



**SPRINGFIELD PUBLIC SCHOOLS**  
ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21<sup>ST</sup> CENTURY GLOBAL SKILLS

# **Springfield Public Schools** **Springfield, New Jersey 07081**

## **Pre-Calculus Honors**

### **Curriculum Guide**

**2022**

**This curriculum may be modified through varying materials, techniques and strategies, as per a student's IEP.**

Springfield Board of Education  
Approved on June 6, 2022



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**Springfield Public Schools  
Springfield, New Jersey**

**Dr. Rachel Goldberg  
Superintendent of Schools**

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*Springfield Board of Education  
PO Box 210  
Springfield, New Jersey 07081*

*"Cultivating compassionate and extraordinary learners"*

**Rachel Goldberg**  
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Resolution: Buildings and Grounds  
Introduced By: Vice President Saha  
Seconded By: Mr. Miller  
Adopted: June 6, 2022

## **Curriculum**

At the recommendation of the Superintendent, to approve the following revised courses:

- a. AP English Language and Composition
- b. Creative Writing and Poetry
- c. DCA Freshman English (Writing/Research was incorporated into this course)
- d. Drama 1 (Modified the description to be combined with Drama II)
- e. Film as Literature
- f. Journalism
- g. Senior Seminar
- h. Yearbook
- i. Kindergarten Mathematics
- j. Grade 1 Mathematics
- k. Grade 2 Mathematics
- l. Grade 3 Mathematics
- m. Grade 4 Mathematics
- n. Grade 5 Mathematics
- o. Algebra II
- p. Calculus
- q. Discrete Math
- r. Financial Algebra
- s. Geometry
- t. Geometry Honors
- u. Pre-Algebra Grade 7
- v. Pre-Calculus
- w. Pre-Calculus Honors

I, Michelle Calas, School Business Administrator/Board Secretary of the Springfield Board of Education, hereby certify that the extract from the minutes of a meeting of the Board of Education duly called and held on June 6, 2022 has been compared by me with the original minute book of said Board of Education, and is a true, complete and exact copy therefor, as far as the same relates to the subject matter referred to said contract.

IN WITNESS THEREOF, I have hereunto set my hand and seal of the Springfield Board of Education, this 7<sup>th</sup> of June 2022.

SEAL



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Michelle Calas  
School Business Administrator/Board Secretary



### Unit Overview Template

**Content Area:** Math      **Grade Level:** 10-12      **Unit Title:** Prerequisites for Precalculus

**Unit Summary:** This unit is a review of topics that students will use throughout the year. All of the material should have been covered in prior courses. The bulk of the material is reviewed by students as part of the summer assignment for the course.

**Primary Interdisciplinary Connections:** *Science and Engineering*

**21<sup>st</sup> Century Themes**   ☐ Creativity and Innovation   ☒ Critical Thinking and Problem Solving   ☐ ICT Literacy  
☐ Information Literacy   ☐ Life and Career Skills   ☐ Communication and Collaboration   ☐ Media Literacy

**Unit Rationale:** The skills in this unit are necessary for the successful completion of all levels of higher mathematics.

**Career Readiness (Standard 9):**

- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.
- 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

### Learning Targets

**Standards:** A-CED; A-REI; F-IF; F-BF; S-ID

**Content Statements:** Create equations that describe numbers or relationships; Understand solving equations as a process of reasoning and explain the reasoning; Solve equations and inequalities in one variable; Solve systems of equations; Represent and solve equations and inequalities graphically; Understand the concept of a function and use function notation; Interpret functions that arise in applications in terms of the context; Analyze functions using different representations; Build a function that models a relationship between two quantities.

| CPI#:     | Cumulative Progress Indicator (CPI):                                                                                                                                                            |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. |



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|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A-CED.A.2  | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| A-CED.A.3  | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.                                                                                                                                                                                                                                                                                          |
| A-CED.A.4  | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$ .                                                                                                                                                                                                                                                                                                                                                                                                               |
| A-REI.A.1  | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.                                                                                                                                                                                                                                                                                                                                       |
| A-REI.B.3  | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| A-REI.B.4  | Solve quadratic equations in one variable.<br>a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.<br>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ . |
| A-REI.C.5  | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.                                                                                                                                                                                                                                                                                                                                                                                                         |
| A-REI.C.6  | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).                                                                                                                                                                                                                                                                                                                                                                                                                       |
| A-REI.D.11 | Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.                                                                                                                                            |
| A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.                                                                                                                                                                                                                                                                                                                   |
| F-IF.A.1   | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .                                                                                                                                                                                                                               |



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|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| F-IF.A.2  | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| F-IF.B.4  | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| F-IF.C.7  | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <ol style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> <li>Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> <li>(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</li> <li>Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</li> </ol> |
| F-IF.C.8  | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. <ol style="list-style-type: none"> <li>Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</li> <li>Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</li> </ol>                                                                                                                               |
| F-BF.A.1  | Write a function that describes a relationship between two quantities. <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</li> <li>Compose functions. For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</li> </ol>                                                           |
| S-ID.B.6  | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |



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|               |                                                                                                                                                      |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | choose a function suggested by the context. Emphasize linear and exponential models.                                                                 |
| S-ID.B.6c     | Fit a linear function for a scatter plot that suggests a linear association.                                                                         |
| SEL.PK-12.2.2 | Recognize the skills needed to establish and achieve personal and educational goals.                                                                 |
| SEL.PK-12.4.1 | Develop, implement and model effective problem-solving, and critical thinking skills.                                                                |
| SEL.PK-12.5.2 | Utilize positive communication and social skills to interact effectively with others.                                                                |
| HS-ETS1-2     | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering. |

### Unit Essential Questions:

- How are equations of a line written given two points or given one point and the slope?
- How are linear equations and inequalities graphed?
- How are systems of equations and inequalities solved?
- What is meant by the domain and range of a function?
- How can functions be combined with other functions?
- What are the properties of parallel and perpendicular lines?

### Unit Enduring Understandings:

- Many real life situations can be modeled using linear functions.
- Linear models can be used to forecast future events.
- Algebraic functions can be used to solve problems in business, chemistry and other disciplines.

### Unit Learning Targets:

*Students will....*

- generate the different forms of linear equation: point-slope, slope-intercept and standard form.
- identify the domain and range of a function.
- solve and graph linear equations, linear inequalities and systems of linear equations and linear inequalities.
- combine functions using various arithmetic operations and composition of functions.
- solve complex, real-life word problems involving linear equations and inequalities.
- generate a linear regression model for a given set of data.
- identify and generate equations for parallel and perpendicular lines.



### Evidence of Learning

**Summative Assessment:** Unit Test

**Equipment Needed:** Graph paper, laptops, TI-84 Graphing Calculator

**Teacher Resources:** Teacher's text, projector, TI-84 emulator software, TI-84 Graphing Calculator

**Formative Assessments:**

- Homework
- Class discussions
- Q&A with students
- Student presentation of problems
- Paired student work



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**Lesson Plans**

| Lesson                               | Time Frame      |
|--------------------------------------|-----------------|
| Lesson 1 <b>Linear Regression</b>    | <b>1 period</b> |
| Lesson 2 <b>Systems of Equations</b> | <b>1 period</b> |
| Lesson 3 <b>Linear Programming</b>   | <b>1 period</b> |

**Teacher Notes:** Ensure that students understand proper use of advanced features of the TI-84.

**Curriculum Development Resources/Technology Resources:**

*Click the links below to access additional resources used to design this unit:*

[https://college.cengage.com/mathematics/brase/understandable\\_statistics/7e/students/datasets/slr/frames/frame.html](https://college.cengage.com/mathematics/brase/understandable_statistics/7e/students/datasets/slr/frames/frame.html)

<http://www.purplemath.com/modules/linprog.htm>

<https://www.youtube.com/watch?v=YjT3QYfoy4Q> (Khan Academy video)

<https://tasks.illustrativemathematics.org/content-standards>

<https://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrig.aspx>



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**Lesson Plan 1 Template**

Content Area: Math      Lesson Title: Linear Regression      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving      ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy      ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos, Excel, Google Sheets

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                      | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Formative Assessment Tasks                                                                                                                                                                             |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>• create a scatterplot of a data set.</li> <li>• find the least-squares regression line (line of best fit).</li> <li>• use the regression model to predict other values.</li> <li>• calculate the value of residuals.</li> <li>• interpret the value of the correlation coefficient.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>1. Teacher demonstrates the use of the TI-84 for calculating the equation of the least-squares regression line.</li> <li>2. Students go to the website and find a data set that interests them.</li> <li>3. Students generate regression lines and answer a series of questions.</li> <li>4. Teacher demonstrates how to use MS Excel or Google Sheets to find the regression equation.</li> <li>5. Students confirm their regression equation using Excel or Google Sheets.</li> </ol> | <ul style="list-style-type: none"> <li>• Q&amp;A with students</li> <li>• Circle the room while students are working</li> <li>• Class discussion</li> <li>• Spot check answers to questions</li> </ul> |
| <b>Differentiation:</b><br>Students are able to choose a data set that interests them. Students can work in pairs.                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                        |



