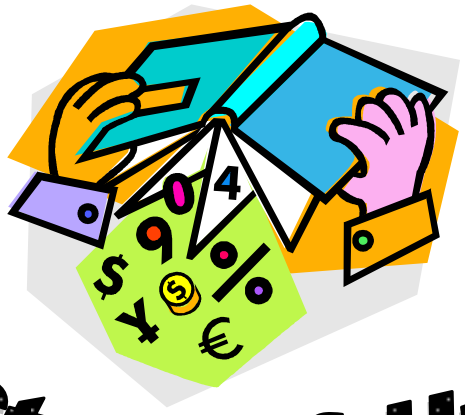
Mathematics:



"A Story of Units"

Parent Handbook

Grade 4
Module 6

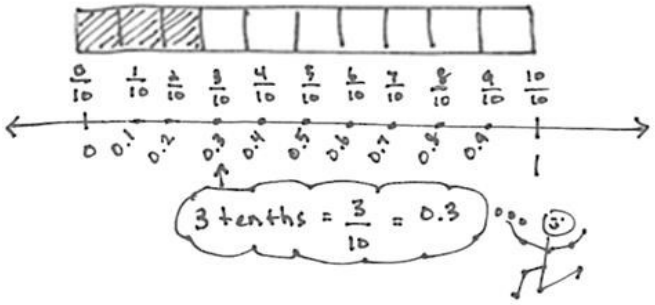
Grade 4 • Module 6

Decimal Fractions

OVERVIEW

This 20-day module gives students their first opportunity to explore decimal numbers via their relationship to decimal fractions, expressing a given quantity in both fraction and decimal forms. Utilizing the understanding of fractions developed throughout Module 5, students apply the same reasoning to decimal numbers, building a solid foundation for Grade 5 work with decimal operations. Previously referred to as whole numbers, all numbers written in the base ten number system with place value units that are powers of 10 are henceforth referred to as decimal numbers, a set which now includes tenths and hundredths, e.g. 1, 15, 248, 0.3, 3.02, and 24.34.

In Topic A, students use their understanding of fractions to explore tenths. At the opening of the topic, they use metric measurement to see tenths in relationship to different whole units: centimeters, meters, kilograms, and liters. Students explore, creating and identifying tenths of various wholes, as they draw lines of specified length, identify the weight of objects, and read the level of liquid measurements. Students connect these concrete experiences pictorially as tenths are represented on the number line and with tape diagrams as pictured to the right. Students express tenths as decimal fractions and are introduced to decimal notation. They write statements of equivalence in unit, fraction, and decimal forms, e.g., 3 tenths = 3/10 = 0.3. Next, students return to the use of metric measurement to investigate decimal fractions greater than 1. Using a centimeter ruler, they draw lines that measure, for example, 2 4/10 or 6 8/10 centimeters. Using the area model, students see that numbers containing a whole number and fractional part, i.e., mixed numbers, can also be expressed using decimal notation provided that the fractional part can be converted to a decimal number. Students use place value disks to represent the value of each digit in a decimal number. Just as they wrote whole numbers in expanded form using multiplication, students write the value of a decimal number in expanded form using fractions and decimals, e.g., 2 ones 4 tenths = 2 4/10 = (2 x 1) + (4 x 1/10) and 2.4 = (2 x 1) + (4 x 0.1). Additionally, students plot decimal numbers on the number line.



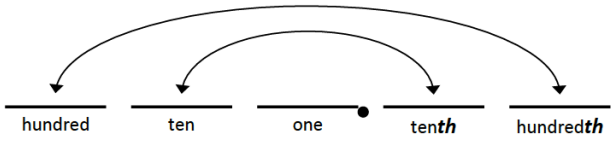
$$31\frac{46}{100} = (3 \times 10) + (1 \times 1) + (4 \times \tfrac{1}{10}) + (6 \times \tfrac{1}{100})$$

$$31.46 = (3 \times 10) + (1 \times 1) + (4 \times 0.1) + (6 \times 0.01)$$

Students decompose tenths into 10 equal parts to create hundredths in Topic B. Through the decomposition of a meter, students identify 1 centimeter as 1 hundredth of a meter. As they count up by hundredths, they realize the equivalence of 10 hundredths and 1 tenth and go on to represent them as both decimal fractions and as decimal numbers. Students use area models, tape diagrams, and number disks on a place value chart to see and model the equivalence of numbers involving units of tenths and hundredths. They express the value of the number in both decimal and fraction expanded forms.

