. $\frac{x}{8} - 12 = -4$

19. $9 - \frac{x}{5} = 0$

20. $17 = \frac{x}{7} + 10$

21. $\frac{2}{5}x + 4 = 8$

22. $-10 - \frac{9}{7}x = 8$

23. $-3 = 15 - \frac{5}{6}x$

24. $\frac{2}{3}x - 18 = 18$

25. $13 - \frac{5}{4}x = 38$

26. $12 = \frac{9}{8}x - 15$

Solve. Leave your answers as reduced fractions. Do not use a calculator.

27. $8x - 2 = 10$

28. $7 - 6y = 11$

29. $-19 = 3x - 11$

To work more complex equations, we distribute first and then combine like terms:

Example

$4x - 8 + 7x - 2 = 100$

$4x + 7x \quad -8 - 2 \quad = 100$

$11x - 10 = 100$

$11x = 110$

$x = 10$

Example

$3x - 2(9 + 5x) = 17$

$3x - 18 - 10x = 17$

$-7x - 18 = 17$

$-7x = 35$

$x = -5$

Practice. Do not use a calculator.

30. $13 - 8y + 4y - 12 = 15$

31. $9x + 3 - 7(12 + 2x) = 11$

32. $\frac{3}{2}x - 9 + 4x - 1 = 10$

33. $x + 12 + 3(x - 8) = 18$

34. $21 = 3x - 4 + 9x - 7$

35. $-13 = 9x + 3 - \frac{1}{2}(2x - 8)$

Variables on both sides. Do not use a calculator.

Use inverse operations so that the variables are on one side of the equation and the constants are on the other side.

Ex: $3x - 8 = 7x + 22$

$3x - 3x - 8 = 7x - 3x + 22$

$-8 = 4x + 22$

$-8 - 22 = 4x + 22 - 22$

$-30 = 4x$

$-7\frac{1}{2} = x$

Practice:

36. $-9x + 2 = 3x + 50$

37. $7 - 3x = 9 - x$

38. $3(2x - 8) = -2(7 + 2x)$

39. $12 - (x + 4) = 9x + 5(3 - x)$

Using one variable equations. Do not use a calculator.

Example

The length of rectangle is 8cm more than its width. If its perimeter is 48 cm, what are the dimensions of the rectangle?

Define the variable: let w = the width; then w + 8 = length

Perimeter is the sum of the sides. $P = 2l + 2w$

Substitute: $48 = 2(w + 8) + 2w$

Solve: $48 = 2w + 16 + 2w$

$48 = 4w + 16$

$32 = 4w$

$8 = w$

The width is 8 cm and our length is 8 + 8 = 16; the dimensions are 8cm by 16cm.

Check: 8 + 8 + 16 + 16 = 48

Practice:

40. If the width is 3 less than the length of a rectangle, and its perimeter is 98 inches, what are the dimensions of the rectangle?

41. The perimeter of a triangle is 50yd. The longest side is twice the length of the shortest side, and the second side is two yards more than the shortest side. Find the lengths of the sides.

Literal Equations/Transforming/Solving for the variable

Sometimes you will need a different form of an equation than the one given. Isolate a variable using inverse operations. The final answer will have an equation as your answer, not a numerical solution.

Example: Given the equation $3x - 4y = -8$, transform to slope-intercept form.

Slope-intercept form is $y = mx + b$, so we need to isolate the y.

$3x - 4y = -8$

$-4y = -3x - 8$

$y = \frac{-3}{-4}x - \frac{8}{-4}$

$y = \frac{3}{4}x + 2$

Practice. Do not use a calculator.

42. $-8x + 3y = 12$

43. $15x + 20y = -60$

44. $-4x - 6y = 36$

Literal Equations

Example: Solve for the given variable:

$P = 2l + 2w$; solve for l

$P - 2w = 2l$

$\frac{P-2w}{2} = l$

Practice. Do not use a calculator.

45. $A = \frac{1}{2} b h$; solve for b

46. $A = \frac{1}{2} b h$; solve for h

47. $V = r^2 h \pi$; solve for h

48. $V = r^2 h \pi$; solve for r

PROPORTIONS

A proportion is a relationship between two equivalent ratios. Cross multiply to solve proportions.

