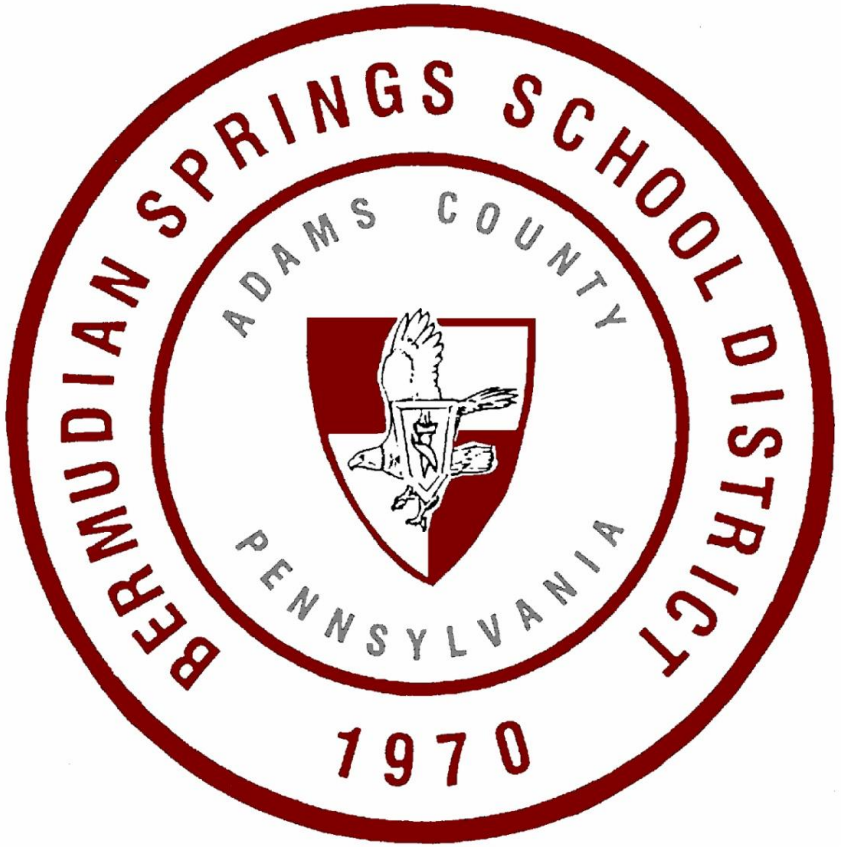
Bermudian Springs PA Core Standards
Math Framework
Sixth Grade



Introduction

Bermudian Springs School District, in partnership with all stakeholders, recognizes the importance of our students being able to use mathematics in everyday life and in the workplace. New knowledge, tools, and ways of solving math problems will significantly enhance opportunities for shaping our students future. Math competencies open doors to productive futures. All students should have the opportunity and support necessary to learn significant math with depth and understanding. Common Core has provided critical areas designed to bring focus to the standards at each grade by describing key concepts in order to guide instruction. The critical areas for instructional focus for sixth grade math outlined by the *Common Core* include the following four areas:

1. **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;** Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. **Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers;** Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. **Writing, interpreting, and using expressions and equations;** Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. **Developing understanding of statistical thinking.** Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area from elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles by decomposing shapes. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Adapted from: commoncore.org, 2013; parcconline.org, 2013; pdesas.org, 2013

Standards for Mathematical Practice in Sixth Grade

Bermudian Springs School District incorporated the following Mathematical Practices which are expected to be integrated into every mathematics lesson for all students as outlined in the Pennsylvania Core Standards. Below are a few examples of how these mathematical practices may be integrated into some tasks that Bermudian students will apply in sixth grade.