Close work with the place value chart helps students see that place value units are not symmetric about the decimal point—a common misconception that often leads students to mistakenly believe there is a "oneths" place. They explore the placement of decimal numbers to hundredths and recognize that the place value chart is symmetric about the ones column. This understanding helps students recognize that, even as we move to the units on the right side of the decimal on the place value chart, a column continues to represent a unit 10 times as large as that of the column to its right. This understanding builds on the place value work done in Module 1 and enables students to understand that 3.2, for example, might be modeled as 3 ones 2 tenths, 32 tenths, or 320 hundredths. Topic B concludes with students using their knowledge of fraction equivalence to work with decimal numbers expressed in unit form, fraction form, and decimal form.

Symmetry with respect to the ones place

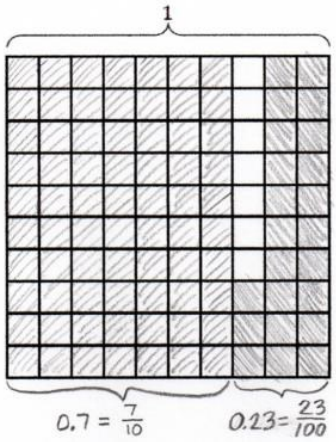


The focus of Topic C is comparison of decimal numbers. To begin, students work with concrete representations of measurements. They see measurement of length on meter sticks, of mass using a scale, and of volume using graduated cylinders. In each case, students record the measurements on a place value chart and then compare them. They use their understanding of metric measurement and decimals to answer questions such as, "Which is greater? Less? Which is longer? Shorter? Which is heavier? Lighter?" Comparing the decimals in the context of measurement supports students' justification of their comparisons and grounds their reasoning, while at the same time setting them up for work with decimal comparison at a more concrete level. Next, students use area models and number lines to compare decimal numbers and use the <, >, and = symbols to record their comparisons. All of their work with comparisons at the pictorial level helps to eradicate the common misconception that is often made when students assume a greater number of hundredths must be greater than a lesser number of tenths. For example, when comparing 7 tenths and 27 hundredths, students recognize that 7 tenths is greater than 27 hundredths because, in any comparison, one must consider the *size of the units*. Students go on to arrange mixed groups of decimal fractions in unit, fraction, and decimal forms in order from greatest to least or least to greatest. They use their understanding of different ways of expressing equivalent values in order to arrange a set of decimal fractions as pictured below.

Rice Bag	ones (kilograms)	.	tenths	hundredths
A	0	.	1	0
B	0	.	6	5
C	0	.	7	
D	0	.	4	6

0.7 kg, 0.65 kg, 0.46 kg, 0.1 kg

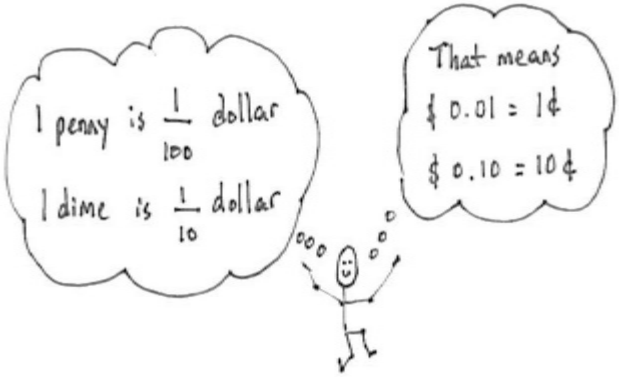
Topic D introduces the addition of decimals by way of finding equivalent decimal fractions and adding fractions. Students add tenths and hundredths, recognizing that they must convert the addends to the same units. The sum is then converted back into a decimal. They use their knowledge of like denominators and understanding of fraction equivalence to do so. Students use the same process to add and subtract mixed numbers involving decimal units. They then apply their new learning to solve word problems involving metric measurements.



