AP CALCULUS SUMMER REVIEW

Welcome to AP Calculus! These questions are intended as review and are due to me by the first day of class.

If a particular topic is unknown or you have forgotten details email me immediately so that I may explain and/or give hints.

ASK ALL YOUR QUESTIONS BEFORE IT IS DUE. DO NOT hesitate, for any reason, to ask about any of these important ideas via email over the summer: cpomije@woodlandsacademy.org

SHOW YOUR WORK IN AN ORGANIZED FASHION WITH PROBLEMS IN NUMERICAL ORDER. You may use a calculator only on problems marked with an *.

If you have a problem partially done and cannot finish then scan it and email it to me and I will supply hints.

If you find you are spending too much time on these or can't get started on a particular type, PLEASE ASK for assistance! I AM HERE TO HELP.

You must show work for all problems to receive full credit. You will be tested on the material the 2nd full day of class.

I Simplify 1-4. Show the work that leads to your answer.

1. $\dfrac{x-4}{x^2-3x-4}$ 2. $\dfrac{x^3-8}{x-2}$ 3. $\dfrac{5-x}{x^2-25}$ 4. $\dfrac{x^2-4x-32}{x^2-16}$

5. Expand $x^{\frac{3}{2}}(x+x^{\frac{5}{2}}-x^2)$

6. Factor and answer with only one negative exponent: $7x^{-5}+4x^{-3}+2x^3$

7. Factor and answer with only one negative exponent: $2x^{-\frac{3}{4}}+8x^{-\frac{7}{4}}-6x^{\frac{1}{4}}$

II

1. Is $\left\{(x,y): y=\sqrt{4-x^2}\right\}$ a function and if so, what is its domain?

2. Given $F(x)=\sqrt{x+9}$ find and simplify $\dfrac{F(x+h)-F(x)}{h}, h\neq 0$

3. Sketch the graph and determine its **DOMAIN**:

a: $\dfrac{(x+3)(x+2)}{(x-4)(x+1)}$. **b:** $f(x)=\dfrac{x^2-25}{x+5}$. **c:** $f(x)=\dfrac{(x^2-4)(x-3)}{x^2-x-6}$.

Include the DOMAIN and RANGE also for the rest:

d: $f(x)=\begin{cases}9-x^2 & x\neq -3\\ 10 & x=-3\end{cases}$ **e:** $f(x)=\lfloor x\rfloor$ ($\lfloor x\rfloor$ means the greatest integer less than or equal to x)

f: $f(x)=\begin{cases}x+3\ , x<-5\\ \sqrt{25-x^2}\ , -5\le x\le 5\\ 3-x,\ x>5\end{cases}$

4. The **unit step function** is defined to be $U(x)=\begin{cases}0,\ x<0\\ 1,\ x\ge 0\end{cases}$. Sketch the graph of

   a: U(x) b: $xU(x)$ c: $(x+1)U(x+1)-xU(x)$

III

1. If $f(x)=\dfrac{x+1}{x-1}$ *and* $g(x)=\dfrac{1}{x}$, write a **formula** for and give the **domain** of:

   a: $f\cdot g$ b: $f\circ g$ c: $f\circ f$

2. If $f(x)=\sqrt{x-2}$ *and* $g(x)=x^2-2$, write a **formula** for and give the **domain** of:

   a: $f\circ g$ b: $g\circ f$ c: $f\circ f$

3. Express $h(x)=\sqrt{x^2-4}$ as the composition of two functions, f and g $(f\circ g)(x)$, in **THREE** different ways.

4. Is $f(x)=\sqrt[3]{x}$ even, odd or neither? Prove analytically (algebraically) using the definition (**NOT** graphically/geometrically). Look up the definition if necessary.

5. Is $f(x)=\dfrac{x^2-5}{2x^3+x}$ even, odd or neither? Prove analytically (**NOT** graphically).

6. Write the function $f(x)=|x-2|-|x+2|$ without using absolute value bars AND state whether f(x) is even, odd or neither with a defense for your answer.

7. If *f* and *g* are two functions such that when composed in either order, the result is the identity function then f and g are inverses of each other. If $f(x) = x^2,\ x \le 0$ *and* $g(x) = -\sqrt{x}$, are 'f' and 'g' inverses? Show that they are or are not inverses analytically via the above definition. Be SURE the details are clear!

8. If $f(x)=x^2$, find TWO functions, *g*, for which $(f \circ g)(x) = 4x^2 - 12x + 9$.

9. If; ***f(x)*** = {(3,5), (2,4), (1,7)}, ***g(x)*** = $\sqrt{x-3}$, ***h(x)***= {(3,2), (4,3), (1,6)}, ***k(x)*** = $x^2 + 5$, determine each of the following:

   a: (f + h)(1) = b: (k – g)(5) = c: (f ∘ h)(3) = d: (g ∘ k)(7) =

   e: $f^{-1}(x)$ = f: $g^{-1}(x)$ = g: $\frac{1}{f(x)}$ =

10. Write the inequality $|A| < B$ without absolute value bars.
11. Write the inequality $|A| > B$ without absolute value bars.
12. Solve for x: $|2x-3| < 5$
13. Solve for x: $|3x-2| > 5$

IV

1. The surface area of a sphere is given by: $A = 4\pi r^2$. Suppose a balloon maintains the shape of a sphere as it is being inflated so that the radius is changing at the constant rate of 3 cm per second. If f(t) centimeters is the radius of the balloon after t seconds:
   a: Compute $(A \circ f)(t)$ and interpret the result (what does it tell you?)
   b: Find the surface area of the balloon after 4 seconds using $(A \circ f)(t)$.