Example

$\frac{4}{12} = \frac{x}{10}$

Cross Multiply $4 \cdot 10 = 12 \cdot x$

Solve $\frac{40}{12} = \frac{12x}{12}$

$x = \frac{10}{3}$

Example

$\frac{4-x}{4x+10} = \frac{3}{1}$

Cross Multiply $(4 - x) \cdot 1 = (4x + 10) \cdot 3$

Solve $4 - x = 12x + 30$

$\quad +x \qquad +x$

$4 = 13x + 30$

$-30 \qquad - 30$

$\frac{-26}{13} = \frac{13x}{13}$

$-2 = x$

Practice. Solve for the variable. Do not use a calculator.

49) $\frac{n}{8} = \frac{12}{16}$

50) $\frac{3}{k} = \frac{5}{15}$

51) $\frac{18}{30} = \frac{y}{4}$

52) $\frac{2.8}{4} = \frac{7}{x}$

53) $\frac{8}{20} = \frac{30}{c}$

54) $\frac{24}{n} = \frac{30}{100}$

55) $\frac{1}{c+5} = \frac{2}{3}$

56) $\frac{8}{b+10} = \frac{4}{2b-7}$

57) $\frac{0.24}{a} = \frac{3}{9.6}$

58) $\frac{17}{8.5} = \frac{z}{0.01}$

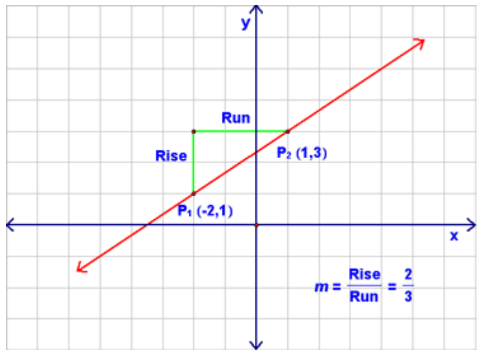
59) $\frac{x+3}{4} = \frac{7}{8}$

60) $\frac{n+12}{4} = \frac{n}{16}$

SLOPE

Slope is the ratio, *m*, of the rise to the run as you move from one point to another along a line.

Rise: Vertical change (Change in the Y-Values)
Run: Horizontal Change (Change in the X-Values)



Determining Slope Given Two Points:

Given the coordinates of two points (x_1, y_1) and (x_2, y_2) on a line the slope can be found using the following equation: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example: Determine the slope of the line that passes through (2, -5) and (7, -10)

First: let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (7, -10)$

Then:

$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{(-10)-(-5)}{(7)-(2)} \Rightarrow m = \frac{-5}{5} \Rightarrow$ m = -1 (this is a negative slope)

Practice. Determine the slope of the line that passes through each pair of points. Do not use a calculator.

61. (0, 0) and (3, 7)

62. (-2, 4) and (4, -1)

63. (2, 4) and (4, -4)

64. (-5, -2) and (-5, 3)

65. (9, -8) and (0, 0)

66. (11, 15) and (-19, 15)

67. $(\frac{1}{2}, \frac{3}{2})$ and $(\frac{9}{2}, \frac{5}{2})$

68. $(\frac{-1}{3}, \frac{17}{3})$ and $(\frac{2}{3}, \frac{11}{3})$

Challenge. There are four types of slope. We figure this out by looking at the final answer for slope.

Positive (any positive number), **Negative** (any negative number), **Zero** (a fraction with a zero in the numerator), and **Undefined** (a fraction with a zero in the denominator).

69. For #61-68 (above), label each slope as either *positive, negative, zero, or undefined.* Label next to the final reduced slope.

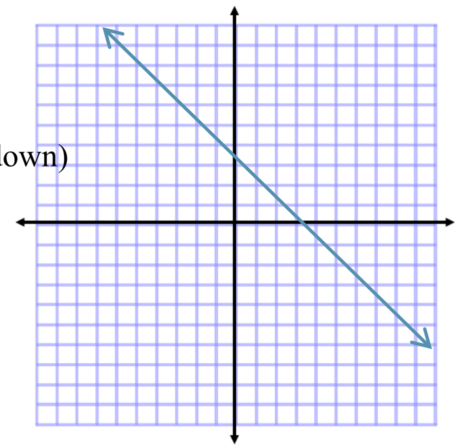
LINEAR EQUATIONS

Slope-Intercept Form

The slope-intercept form of an equation of a line is y = mx + b where m is the slope and b is the y-intercept. If the equation is not in slope-intercept form, manipulate it (see equations) so that the y variable is isolated.