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**ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21<sup>ST</sup> CENTURY GLOBAL SKILLS**

### **Resources Provided:**

- Graphing calculators
- Website for data:  
[https://college.cengage.com/mathematics/brase/understandable\\_statistics/7e/students/datasets/slr/frames/frame.html](https://college.cengage.com/mathematics/brase/understandable_statistics/7e/students/datasets/slr/frames/frame.html)



### Lesson Plan 2 Template

Content Area: Math      Lesson Title: Systems of Equations      Time Frame: 1 period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                            | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                              | Formative Assessment Tasks                                                                                                                                                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>• solve systems of equations in two variables using a variety of methods.</li> <li>• confirm their solutions using Desmos.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>1. The students lead the class discussion on solving systems using substitution and elimination.</li> <li>2. The teacher reminds the students how to use Desmos.</li> <li>3. Students work several problems from a problem set to find solutions.</li> <li>4. Students confirm their solutions graphically using Desmos.</li> <li>5. Students share answers with the class.</li> </ol> | <ul style="list-style-type: none"> <li>• Q&amp;A with students</li> <li>• Teacher circles classroom as students work</li> <li>• Students share solutions with the class</li> </ul> |
| <b>Differentiation:</b><br>Students can select the appropriate number of problems from the problem set. The teacher gives a variety of question types to students.                          |                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                    |
| <b>Resources Provided:</b> <ul style="list-style-type: none"> <li>• Problem set</li> </ul>                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                    |



### Lesson Plan 3 Template

Content Area: Math      Lesson Title: Linear Programming      Time Frame: 1 period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                         | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                     | Formative Assessment Tasks                                                                                                                                |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify constraints in a linear programming problem.</li> <li>generate an objective function.</li> <li>graph a system of linear inequalities.</li> <li>find the intersection point of two lines.</li> <li>find the maximum or minimum value of the objective function.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students break into groups to discover how to solve linear programming problems.</li> <li>Students lead the discussion and demonstration of the steps in solving a linear programming problem.</li> <li>Students work out example problems with intermittent Q&amp;A to ensure understanding.</li> <li>Students present solutions to their groups.</li> </ol> | <ul style="list-style-type: none"> <li>Q&amp;A with students</li> <li>Teacher circles room as students work</li> <li>Students present solution</li> </ul> |
| <b>Differentiation:</b><br>The discovery problems are scaffolded for different levels.                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                           |
| <b>Resources Provided:</b> <ul style="list-style-type: none"> <li>Problem set</li> <li>Rulers and graph paper</li> </ul>                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                           |



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### Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly, Moderately* or *Weakly* meets the criteria below.

| Lesson Activities                                                                                                    | Strongly                            | Moderately                          | Weakly                   |
|----------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|--------------------------|
| Are challenging <b>Strongly</b> and require higher order thinking and problem solving skills                         | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Allow for student choice                                                                                             | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide scaffolding for acquiring targeted knowledge/skills                                                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Integrate global perspectives                                                                                        | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Integrate 21st century skills                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for interdisciplinary connection and transfer of knowledge and skills                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are varied to address different student learning styles and preferences                                              | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are differentiated based on student needs                                                                            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for student reflection and self- assessment                                                    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide data to inform and adjust instruction to better meet the varying needs of learners                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |

### Curriculum Design Template

|                           |                           |                                                  |
|---------------------------|---------------------------|--------------------------------------------------|
| <b>Content Area:</b> Math | <b>Grade Level:</b> 10-12 | <b>Unit Title:</b> Prerequisites for Precalculus |
| <b>Created:</b> 2022      | <b>Board Approved on:</b> |                                                  |



### Unit Overview Template

**Content Area:** Math      **Grade Level:** 10-12      **Unit Title:** The Nature of Graphs

**Unit Summary:** Students will explore the graphs of many families of functions: quadratics, cubics, square root, cube root, absolute value and rational functions.

**Primary Interdisciplinary Connections:** *Science and Engineering*

**21<sup>st</sup> Century Themes**   ☐ Creativity and Innovation   ☒ Critical Thinking and Problem Solving   ☐ ICT Literacy  
☐ Information Literacy   ☐ Life and Career Skills   ☐ Communication and Collaboration   ☐ Media Literacy

**Unit Rationale:** The understanding of basic families of functions is fundamental to the study of mathematics.

**Career Readiness (Standard 9):**

- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.
- 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12.prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

### Learning Targets

**Standards:** A-APR; F-IF; F-BF

**Content Statements:** Rewrite rational expressions; Understand the concept of a function and use function notation; Interpret functions that arise in applications in terms of the context; Analyze functions using different representations; Build a function that models a relationship between two quantities; Build new functions from existing functions.

| CPI#:     | Cumulative Progress Indicator (CPI):                                                                                                                                                                                                                                                                                                 |
|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A-APR.D.6 | Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system. |
| A-APR.D.7 | Understand that rational expressions form a system analogous to the rational                                                                                                                                                                                                                                                         |



## SPRINGFIELD PUBLIC SCHOOLS

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|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|          | numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.                                                                                                                                                                                                                                                                                                                                                                                                                              |
| F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <ol style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> <li>Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> <li>Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</li> <li>Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</li> </ol> |
| F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| F-BF.A.1 | Write a function that describes a relationship between two quantities. <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</li> <li>Compose functions. For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</li> </ol>                                                       |
| F-BF.B.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| F-BF.B.4 | Find inverse functions. <ol style="list-style-type: none"> <li>Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse</li> </ol>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |



|               |                                                                                                                                                                                                                                                                                                                            |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | and write an expression for the inverse.<br>b. Verify by composition that one function is the inverse of another.<br>c. Read values of an inverse function from a graph or a table, given that the function has an inverse.<br>d. Produce an invertible function from a non-invertible function by restricting the domain. |
| SEL.PK-12.2.2 | Recognize the skills needed to establish and achieve personal and educational goals.                                                                                                                                                                                                                                       |
| SEL.PK-12.4.1 | Develop, implement and model effective problem-solving, and critical thinking skills.                                                                                                                                                                                                                                      |
| SEL.PK-12.5.2 | Utilize positive communication and social skills to interact effectively with others.                                                                                                                                                                                                                                      |
| HS-ETS1-2     | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.                                                                                                                                                                       |

**Unit Essential Questions:**

- What are the basic transformations that can be performed on a function?
- How are the graphs of functions affected by changes to the function?
- How is limit notation used to describe the end behavior of a function?
- How do changes to one variable in a relation affect the value of other variables in the relation?

**Unit Enduring Understandings:**

- There are common modifications that can be made to all types of functions that have similar effects on the graphs of these functions (translations, reflections, etc.)
- The value of a limit does not describe what happens at a specific x-value, rather what happens to the function as x-value approaches a specific value.
- Functions have many important features that if known, can yield an accurate sketch of the function.

**Unit Learning Targets:**

*Students will....*

- identify a function as even, odd or neither.
- sketch the graphs of linear, quadratic, cubic, square root, cube root, rational and absolute value functions.
- describe and graphically represent  $f(x)$  under any combination of the following transformations:  $f(x+h)$ ,  $f(x)+k$ ,  $-f(x)$ ,  $af(x)$ ,  $f(ax)$ ,  $f(|x|)$ ,  $|f(x)|$ .
- graph nonlinear inequalities.
- describe the end behavior of a function using limit notation.
- identify the following features of a function: y-intercept, x-intercept(s), end behavior, absolute/local minima and maxima, intervals where the function is increasing/decreasing, point(s) of inflection, intervals where the function is concave up/concave down.
- find and verify inverses of functions.
- graph rational functions by identifying x and y-intercepts, vertical, horizontal and slant asymptotes and identifying the behavior of the function near all vertical asymptotes.
- determine whether a function is continuous at a given value of x.
- differentiate between removable and non-removable discontinuities.
- remove a point discontinuity from a piecewise or rational function.
- solve problems involving direct, inverse and joint variation.



### Evidence of Learning

**Summative Assessment:** Unit Test

**Equipment Needed:** Graph paper, laptops, TI-84 Graphing Calculator

**Teacher Resources:** Teacher's text, projector, TI-84 emulator software, TI-84 Graphing Calculator

**Formative Assessments:**

- Homework
- Class discussions
- Q&A with students
- Student presentation of problems
- Paired student work



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### Lesson Plans

| Lesson                                       | Time Frame      |
|----------------------------------------------|-----------------|
| Lesson 1 <b>Families of Functions</b>        | <b>1 period</b> |
| Lesson 2 <b>Transforming Functions</b>       | <b>1 period</b> |
| Lesson 3 <b>Seven Features of a Function</b> | <b>1 period</b> |

**Teacher Notes:** Give students one question pop quizzes on the different families of functions periodically.

### Curriculum Development Resources/Technology Resources:

*Click the links below to access additional resources used to design this unit:*

<https://tasks.illustrativemathematics.org/content-standards>

<https://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrig.aspx>



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**Lesson Plan 1 Template**

Content Area: Math      Lesson Title: Families of Functions      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                        | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Formative Assessment Tasks                                                                                                                                            |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify at least a dozen basic families of functions.</li> <li>compare basic features of the families of functions.</li> <li>analyze various patterns that occur within and across function families.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students brainstorm in small groups to come up with as many different types of graphs (families) as they can.</li> <li>Students share results of brainstorming with class.</li> <li>Students demonstrate graphs of functions using graphing calculators.</li> <li>Teacher guides Q&amp;A regarding basic features of functions (intercepts, increasing/decreasing, end behavior)</li> <li>Students complete the activity in which they group families of functions by basic features.</li> </ol> | <ul style="list-style-type: none"> <li>Student participation in brainstorming</li> <li>Students participate in Q&amp;A</li> <li>Students complete activity</li> </ul> |



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### **Differentiation:**

Students of differing abilities may be placed in specific groups for the brainstorming portion of the lesson.

