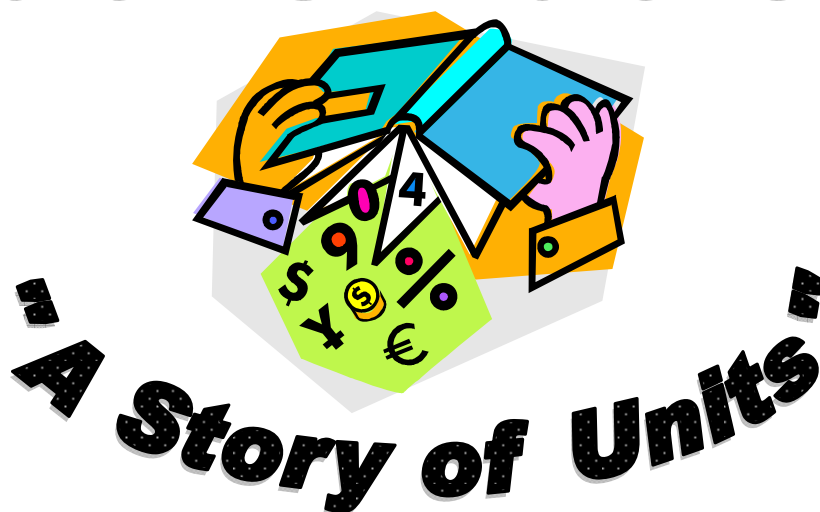


Mathematics!



Parent Handbook

Grade 4
Module 3

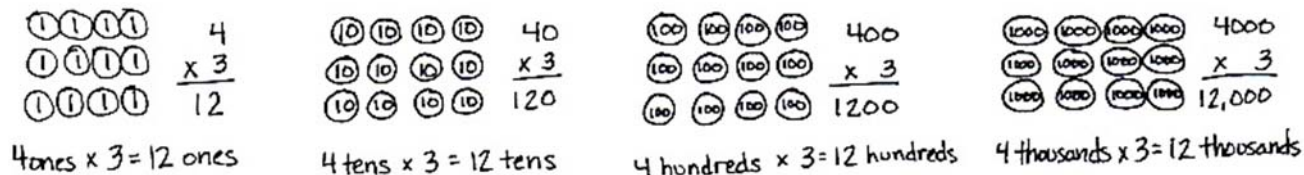
Multi-Digit Multiplication and Division

OVERVIEW

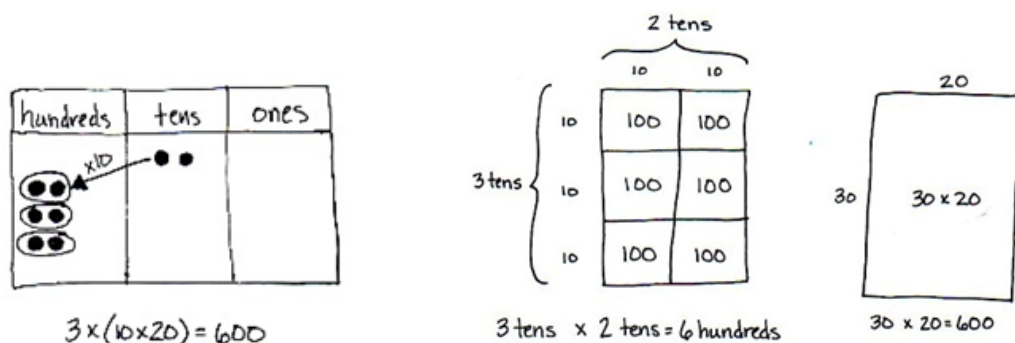
Students use place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. As a key area of focus for Grade 4, this module moves slowly but comprehensively to develop students' ability to reason about the methods and models chosen to solve problems with multi-digit factors and dividends.

Students begin in Topic A by investigating the formulas for area and perimeter. They then solve multiplicative comparison problems including the language of *times as much* with a focus on problems using area and perimeter as a context (e.g., "A field is 9 feet wide. It is 4 times as long as it is wide. What is the perimeter of the field?"). Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities. This Grade 4 module, beginning with area and perimeter, allows for new and interesting word problems as students learn to calculate with larger numbers and interpret more complex problems.

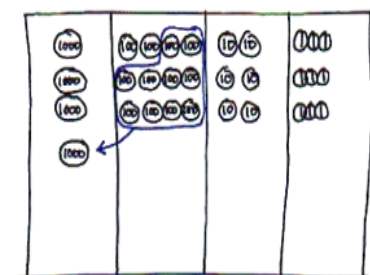
In Topic B, students use number disks to multiply single-digit numbers by multiples of 10, 100, and 1,000 and two-digit multiples of 10 by two-digit multiples of 10. Reasoning between arrays and written numerical work allows students to see the role of place value units in multiplication (as pictured below). Students also practice the language of units to prepare them for multiplication of a single-digit factor by a factor with up to four digits and multiplication of 2 two-digit factors.



In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying 2 two-digit multiples of 10. For example, students have multiplied 20 by 10 on the place value chart and know that it shifts the value one place to the left, $10 \times 20 = 200$. To multiply 20 by 30, the associative property allows for simply tripling the product, $3 \times (10 \times 20)$, or multiplying the units, $3 \text{ tens} \times 2 \text{ tens} = 6 \text{ hundreds}$ (alternatively, $(3 \times 10) \times (2 \times 10) = (3 \times 2) \times (10 \times 10)$). Introducing this early in the module allows students to practice during fluency so that by the time it is embedded within the two-digit by two-digit multiplication in Topic H, understanding and skill are in place.



Building on their work in Topic B, students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students use the distributive property and multiply using number disks to model. Practice with number disks is used for two-, three-, and four-digit by one-digit multiplication problems with recordings as partial products. Students bridge partial products to the recording of multiplication via the standard algorithm. Finally, the partial products method, the standard algorithm, and the area model are compared and connected by the distributive property.