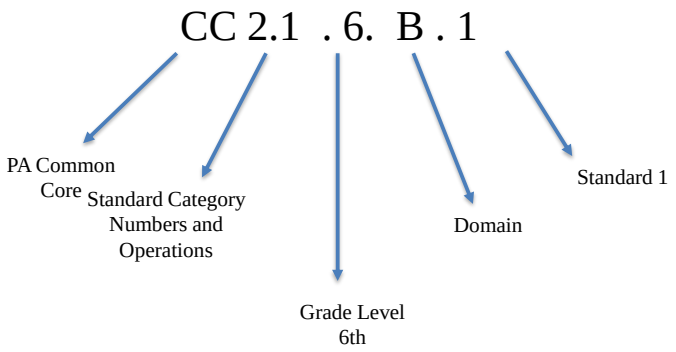
Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In grade 6, students solve real world problems through the application of algebraic and geometric concepts. These problems involve ratio, rate, area and statistics. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" Students can explain the relationships between equations, verbal descriptions, tables and graphs. Mathematically proficient students check answers to problems using a different method.
2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variables as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?'', "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	In grade 6. Students model problem situations symbolically, graphically, tabularly, and contextually. Students from expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to understand division and to create dot plots, histograms and box plots to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric

	figures, data displays, and components of expressions, equations or inequalities.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality, $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

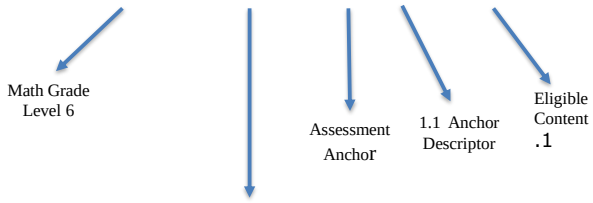
commoncore.org

Mathematical Standards: Development and Progression												
	Pre K	K	1	2	3	4	5	6	7	8	HS
2.1 Numbers and Operations	(A) Counting & Cardinality										
		(B) Number and Operations in Base Ten						(D) Ratios and Proportional Relationships			(F) Number and Quantity
					(C) Number and Operations - Fractions			(E) The Number System			
2.2 Algebraic Concepts	(A) Operations and Algebraic Thinking							(B) Expressions and Equations			(D) Algebra
											(C) Functions
2.3 Geometry	(A) Geometry										
2.4 Measurement, Data and Probability	(A) Measurement and Data							(B) Statistics and Probability			

Standard Taxonomy



Anchor and Eligible Content



- A-N = The Number System (A – Numbers and
- B-E = Expressions and Equations
- B-F = Functions
- **C-G = Geometry**
- D-S = Statistics and Probability

2.1 Number and Operations

Domain: (D) Ratios & Proportional Relationships

Standard: CC.2.1.6.D.1 Understand ratio concepts and use ratio reasoning to solve problems.

Anchor Descriptor*:* **M06.A-R.1.1** Represent and/or solve real-world mathematical problems using rates, ratios and/or percents.

- **M06.A-R.1.1.1** Use ratio language and notation (such as 3 to 4, world and 3:4, ¾) to describe a ratio relationship between two quantities. *Example 1: "The ratio of girls to boys in a math class is 2:3, because for every 2 girls there are 3 boys." Example 2: "For every five votes candidate A received, candidate B received four votes."*
- **M06.A-R.1.1.2** Find the unit rate a/b associated with a ratio a:b (with $b \neq 0$), and use rate language in the context of a ratio relationship. *Example 1: "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." Example 2: "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
- **M06.A-R.1.1.3** Construct tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and/or plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- **M06.A-R.1.1.4** Solve unit rate problems including those involving unit pricing and constant speed. *Example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
- **M06.A-R.1.1.5** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Key Concepts	**Key Vocabulary**
Represent and/or solve real-world mathematical problems using rates, ratios and/or percents	ratio, percents, proportions, unit rates, rates, quantity, equivalent

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Solve rate and ratio word problems
- Solve problems involving ratios and/or percents
- Solve unit rate problems including unit pricing & constant speed
- Solve percent problems finding the whole, given the part
- Use ratio reasoning to convert

2.1 Number and Operations

Domain: (D) Ratios & Proportional Relationships

Standard: CC.2.1.6.E.1 Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Anchor Descriptor: M06.A-N.1.1 Solve real-world and mathematical problems involving division of fractions.

