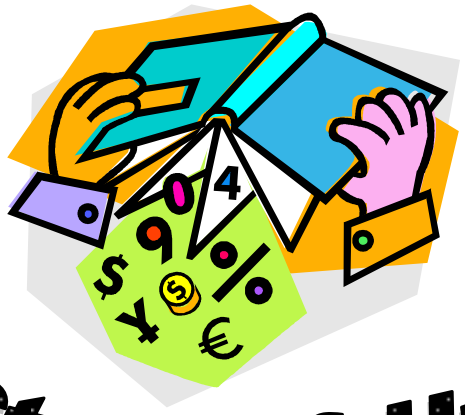
Mathematics:



"A Story of Units"

Parent Handbook

Grade 4
Module 5

Grade 4 • Module 5

Fraction Equivalence, Ordering, and Operations

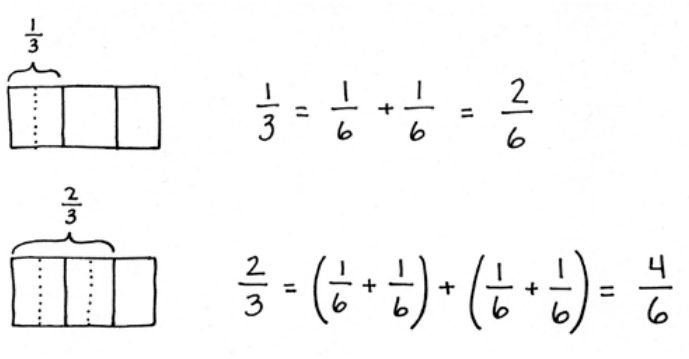
OVERVIEW

In this 40-day module, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations.

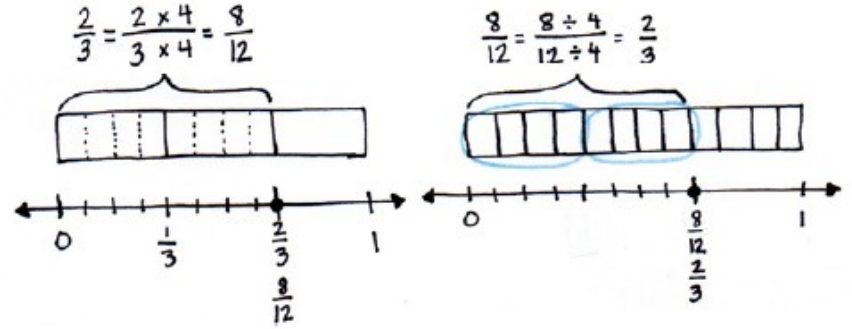
Students begin Topic A by decomposing fractions and creating tape diagrams to represent them as sums of fractions with the same denominator in different ways (e.g., 3/5 = 1/5 + 1/5 + 1/5 = 1/5 + 2/5). They go on to see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts.

For example, just as 3 twos = 2 + 2 + 2 = 3 × 2, so does 3/4 = 1/4 + 1/4 + 1/4 = 3 × 1/4.

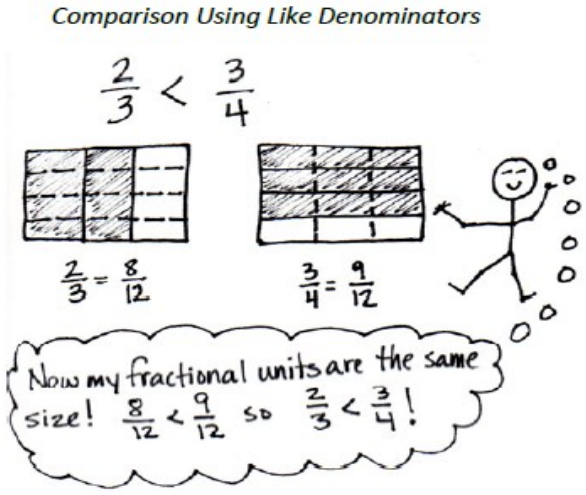
The introduction of multiplication as a record of the decomposition of a fraction early in the module allows students to become familiar with the notation before they work with more complex problems. As students continue working with decomposition, they represent familiar unit fractions as the sum of smaller unit fractions. A folded paper activity allows them to see that when the number of fractional parts in a whole increases, the size of the pieces decreases. They go on to investigate this concept with the use of tape diagrams and area models. Reasoning enables them to explain why two different fractions can represent the same portion of a whole.