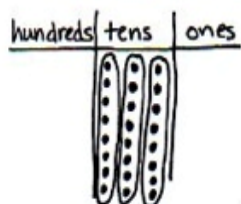
$$\begin{array}{r} 1423 \\ \times 3 \\ \hline 9 \leftarrow 3 \times 3 \text{ ones} \\ 60 \leftarrow 3 \times 2 \text{ tens} \\ 1200 \leftarrow 3 \times 4 \text{ hundreds} \\ 3000 \leftarrow 3 \times 1 \text{ thousand} \\ \hline 4269 \end{array}$$

$$\begin{array}{r} 1423 \\ \times 3 \\ \hline 4269 \end{array}$$

	1000	+	400	+	20	+	3
3	3000		1200		60		9

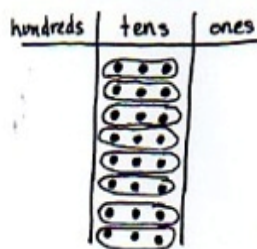
Topic D gives students the opportunity to apply their new multiplication skills to solve multi-step word problems and multiplicative comparison problems. Students write equations from statements within the problems and use a combination of addition, subtraction, and multiplication to solve. In Topic E, students synthesize their Grade 3 knowledge of division types (*group size unknown* and *number of groups unknown*) with their new, deeper understanding of place value.

Group Size Unknown



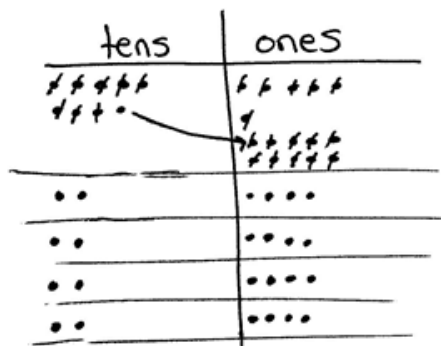
$$\begin{array}{l} 24 \text{ tens} \div 3 = 8 \text{ tens} \\ \uparrow \quad \quad \uparrow \\ \# \text{ of groups} \quad \text{size of groups} \\ 240 \div 3 = 80 \end{array}$$

Number of Groups Unknown



$$\begin{array}{l} 24 \text{ tens} \div 3 \text{ tens} = 8 \\ \uparrow \quad \quad \uparrow \\ \text{size of groups} \quad \# \text{ of groups} \\ 240 \div 30 = 8 \end{array}$$

Students focus on interpreting the remainder within division problems both in word problems and within long division. Some scenarios would not call for a remainder to be subdivided, while others may. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided. Students represent division with single-digit divisors using arrays and the area model before practicing with place value disks. The standard division algorithm is practiced using place value knowledge decomposing unit by unit. Finally, students use the area model to solve division problems, first with and then without remainders.



$$\begin{array}{r} 24 \\ 4 \overline{) 96} \\ \underline{-8} \\ 16 \\ \underline{-16} \\ 0 \end{array}$$

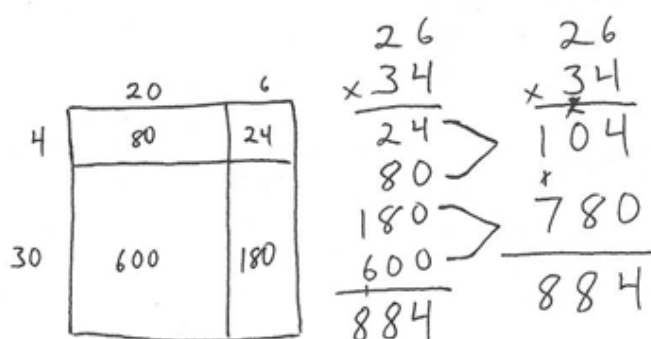
	20	4
4	80	16

$$\begin{array}{c} 96 \\ \swarrow \quad \searrow \\ 80 \quad 16 \\ (80 \div 4) + (16 \div 4) \\ = 20 + 4 \\ = 24 \end{array}$$

In Topic F, armed with an understanding of remainders, students explore factors, multiples, and prime and composite numbers within 100, gaining valuable insights into patterns of divisibility as they test for primes and find factors and multiples. This prepares them for Topic G's work with multi-digit dividends.

Topic G extends the practice of division with three- and four-digit dividends using place value understanding. A connection to Topic B is made initially with dividing multiples of 10, 100, and 1,000 by single-digit numbers. Number disks support students visually as they decompose each unit before dividing. Students then practice using the standard algorithm to record long division, including solving word problems and making connections to the area model as was done with two-digit dividends.

Finally, students multiply two-digit by two-digit numbers. Students use their place value understanding and understanding of the area model to empower them to multiply by larger numbers as pictured to the right. Topic H culminates at the most abstract level by explicitly connecting the partial products appearing in the area model to the distributive property and recording the calculation vertically. Students see partial products written vertically are the same as those obtained via the distributive property: $4 \text{ twenty-sixes} + 30 \text{ twenty-sixes} = 104 + 780 = 884$.



As students progress through this module, they are able to apply the multiplication and division algorithm because of their in-depth experience with the place value system and multiple conceptual models. This prepares them for fluency with the multiplication algorithm in Grade 5 and the division algorithm in Grade 6. Students are encouraged in Grade 4 to continue using models to solve when appropriate.

****The sample questions/responses contained in this manual are straight from <http://www.engageny.org/>. They are provided to give some insight into the kinds of skills expected of students as the lesson is taught.**

Terminology

New or Recently Introduced Terms

Associative property [$96 = 3 \times (4 \times 8) = (3 \times 4) \times 8$]

Composite number (positive integer having three or more whole number factors)

Distributive Property [$64 \times 27 = (60 \times 20) + (60 \times 7) + (4 \times 20) + (4 \times 7)$]

Divisor (the number by which another number is divided)

Partial product (e.g. $24 \times 6 = (20 \times 6) + (4 \times 6) = 120 + 24$)

Prime number (positive integer only having whole number factors of one and itself)

Remainder (the number left over when one integer is divided by another)

Familiar Terms and Symbols

Algorithm (steps for base ten computations with the four operations)

Area (the amount of two-dimensional space in a bounded region)

Area model (a model for multiplication problems, in which the length and width of a rectangle represent the factors)

Bundling, grouping, renaming, changing (replaces “regrouping” term)

Compare (to find the similarity or dissimilarity between)

Distribute (decompose an unknown product in terms of two known products to solve)

Divide/Division (e.g., $15 \div 5 = 3$)