- **M06.A-N.1.1.1** Interpret and compute quotients of fractions (including mixed numbers), and solve word problems involving division of fractions by fractions. *Example 1: Given a story context for (2/3) ÷ (3/4), explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = (a/b) × (d/c) = ad/bc.) Example 2: How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Example 3: How many 2 1/4-foot pieces can be cut from a 15 1/2-foot board?*

Key Concepts	Key Vocabulary
Solve real-world and mathematical problems involving division of fractions.	quotients, mixed numbers, fraction, division, multiplication

Competencies
Describe what students should be able to do (key skills) as a result of this instruction

- Extend understanding of fractions
- Understand reciprocals
- Solve fraction word problems by using visual fraction models and equations

2.1 Number and Operations

Domain: (E) The Number System

Standard: CC.2.1.6.E.2 Identify and choose appropriate processes to compute fluently with multi-digit numbers.

Anchor Descriptor: M06.A-N.2.1 Compute with multi-digit numbers using the four arithmetic operations with or without a calculator.

- **M06.A-N.2.1.1** Solve problems involving operations (+, –, ×, ÷) with whole numbers, decimals (through thousandths), straight computation, or word problems.

Key Concepts	Key Vocabulary
Compute with multi-digit numbers using the four arithmetic operations with or without a calculator.	add, subtract, multiply, divide, compute

Competencies
Describe what students should be able to do (key skills) as a result of this instruction

- Identify and choose appropriate processes to compute fluently with multi-digit numbers
- Divide multi-digit numbers using the standard algorithm
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm

2.1 Number and Operations

Domain: (E) The Number System

Standard: CC.2.1.6.E.3 Develop and/or apply number theory concepts to find common factors and multiples.

Anchor Descriptor: **M06.A-N.2.2** Apply number theory concepts (specifically, factors and multiples).

- **M06.A-N.2.2.1** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
- **M06.A-N.2.2.2** Apply the distributive property to express a sum of two whole numbers, 1 through 100, with a common factor as a multiple of a sum of two whole numbers with no common factor. *Example: Express 36 + 8 as 4(9 + 2).*

Key Concepts	**Key Vocabulary**
Apply number theory concepts (specifically, factors and multiples)	greatest common factor (GCF), least common multiple (LCM), distributive property

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Find the Greatest Common Factor of two whole numbers less than or equal to 100.
- Find the Least Common Multiple of two whole numbers less than or equal to 12.
- Use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.

2.1 Number and Operations

Domain: (E) The Number System

Standard: CC.2.1.6.E.4 Apply and extend previous understandings of numbers to the system of rational numbers.

Anchor Descriptor: M06.A-N.3.1 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values and locations on the number line and coordinate plane.

- **M06.A-N.3.1.1** Represent quantities in real-world contexts using positive and negative numbers, explaining the meaning of 0 in each situation (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge).
- **M06.A-N.3.1.2** Determine the opposite of a number and recognize that the opposite of the opposite of a number is the number itself (e.g., – (– 3) = 3, and that 0 is its own opposite).
- **M06.A-N.3.1.3** Locate and plot integers and other rational numbers on a horizontal or vertical number line; locate and plot pairs of integers and other rational numbers on a coordinate plane.

Anchor Descriptor: M06.A-N.3.2 Understand ordering and absolute value of rational numbers.