### **Resources Provided:**

- Graphing calculators
- Activity worksheet



### Lesson Plan 2 Template

Content Area: Math      Lesson Title: Transforming Functions      Time Frame: 1 period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Formative Assessment Tasks                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify the transformations that occur to the graph of a function when certain changes are made to the equation of a parent function (translation, reflection, compression, etc.).</li> <li>generate a function that has a specified transformation.</li> <li>generate a function based on examination of a graph.</li> <li>generate the graph of a transformed function with and without the use of a graphing calculator.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students begin the lesson by revisiting the parent graphs of major function families.</li> <li>Groups of students are given a variety of functions from different families with similar transformations to explore using the graphing calculator.</li> <li>Teacher circles the room, asking students what they have discovered about the various transformations.</li> <li>Students share their learnings with the class, which will enable generalizations to be made regarding the effect of each transformation.</li> </ol> | <ul style="list-style-type: none"> <li>Participation in review of parent graphs</li> <li>Participation in group activity</li> <li>Presentation of findings to the class</li> <li>Completion of summary assignment</li> </ul> |



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- |  |                                                                                   |  |
|--|-----------------------------------------------------------------------------------|--|
|  | 5. Students complete a summary exercise that enables them to demonstrate mastery. |  |
|--|-----------------------------------------------------------------------------------|--|

### **Differentiation:**

Students of differing abilities may be placed in differentiated groups for the brainstorming portion of the lesson.

### **Resources Provided:**

- Graphing calculator
- List of functions
- Summary assignment



### Lesson Plan 3 Template

Content Area: Math      Lesson Title: Seven Features of a Function      Time Frame: 1 period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                                                                                         | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Formative Assessment Tasks                                                                                                                                                                          |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify x-intercepts, y-intercepts and zeros of a function.</li> <li>describe the end behavior of a function using limit notation.</li> <li>locate intervals where a function is increasing, decreasing, concave up and concave down.</li> <li>identify local and absolute minima and maxima of a function.</li> <li>approximate the location of points of inflection.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students brainstorm important features of a function. Teacher guides towards seven important features: x-intercepts, y-intercepts, end behavior, minima/maxima, increasing/decreasing, points of inflection, concavity.</li> <li>Given a cubic function, students provide as much information as they can based on prior knowledge.</li> <li>Teacher demonstrates the use of the graphing calculator to find zeros, minima and maxima, and points of inflection.</li> </ol> | <ul style="list-style-type: none"> <li>Participation in brainstorming</li> <li>Participation in discussion</li> <li>Completion of task of finding all 7 major features of given function</li> </ul> |



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4. Teacher leads discussion regarding proper notation, including limit notation.
5. Students work in pairs to find all 7 features of a given function (cubic or quartic).

### **Differentiation:**

Students may be paired based on ability and may be given different functions to analyze in the final activity.

### **Resources Provided:**

- Graphing calculator
- List of functions



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### Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly, Moderately* or *Weakly* meets the criteria below.

| Lesson Activities                                                                                                    | Strongly                            | Moderately                          | Weakly                   |
|----------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|--------------------------|
| Are challenging <b>Strongly</b> and require higher order thinking and problem solving skills                         | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Allow for student choice                                                                                             | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide scaffolding for acquiring targeted knowledge/skills                                                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Integrate global perspectives                                                                                        | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Integrate 21st century skills                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for interdisciplinary connection and transfer of knowledge and skills                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are varied to address different student learning styles and preferences                                              | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are differentiated based on student needs                                                                            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process    | <input type="checkbox"/>            | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for student reflection and self- assessment                                                    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide data to inform and adjust instruction to better meet the varying needs of learners                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |

### Curriculum Design Template

**Content Area:** Math      **Grade Level:** 10-12      **Unit Title:** The Nature of Graphs

**Created:** 2022

**Board Approved on:**



### Unit Overview Template

**Content Area:** Math    **Grade Level:** 10-12    **Unit Title:** Polynomial and Rational Functions

**Unit Summary:** Students will use various methods, including technology, to locate and approximate the zeros of polynomial functions. They examine the relationship locating zeros of a function to solving for the roots of an equation. The ongoing practice of modeling real-world data is extended to include polynomial functions.

**Primary Interdisciplinary Connections:** *Science and Engineering*

**21<sup>st</sup> Century Themes**    ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ ICT Literacy  
☐ Information Literacy    ☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy

**Unit Rationale:** Polynomial functions are used in many ways to model real-world phenomena. An understanding of these functions is critical to the study of mathematics.

**Career Readiness (Standard 9):**

- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.
- 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12.prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

### Learning Targets

**Standards:** A-SSE; A-APR; A-REI; F-IF; F-LE; N-CN

**Content Statements:** Write expressions in equivalent forms to solve problems; Perform arithmetic operations on polynomials; Understand the relationship between zeros and factors of polynomials; Use polynomial identities to solve problems; Rewrite rational expressions; Solve equations and inequalities in one variable; Represent and solve equations and inequalities graphically; Analyze functions using different representations; Construct and compare linear and exponential models and solve problems.

| CPI#:     | Cumulative Progress Indicator (CPI):                                                                                                   |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------|
| A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |



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|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|            | <p>a. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>c. Use the properties of exponents to transform expressions for exponential functions.</p>                                                                                                                                                                                                                                                                                                                                                               |
| A-APR.A.1  | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| A-APR.B.2  | Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| A-APR.B.3  | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| A-APR.C.4  | Prove polynomial identities and use them to describe numerical relationships.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| A-APR.D.6  | Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.                                                                                                                                                                                                                                                                                                                                           |
| A-REI.B.4  | <p>Solve quadratic equations in one variable.</p> <p>a. Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</p> <p>b. Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</p> |
| A-REI.D.11 | Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.                                                                                                                                                                                                                           |
| F-IF.C.7   | <p>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p>                                                                                                                                                    |
| F-IF.C.8   | <p>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of</p>                                                                                                                                                                                                                                                                                                                                              |



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|               |                                                                                                                                                      |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | a context.                                                                                                                                           |
| N-CN.C.7      | Solve quadratic equations with real coefficients that have complex solutions.                                                                        |
| N-CN.C.8      | Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .                                          |
| N-CN.C.9      | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.                                                             |
| SEL.PK-12.2.2 | Recognize the skills needed to establish and achieve personal and educational goals.                                                                 |
| SEL.PK-12.4.1 | Develop, implement and model effective problem-solving, and critical thinking skills.                                                                |
| SEL.PK-12.5.2 | Utilize positive communication and social skills to interact effectively with others.                                                                |
| HS-ETS1-2     | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering. |

**Unit Essential Questions:**

- What is the significance of the roots of polynomial functions?
- How can roots of polynomial functions be found mathematically and using technology?
- How are rational and radical equations solved?
- How can polynomial functions be used to model real-world phenomena?

**Unit Enduring Understandings:**

- There are several mathematical processes that can help locate roots of polynomial functions.
- Roots of polynomial functions are the x-values that give the function a value of zero.
- Polynomial functions can be used to model a wide array of real -world phenomena.
- Manipulation of radical and rational expressions is vital to the solution of complex problems.

**Unit Learning Targets:**

*Students will....*

- find the roots of quadratic functions by factoring, completing the square and by using the quadratic formula.
- use the rational root test, Descartes' Rule of Signs, the upper and lower bound theorems and the Intermediate Value theorem to locate roots of polynomial functions.
- solve rational equations and inequalities.
- solve radical equations and inequalities.
- decompose an expression into partial fractions.
- sketch the graph of polynomial functions with single, double and triple roots.
- generate a function given a variety of real and imaginary roots.
- generate quadratic, cubic and quartic regressions to model real-world data.