Equation (a statement that the values of two mathematical expressions are equal using the = sign)

Factors (numbers that can be multiplied together to get other numbers)

Mixed units (e.g., 1 ft 3 in, 4 lb 13 oz)

Multiple (product of a given number and any other whole number)

Multiply/Multiplication (e.g., $5 \times 3 = 15$)

Perimeter (length of a continuous line forming the boundary of a closed geometric figure)

Place value (the numerical value that a digit has by virtue of its position in a number)

Product (the result of multiplication)

Quotient (the result of division)

Rectangular array (an arrangement of a set of objects into rows and columns)

___ *times as many* ___ *as* ___ (sentence frame)

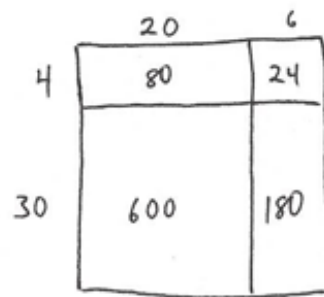
Suggested Tools and Representations

Area model

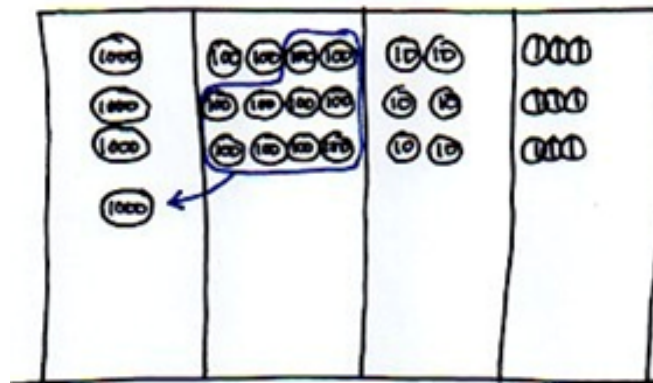
Place value disks

Place value mats

Area Model: This area model shows a visual representation of 26×34 , breaking apart the 26 by tens and ones (20 and 6) and the 34 by tens and ones (30 and 4).



Place Value Mats and Number Disks: The mat allows students to isolate place values and relate values of digits. It may be used with number disks (no labels) or with number dots (with place value labels) to represent numbers. The value 4,269 is shown using number disks. Notice how a group of ten hundreds is bundled together to form one thousand.



Lesson 1

Objective: Investigate and use the formulas for area and perimeter of rectangles.

2. Determine the perimeter and area of each rectangle.

a.

6 cm

5 cm

$$P = 2 \times (l + w)$$

$$P = 2 \times 11$$

$$P = 22$$



$$P = 22 \text{ cm}$$

$$A = 30 \text{ sq. cm}$$

$$A = l \times w \quad A = 6 \times 5 = 30$$

4. Given the rectangle's area, find the unknown side length.

b.

8 cm



x cm

$$A = l \times w$$

$$A = 8 \times w$$

$$80 = 8 \times x$$

$$x = 10$$

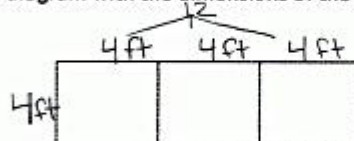
$$x = 10$$

Lesson 2

Objective: Solve multiplicative comparison word problems by applying the area and perimeter formulas.

1. A rectangular porch is 4 feet wide. It is 3 times as long as it is wide.

a. Label the diagram with the dimensions of the porch.



b. Find the perimeter of the porch.

$$P = 2 \times (l + w)$$

$$= 2 \times (12 + 4)$$

$$= 2 \times 16$$

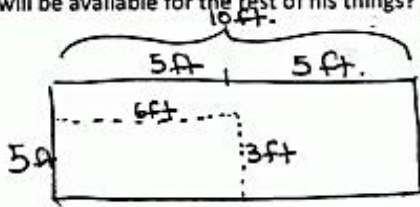
$$= 32$$

$$P = 32 \text{ feet}$$

Lesson 3

Objective: Demonstrate understanding of area and perimeter formulas by solving multi-step real world problems.

2. The width of David's tent is 5 feet. The length is twice the width. David's rectangular air mattress measures 3 feet by 6 feet. If David puts the air mattress in the tent, how many square feet of floor space will be available for the rest of his things?



$$\begin{aligned} \text{Tent} \\ A &= l \times w \\ &= 10 \times 5 \\ &= 50 \\ A &= 50 \text{ square ft.} \end{aligned}$$

$$\begin{aligned} \text{air mattress} \\ A &= l \times w \\ &= 6 \times 3 \\ &= 18 \\ A &= 18 \text{ square ft.} \end{aligned}$$

$$\begin{array}{r} 4 \text{ } 10 \\ 50 \\ - 18 \\ \hline 32 \end{array}$$

32 square feet of floor space will be available.

Lesson 4

Objective: Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically.

Directions: Draw number disks and arrows as shown to represent each product.

4. $12 \times 10 = 120$

(1 ten 2 ones) $\times 10 = 12 \text{ tens}$

thousands	hundreds	tens	ones
		1 ten disk, 2 one disks	

5. $18 \times 100 = 1800$

$18 \times 10 \times 10 = 1800$

(1 ten 8 ones) $\times 100 = 18 \text{ hundreds}$

thousands	hundreds	tens	ones
	1 hundred disk, 8 ten disks		

Decompose each multiple of 10, 100, or 1000 before multiplying.

$$\begin{aligned} 7. \quad 3 \times 40 &= 3 \times 4 \times 10 \\ &= 12 \times 10 \\ &= 120 \end{aligned}$$

$$\begin{aligned} 8. \quad 3 \times 200 &= 3 \times 2 \times 100 \\ &= 6 \times 100 \\ &= 600 \end{aligned}$$

Lesson 5

Objective: Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns.