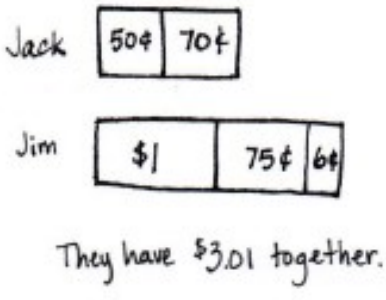
$$\frac{7}{10} + \frac{23}{100} = \frac{70}{100} + \frac{23}{100} = \frac{93}{100}$$

$$\frac{93}{100} = 0.93$$

Students conclude their work with decimal fractions in Topic E by applying their knowledge to the real world context of money. They recognize 1 penny as 1/100 dollar, 1 dime as 1/10 dollar, and 1 quarter as 25/100 dollar. They apply their understanding of tenths and hundredths to write given amounts of money in both fraction and decimal forms. To do this, students decompose a given amount of money into dollars, quarters, dimes, and pennies, and express the amount as a decimal fraction and decimal number. Students then add various numbers of coins and dollars using Grade 2 knowledge of the equivalence of 100 cents to 1 dollar. Addition and subtraction word problems are solved using unit form, adding dollars and cents. Multiplication and division word problems are solved using cents as the unit. The final answer in each word problem is converted from cents into a decimal using a dollar symbol for the unit.



For example: *Jack has 2 quarters and 7 dimes. Jim has 1 dollar, 3 quarters, and 6 pennies. How much money do they have together? Write your answer as a decimal.*



1 dollar 20 cents + 1 dollar 81 cents

= 2 dollars 101 cents

100 1

= 3 dollars 1 cent

= $3.01

**The sample questions/responses contained in this manual are straight from http://www.engageny.org/. They are provided to give some insight into the kinds of skills expected of students as the lesson is taught.

Terminology

New or Recently Introduced Terms

- Decimal number (number written using place value units that are powers of 10)
- Decimal expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times 0.1) + (9 \times 0.01) = 24.59$)
- Decimal fraction (fraction with a denominator of 10, 100, 1,000, etc.)
- Decimal point (period used to separate the whole number part from the fractional part of a decimal number)
- Fraction expanded form (e.g., $(2 \times 10) + (4 \times 1) + \left(5 \times \frac{1}{10}\right) + \left(9 \times \frac{1}{100}\right) = 24\frac{59}{100}$)
- Hundredth (place value unit such that 100 hundredths equals 1 one)
- Tenth (place value unit such that 10 tenths equals 1 one)

Familiar Terms and Symbols

- Expanded form (e.g., 100 + 30 + 5 = 135)
- Fraction (numerical quantity that is not a whole number, e.g., $\frac{1}{3}$)

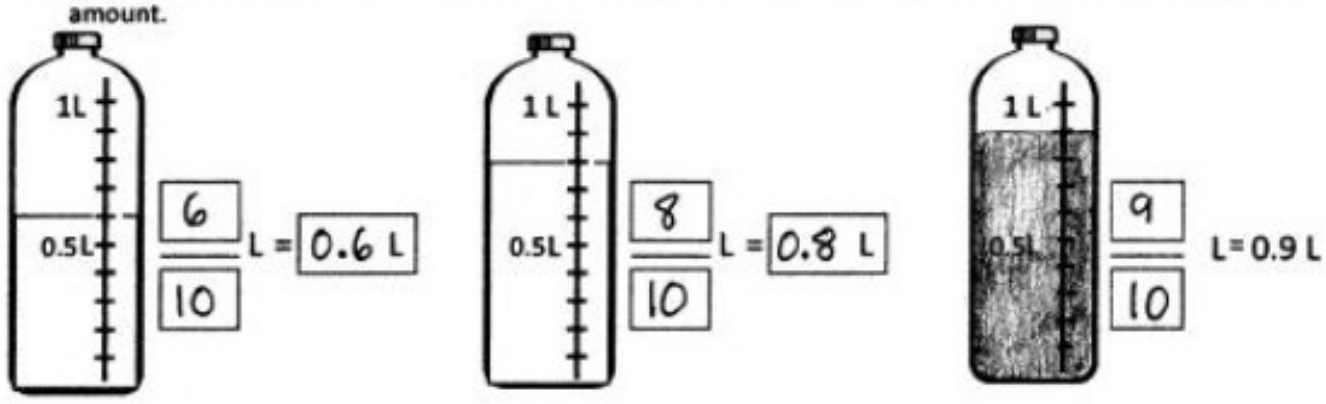
Suggested Tools and Representations

- 1-liter container with milliliters marks
- Area model
- Centimeter ruler
- Digital scale
- Meter stick
- Number disks (including decimal number disks to hundredths)
- Number line
- Place value chart with decimals to hundredths
- Tape diagram

Lesson 1

Objective: Use metric measurement to model the decomposition of one whole into tenths.