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### Evidence of Learning

**Summative Assessment:** Unit Test

**Equipment Needed:** Graph paper, laptops, TI-84 Graphing Calculator

**Teacher Resources:** Teacher's text, projector, TI-84 emulator software, TI-84 Graphing Calculator

**Formative Assessments:**

- Homework
- Class discussions
- Q&A with students
- Student presentation of problems
- Paired student work



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### Lesson Plans

| Lesson                                     | Time Frame      |
|--------------------------------------------|-----------------|
| Lesson 1 <b>Rational Root Test</b>         | <b>1 period</b> |
| Lesson 2 <b>Intermediate Value Theorem</b> | <b>1 period</b> |
| Lesson 3 <b>Rational Inequalities</b>      | <b>1 period</b> |

**Teacher Notes:** Remind students the importance of properly applying the theorems when solving problems.

### Curriculum Development Resources/Technology Resources:

*Click the links below to access additional resources used to design this unit:*

<https://tasks.illustrativemathematics.org/content-standards>

<https://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrig.aspx>



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**Lesson Plan 1 Template**

Content Area: Math      Lesson Title: Rational Root Test      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills ☒ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                     | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Formative Assessment Tasks                                                                                                                                 |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify possible rational roots of a function.</li> <li>use synthetic division to locate actual roots.</li> <li>use technology to assist in the process of locating zeros.</li> <li>use upper and lower bound theorems to shorten the process of locating zeros.</li> <li>confirm their solutions.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>State and explain the Rational Root Test--that any rational roots must be in the form of p/q.</li> <li>Discuss and work through an example, helping students find possible values for p and q.</li> <li>Demonstrate the use of the graphing calculator and Desmos to find roots of functions by graphing .</li> <li>Students are given problems to work through individually or in pairs.</li> <li>Students present work to class.</li> <li>Students complete an exit ticket at the end of class.</li> </ol> | <ul style="list-style-type: none"> <li>Question-and-answer</li> <li>Class discussion</li> <li>Problem sets</li> <li>Exit card</li> <li>Homework</li> </ul> |



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### **Differentiation:**

Students of different ability levels can be given easier/harder problems to solve.

### **Resources Provided:**

- Sample problems
- Graphing calculators
- Exit ticket



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**Lesson Plan 2 Template**

Content Area: Math      Lesson Title: Intermediate Value Theorem      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving      ☐ Information Literacy  
☐ Life and Career Skills ☐ Communication and Collaboration    ☐ Media Literacy      ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                            | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Formative Assessment Tasks                                                                                                                                 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify the minimum number of zeros for a function.</li> <li>locate the zeros of a function between integers on a table of values.</li> <li>justify their answers with a coherent, mathematical argument.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Teacher gives the abstract version of the IVT.</li> <li>Teacher immediately follows this with a concrete example, connecting the theorem to the example on a clear, step-by-step basis.</li> <li>Students are given a paper-based sample table of values and asked to find the number and location of all zeros.</li> <li>Students write a coherent justification of their answers using the IVT.</li> <li>Students use the table feature of the TI-84 to do the same.</li> <li>Exit card to demonstrate mastery.</li> </ol> | <ul style="list-style-type: none"> <li>Question-and-answer</li> <li>Class discussion</li> <li>Problem sets</li> <li>Exit card</li> <li>Homework</li> </ul> |



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### **Differentiation:**

Students of different ability levels can be given easier/harder problems to solve.

### **Resources Provided:**

- Graphing calculator
- Problem set
- Sample problems



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**Lesson Plan 3 Template**

Content Area: Math                      Lesson Title: Rational Inequalities                      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy            ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving                      ☐ Information Literacy  
☐ Life and Career Skills ☐ Communication and Collaboration    ☐ Media Literacy                      ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                                                                                 | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Formative Assessment Tasks                                                                                                                                 |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>identify values that make a rational expression equal zero.</li> <li>connect rational inequalities to graphs of rational functions.</li> <li>identify values that make a rational expression undefined.</li> <li>find solutions to rational inequalities using the "number line" technique.</li> <li>express solutions to rational inequalities mathematically.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students begin by graphing a given rational inequality by applying previous knowledge.</li> <li>Students brainstorm locations on the graph where the value of the function switches from greater than zero to less than zero or vice versa.</li> <li>Teacher allows students to realize (with guided questions if necessary) that changes occur at vertical asymptotes and x-intercepts.</li> <li>Students locate the vertical asymptotes and x-intercepts of another function without graphing and plot values on a number line.</li> </ol> | <ul style="list-style-type: none"> <li>Question-and-answer</li> <li>Class discussion</li> <li>Problem sets</li> <li>Exit card</li> <li>Homework</li> </ul> |



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- |  |                                                                                                                                                                                           |  |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|  | <ol style="list-style-type: none"><li>5. Teacher introduces and explains the number line technique.</li><li>6. Students work out an additional example.</li><li>7. Exit ticket.</li></ol> |  |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|

### **Differentiation:**

Students of different ability levels can be given easier/harder problems to solve.

### **Resources Provided:**

- Graphing calculator



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### Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly, Moderately* or *Weakly* meets the criteria below.

| Lesson Activities                                                                                                    | Strongly                            | Moderately                          | Weakly                   |
|----------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|--------------------------|
| Are challenging <b>Strongly</b> and require higher order thinking and problem solving skills                         | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Allow for student choice                                                                                             | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide scaffolding for acquiring targeted knowledge/skills                                                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Integrate global perspectives                                                                                        | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Integrate 21st century skills                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for interdisciplinary connection and transfer of knowledge and skills                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are varied to address different student learning styles and preferences                                              | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are differentiated based on student needs                                                                            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for student reflection and self- assessment                                                    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide data to inform and adjust instruction to better meet the varying needs of learners                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |

### Curriculum Design Template

|                           |                           |                                                      |
|---------------------------|---------------------------|------------------------------------------------------|
| <b>Content Area:</b> Math | <b>Grade Level:</b> 10-12 | <b>Unit Title:</b> Polynomial and Rational Functions |
| <b>Created:</b> 2022      | <b>Board Approved on:</b> |                                                      |



### Unit Overview Template

**Content Area:** Math    **Grade Level:** 10-12    **Unit Title:** Exponential and Logarithmic Functions

**Unit Summary:** Students will use various methods, including technology, to solve and graph exponential and logarithmic functions.

**Primary Interdisciplinary Connections:** Science and Engineering

**21<sup>st</sup> Century Themes**    ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ ICT Literacy  
☐ Information Literacy    ☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy

**Unit Rationale:** Exponential and logarithmic functions are used in many ways to model real-world phenomena. An understanding of these functions is critical to the study of mathematics.

**Career Readiness (Standard 9):**

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.

9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12.prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

### Learning Targets

**Standards:** N-RN; A-REI; F-IF; F-BF; F-LE

**Content Statements:** Extend the properties of exponents to rational exponents; Represent and solve equations and inequalities graphically; Interpret functions that arise in applications in terms of the context; Analyze functions using different representations; Build new functions from existing functions; Construct and compare linear and exponential models and solve problems; Interpret expressions for functions in terms of the situation they model.

| <b>CPI#:</b> | <b>Cumulative Progress Indicator (CPI):</b>                                                                                                                                                                      |
|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| N-RN.A.2     | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. |
| N-RN.A.2     | Rewrite expressions involving radicals and rational exponents using the properties of exponents.                                                                                                                 |



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|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A-REI.D.11 | Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.                                                                         |
| F-IF.B.4   | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.                                                                                              |
| F-IF.B.5   | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.                                                                                                                                                                                                        |
| F-IF.C.7   | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.<br>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.                                                                                                                                                                                                          |
| F-BF.B.3   | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.                                                                                                              |
| F-BF.B.4   | Find inverse functions.<br>a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse.<br>b. Verify by composition that one function is the inverse of another.<br>c. Read values of an inverse function from a graph or a table, given that the function has an inverse.<br>d. Produce an invertible function from a non-invertible function by restricting the domain.                                                                           |
| F-BF.B.5   | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.                                                                                                                                                                                                                                                                                                                                                                                                       |
| F-LE.A.1   | Distinguish between situations that can be modeled with linear functions and with exponential functions.<br>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.<br>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.<br>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| F-LE.A.2   | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output                                                                                                                                                                                                                                                                                                                                                             |



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|               |                                                                                                                                                                                                                                                                       |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | pairs (include reading these from a table).                                                                                                                                                                                                                           |
| F-LE.A.3      | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.                                                                              |
| F-LE.A.4      | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology. |
| F-LE.B.5      | Interpret the parameters in a linear or exponential function in terms of a context.                                                                                                                                                                                   |
| SEL.PK-12.2.2 | Recognize the skills needed to establish and achieve personal and educational goals.                                                                                                                                                                                  |
| SEL.PK-12.4.1 | Develop, implement and model effective problem-solving, and critical thinking skills.                                                                                                                                                                                 |
| SEL.PK-12.5.2 | Utilize positive communication and social skills to interact effectively with others.                                                                                                                                                                                 |
| HS-ETS1-2     | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.                                                                                                                  |

**Unit Essential Questions:**

- What are the different ways to express exponential and radical quantities?
- How are exponential and logarithmic functions related?
- What are the different models for growth and decay and when is each appropriate?
- How can the properties of logarithms be used to solve certain types of equations?

**Unit Enduring Understandings:**

- Exponential and logarithmic functions are inverses of one another.
- Exponential growth and decay ultimately result in larger (or smaller) values than linear growth or decay.
- Using properties of logarithms can simplify the solution process for a number of problems.
- Exponential and logistic growth can be used to model numerous real-world situations.