- **M06.A-N.3.2.1** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *Example: Write – 3°C > – 7°C to express the fact that – 3°C is warmer than – 7°C.*
- **M06.A-N.3.2.2** Interpret the absolute value of a rational number as its distance from 0 on the number line and as a magnitude for a positive or negative quantity in a real-world situation. *Example: For an account balance of – 30 dollars, write | – 30| = 30 to describe the size of the debt in dollars, and recognize that an account balance less than – 30 dollars represents a debt greater than 30 dollars.*
- **M06.A-N.3.2.3** Solve real-world and mathematical problems by plotting points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Key Concepts	**Key Vocabulary**
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values and locations on the number line and coordinate plane, Understand ordering and absolute value of rational numbers.	quantities, integers, opposite, rational numbers, coordinate plane, quadrant, number line, absolute value, rational numbers, integers quadrants, coordinate plane

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Understand and represent positive and negative numbers in real world situations, explaining the meaning of 0.
- Recognize opposite signs of numbers as being placed on opposite sides of the number line.
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.
- Temperatures
- Money (-) withdrawal (+) deposit
- Interpret statements of inequality as statements about the relative position of two numbers on a number line.
- Write, interpret, and explain statements of order for rational numbers in real-world contexts
- Understand and interpret absolute value as a magnitude for positive and negative quantity in a real-world situation.
- Solve real-world problems by graphing points in all four quadrants of the coordinate plane

2.2 Algebraic Concepts

Domain: (B) Expressions and Equations

Standard: CC.2.2.6.B.1 Apply and extend previous understandings of arithmetic to algebraic expressions.

Anchor Descriptor: M06.B-E.1.1 Identify, write, and evaluate numerical and algebraic expressions.

- **M06.B-E.1.1.1** Write and evaluate numerical expressions involving whole-number exponents.
- **M06.B-E.1.1.2** Write algebraic expressions from verbal descriptions. *Example: Express the description "five less than twice a number" as 2y – 5.*
- **M06.B-E.1.1.3** Identify parts of an expression using mathematical terms (e.g., sum, term, product, factor, quotient, coefficient, quantity). *Example: Describe the expression 2(8 + 7) as a product of two factors.*
- **M06.B-E.1.1.4** Evaluate expressions at specific values of their variables, including expressions that arise from formulas used in real-world problems. *Example: Evaluate the expression b2 – 5 when b = 4.*
- **M06.B-E.1.1.5** Apply the properties of operations to generate equivalent expressions. *Example 1: Apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x. Example 2: Apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y). Example 3: Apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Key Concepts	**Key Vocabulary**
Identify, write, and evaluate numerical and algebraic expressions.	exponents, whole numbers, algebraic expressions, sum, term, product, factor, quotient, coefficient, quantity, variable, formula, distributive property, equivalent expression

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Apply the properties of operations to generate equivalent expressions.
- Identify when two expressions are equivalent.

2.2 Algebraic Concepts

Domain: (B) Expressions and Equations

Standard: CC.2.2.6.B.2 Understand the process of solving a one-variable equation or inequality and apply to real-world and mathematical problems.

Anchor Descriptor: M06.B-E.2.1 Create, solve, and interpret one-variable equations or inequalities in real-world and mathematical problems.

- **M06.B-E.2.1.1** Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **M06.B-E.2.1.2** Write algebraic expressions to represent real-world or mathematical problems.
- **M06.B-E.2.1.3** Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all non-negative rational numbers.
- **M06.B-E.2.1.4** Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem and/or represent solutions of such inequalities on number lines.

Key Concepts	**Key Vocabulary**
Create, solve, and interpret one-variable equations or inequalities in real world and mathematical problems.	equation, inequality, algebraic expression, rational number, constraint, number line

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Understand solving an equation or inequality as a process of answering a question
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem
- Solve real-world and mathematical problems by writing and solving equations of the form $x_{+p=q}$ and px=q for cases in which p, q, and x are all nonnegative rational numbers
- Write an inequality of the form x>c or x<c to represent a constraint or condition in a real-world or mathematical problem

2.2 Algebraic Concepts

Domain: (B) Expressions and Equations

Standard: CC.2.2.6.B.3 Represent and analyze quantitative relationships between dependent and independent variables.