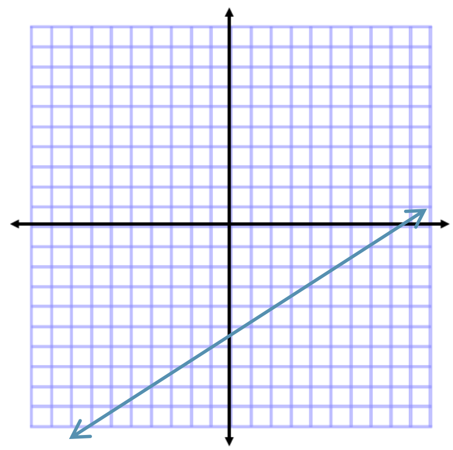
Examples:

Graph the line $y = -\frac{3}{4}x + 4$



1. Graph the y-intercept (0, 4)
2. The slope is $-\frac{3}{4}$ (since it is negative the line should be going down)
3. From the y-intercept, use the slope to plot another point.
   Go down three and the right four.

Graph the line -2x + 3y = -6.



1. Rewrite the equation in slope-intercept form.

   -2x + 3y = -6
   +2x   +2x

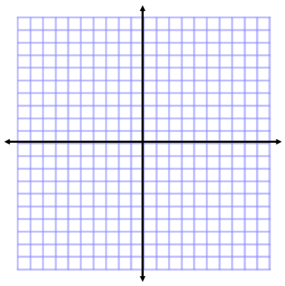
   $\frac{3y}{3} = \frac{2x}{3} + \frac{6}{3}$

   $y = \frac{2}{3}x + 2$

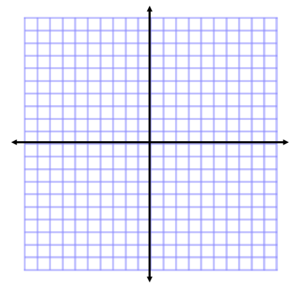
2. Graph the y-intercept. (0, 2)
3. Determine whether the slope is positive or negative: Positive, m = $\frac{2}{3}$
4. From the y-intercept use the slope to plot another point. Up 2, right 3.

Practice. Graph the lines.