**Unit Learning Targets:**

*Students will....*

- convert between expressions using radicals and fractional exponents.
- simplify radical and exponential expressions.
- graph exponential and logarithmic equations and inequalities.
- convert between exponential and logarithmic forms of an equation.
- rewrite expressions using the properties of logarithms.
- solve exponential and logarithmic equations.
- solve problems involving exponential growth and decay including problems involving compound interest (including continuously compounded interest).
- appreciate the derivation of the number  $e$ .
- model real-world situations using both exponential and logistic models.
- identify and explain the carrying capacity in a logistic model.



### Evidence of Learning

**Summative Assessment:** Unit Test

**Equipment Needed:** Graph paper, laptops, TI-84 Graphing Calculator

**Teacher Resources:** Teacher's text, projector, TI-84 emulator software, TI-84 Graphing Calculator

**Formative Assessments:**

- Homework
- Class discussions
- Q&A with students
- Student presentation of problems
- Paired student work



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### Lesson Plans

| Lesson                                   | Time Frame      |
|------------------------------------------|-----------------|
| Lesson 1 <b>Compound Interest</b>        | <b>1 period</b> |
| Lesson 2 <b>Properties of Logarithms</b> | <b>1 period</b> |
| Lesson 3 <b>Logistic Growth</b>          | <b>1 period</b> |

**Teacher Notes:** After playing the “zombie game,” ask students for more realistic examples where logistic growth occurs.

### Curriculum Development Resources/Technology Resources:

*Click the links below to access additional resources used to design this unit:*

<https://tasks.illustrativemathematics.org/content-standards>

<https://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrig.aspx>



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**Lesson Plan 1 Template**

Content Area: Math                      Lesson Title: Compound Interest                      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy            ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving                      ☐ Information Literacy  
☒ Life and Career Skills ☐ Communication and Collaboration    ☐ Media Literacy                      ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                                                                                                  | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Formative Assessment Tasks                                                                                                                                 |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>calculate the amount of interest earned over a period of time.</li> <li>recognize that the more frequently interest is compounded, the faster it is earned.</li> <li>choose the wisest investment from a list where the options have different interest rates and compounding timeframes.</li> <li>calculate continuously compounded interest.</li> <li>confirm their solutions.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students manually calculate the interest earned on a 5-year investment compounded annually.</li> <li>Based on their work, students derive the formula for calculating interest.</li> <li>Students then are introduced to interest compounded monthly and asked how that will affect their formula.</li> <li>Students are then given the formula for continuously compounded interest and solve a sample problem.</li> <li>Students are then given a variety of investment opportunities and asked</li> </ol> | <ul style="list-style-type: none"> <li>Question-and-answer</li> <li>Class discussion</li> <li>Problem sets</li> <li>Exit card</li> <li>Homework</li> </ul> |



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- to identify the wisest one  
(and justify their choice).
6. Students present their  
solutions.

### **Differentiation:**

Students of different ability levels can be given easier/harder problems to solve.

### **Resources Provided:**

- Sample problems
- Graphing calculators



### Lesson Plan 2 Template

Content Area: Math      Lesson Title: Properties of Logarithms      Time Frame: 1 period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                 | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Formative Assessment Tasks                                                                                                                                           |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>• solve logarithmic equations with logarithms on one side of the equation.</li> <li>• solve logarithmic equations with logarithms on both sides of the equation.</li> <li>• rewrite expressions using properties of logarithms.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>1. Students are given the four main properties of logarithms: addition/multiplication, subtraction/division, exponents/coefficients and change of base.</li> <li>2. Teacher immediately follows this with concrete examples, connecting the theorem to the example on a clear, step-by-step basis.</li> <li>3. Students are given a set of problems and work in pairs to solve.</li> <li>4. Students present their learnings to the class.</li> <li>5. Exit card to demonstrate mastery.</li> </ol> | <ul style="list-style-type: none"> <li>• Question-and-answer</li> <li>• Class discussion</li> <li>• Problem sets</li> <li>• Exit card</li> <li>• Homework</li> </ul> |



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### **Differentiation:**

Students of different ability levels can be given easier/harder problems to solve.

### **Resources Provided:**

- Graphing calculator
- Problem set
- Sample problems



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**Lesson Plan 3 Template**

Content Area: Math      Lesson Title: Logistic Growth      Time Frame: 1 period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                                                                                          | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Formative Assessment Tasks                                                                                                                                 |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>explain why the logistic model is more appropriate than the exponential model in certain situations.</li> <li>identify the carrying capacity of a logistic scenario.</li> <li>identify the time at which a logistic model is growing fastest.</li> <li>describe the end behavior of a logistic function.</li> <li>model a real world scenario using logistic regression.</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Students are given an exponential model for population growth in a city and asked to find the population in 500 years.</li> <li>Students explain why the model is nonsensical, using real-world explanations.</li> <li>Class engages in a "disease spreading" simulation that models the spread of disease where students are infected over the course of "days" based upon the current level of infection.</li> <li>Results from the simulation are graphed on the graphing calculator and discussed.</li> </ol> | <ul style="list-style-type: none"> <li>Question-and-answer</li> <li>Class discussion</li> <li>Problem sets</li> <li>Exit card</li> <li>Homework</li> </ul> |



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- |  |                                                                                                                                                                                                                                                                         |  |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|  | <ol style="list-style-type: none"><li>5. Students find the logistic regression using the calculator.</li><li>6. Properties of the function are discussed.</li><li>7. Students find the logistic model for growth of the population of Ohio as an exit ticket.</li></ol> |  |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|

### **Differentiation:**

Students of different ability levels can be given easier/harder problems to solve.

### **Resources Provided:**

- Graphing calculators



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### Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly*, *Moderately* or *Weakly* meets the criteria below.

| Lesson Activities                                                                                                    | Strongly                            | Moderately                          | Weakly                   |
|----------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|--------------------------|
| Are challenging <b>Strongly</b> and require higher order thinking and problem solving skills                         | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Allow for student choice                                                                                             | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide scaffolding for acquiring targeted knowledge/skills                                                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Integrate global perspectives                                                                                        | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Integrate 21st century skills                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for interdisciplinary connection and transfer of knowledge and skills                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are varied to address different student learning styles and preferences                                              | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are differentiated based on student needs                                                                            | <input type="checkbox"/>            | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for student reflection and self- assessment                                                    | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide data to inform and adjust instruction to better meet the varying needs of learners                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |

### Curriculum Design Template

|                           |                           |                                                          |
|---------------------------|---------------------------|----------------------------------------------------------|
| <b>Content Area:</b> Math | <b>Grade Level:</b> 10–12 | <b>Unit Title:</b> Exponential and Logarithmic Functions |
| <b>Created:</b> 2022      | <b>Board Approved on:</b> |                                                          |



### Unit Overview Template

**Content Area:** Math      **Grade Level:** 11-12      **Unit Title:** Graphs of Trigonometric Functions

**Unit Summary:** Students will review graphs of trigonometric functions and their inverses using radian measure. Students will learn about amplitude, period, phase shift, and vertical shift. Applications include finding linear and angular velocity and writing trigonometric equations to model real-life situations.

**Primary Interdisciplinary Connections:** *Science and Engineering*

**21<sup>st</sup> Century Themes**   ☐ Creativity and Innovation   ☒ Critical Thinking and Problem Solving   ☐ ICT Literacy  
☒ Information Literacy   ☒ Life and Career Skills   ☐ Communication and Collaboration   ☐ Media Literacy

**Unit Rationale:** In this unit, students will learn how to convert from degrees to radians and vice versa; how to draw the graph of different trigonometric functions; how to find amplitude, period, phase shift, and vertical shift of the given trigonometric function.

**Career Readiness (Standard 9):**

- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.
- 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

### Learning Targets

**Standards:** F-TF; F-IF; N-Q

**Content Statements:** Extend the domain of trigonometric functions using the unit circle; Model periodic phenomena with trigonometric functions; Analyze functions using different representations; Reason quantitatively and use units to solve problems

|              |                                             |
|--------------|---------------------------------------------|
| <b>CPI#:</b> | <b>Cumulative Progress Indicator (CPI):</b> |
|--------------|---------------------------------------------|



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|               |                                                                                                                                                                                                                                                                                                            |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| F-TF.A.1      | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.                                                                                                                                                                                                  |
| F-TF.A.2      | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.                                                                                      |
| F-TF.A.3      | Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $\pi x$ , $\pi+x$ , and $2\pi-x$ in terms of their values for $x$ , where $x$ is any real number. |
| F-TF.A.4      | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.                                                                                                                                                                                                         |
| F-IF.C.7E     | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline and amplitude.                                                                                                                                                      |
| F-TF.B.5      | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.                                                                                                                                                                                               |
| F-TF.B.6      | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.                                                                                                                                               |
| F-TF.B.7      | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.                                                                                                                               |
| N-Q.A.1       | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.                                                                                |
| SEL.PK-12.2.2 | Recognize the skills needed to establish and achieve personal and educational goals.                                                                                                                                                                                                                       |
| SEL.PK-12.4.1 | Develop, implement and model effective problem-solving, and critical thinking skills.                                                                                                                                                                                                                      |
| SEL.PK-12.5.2 | Utilize positive communication and social skills to interact effectively with others.                                                                                                                                                                                                                      |
| HS-ETS1-2     | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.                                                                                                                                                       |

**Unit Essential Questions:**

- How are degree measurements converted to radian measurements and vice versa?
- How can the length of an arc or the area of a sector be found given the measure of the central angle?
- How are linear and angular velocity calculated?
- How are graphs of sine, cosine, tangent, cotangent, secant, and cosecant functions produced?
- How are equations of trigonometric functions written?