3. $2 \times 300 = \underline{600}$

2 times 3 hundreds is 6 hundreds.

thousands	hundreds	tens	ones
	●●● ●●●		

$2 \times 3 \text{ hundreds} = 6 \text{ hundreds}$

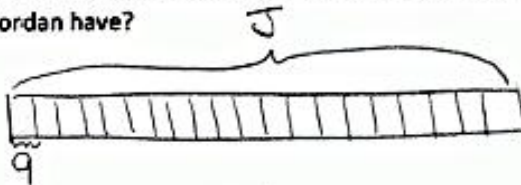
$2 \times 3,000 = \underline{6,000}$

2 times 3 thousands is 6 thousands.

thousands	hundreds	tens	ones
●●● ●●●			

$2 \times 3 \text{ thousands} = 6 \text{ thousands}$

7. Jordan has twenty times as many baseball cards as his brother. His brother has 9 cards. How many cards does Jordan have?



Jordan has 180 cards.

$$\begin{aligned} J &= 20 \times 9 \\ &= 2 \times 9 \times 10 \\ &= 18 \times 10 \\ &= 180 \end{aligned}$$

Lesson 6

Objective: Multiply two-digit multiples of 10 by two-digit multiples of 10 with the area model.

Directions: Represent the following problem by drawing disks in the place value chart.

1. To solve 20×40 , think:

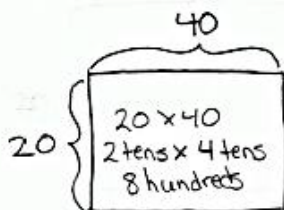
$(2 \text{ tens} \times 4) \times 10 = \underline{800}$

$20 \times (4 \times 10) = \underline{800}$

$20 \times 40 = \underline{800}$

Hundreds	Tens	Ones
●● ●● ●● ●●	●● ●● ●● ●●	

2. Draw an area model to represent 20×40 .



$2 \text{ tens} \times 4 \text{ tens} = \underline{8 \text{ hundreds}}$

Lesson 7

Objective: Use place value disks to represent two-digit by one-digit multiplication.

1. Represent the following expressions with disks, regrouping as necessary, writing a matching expression, and recording the partial products vertically as shown below.

1×43

$1 \times 4 \text{ tens} + 1 \times 3$

tens	ones
● ● ● ●	● ● ●

$$\begin{array}{r} 43 \\ \times 1 \\ \hline 3 \rightarrow 1 \times 3 \text{ ones} \\ + 40 \rightarrow 1 \times 4 \text{ tens} \\ \hline 43 \end{array}$$

2. Represent the following expressions with disks, regrouping as necessary. To the right, record the partial products vertically.

2×36

hundreds	tens	ones
	● ● ● ●	● ● ● ● ● ●
	● ● ● ●	● ● ● ● ● ●

$2 \times 3 \text{ tens} + 2 \times 6 \text{ ones}$

$7 \text{ tens} + 2 \text{ ones} = 72$

$$\begin{array}{r} 36 \\ \times 2 \\ \hline 12 \rightarrow 2 \times 6 \text{ ones} \\ + 60 \rightarrow 2 \times 3 \text{ tens} \\ \hline 72 \end{array}$$

Lesson 8

Objective: Extend use of place value disks to represent three- and four-digit by one-digit multiplication.

1. Represent the following expressions with disks, regrouping as necessary, writing a matching expression, and recording the partial products vertically as shown below.

a. 1×213

hundreds	tens	ones
● ●	●	● ● ●

$1 \times 2 \text{ hundreds} + 1 \times 1 \text{ ten} + 1 \times 3 \text{ ones}$

$2 \text{ hundreds} + 1 \text{ ten} + 3 \text{ ones} = 213$

$$\begin{array}{r} 213 \\ \times 1 \\ \hline 3 \rightarrow 1 \times 3 \text{ ones} \\ 10 \rightarrow 1 \times 1 \text{ ten} \\ + 200 \rightarrow 1 \times 2 \text{ hundreds} \\ \hline 213 \end{array}$$

2. Represent the following expressions with disks, using either method shown during the class, renaming as necessary. To the right, record the partial products vertically.

a. 3×212

hundreds	tens	ones
● ● ● ● ● ●	● ● ● ●	● ● ● ● ● ●

$6 \quad 3 \quad 6 \quad 636$

212

$$\begin{array}{r} 212 \\ \times 3 \\ \hline 6 \\ 30 \\ + 600 \\ \hline 636 \end{array}$$

Lesson 9

Objective: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm.

1. Solve using each method.

Partial Products	Standard Algorithm
a) $ \begin{array}{r} 34 \\ \times 4 \\ \hline 16 \\ + 120 \\ \hline 136 \end{array} $	$ \begin{array}{r} 34 \\ \times 4 \\ \hline 136 \end{array} $

7. A small bag of chips weighs 48 g. A large bag of chips weighs three times as much as the small bag. How much will 7 large bags of chips weigh?

Handwritten solution for problem 7:

small bag: 48 g
large bag: 144 g (3 times 48 g)

Calculation for 7 large bags:

$$\begin{array}{r}
 144 \\
 \times 7 \\
 \hline
 1,008
 \end{array}$$

Seven large bags of chips will weigh 1,008 grams.

Lesson 10

Objective: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm.

1. Solve using the standard algorithm.

a) 3×42 $ \begin{array}{r} 42 \\ \times 3 \\ \hline 126 \end{array} $	b) 6×42 $ \begin{array}{r} 42 \\ \times 6 \\ \hline 252 \end{array} $
---	---

4. Jake ran 2 miles. Jesse ran 4 times as far. There are 5,280 ft. in a mile. How many feet did Jesse run?

Handwritten solution for problem 4:

Jake: 2 miles
Jesse: 8 miles (4 times 2 miles)

Calculation for Jesse's distance in feet:

$$\begin{array}{r}
 5,280 \\
 \times 8 \\
 \hline
 42,240
 \end{array}$$

Jesse ran 42,240 feet.

Lesson 11

Objective: Connect the area model and the partial products method to the standard algorithm.

1. Solve the following expressions using the standard algorithm, the partial products method, and the area model.

$$425 \times 4$$

$$\begin{array}{r} 425 \\ \times 4 \\ \hline 20 \\ 80 \\ + 1,600 \\ \hline 1,700 \end{array}$$

$$\begin{array}{r} 425 \\ \times 4 \\ \hline 1,700 \end{array}$$

	400	20	5
4	1,600	80	20

$$4(400 + 20 + 5)$$

$$(4 \times 400) + (4 \times 20) + (4 \times 5) = 1,600 + 80 + 20 = 1,700$$

5. 3 times as many as 2,805.