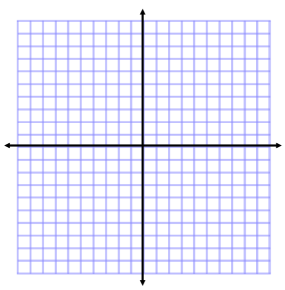
70. $y = \frac{3}{4}x - 2$



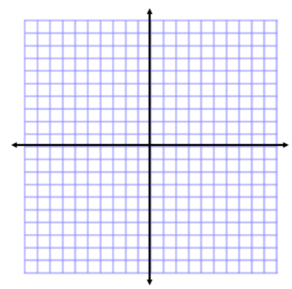
71. $y = -\frac{1}{5}x + 3$



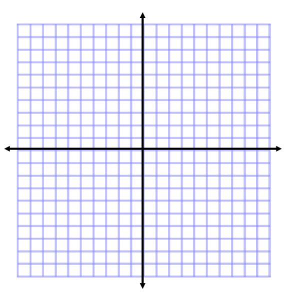
72. $y = -\frac{3}{5}x - 1$



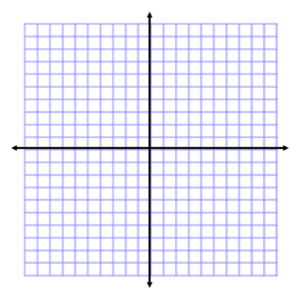
73. $y = \frac{5}{2}x + 3$



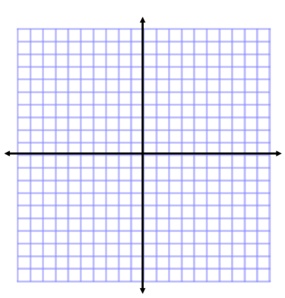
74. $y = \frac{7}{3}x$



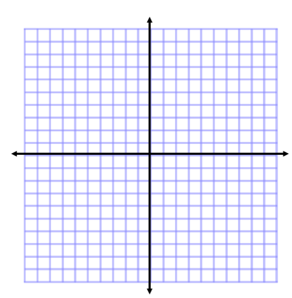
75. 4y = - 2x – 8



76. 3x – 9y = -18



77. -2(x - 5) = y



SYSTEMS OF EQUATIONS

A system of equations is two or more equations that have the same solution. You can use what you have learned about graphing to help you find the solution.

Solve this system of equations graphically.

$$4x - 6y = 12$$
$$2x + 2y = 6$$

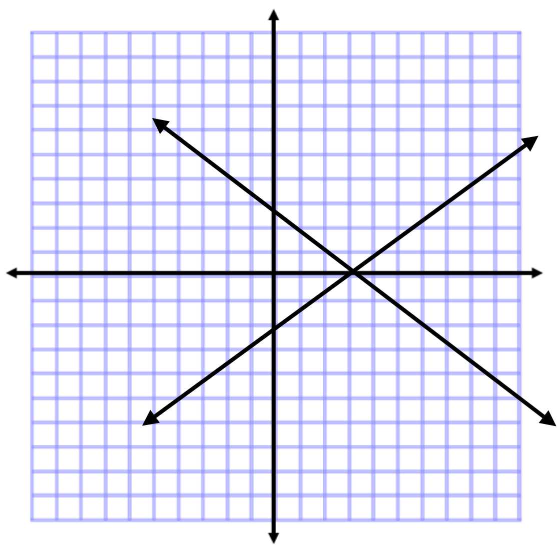
In order to solve the system graphically, you need to first graph the equations. Change each equation to slope-intercept form and graph.

Equation 1: $4x - 6y = 12$

$y = \frac{2}{3}x - 2$

Equation 2: $2x + 2y = 6$

$y = -x + 3$



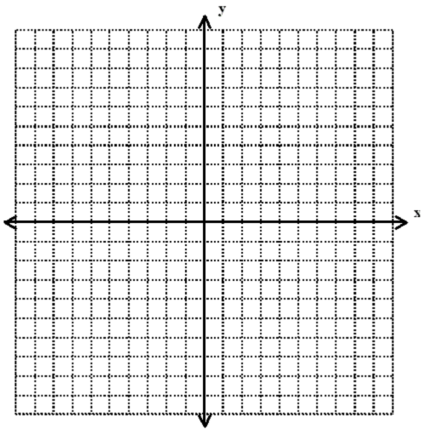
Check: Since the two lines cross at (3,0), the solution is $x = 3$ and $y = 0$. Checking these value shows that this answer is correct. Substitute these values into the ORIGINAL equations and get a true result.

4*x* - 6*y* = 12	**2*x* + 2*y* = 6**
4(3) - 6(0) = 12	**2(3) + 2(0) = 6**
12 - 0 = 12	**6 + 0 = 6**
12 = 12 (YES!)	**6 = 6 (YES!)**

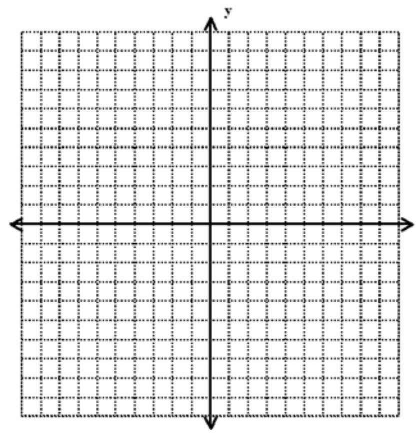
If the lines intersect, there will be one solution.
If the lines are parallel, there will be no solution.
If the lines coincide, there will be an infinite number of solutions.

Practice. Solve the following system of equations using the graphing method. Check your work.