**Unit Enduring Understandings:**

- There is a strong connection between the formulas we learned in geometry and trigonometry
- We can use trigonometric functions to model real- life data such as sound waves, seasonal temperature change, etc.



### Unit Learning Targets:

*Students will....*

- Find the amplitude, period, phase shift, vertical shift for sine, cosine, tangent, cotangent, secant, and cosecant functions.
- Write equations of trigonometric functions given the amplitude, period, phase shift, vertical shift.
- Model real-life data using sine and cosine functions.
- Graph inverse trigonometric functions.
- Find principal values of inverse trigonometric functions.

### Evidence of Learning

**Summative Assessment:** Two quizzes, one unit test

**Equipment Needed:** Teacher laptop, student laptops, graphing calculator

**Teacher Resources:** Textbook, Notes on Chapter 6, Workbook Worksheets, Online Student Text, Glencoe Test Generator

### Formative Assessments:

- Students will be assessed on classwork assignments.
- Students will be assessed on group and individual projects.
- Students will be assessed on quizzes and tests.
- Students will be assessed on homework assignments.
- Students will be assessed on exit/entry tickets.



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**Lesson Plans**

| Lesson                                             | Time Frame            |
|----------------------------------------------------|-----------------------|
| Lesson 1 <b>Angles and Radian Measures</b>         | <b>1 class period</b> |
| Lesson 2 <b>Linear and Angular Velocity</b>        | <b>1 class period</b> |
| Lesson 3 <b>Graphing Sine and Cosine Functions</b> | <b>1 class period</b> |

**Teacher Notes:**

**Curriculum Development Resources/Technology Resources:**

*Click the links below to access additional resources used to design this unit:*

<http://app.uncoursesystems.com/>

<http://www.wikipedia.com> for climate information

<https://tasks.illustrativemathematics.org/content-standards>

<https://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrig.aspx>



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**Lesson Plan 1 Template**

Content Area: Math      Lesson Title: Angles and Radian Measures      Time Frame: 1 class period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☒ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☒ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☒ Life and Career Skills    ☐ Communication and Collaboration    ☒ Media Literacy    ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                       | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Formative Assessment Tasks                                                                                                                                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>Be able to change from radian measure to degree measure, and vice versa</li> <li>Find the length of an arc given the measure of the central angle.</li> <li>Find the area of a sector</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Define the radian (use string cut to the length of the circle radius).</li> <li>Have students derive the connection between radian and degree measures using Desmos.</li> <li>Show an example how to convert from degree measure to radian measure and vice versa</li> <li>Review how to find reference angles.</li> <li>Show how to determine values of trigonometric functions using reference angles.</li> <li>Discuss the Length of an Arc and the Area of a Circular Sector Formulas.</li> </ol> | <ul style="list-style-type: none"> <li>Students respond to verbal prompts during initial discussion.</li> <li>Circle the room as students are working to check for understanding</li> <li>Talk to each pair during the process or after they finish to assess understanding</li> <li>Collect problem set to assess mastery</li> </ul> |



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- |  |                                                                                                                                                                                                 |  |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|  | <ol style="list-style-type: none"><li>7. Students will be asked to work in pairs on Desmos problems</li><li>8. Students then show their work to the partners and discuss the results.</li></ol> |  |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|

### **Differentiation:**

Students may be grouped so that they are working with partners at similar ability levels. Problem sets may be modified to address needs of individual learners.

### **Resources Provided:**

- Guided notes
- Exit Ticket
- Access to the online textbook for application problem access as well as textbook for student's preference.



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**Lesson Plan 2 Template**

Content Area: Math      Lesson Title: Linear and Angular Velocities      Time Frame: 1 class period

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving      ☐ Information Literacy  
☒ Life and Career Skills ☐ Communication and Collaboration    ☒ Media Literacy      ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                            | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Formative Assessment Tasks                                                                                                                                                                                                                                                                                                                                                                        |
|---------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>Find linear and angular velocities</li> <li>Determine angular displacement</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Present “The Children’s Museum carousel” problem</li> <li>Ask students to derive how to calculate angular displacement</li> <li>Discuss definitions of Angular Velocity and Linear Velocity and show some examples on Desmos.</li> <li>Ask students to work in pairs and determine the angular and linear velocities of someone riding an animal in the inner circle and someone riding an animal in the same row in the outer circle (“Carousel Problem”).</li> <li>Ask students to present their conclusions.</li> </ol> | <ul style="list-style-type: none"> <li>Students respond to verbal prompts during initial discussion.</li> <li>Circle the room as students are working to check for understanding</li> <li>Talk to students during the process or after they finish to assess understanding.</li> <li>Have students present solutions.</li> <li>Collect application problems to assess student mastery.</li> </ul> |



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### Differentiation:

Extension problem:

Joe is running one lap on the outside of a circular track with a diameter of 1000 ft. His constant rate of speed is 12 mph. Sunil is running one lap on the outside of a nearby circular track with a diameter of 1125 ft. He too is running at 12 mph. Find the time in minutes for each of the two runners, and compare their times with the radius of each other circle.

### Resources Provided:

- Problem Set
- Access to online textbook
- Graphing Calculators



### Lesson Plan 3 Template

Content Area: Math Lesson Title: Graphing Sine and Cosine Functions Time Frame: 1 class period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy    ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☒ Life and Career Skills    ☐ Communication and Collaboration    ☐ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science, Computer Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                                                                                                                                                                                  | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Formative Assessment Tasks                                                                                                                                                                                                                                                                                                                                                                |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>• Generate the graphs of the sine and cosine functions</li> <li>• Be to identify the amplitude, phase shift, vertical shift and period of a sine or cosine function</li> <li>• Generate the graph of a sine or cosine function from a graph</li> <li>• Use sine functions to model naturally occurring phenomena</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>1. Discuss the “Meteorology” problem.</li> <li>2. Have students access climate data for two US cities: Baltimore, Maryland, and Asheville, North Carolina</li> <li>3. Explain what a periodic function is.</li> <li>4. Discuss with the class what makes these functions periodic, then find the period.</li> <li>5. Ask students to give examples of periodic functions in real life.</li> <li>6. Show students a graph of sine and its connection with the unit circle.</li> <li>7. Review properties of the sine functions.</li> </ol> | <ul style="list-style-type: none"> <li>• Students respond to verbal prompts during initial discussion.</li> <li>• Circle the room to check for understanding.</li> <li>• Talk to each group during the process or after they finish to assess understanding</li> <li>• Have students present solutions to interesting problems</li> <li>• Collect exit cards to assess mastery</li> </ul> |



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- |  |                                                                                                                                                                                   |  |
|--|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|  | <ol style="list-style-type: none"><li>8. Show students a graph of cosine and its connection with the unit circle.</li><li>9. Review properties of the cosine functions.</li></ol> |  |
|--|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|

### Differentiation:

Students will research and observe what the graph of the sound pattern of a person's live voice, a song recorded by Frank Sinatra, a rock concert, and a healthy and an unhealthy heart would look like. Students will be asked to decide whether these patterns are periodic functions.

### Resources Provided:

- Laptops
- Graphing calculators
- Problem sets



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### Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly*, *Moderately* or *Weakly* meets the criteria below.

| Lesson Activities                                                                                                    | Strongly                            | Moderately                          | Weakly                   |
|----------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|--------------------------|
| Are challenging <b>Strongly</b> and require higher order thinking and problem solving skills                         | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Allow for student choice                                                                                             | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide scaffolding for acquiring targeted knowledge/skills                                                          | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Integrate global perspectives                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Integrate 21st century skills                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for interdisciplinary connection and transfer of knowledge and skills                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are varied to address different student learning styles and preferences                                              | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Are differentiated based on student needs                                                                            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process    | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide opportunities for student reflection and self- assessment                                                    | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide data to inform and adjust instruction to better meet the varying needs of learners                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |

### Curriculum Design Template

|                           |                           |                                                      |
|---------------------------|---------------------------|------------------------------------------------------|
| <b>Content Area:</b> Math | <b>Grade Level:</b> 11-12 | <b>Unit Title:</b> Graphs of Trigonometric Functions |
| <b>Created:</b> 2022      | <b>Board Approved on:</b> |                                                      |



### Unit Overview Template

**Content Area:** Math      **Grade Level:** 11-12      **Unit Title:** Trigonometric Identities and Equations

**Unit Summary:** Students will review the basic trigonometric identities, use sum and difference identities, and use double- and half-angle identities. Students will also solve trigonometric equations.