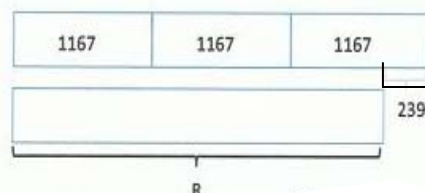
	2,000	800	5
3	6,000	2,400	15

$$(3 \times 2,000) + (3 \times 800) + (3 \times 5) = 6,000 + 2,400 + 15 = 8,415$$

Lesson 12

Objective: Solve two-step word problems, including multiplicative comparison.

4. a. Write an equation that would allow someone to find the value of R.

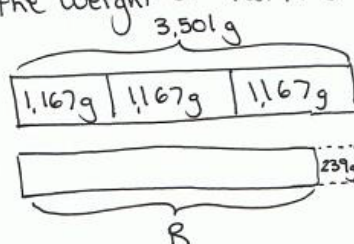


$$R + 239 = 3 \times 1,167$$

$$R = 3 \times 1,167 - 239$$

- b. Write your own word problem to correspond to this tape diagram, then solve.

Patti's sandals weigh 1,167 grams. She bought 3 pairs, all different colors. All 3 pairs of sandals weigh 239 grams more than her winter boots. What is the weight of Patti's winter boots?



$$\begin{array}{r} 1,167 \quad 3,501 \\ \times 3 \\ \hline 3,501 \end{array} \quad \begin{array}{r} 3,501 \\ - 239 \\ \hline 3,262 \end{array}$$

Patti's winter boots weigh 3,262 grams.

Lesson 13

Objective: Use multiplication, addition, or subtraction to solve multi-step word problems.

3. Three boxes weighing 128 pounds each and one box weighing 254 pounds were loaded onto the back of an empty truck. A crate of apples was then loaded onto the same truck. If the total weight loaded onto the truck was 2,000 pounds, how much did the crate of apples weigh?

Handwritten solution for Lesson 13, Question 3:

First, calculate the weight of the three boxes: $128 \times 3 = 384$.

Then, add the weight of the single box: $384 + 254 = 638$.

The total weight of the boxes is 638 pounds.

The total weight loaded onto the truck is 2,000 pounds.

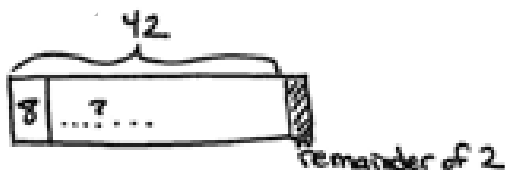
To find the weight of the crate of apples, subtract the weight of the boxes from the total weight: $2,000 - 638 = 1,362$.

The crate of apples weighed 1,362 pounds.

Lesson 14

Objective: Solve division word problems with remainders.

4. The baker has 42 kilograms of flour. She uses 8 kilograms each day. After how many days will she need to buy more flour?



8, 16, 24, 32, 40, 48

$$42 \div 8$$

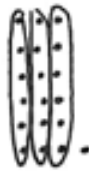
The quotient is 5 and the remainder is 2.

The baker will need to buy flour after 5 days.

Lesson 15

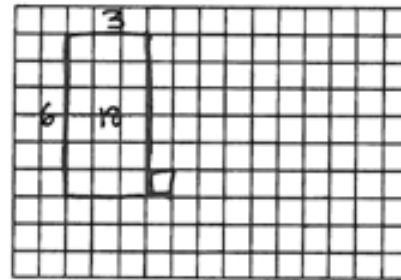
Objective: Understand and solve division problems with a remainder using the array and area models.

$$19 \div 6$$



Quotient = 3

Remainder = 1



Can you show $19 \div 6$ with one rectangle? no
Explain how you showed the remainder:

I outlined one more square unit.

Solve using an array and an area model.

$$29 \div 3$$



Quotient = 9 Remainder = 2

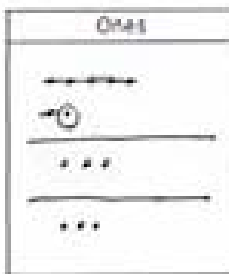


Lesson 16

Objective: Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.

Show the division using disks. Relate your work on the place value chart to long division. Check your quotient and remainder by using multiplication and addition.

$$7 \div 2$$



$$\begin{array}{r} 3 \overline{) 7} \\ \underline{6} \\ 1 \end{array}$$

3 ones

quotient = 3

remainder = 1

Check Your Work

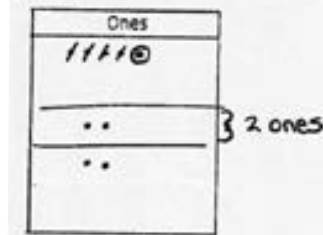
$$\begin{array}{r} 3 \times 2 = 6 \\ 6 + 1 = 7 \end{array}$$

Lesson 17

Objective: Represent and solve division problems requiring decomposing a remainder in the tens.

Show the division using disks. Relate your model to long division. Check your quotient and remainder by using multiplication and addition.

$$5 \div 2$$



$$\begin{array}{r} 2 \overline{) 5} \\ \underline{-4} \\ 1 \end{array}$$

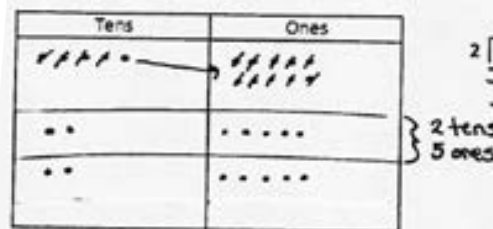
quotient = 2
remainder = 1

Check Your Work

$$\begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 2 \\ + 1 \\ \hline 3 \end{array}$$

$$50 \div 2$$



$$\begin{array}{r} 2 \overline{) 50} \\ \underline{-40} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

quotient = 25
remainder = 0

Check Your Work

$$\begin{array}{r} 25 \\ \times 2 \\ \hline 50 \end{array}$$

Lesson 18

Objective: Find whole number quotients and remainders.