Anchor Descriptor: M06.B-E.3.1 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

- **M06.B-E.3.1.1** Write an equation to express the relationship between the dependent and independent variables. Example: In a problem involving motion at a constant speed of 65 units, write the equation d = 65t to represent the relationship between distance and time.
- **M06.B-E.3.1.2** Analyze the relationship between the dependent and independent variables using graphs and tables, and/or relate these to an equation.

Key Concepts	**Key Vocabulary**
Use variables to represent two quantities in a real-world problem that change in relationship to one another.	equation, dependent variable, independent variable

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Use variables to represent two quantities in a real-world problem that change in relationship to one another
- Write equations to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable
- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate those to the equation

2.3 Geometry

Domain: (A) Geometry

Standard: CC.2.3.6.A.1 Apply appropriate tools to solve real-world and mathematical problems involving area, surface area, and volume.

Anchor Descriptor: M06.C-1.1.1 Find area, surface area, and volume by applying formulas and using various strategies.

- **M06.C-G.1.1.1** Determine the area of triangles and special quadrilaterals (i.e., square, rectangle, parallelogram, rhombus, and trapezoid). **Formulas will be provided.**
- **M06.C-G.1.1.2** Determine the area of irregular or compound polygons.
- Example: Find the area of a room in the shape of an irregular polygon by composing and/or decomposing.
- **M06.C-G.1.1.3** Determine the volume of right rectangular prisms with fractional edge lengths. **Formulas will be provided.**
- **M06.C-G.1.1.4** Given coordinates for the vertices of a polygon in the plane, use the coordinates to find side lengths and area of the polygon (limited to triangles and special quadrilaterals). **Formulas will be provided.**
- **M06.C-G.1.1.5** Represent three-dimensional figures using nets made up of rectangles and triangles.
- **M06.C-G.1.1.6** Determine the surface area of triangular and rectangular prisms (including cubes). **Formulas will be provided.**

Key Concepts	**Key Vocabulary**
Find area, surface area, and volume by applying formulas and using various strategies.	area, quadrilaterals, square, rectangle, parallelogram, rhombus, trapezoid, irregular polygon, compound polygon, formula, rectangular prism, fractional edge length, coordinates, vertex, three-dimensional figure, surface area, triangular prism, volume, cube

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes
- Apply these techniques in the context of solving real-world and mathematical problems
- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths
- Apply the formulas V=l w h and V= B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems
- Draw polygons in the coordinate plane given coordinates for the vertices
- Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate

2.4 Measurement, Data and Probability

Domain: (B) Statistics and Probability

Standard: CC.2.4.6.B.1 Demonstrate an understanding of statistical variability by displaying, analyzing, and summarizing distributions.

Anchor Descriptor: M06.D-S.1.1 Display, analyze, and summarize numerical data sets in relation to their context.

- **M06.D-S.1.1.1** Display numerical data in plots on a number line, including dot plots, histograms, and box-and whisker plots.
- **M06.D-S.1.1.2** Determine quantitative measures of center (e.g., median, mean, and/or mode) and variability (e.g., range, interquartile range, and/or mean absolute deviation).
- **M06.D-S.1.1.3** Describe any overall pattern and any deviations from the overall pattern with reference to the context in which the data were gathered.
- **M06.D-S.1.1.4** Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Key Concepts	**Key Vocabulary**
Display, analyze, and summarize numerical data sets in relation to their context.	number line, dot plots, histograms, box-and-whisker plot, mean, median, mode, range, interquartile range, mean absolute deviation

Competencies

Describe what students should be able to do (key skills) as a result of this instruction

- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. Example: "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.
- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number
- Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- Summarize numerical data sets in relation to their context, such as: giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation)
- Summarize numerical data sets in relation to their context such as: relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- Summarize numerical data sets in relation to their context such as: reporting the number of observations and describing the nature of the attribute under investigation, including how it was measured and its units of measurement.