**Primary Interdisciplinary Connections:** *Science and Engineering*

**21<sup>st</sup> Century Themes**   ☐ Creativity and Innovation   ☒ Critical Thinking and Problem Solving   ☐ ICT Literacy  
☒ Information Literacy   ☒ Life and Career Skills   ☐ Communication and Collaboration   ☐ Media Literacy

**Unit Rationale:** In this unit, students will learn how to use different trigonometric identities and solve trigonometric equations.

**Career Readiness (Standard 9):**

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.

9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

### Learning Targets

**Standards:** F-TF; A-REI

**Content Statements:** Prove and apply trigonometric identities; Represent and solve equations and inequalities graphically

| CPI#:    | Cumulative Progress Indicator (CPI):                                                                                                                                                 |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| F-TF.C.8 | Prove the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ and use it to find $\sin x$ , $\cos x$ , or $\tan x$ given $\sin x$ , $\cos x$ or $\tan x$ and the quadrant of the angle. |
| F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.                                                                             |



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|               |                                                                                                                                                                                                                                                                                                               |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A-REI.D.11    | Explain why the x-coordinates of the points where the graphs of the equations of $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. |
| SEL.PK-12.2.2 | Recognize the skills needed to establish and achieve personal and educational goals.                                                                                                                                                                                                                          |
| SEL.PK-12.4.1 | Develop, implement and model effective problem-solving, and critical thinking skills.                                                                                                                                                                                                                         |
| SEL.PK-12.5.2 | Utilize positive communication and social skills to interact effectively with others.                                                                                                                                                                                                                         |
| HS-ETS1-2     | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.                                                                                                                                                          |

### Unit Essential Questions:

- What does it mean to verify a trigonometric identity and what are some methods for simplifying the process?
- How are trigonometric equations solved?

### Unit Enduring Understandings:

- There is a strong connection between the formulas we learned in geometry and trigonometry.
- We can use trigonometric functions to model real- life data such as sound waves, seasonal temperature change, etc.

### Unit Learning Targets:

*Students will....*

- Identify and use reciprocal, quotient, Pythagorean, symmetry, and opposite-angle identities
- Use the basic trigonometric identities to verify other identities
- Find numerical values of trigonometric functions
- Use the sum and difference identities for the sine, cosine, and tangent functions
- Identify equivalent values for trigonometric functions involving quadrantal angles
- Use the double- and half-angle identities for the sine, cosine, and tangent functions
- Solve trigonometric equations and inequalities



### Evidence of Learning

**Summative Assessment:** Two quizzes, one unit test

**Equipment Needed:** Teacher laptop, student laptops, graphing calculator

**Teacher Resources:** Textbook, Notes on Chapter 7, Workbook Worksheets, Online Student Text, Glencoe Test Generator

**Formative Assessments:**

- Students will be assessed on classwork assignments.
- Students will be assessed on group and individual projects.
- Students will be assessed on quizzes and tests.
- Students will be assessed on homework assignments.
- Students will be assessed on exit/entry tickets.



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**Lesson Plans**

| Lesson                                             | Time Frame             |
|----------------------------------------------------|------------------------|
| Lesson 1 <b>Basic Trigonometric Identities</b>     | <b>2 class periods</b> |
| Lesson 2 <b>Verifying Trigonometric Identities</b> | <b>1 class period</b>  |
| Lesson 3 <b>Solving Trigonometric Equations</b>    | <b>2 class periods</b> |

**Teacher Notes:**

**Curriculum Development Resources/Technology Resources:**

*Click the links below to access additional resources used to design this unit:*

<http://app.uncoursesystems.com/>

<http://www.wikipedia.com> for climate information

<https://tasks.illustrativemathematics.org/content-standards>

<https://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrig.aspx>



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**Lesson Plan 1 Template**

Content Area: Math      Lesson Title: Basic Trigonometric Identities      Time Frame: 2 class periods

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☒ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☒ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☐ Life and Career Skills    ☐ Communication and Collaboration    ☒ Media Literacy    ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                        | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Formative Assessment Tasks                                                                                                                                                                                                                                                                                                            |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>Be able to identify and use reciprocal, quotient, Pythagorean, symmetry, and opposite-angle identities</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Introduce students to the “polarized lenses” problem.</li> <li>Explain what an identity is and give some examples from Algebra 2, then ask students to get in pairs and derive a trigonometric identity. Work through other identities as a whole class.</li> <li>Have pairs of students prove that a given statement is not an identity.</li> <li>Introduce students to Reciprocal, Quotient, and Pythagorean Identities.</li> <li>Use the Unit Circle to present Symmetry and</li> </ol> | <ul style="list-style-type: none"> <li>Students respond to verbal prompts during initial discussion.</li> <li>Circle the room as students are working to check for understanding</li> <li>Talk to each pair during the process or after they finish to assess understanding</li> <li>Collect problem set to assess mastery</li> </ul> |



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ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21<sup>ST</sup> CENTURY GLOBAL SKILLS

Opposite-Angle  
Identities.

6. Ask students to use these basic identities and give information to find the exact value of other trigonometric functions.
7. Students then show their work to the partners and discuss the results.

### **Differentiation:**

Students may be grouped so that they are working with partners at similar ability levels. Problem sets may be modified to address needs of individual learners.

### **Resources Provided:**

- Guided notes
- Exit Ticket
- Access to the online textbook for application problem access as well as textbook for student's preference.



### Lesson Plan 2 Template

Content Area: Math      Lesson Title: Verifying Trigonometric Identities Time Frame: 1 class period

### Lesson Components

#### 21<sup>st</sup> Century Themes

- ☐ Global Awareness    ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

#### 21<sup>st</sup> Century Skills

- ☐ Creativity and Innovation    ☒ Critical Thinking and Problem Solving    ☐ Information Literacy  
☒ Life and Career Skills    ☐ Communication and Collaboration    ☒ Media Literacy    ☐ ICT Literacy

#### **Interdisciplinary Connections:**

Science and Engineering

#### **Integration of Technology:**

Graphing Calculators, Desmos

#### **Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                                                                                                              | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                  | Formative Assessment Tasks                                                                                                                                                                                                                                                                                                                                                                                  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>• Use the basic trigonometric identities to verify other identities</li> <li>• Find numerical value of trigonometric functions</li> </ul>                               | Lesson Sequence <ol style="list-style-type: none"> <li>1. Introduce the students to the “ladder problem.”</li> <li>2. Students will work in groups to derive an expression for the length of a ladder.</li> <li>3. Choose two different (correct) formulas and show students that they are equivalent.</li> <li>4. Give students some problems to verify.</li> <li>5. Ask students to present work in pairs and work on their proofs. They could use “identity cards.”</li> </ol> | <ul style="list-style-type: none"> <li>• Students respond to verbal prompts during initial discussion.</li> <li>• Circle the room as students are working to check for understanding</li> <li>• Talk to students during the process or after they finish to assess understanding.</li> <li>• Have students present solutions.</li> <li>• Collect application problems to assess student mastery.</li> </ul> |
| <b>Differentiation:</b><br>Kinesthetic: Students will work in groups to model signs of trigonometric functions. Each group should use masking tape or rope on the ground to model the x- and y-axis. One student will place a |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                             |



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meter stick on the coordinate plane with one end on the origin. Other students will state the signs of the six trigonometric functions for the angle formed.

### Resources Provided:

- Problem Set
- Access to online textbook
- Graphing Calculators



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**Lesson Plan 3 Template**

Content Area: Math      Lesson Title: Solving Trigonometric Equations      Time Frame: 2 class periods

**Lesson Components**

**21<sup>st</sup> Century Themes**

- ☐ Global Awareness      ☒ Financial, Economic, Business and Entrepreneurial Literacy  
☐ Civic Literacy      ☐ Health Literacy

**21<sup>st</sup> Century Skills**

- ☐ Creativity and Innovation      ☒ Critical Thinking and Problem Solving      ☐ Information Literacy  
☒ Life and Career Skills      ☐ Communication and Collaboration      ☐ Media Literacy      ☐ ICT Literacy

**Interdisciplinary Connections:**

Science and Engineering

**Integration of Technology:**

Graphing Calculators, Desmos

**Equipment Needed:**

Laptops, graphing calculators

| Goals/Objectives                                                                                                                              | Learning Activities/<br>Instructional Strategies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Formative Assessment Tasks                                                                                                                                                                                                                                                                                                                                                      |
|-----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Students will: <ul style="list-style-type: none"> <li>Solve trigonometric equations</li> <li>Solutions using a graphing calculator</li> </ul> | Lesson Sequence <ol style="list-style-type: none"> <li>Have students conduct internet searches to find size and period for a famous Ferris wheel such as the London Eye.</li> <li>Ask students how to solve an equation for the principal values of <math>x</math>.</li> <li>Ask students how to solve an equation for <math>x</math> between 0 and 360 degrees.</li> <li>Ask students how to solve an equation for all real values of <math>x</math>.</li> <li>Discuss the difference and also show that the answer can be presented either in degrees or radians.</li> <li>Students will be working in pairs to complete a Desmos activity.</li> </ol> | <ul style="list-style-type: none"> <li>Students respond to verbal prompts during initial discussion.</li> <li>Circle the room to check for understanding.</li> <li>Talk to each group during the process or after they finish to assess understanding</li> <li>Have students present solutions to interesting problems</li> <li>Collect exit cards to assess mastery</li> </ul> |



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7. Students will check answers using graphing calculators.
8. Ask some students to show their solutions on the board.
9. Give students Exit Cards to complete.