Solve using the standard algorithm. Check your quotient and remainder by using multiplication and addition.

$$46 \div 2$$

$$\begin{array}{r} 23 \\ 2 \overline{) 46} \\ \underline{-4} \\ 06 \\ \underline{-6} \\ 0 \end{array}$$

Quotient = 23
Remainder = 0

$$\begin{array}{r} 23 \\ \times 2 \\ \hline 46 \end{array}$$

$$96 \div 3$$

$$\begin{array}{r} 32 \\ 3 \overline{) 96} \\ \underline{-9} \\ 06 \\ \underline{-6} \\ 0 \end{array}$$

Quotient = 32
Remainder = 0

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$

Lesson 19

Objective: Explain remainders by using place value understanding and models.

2. Cayman says that $94 \div 3$ is 30 with a remainder of 4. He reasons it is correct because $(3 \times 30) + 4 = 94$.

What mistake has Cayman made? Explain how he can correct his work.

*Cayman's mistake is that his remainder is greater than his divisor. That means he can divide even more. Instead of 30, he can make 31 groups.
 $94 \div 3 = 31$ with a remainder of 1. $(3 \times 31) + 1 = 94$.*

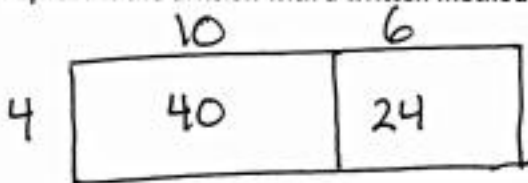
3. The number disk model is showing $72 \div 3$. Complete the model. Explain what happens to the 1 ten that is remaining in the tens column.



Lesson 20

Objective: Solve division problems without remainders using the area model.

3. Solve $64 \div 4$ using an area model. Draw a number bond to show how you partitioned the area, and represent the division with a written method.



$$\begin{array}{c}
 (64) \\
 \swarrow \quad \searrow \\
 (40) \quad (24) \\
 (40 \div 4) + (24 \div 4) \\
 = 10 + 6 \\
 = 16
 \end{array}$$

Lesson 21

Objective: Solve division problems with remainders using the area model.

1. Solve $37 \div 2$ using an area model. Use long division and the distributive property to record your work.

Area Model for $37 \div 2$:

2 $\left[\begin{array}{|c|c|} \hline \text{20 square units} & \text{16 square units} \\ \hline \end{array} \right]$ 1 square unit remaining

$(20 \div 2) + (16 \div 2)$
 $= 10 + 8$
 $= 18$
 $(18 \times 2) + 1 = 37$

Long Division for $37 \div 2$:

$$\begin{array}{r} 18 \text{ R} 1 \\ 2 \overline{) 37} \\ \underline{- 2} \\ 17 \\ \underline{- 16} \\ 1 \end{array}$$

Lesson 22

Objective: Find factor pairs for numbers to 100 and use understanding of factors to define prime and composite.

1. Record the factors of the given numbers as multiplication sentences and as a list in order from least to greatest. Classify each as prime (P) or composite (C).

	Multiplication Sentences	Factors	Prime or Composite
a.	4 $1 \times 4 = 4$ $2 \times 2 = 4$	The factors of 4 are: 1, 2, and 4	C
b.	6 $1 \times 6 = 6$ $2 \times 3 = 6$	The factors of 6 are: 1, 2, 3, 6	C

2. Find all factors for the following numbers and classify as prime or composite. Explain your classification of each as prime or composite.

Factor Pairs for 28	
1	28
2	14
4	7

Composite
more than 2 factors

Factor Pairs for 29	
1	29

Prime
only 2 factors

Lesson 23

Objective: Use division and the associative property to test for factors and observe patterns.

1. Explain your thinking or use division to answer the following.

a. Is 2 a factor of 84? Yes. 84 is an even number. 2 is a factor of every even number.	b. Is 2 a factor of 83? No. 83 is an odd number. 2 is not a factor of odd numbers.
---	---

2. Use the associative property to find more factors of 24 and 36.

a. $24 = 12 \times 2$

$= (4 \times 3) \times 2$

$= 4 \times (3 \times 2)$

$= 4 \times 6$

$= 24$

b. $36 = 9 \times 4$

$= (3 \times 3) \times 4$

$= 3 \times (3 \times 4)$

$= 3 \times 12$

$= 36$

Lesson 24

Objective: Determine whether a whole number is a multiple of another number.

3. Use mental math, division, or the associate property to solve.

a. Is 12 a multiple of 4? yes Is 4 a factor of 12? yes

b. Is 42 a multiple of 8? no Is 8 a factor of 42? no

c. Is 84 a multiple of 6? yes Is 6 a factor of 84? yes

4. Can a prime number be a multiple of any other number except itself? Explain your reasons why.

Yes. A prime number is also a multiple of 1.
A prime number has two factors; itself and 1.

Lesson 25

Objective: Explore properties of prime and composite numbers to 100 by using multiples.

2. List the circled numbers. (From a previous question...numbers after 2 that were not multiples of 2)

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

- a. Why weren't the circled numbers crossed off along the way? (From a previous question...numbers that were crossed off were numbers that were multiples of particular numbers)

The circled numbers were not crossed off along the way because they are not multiples of any other numbers besides 1 and themselves.

Lesson 26

Objective: Divide multiples of 10, 100, and 1,000 by single-digit numbers.

2. Draw number disks to represent each problem. Rewrite each in unit form and solve.

$$12 \div 3 = \underline{4}$$

$$12 \text{ ones} \div 3 = \underline{4 \text{ ones}}$$



3. Rewrite each in unit form. Solve for the quotient.

a. $800 \div 2 = 400$ 8 hundreds $\div 2 =$ 4 hundreds	b. $600 \div 2$ 6 hundreds $\div 2 =$ 3 hundreds	c. $800 \div 4$ 8 hundreds $\div 4 =$ 2 hundreds	d. $900 \div 3$ 9 hundreds $\div 3 =$ 3 hundreds
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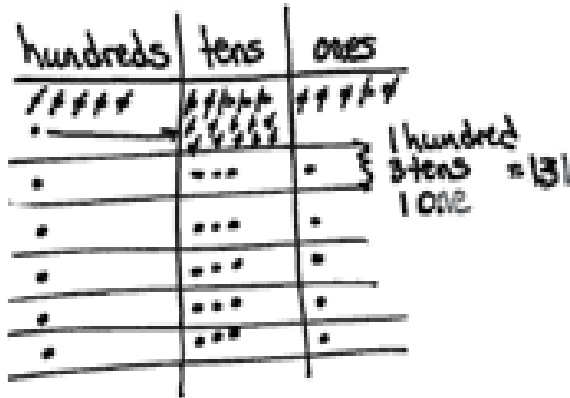
Lesson 27

Objective: Represent and solve division problems with up to a three-digit dividend numerically and with number disks requiring decomposing a remainder in the hundreds place.