*2. A rectangular field is to be enclosed with 240m of fence but one side of the rectangle is a river so the fencing only needs to be used on the other three sides. Express the area of the field as a function of the length (the dimension parallel to the river), graph the function and give, to the nearest tenth of a meter, the dimensions of the field having the greatest area.

*3. A manufacturer makes open tin boxes from pieces of tin that are 12cm by 15cm by cutting squares out of the corners and bending up the sides. Find the size, to the nearest .01cm, of the cut-out squares in order that the volume of the boxes is as great as possible.

V Simplify:

1. $\frac{\sqrt{x}}{x}$
2. $e^{\ln 3}$
3. $e^{(1+\ln x)}$
4. $\ln 1$
5. $\ln e^{7}$
6. $\log_3(1/3)$
7. $\log_{1/2} 8$
8. $\ln \frac{1}{2}$
9. $e^{3\ln x}$
10. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$
11. $27^{2/3}$
12. $(5a^{2/3})(4a^{3/2})$
13. $(4a^{5/3})^{3/2}$
14. $\frac{3(n+2)!}{5n!}$ [NOTE: $5n! \neq (5n)!$]

VI

1. Write in slope intercept form the line perpendicular to $2x - 3y = 7$ and passing through (5,1) by using the **POINT SLOPE** form of a straight line. (look this up if you need to, we will use it!)

2. Find the equation of a straight line (in slope intercept form) that is tangent to the circle of radius 2, centered at the origin at a point that is **in** the center of the second quadrant using the **POINT SLOPE** form of a straight line.

VII. Without a calculator, determine the **exact** value of each expression. (Assume principal inverse values). BE SURE YOU **KNOW** THESE!

1. $\sin 0$
2. $\sin \frac{\pi}{2}$
3. $\sin \frac{3\pi}{4}$
4. $\cos \pi$
5. $\cos \frac{7\pi}{6}$
6. $\cos \frac{\pi}{3}$
7. $\tan \frac{7\pi}{4}$
8. $\tan \frac{\pi}{6}$
9. $\sec \frac{2\pi}{3}$
10. $\cos 0$
11. $\cos(\sin^{-1} \frac{1}{2})$
12. $\sin^{-1}(\sin \frac{7\pi}{6})$

VIII Solve for x, where x is a real number. Show the work that leads to your solution.

1. $x^2 + 3x - 4 = 14$
2. $\frac{x^4 - 1}{x^3} = 0$
3. $(x - 5)^2 = 9$
4. $2x^2 + 5x = 8$
5. $(x + 3)(x - 3) > 0$
6. $x^2 - 2x - 15 \leq 0$
7. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$
8. $(x-2)(x+3)^7(x-14)^{18}(x+11)^{29}(x)^{34} > 0$
9. $27^{2x} = 9^{x-3}$
10. $\log x + \log(x - 3) = 1$
11. $e^{3x} = 5$
12. $\ln y = 2x - 3$

IX

From Memory, without a calculator, state the exact values of the following:

1. a: $\sin\frac{17\pi}{3}=$ b: $\tan\frac{43\pi}{6}=$ c: $\cos\frac{97\pi}{4}=$ d: $\sin(-\frac{55\pi}{3})=$

e: $\cos(-\frac{71\pi}{6})=$ f: $\cot(-\frac{213\pi}{4})=$ g: $\sec(-\frac{137\pi}{3})=$ h: $\csc\frac{1000\pi}{6}=$

i: $\tan^{-1}(\cot(-\frac{13\pi}{4}))=$ j: $\sin(\cot^{-1}(-\sqrt{3}))=$ k: $\csc(\cos^{-1}(-\frac{\sqrt{3}}{2}))=$

X - Challenge section- try your best!

Suppose that the trigonometric functions were defined in exactly the way you have learned based on the unit circle **BUT** instead were based on a **UNIT SQUARE** instead of a unit circle. A unit square is ONE unit on each side, centered on the origin with sides parallel to the axes. With this **one** modification, what would the values of these trig functions now be? (NO Calculator)

1. $\sin\frac{\pi}{4}=$ 2. $\tan\frac{\pi}{4}=$ 3. $\cos\frac{3\pi}{4}=$ 4. $\csc(-\frac{19\pi}{4})$ 5. $\sin 30^o=$

6. $\cos(120^o)=$ 7. $\tan(-210^o)=$

8. $\sin(-948.6671^0=)$ 11. $\sin^2 315^o+\cos^2 315=$

Check your answers carefully and question anything for which you may need a reminder by emailing me.