2. . Write the total amount of water in fraction and decimal form. Shade the last bottle to show the correct amount.



Lesson 2

Objective: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

For each length given below, draw a line segment to match. Express each measurement as an equivalent mixed number.

a. 2.6 cm _____ $2.6\text{ cm} = 2\frac{6}{10}\text{ cm}$

Write the following as equivalent decimals. Then model and rename the number as shown below.

4 ones and 2 tenths = 4.2

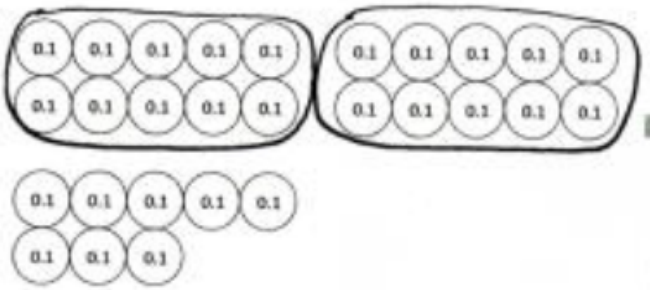
$4\frac{2}{10} = 4 + \frac{2}{10} = 4 + 0.2 = 4.2$

Lesson 3

Objective: Represent mixed numbers with units of tens, ones, and tenths with number disks, on the number line, and in expanded form.

1. Circle groups of tenths to make as many ones as possible.

a. How many tenths in all?



There are 28 tenths.

Write and draw the same number using ones and tenths.

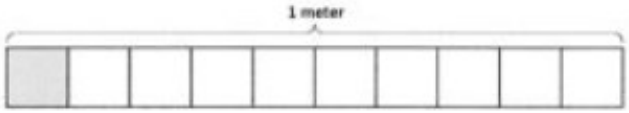
Decimal Form: 2.8

How much more is needed to get to 3? 0.2

Lesson 4

Objective: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

a. What is the length of the shaded part of the meter stick in centimeters?

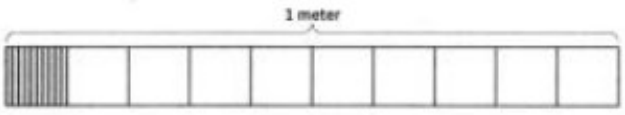


10 cm

b. What fraction of a meter is 1 centimeter?

$\frac{1}{100}$ m

c. In fraction form, express the length of the shaded portion of the meter stick.



$\frac{10}{100}$ m

d. In decimal form, express the length of the shaded portion of the meter stick.

0.10 m

e. What fraction of a meter is 10 centimeters?

$\frac{1}{10}$ m

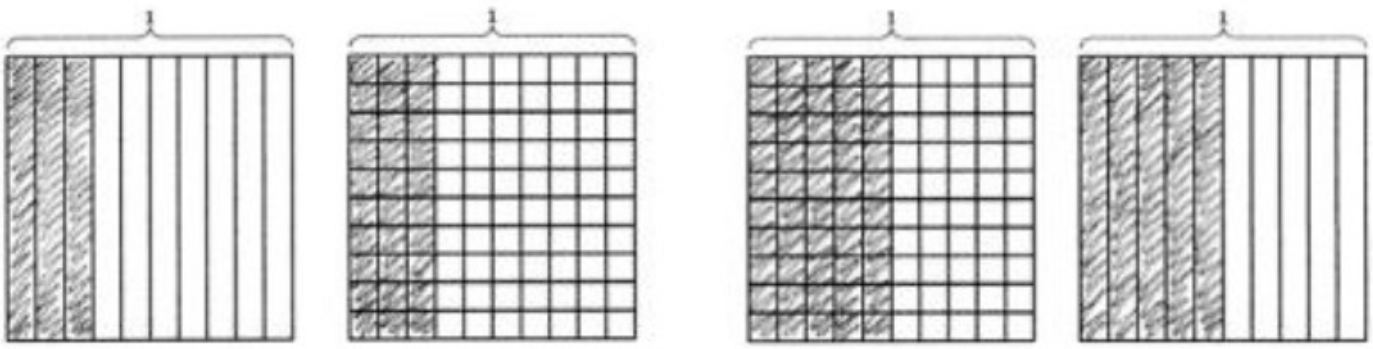
Lesson 5

Objective: Model the equivalence of tenths and hundredths using the area model and number disks.