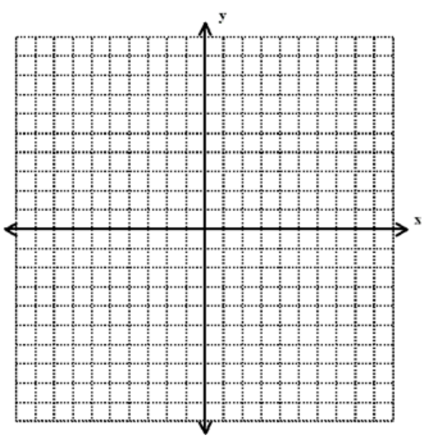
78. $y = 3x - 4$
$y = -3x + 2$



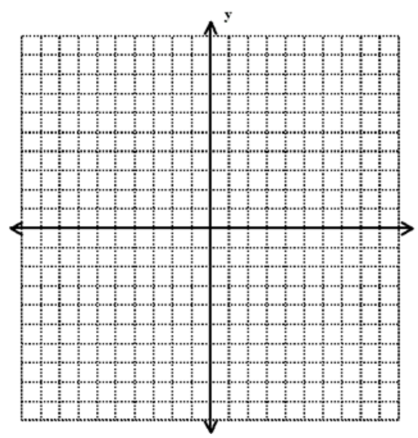
79. $y = 5x - 8$
$y = 5x + 3$



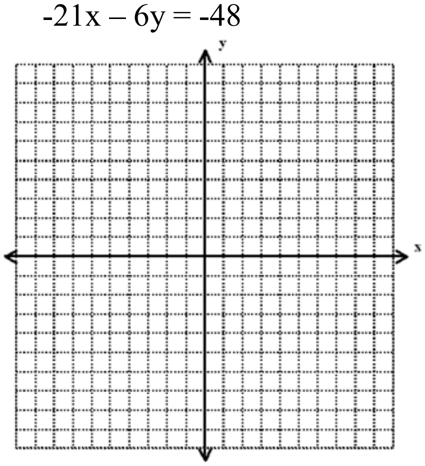
80. $y = \frac{4}{3}x + 3$
$y = -\frac{2}{3}x + 3$



81. $2x - 3y = 9$
$x = 3$



82. $7x + 2y = 16$
$-21x - 6y = -48$



Challenge. Try setting up two separate equations and solve.

83. An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two point questions are there? How many five point questions are there? You may use a calculator.

RADICALS

Radical expressions contain numbers under a radical sign. A radical sign tells you to take the square root of the value under the symbol. The radicand is the expression under the radical sign.

$\sqrt{100} = 10$ because $10 \cdot 10 = 100$

When simplifying radicals, rewrite the radical in such a way that the radicand contains no factors that are perfect squares.

Example 1 Simplify $\sqrt{18}$

$$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

Example 2: Simplify $\sqrt{700}$

$$\sqrt{700} = \sqrt{100 \cdot 7} = 10\sqrt{7}$$

Example 3: Solve for the variable

$n^2 = 75$

The inverse operation of squaring a number is taking the square root.

$n = \sqrt{75} = 5\sqrt{3}$

For examples and practice:

Practice: Simplify. Do not use a calculator.

84. $\sqrt{126}$

85. $\sqrt{180}$

86. $\sqrt{324}$

87. $\sqrt{39}$

88. $4\sqrt{12}$

89. $\sqrt{\frac{1}{9}}$

90. $\sqrt{22050}$

Solve for the variable. Simplify radicals. Do not use a calculator.

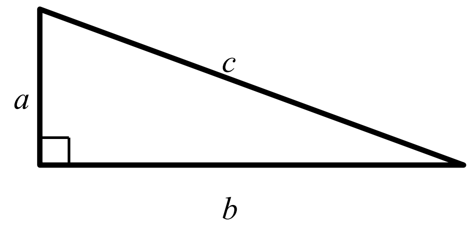
91. $x^2 = 49$

92. $b^2 = 128$

93. $n^2 = 625$

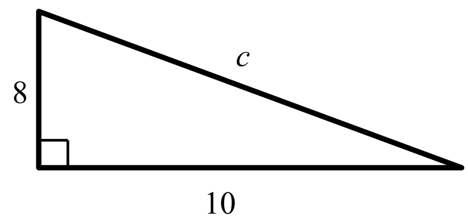
PYTHAGOREAN THEOREM

If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $a^2 + b^2 = c^2$



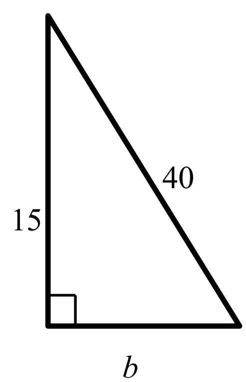
Use the Pythagorean Theorem to find the length of the missing sides in a right triangle.