2. Model using number disks and record using the algorithm.

$$655 \div 5$$

Number Disks



Algorithm

$$\begin{array}{r} 131 \\ 5 \overline{) 655} \\ \underline{-5} \\ 15 \\ \underline{-15} \\ 05 \\ \underline{-5} \\ 0 \end{array}$$

Lesson 28

Objective: Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically.

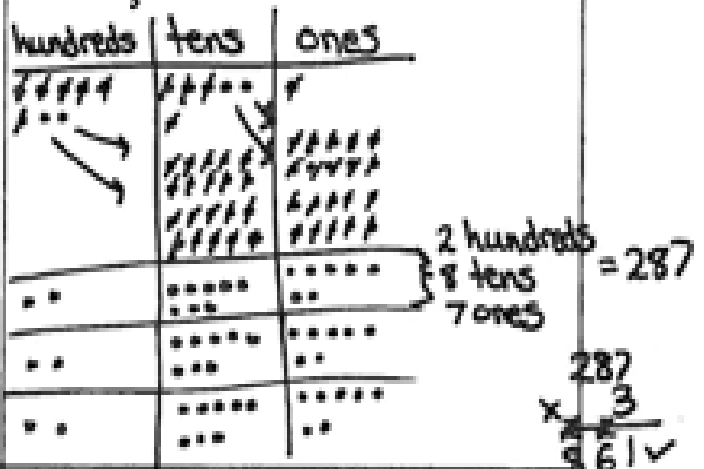
1. Divide. Check your work by multiplying. Draw disks on a place value chart as needed.

$$574 \div 2$$

$$\begin{array}{r} 287 \\ 2 \overline{) 574} \\ \underline{-4} \\ 17 \\ \underline{-16} \\ 14 \\ \underline{-14} \\ 0 \end{array}$$

$$\begin{array}{r} 287 \\ \times 2 \\ \hline 574 \checkmark \end{array}$$

$$861 \div 3$$



Lesson 29

Objective: Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times.

1. Divide, then check using multiplication.

$1,672 \div 4$ $\begin{array}{r} 4 \overline{) 1672} \\ \underline{-16} \\ 07 \\ \underline{-4} \\ 32 \\ \underline{-32} \\ 0 \end{array}$ $\begin{array}{r} 418 \\ \times 4 \\ \hline 1672 \checkmark \end{array}$	$1,578 \div 4$ $\begin{array}{r} 394 R2 \\ 4 \overline{) 1578} \\ \underline{-12} \\ 37 \\ \underline{-36} \\ 18 \\ \underline{-16} \\ 2 \end{array}$ $\begin{array}{r} 394 1576 \\ \times 4 + 2 \\ \hline 1576 1578 \checkmark \end{array}$ Q = 394 R = 2
--	---

Lesson 30

Objective: Solve division problems with a zero in the dividend or with a zero in the quotient.

Divide. Check your solutions by multiplying.

$204 \div 4$

hundreds	tens	ones
.. →		

} 5 tens = 51
1 one = 51

$$\begin{array}{r} 51 \\ \times 4 \\ \hline 204 \checkmark \end{array}$$

$704 \div 3$

$$\begin{array}{r} 234 R2 \\ 3 \overline{) 704} \\ \underline{-6} \\ 10 \\ \underline{-9} \\ 14 \\ \underline{-12} \\ 2 \end{array}$$

$$\begin{array}{r} 234 702 \\ \times 3 + 2 \\ \hline 702 704 \checkmark \end{array}$$

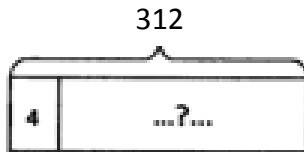
Q = 234
R = 2

Lesson 31

Objective: Interpret division word problems as either *number of groups unknown* or *size unknown*.

Draw a tape diagram and solve. The first two tape diagrams have been drawn for you. Identify if the group size or the number of groups is unknown.

1. Monique needs exactly 4 plates on each table for the banquet. If she has 312 plates, how many tables is she able to prepare?

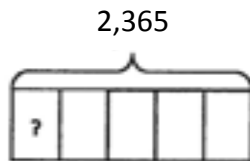


$$\begin{array}{r} 78 \\ 4 \overline{)312} \\ \underline{-28} \\ 32 \\ \underline{-32} \\ 0 \end{array}$$

Monique is able to prepare 78 tables.

The number of groups is unknown.

2. 2,365 books were donated to an elementary school. If 5 classrooms shared the books equally, how many books did each class receive?



$$\begin{array}{r} 473 \\ 5 \overline{)2365} \\ \underline{-20} \\ 36 \\ \underline{-35} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

Each class received 473 books.

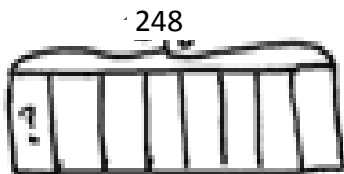
The group size is unknown.

Lesson 32

Objective: Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9.

Solve the following problems. Draw tape diagrams to help you solve. If there is a remainder, shade in a small portion of the tape diagram to represent that portion of the whole.

1. A concert hall contains 8 sections of seats with the same number of seats in each section. If there are 248 seats, how many seats are in each section?



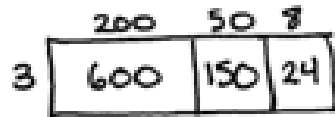
$$\begin{array}{r} 31 \\ 8 \overline{)248} \\ \underline{-24} \\ 08 \\ \underline{-8} \\ 0 \end{array}$$

There are 31 seats in each section.