Find the equivalent fraction using multiplication or division. Shade the area models to show the equivalency. Record it as a decimal.

a. $\frac{3 \times 10}{10 \times 10} = \frac{30}{100} = 0.30$

b. $\frac{50 \div 10}{100 \div 10} = \frac{5}{10} = 0.5$

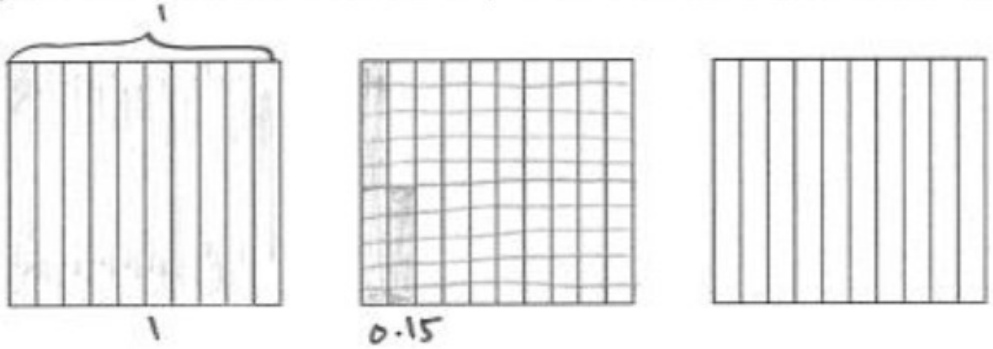


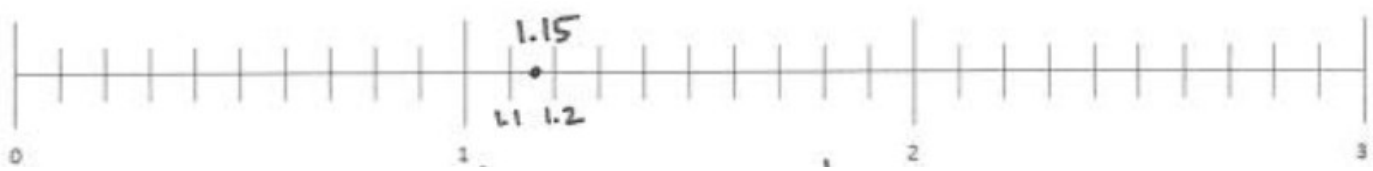
Lesson 6

Objective: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and

Shade the area models to represent the number, drawing horizontal lines to make hundredths as needed. Locate the corresponding point on the number line. Label with a point and record the mixed number as a decimal.

a. $1\frac{15}{100} = 1.15$





Lesson 7

Objective: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

Write a decimal number sentence to identify the total value of the number disks.

a.

2 tens 5 tenths 3 hundredths

20 + 0.5 + 0.03 = 20.53

Write each number in expanded form, using both decimal and fraction notation. The first one has been done for you.

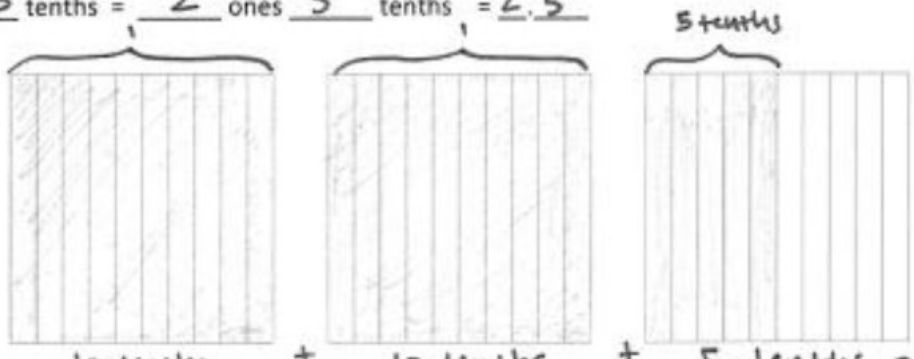
	Expanded Form	
Decimal and Fraction Form	Fraction Notation	Decimal Notation
$15.43 = 15\frac{43}{100}$	$(1 \times 10) + (5 \times 1) + (4 \times \frac{1}{10}) + (3 \times \frac{1}{100})$ $10 + 5 + \frac{4}{10} + \frac{3}{100}$	$(1 \times 10) + (5 \times 1) + (4 \times 0.1) + (3 \times 0.01)$ $10 + 5 + 0.4 + 0.03$
$21.4 = 21\frac{4}{10}$	$(2 \times 10) + (1 \times 1) + (4 \times \frac{1}{10})$ $20 + 1 + \frac{4}{10}$	$(2 \times 10) + (1 \times 1) + (4 \times 0.1)$ $20 + 1 + 0.4$