Example 1: Find the length of the hypotenuse of the right triangle.



$8^2 + 10^2 = c^2$
$64 + 100 = c^2$
$164 = c^2$
$\sqrt{164} = c$
$2\sqrt{41} = c$

Example 2: Find the length of the leg of the right triangle.



$15^2 + b^2 = 40^2$
$225 + b^2 = 1600$
$b^2 = 1375$
$b = \sqrt{1375}$
$b = 5\sqrt{55}$

Practice

Find the missing side of the right triangle described. Radicals should be simplified. Do not use a calculator.

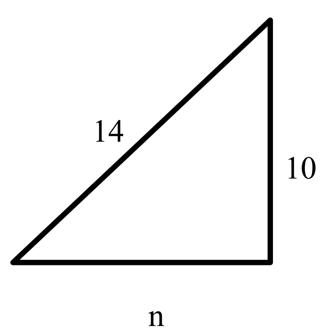
94. $a = 8$, $b = 15$ find the hypotenuse

95. $a = 12$, $c = 25$, find the leg

96. $a = 2\sqrt{3}$, $b = \sqrt{5}$, find the hypotenuse

97. leg = 32, hypotenuse = 50, find the other leg

98.



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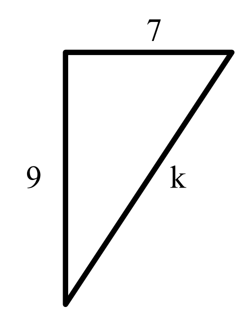

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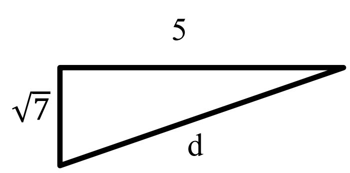
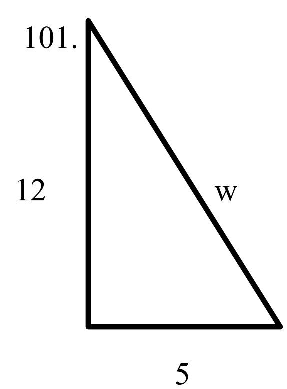


101.



Practice. You may use a calculator.

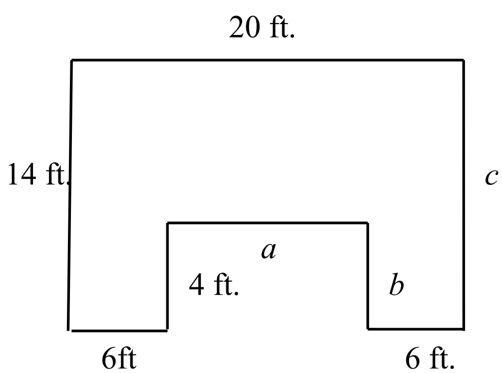
102. Mike accidentally threw his Frisbee on top of his shed roof, 10 feet above the ground. Since he works part-time at the local hardware store, he went down to the store to borrow a ladder. There is a row of bushes along the edge of the shed, so he will have to place the ladder 3 feet from the shed. What is the minimum length of ladder that he needs to reach the top of the school?

103. Two ships leave port at the same time. Ship X is heading due north and Ship Y is heading due east. Thirteen hours later they are 650 miles apart, diagonally. If the Ship X had traveled 520 miles from the port, how many miles had Ship Y traveled?

AREA AND PERIMETER OF COMPOSITE SHAPES

Composite shapes are figures that are constructed using a variety of shapes such as circle, semicircles, rectangles or triangles.
To calculate the perimeter, add all the side lengths together. Do not include any segments on the interior.
To calculate the area, divide the shape into basic shapes, and apply appropriate area formulas. Add the areas together for a total.
Remember your labels. Perimeter is a linear value; its label is in units. Area is two-dimensional; its label is in units squared.