### **Differentiation:**

Students will create a set of trigonometric equations and inequalities and provide solutions for each.

### **Resources Provided:**

- Laptops
- Graphing calculators
- Problem sets



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### Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly, Moderately* or *Weakly* meets the criteria below.

| Lesson Activities                                                                                                    | Strongly                            | Moderately                          | Weakly                   |
|----------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------------------------------------|--------------------------|
| Are challenging <b>Strongly</b> and require higher order thinking and problem solving skills                         | <input type="checkbox"/>            | <input type="checkbox"/>            | <input type="checkbox"/> |
| Allow for student choice                                                                                             | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide scaffolding for acquiring targeted knowledge/skills                                                          | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Integrate global perspectives                                                                                        | <input type="checkbox"/>            | <input type="checkbox"/>            | <input type="checkbox"/> |
| Integrate 21st century skills                                                                                        | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Provide opportunities for interdisciplinary connection and transfer of knowledge and skills                          | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are varied to address different student learning styles and preferences                                              | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Are differentiated based on student needs                                                                            | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |
| Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process    | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide opportunities for student reflection and self- assessment                                                    | <input type="checkbox"/>            | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| Provide data to inform and adjust instruction to better meet the varying needs of learners                           | <input checked="" type="checkbox"/> | <input type="checkbox"/>            | <input type="checkbox"/> |

### Curriculum Design Template

|                           |                           |                                                           |
|---------------------------|---------------------------|-----------------------------------------------------------|
| <b>Content Area:</b> Math | <b>Grade Level:</b> 11-12 | <b>Unit Title:</b> Trigonometric Identities and Equations |
| <b>Created:</b> 2022      | <b>Board Approved on:</b> |                                                           |



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GLOBAL SKILLS**

### **IEP AND 504 ACCOMMODATIONS AND MODIFICATIONS FOR EARLY CHILDHOOD AND ELEMENTARY SCHOOL STUDENTS:**

Large print textbooks  
Textbooks for at-home use  
Additional time for assignments  
Review of directions  
Use of mnemonics  
Have student restate information  
Concrete examples  
Adaptive writing utensils  
Support auditory presentations with visuals  
Use of a study carrel  
Assistance in maintaining uncluttered space  
Weekly home-school communication tools (notebook, daily log, phone calls or email messages)  
Space for movement or breaks  
Study sheets and teacher outlines  
Extra visual and verbal cues and prompts  
Math sheets with highlighted instructions  
Graph paper to assist in organizing or lining up math problems  
Use of computers and calculators  
Books on tape  
Graphic organizers  
Quiet corner or room to calm down and relax when anxious  
Preferential seating  
Alteration of the classroom arrangement  
Reduction of distractions  
Answers to be dictated  
Hands-on activities  
Use of manipulatives  
No penalty for spelling errors or sloppy handwriting  
Follow a routine/schedule  
Alternate quiet and active time  
Teach time management skills  
Rest breaks  
Verbal and visual cues regarding directions and staying on task  
Agenda book and checklists  
Adjusted assignment timelines  
Visual daily schedule



## **SPRINGFIELD PUBLIC SCHOOLS**

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GLOBAL SKILLS**

Varied reinforcement procedures  
Immediate feedback  
Work-in-progress check  
Personalized examples

### **TESTING AND ASSESSMENT ACCOMMODATIONS AND MODIFICATIONS:**

Answers to be dictated  
Frequent rest breaks  
Additional time  
Oral testing  
Untimed tests  
Choice of test format (multiple-choice, essay, true-false)  
Alternate ways to evaluate (projects or oral presentations instead of written tests)  
Accept short answers  
Open-book tests  
Read test and directions to student  
Provide study outline prior to tests  
Highlight key directions  
Test in alternative site  
Use of calculator  
Extra credit option  
Preview test procedures  
Simplified test wording; rephrased test questions and/or directions

### **GENERAL ACCOMMODATIONS AND MODIFICATIONS:**

Use of alternative books or materials on the topic being studied  
Computerized spell-check support  
Word bank of choices for answers to test questions  
Provision of calculator and/or number line for math tests  
Reworded questions in simpler language  
Projects instead of written reports  
Highlighting important words or phrases in reading assignments  
Modified workload or length of assignments/tests  
Modified time demands  
Modified grades based on IEP

### **BEHAVIOR ACCOMMODATIONS MODIFICATIONS:**



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GLOBAL SKILLS**

Breaks between tasks  
Cue expected behavior  
Daily feedback to student  
Have contingency plans  
Use de-escalating strategies  
Use positive reinforcement  
Use proximity/touch control  
Use peer supports and mentoring  
Model expected behavior by adults  
Have parent sign homework  
Have parent sign behavior chart  
Set and post class rules  
Chart progress and maintain data



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GLOBAL SKILLS

### **IEP AND 504 ACCOMODATIONS AND MODIFICATIONS FOR MIDDLE AND HIGH SCHOOL STUDENTS:**

Large print textbooks  
Textbooks for at-home use  
Additional time for assignments  
A locker with adapted lock  
Review of directions  
Review sessions  
Use of mnemonics  
Have student restate information  
Provision of notes or outlines  
Concrete examples  
Adaptive writing utensils  
Support auditory presentations with visuals  
Weekly or bi-weekly home-school communication  
Space for movement or breaks  
Study sheets and teacher outlines  
Extra visual and verbal cues and prompts  
Lab and math sheets with highlighted instructions  
Graph paper to assist in organizing or lining up math problems  
Use of tape recorder for lectures  
Use of Interactive notebook  
Use of a scribe  
Use of computers and calculators  
Books on tape  
Graphic organizers  
Quiet corner or room to calm down and relax when anxious  
Preferential seating  
Alteration of the classroom arrangement  
Reduction of distractions  
Answers to be dictated  
Hands-on activities  
Ability to type all assignments  
Follow a routine/schedule  
Teach time management and organizational skills  
Rest breaks  
Verbal and visual cues regarding directions and staying on task  
Agenda book and checklists  
Daily check-in with case manager or special education teacher  
Adjusted assignment timelines  
Visual daily schedule



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GLOBAL SKILLS**

Varied reinforcement procedures  
Immediate feedback  
Work-in-progress check  
Personalized examples

### **TESTING AND ASSESSMENT ACCOMMODATIONS AND MODIFICATIONS:**

Answers to be dictated  
Frequent rest breaks  
Additional time  
Oral testing  
Choice of test format (multiple-choice, essay, true-false)  
Alternate ways to evaluate (projects or oral presentations instead of written tests)  
Accept short answers  
Open-book or open-note tests  
Read test and directions to student  
Provide study guides prior to tests  
Highlight key directions  
Test in alternative site  
Use of calculator  
Extra credit option  
Pace long-term projects  
Preview test procedures  
Simplified test wording; rephrased test questions and/or directions

### **GENERAL ACCOMMODATIONS AND MODIFICATIONS:**

Allow outlining, instead of writing for an essay or major project  
Use of alternative books or materials on the topic being studied  
Computerized spell-check support  
Word bank of choices for answers to test questions  
Provision of calculator for math tests  
Film or video supplements in place of reading text  
Reworded questions in simpler language  
Projects instead of written reports  
Highlighting important words or phrases in reading assignments  
Modified workload or length of assignments/tests  
Modified time demands



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GLOBAL SKILLS**

Pass/no pass option  
Modified grades based on IEP

### **BEHAVIOR ACCOMMODATIONS FOR STUDENTS:**

- Breaks between tasks
- Cue expected behavior
- Daily feedback to student
- Have contingency plans
- Use de-escalating strategies
- Use positive reinforcement
- Use proximity/touch control
- Use peer supports and mentoring
- Model expected behavior by adults
- Have parent sign homework
- Set and post class rules
- Chart progress and maintain data



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GLOBAL SKILLS

### Accommodations and Modifications for ESL Students

- Give tests orally
- Give instructions/directions in writing and orally
- Assign a peer tutor, same language or English speaking
- Allow use of bilingual/translation dictionary
- Provide summaries of text or study guides of particular chapters
- Allow errors in speaking
- Allow errors in writing
- Accept writing in first language
- Highlight key vocabulary
- Reduce amount of work required
- Assess comprehension through demonstration or other alternative means (gestures, drawings)
- Rephrase questions, directions, and explanations
- Reduce multiple choices on assessments
- Provide study guides and/ or outlines
- Shorten reading assignments
- Allow extended time for test/project completion
- Use of audio books
- Use of graphic organizers
- Use reduced text, so that print is not dense
- Use visuals/pictures throughout lesson/unit/assignment
- Adapt homework to reflect language proficiency/home support
- Adapt homework to reflect home support
- Provide hands-on activities and explanations
- Allow extended time to answer questions, and permit drawing, as an explanation
- Use of translation dictionaries to locate words in the native language
- Use anecdotal records and portfolio of work as forms of assessment
- Use story retellings to assess comprehension