Lesson 8

Objective: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

Use the area model to represent $\frac{250}{100}$. Complete the number sentence.

a. $\frac{250}{100}$ = 25 tenths = 2 ones 5 tenths = 2.5



10 tenths + 10 tenths + 5 tenths = 25 tenths = 2.5

b. In the space below, explain how you determined your answer to (a).

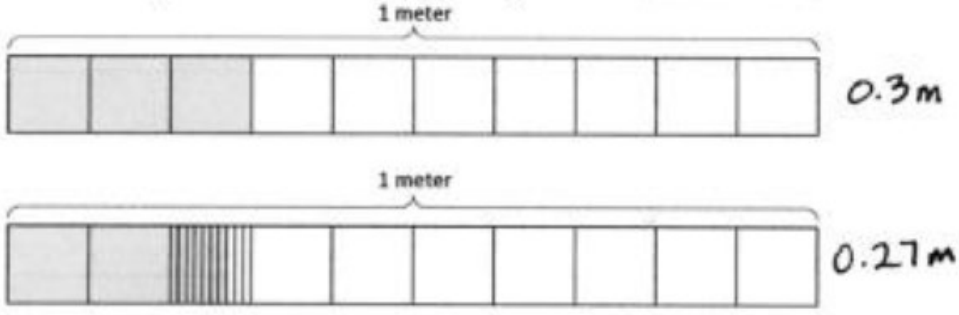
I shaded in 25 tenths which was 25 bars. 20 bars equals 2 ones and the 5 bars left equals 5 tenths so I got 2.5.

Lesson 9

Objective: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

Express the lengths of the shaded parts in decimal form. Write a sentence that compares the two lengths. Use the expressions *shorter than* or *longer than* in your sentence.

a.



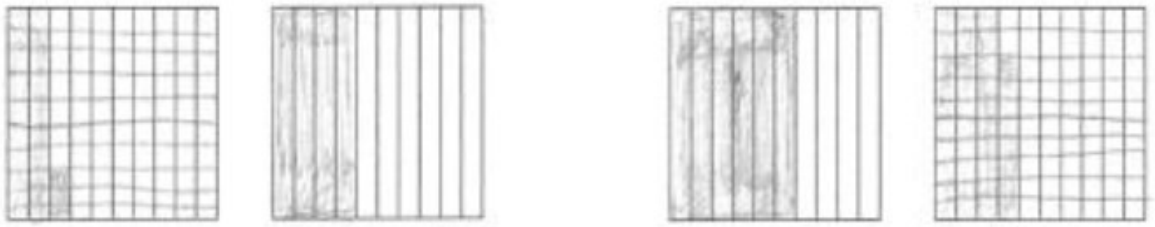
0.3m is longer than 0.27 m.

Lesson 10

Objective: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

Shade the area models below, decomposing tenths as needed, to represent the pairs of decimal numbers. Fill in the blank with <, >, or = to compare the decimal numbers.

a. 0.23 ___<___ 0.4 b. 0.6 ___>___ 0.38



Locate and label the points for each of the decimal numbers on the number line. Fill in the blank with <, >, or = to compare the decimal numbers.

a. 10.03 ___<___ 10.3

Lesson 11

Objective: Compare and order mixed numbers in various forms.

Plot the following points on the number line.

$6\frac{3}{10}$, 6.31, $\frac{628}{100}$, $\frac{62}{10}$, 6.43, 6.40

Arrange the following numbers in order from greatest to least using decimal form. Use the > symbol between each number.