Example 1: Find the area and perimeter of the composite shape.

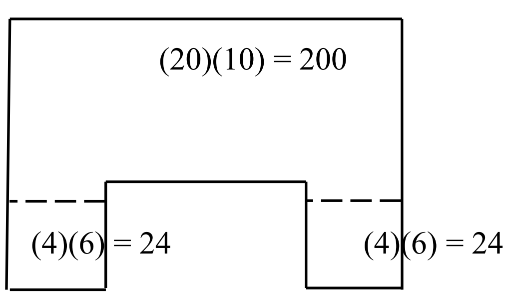


Perimeter: Determine the lengths of sides labeled *a, b, c*

a = 20 -6-6 = 8 (total length minus two parts known)
b = 4
c = 14

Perimeter = 20 + 14+ 6 +4 + 8 +4 +6 +14 = 76 ft.

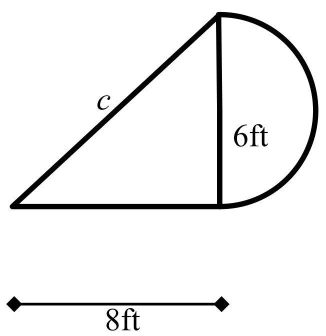
Find the area of each rectangle.



Area: divide the composite shape into rectangles.

Area = 200 + 24 +24 = 248 ft²

Sometimes it is useful to use the Pythagorean Theorem.



Perimeter:

Side length

$8^2 + 6^2 = c^2$

$64 + 36 = c^2$

$100 = c^2$

$10 = c$

Half circle = half of the circumference

$C = 2\pi r$

$= 2\pi(3)$

$= 18.85$

half circle = 9.42

Perimeter = 8 + 10 + 9.42 = 27.42 ft.

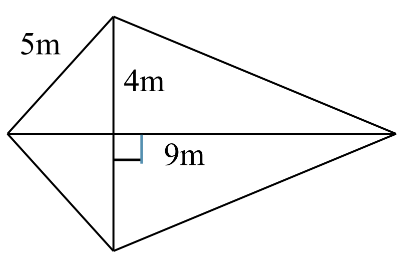
Area = area of triangle + area of half circle

$= \frac{1}{2}(8)(6) + \frac{1}{2}(\pi)(3^2)$

$= 38.14 \text{ ft}^2$

Practice. Find area and perimeter. You may use a calculator.

104



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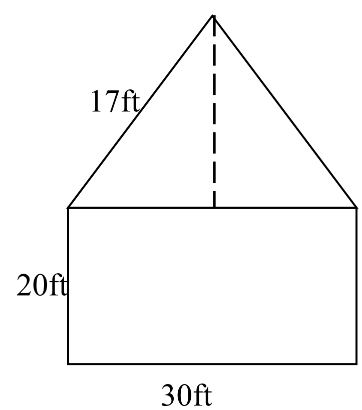

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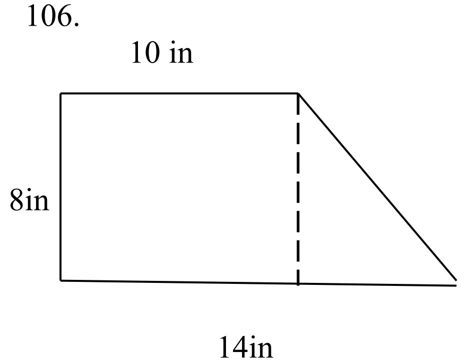

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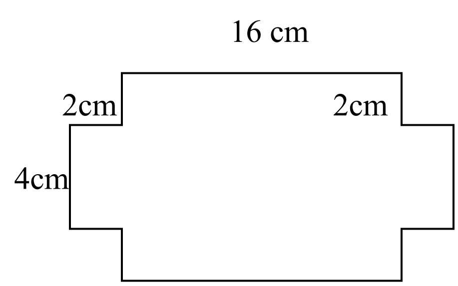