Lesson 33

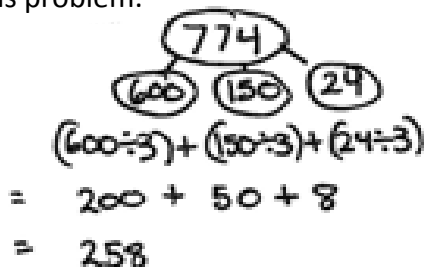
Objective: Explain the connection of the area model of division to the long division algorithm for three- and four-digit dividends.

Draw an area model to solve $774 \div 3$.



$$774 \div 3 = 258$$

Draw a number bond to represent this problem.



Record your work using the long division algorithm.

$$\begin{array}{r} 258 \\ 3 \overline{) 774} \\ \underline{-6} \\ 17 \\ \underline{-15} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

Lesson 34

Objective: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.

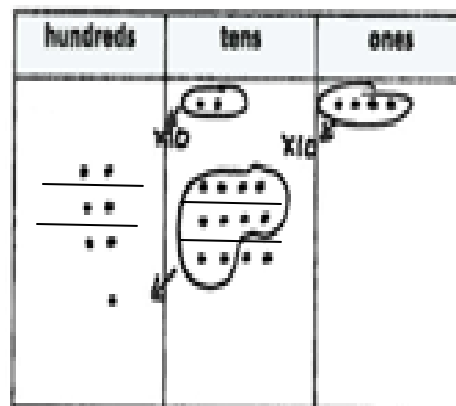
Use the associative property to rewrite each expression. Solve using disks and then complete the number sentences.

a. 30×24

$$= (\underline{3} \times 10) \times 24$$

$$= \underline{3} \times (10 \times 24)$$

$$= \underline{720}$$

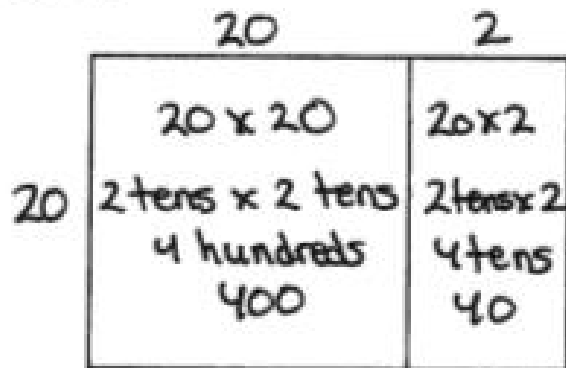


Lesson 35

Objective: Multiply two-digit multiples of 10 by two-digit numbers using the area model.

Use an area model to represent the following expressions. Then record the partial products and solve.

1. 20×22

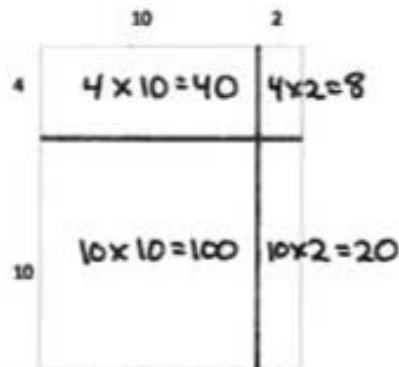
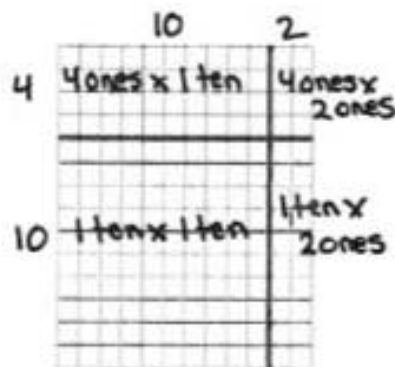


$$\begin{array}{r}
 22 \\
 \times 20 \\
 \hline
 40 \\
 + 400 \\
 \hline
 440
 \end{array}$$

Lesson 36

Objective: Multiply two-digit by two-digit numbers using four partial products.

- a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.



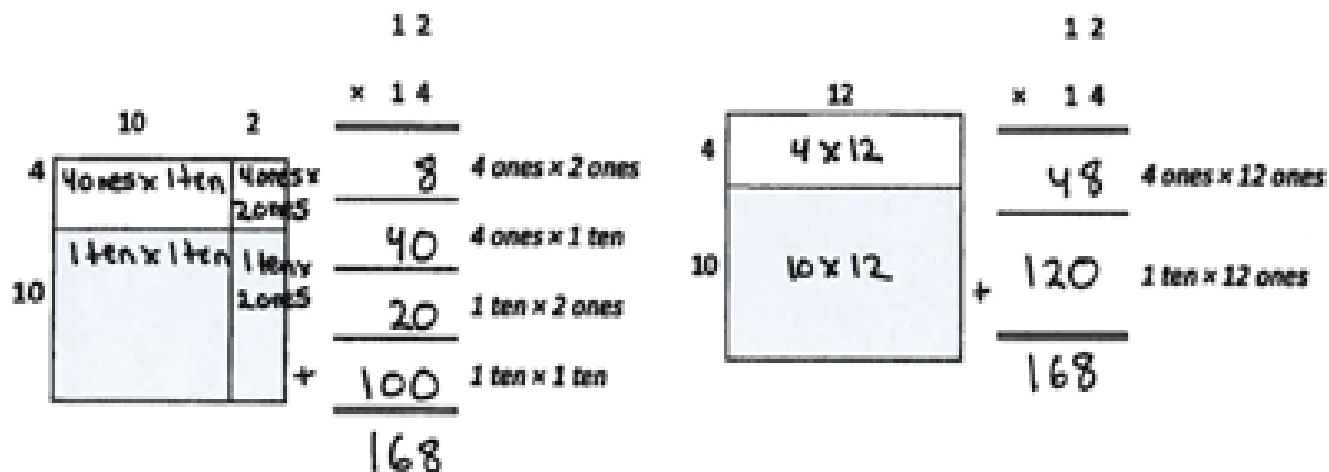
- b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form and then read in unit form.

$$14 \times 12 = (4 \times 2) + (4 \times 10) + (10 \times 2) + (10 \times 10)$$

Lesson 37

Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

1. Solve 14×12 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.



Lesson 38

Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

1. Express 23×54 as two partial products using the distributive property. Solve.