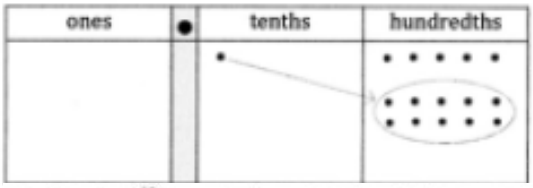
$\frac{27}{10}$, 2.07, $\frac{27}{100}$, $2\frac{71}{100}$, $\frac{227}{100}$, 2.72

2.72 > 2.71 > 2.7 > 2.27 > 2.07 > 0.27

Lesson 12

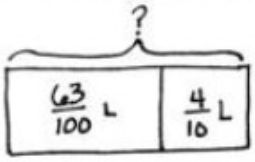
Objective: Apply understanding of fraction equivalence to add tenths and hundredths.

Complete the number sentence by expressing each part using hundredths. Model using the place value chart, as shown in Problem 1(a).



a. 1 tenth + 5 hundredths = 15 hundredths

Beaker A has $\frac{63}{100}$ liter of iodine and is then filled the rest of the way with water up to 1 liter. Beaker B has $\frac{4}{10}$ liter of iodine and is then filled the rest of the way with water up to 1 liter. If both beakers are emptied into a large beaker, how much iodine will be in the large beaker?



$\frac{63}{100} + \frac{4}{10} = \frac{63}{100} + \frac{40}{100} = \frac{103}{100} = 1\frac{3}{100} = 1.03$

$\frac{100}{100}$ $\frac{3}{100}$

The larger beaker will have 1.03 milliliters of iodine.

Lesson 13

Objective: Add decimal numbers by converting to fraction form.

Solve. Convert tenths to hundredths before finding the sum. Rewrite the complete number sentence in decimal form.

a. $2\frac{1}{10}+\frac{3}{100}=2\frac{10}{100}+\frac{3}{100}=2\frac{13}{100}$

$2.1+0.03=2.13$

b. $2\frac{1}{10}+5\frac{3}{100}=2\frac{10}{100}+5\frac{3}{100}=7\frac{13}{100}$

$2.1+5.03=7.13$

Solve by rewriting the number sentence in fraction form. After solving, rewrite the complete number sentence in decimal form.

a. $6.4+5.3$

$6\frac{4}{10}+5\frac{3}{10}=11\frac{7}{10}$

$6.4+5.3=11.7$

b. $6.62+2.98$

$6\frac{62}{100}+2\frac{98}{100}=9\frac{60}{100}$

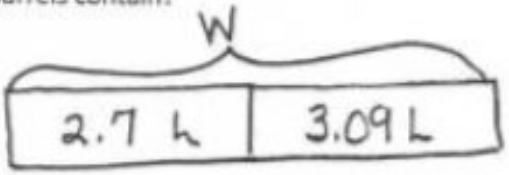
$\frac{60}{100}\ \ \frac{3}{100}$

$6.62+2.98=9.6$

Lesson 14

Objective: Solve word problems involving the addition of measurements in decimal form.

Barrel A contains 2.7 liters of water. Barrel B contains 3.09 liters of water. Together, how much water do the two barrels contain?



$W=2.70+3.09$

$=2\frac{70}{100}L+3\frac{9}{100}L=5\frac{79}{100}L$

The barrels contain 5.79L of water.

Lesson 15

Objective: Express money amounts given in various forms as decimal numbers.

100 pennies = $1.00	100¢ = $\frac{100}{100}$ dollar
1 penny = $0.01	1¢ = $\frac{1}{100}$ dollar
6 pennies = $0.06	6¢ = $\frac{6}{100}$ dollar
10 pennies = $0.10	10¢ = $\frac{10}{100}$ dollar

Solve. Give the total amount of money in fraction and decimal form.

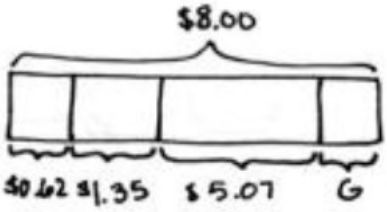
3 dimes and 8 pennies $\frac{38}{100}$ $0.38

8 dimes and 23 pennies $\frac{103}{100}$ $1.03

Lesson 16

Objective: Solve word problems involving money.

Vanessa has 6 dimes and 2 pennies. Joachim has 1 dollar, 3 dimes, and 5 pennies. Jimmy has 5 dollars and 7 pennies. They want to put their money together to buy a game that cost $8.00. Do they have enough money to buy the game? If not, how much more money do they need?



62
135
+507
704

7 9 10
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96

G = 96 cents = 96¢ = $0.96

They don't have enough. They need $0.96 more for the game.