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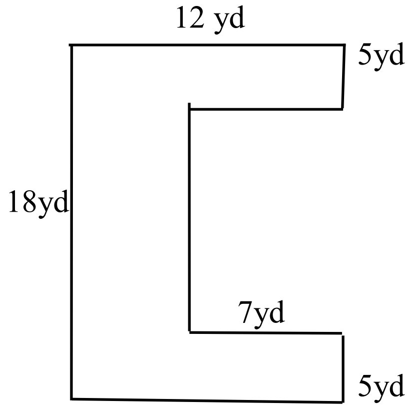

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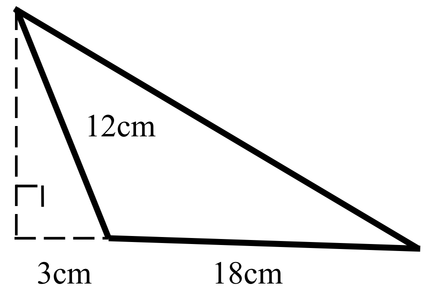

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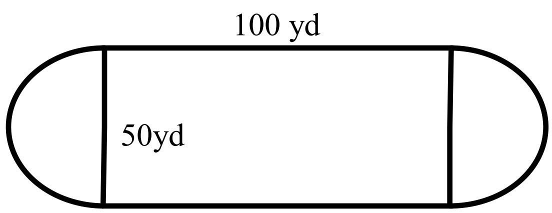

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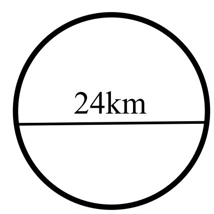

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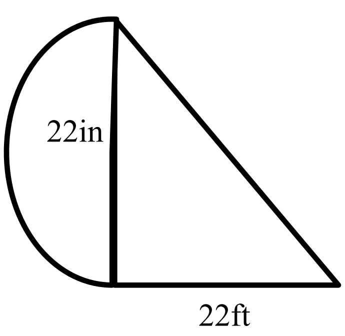

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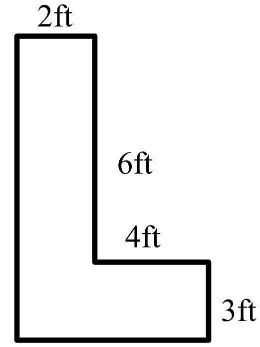

PROBABILITY

Probability measures the likelihood of an event occurring.

P(event) = $\frac{number\ of\ ways\ the\ desired\ event\ can\ occur}{total\ number\ of\ possible\ outcomes}$

Example

A glass jar contains 5 pennies, 7 nickels, 10 dimes and 2 quarters. If a single coin is chosen at random from the jar, what is the probability of choosing a dime?

Probability of choosing a dime = $\frac{The\ amount\ of\ dimes\ possible}{The\ total\ amount\ of\ coins\ in\ the\ jar}$

P (dime) = $\frac{10}{24}$ = $\frac{5}{12}$ = $.41\overline{6}$ = 41.66%

Practice. You may use a calculator.

114. P (penny) =

115. P (nickel) =

116. P (quarter) =

<u>"AND" Problems:</u> probability that two events occur in succession

For "AND" problems, find the probability of two (or more) independent events occurring separately, and then multiply the probabilities.

A glass jar contains 5 pennies, 7 nickels, 10 dimes and 2 quarters. If a single coin is chosen at random from the jar, what is the probability of choosing a dime **and** then a penny? Assume the dime is replaced before the penny is drawn.

Probability of choosing a dime, REPLACING it, and then picking a penny.

P(dime AND penny) = $\frac{10}{24} \cdot \frac{5}{24}$ = $\frac{50}{576}$ = .08681 = 8.68%

Practice. You may use a calculator.

117. P(nickel and dime)

118. P(quarter and penny) =

119. P(dime and dime) =

<u>"OR" Problems:</u> probability that one or both events occur in succession

To find the probability of these "OR" problems, add the probabilities of each event.

There are 52 cards in a deck of cards.

What is the probability of getting a King OR Queen if pulling a card from a deck of cards.

4 cards are Kings, 4 cards are Queens

P(King OR Queen) = $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$ = .1538 = 15.38%

Practice. You may use a calculator.

120. P(First Card is … Red OR 7) =

121. P(First Card is Heart OR Spade) =

122. Looking at the Alphabet P(Getting a Vowel OR One of the Last Three Letters in the Alphabet) =