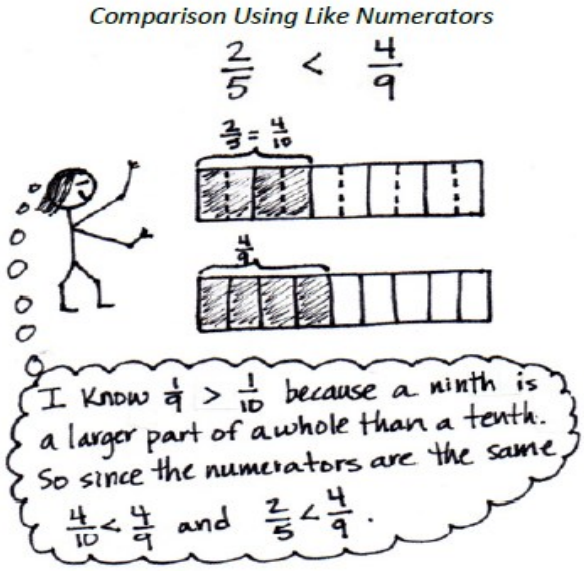


In Topic B, students use tape diagrams and area models to analyze their work from earlier in the module and begin using multiplication to create an equivalent fraction comprised of smaller units, e.g., 2/3 = 2 × 4 / 3 × 4 = 8/12 . Based on the use of multiplication, they reason that division can be used to create a fraction comprised of larger units (or a single unit) that is equivalent to a given fraction, e.g., 8/12 = 8 ÷ 4 / 12 ÷ 4 = 2/3. Their work is justified using area models and tape diagrams and, conversely, multiplication is used to test for and/or verify equivalence. Students use the tape diagram to transition to modeling equivalence on the number line. They see that, by multiplying, any unit fraction length can be partitioned into *n* equal lengths and that doing so multiplies both the total number of fractional units (the denominator) and the number of selected units (the numerator) by *n*. They also see that there are times when fractional units can be grouped together, or divided, into larger fractional units. When that occurs, both the total number of fractional units and the number of selected units are divided by the same number.



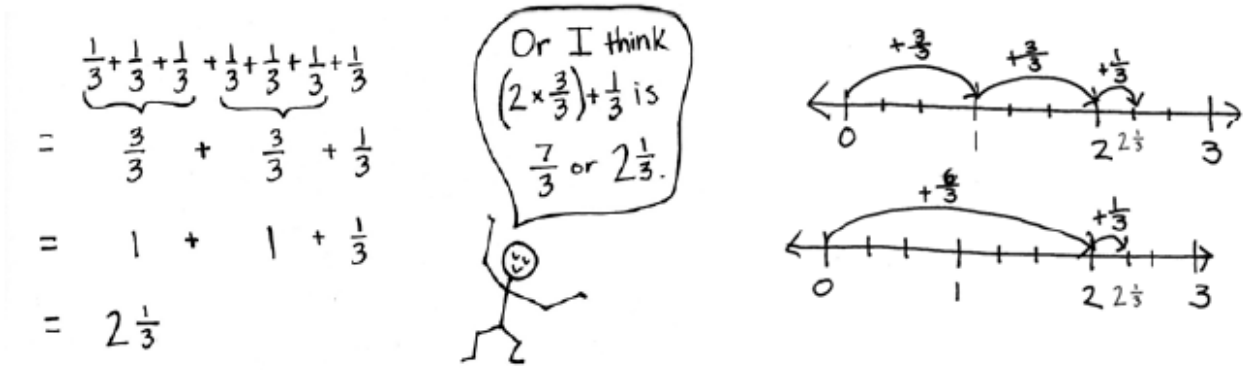
In Grade 3, students compared fractions using fraction strips and number lines with the same denominators. In Topic C, they expand upon comparing fractions by reasoning about fractions with unlike denominators. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, 1/2 , or 1) on the number line. Alternatively, students compare using the same numerators. They find that the fraction with the greater denominator is the lesser fraction, since the size of the fractional unit is smaller as the whole is decomposed into more equal parts, e.g., 1/5 > 1/10 therefore 3/5 >3/10. Throughout, their reasoning is supported using tape diagrams and number lines in cases where one numerator or denominator is a factor of the other, such as 1/5 and 1/10 or 2/3 and 5/6. When the units are unrelated, students use area models and multiplication, the general method pictured below to the left, whereby two fractions are expressed in terms of the same denominators. Students also reason that comparing fractions can only be done when referring to the same whole, and they record their comparisons using the comparison symbols <, >, and =.





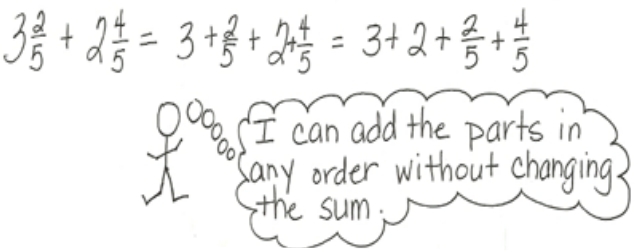
In Topic D, students apply their understanding of whole number addition (the combining of like units) and subtraction (finding an unknown part) to work with fractions. They see through visual models that if the units are the same, computation can be performed immediately, e.g., 2 bananas + 3 bananas = 5 bananas and 2 eighths + 3 eighths = 5 eighths. They see that when subtracting fractions from one whole, the whole is decomposed into the same units as the part being subtracted, e.g., 1 – 3/5 = 5/5 – 3/5 = 2/5. Students practice adding more than two fractions and model fractions in word problems using tape diagrams. As an extension of the Grade 4 standards, students apply their knowledge of decomposition from earlier topics to add fractions with related units using tape diagrams and area models to support their numerical work. To find the sum of 1/2 and 1/4 , for example, one simply decomposes 1 half into 2 smaller equal units, fourths, just as in Topics A and B. Now the addition can be completed: 2/4 + 1/4 = 3/4. Though not assessed, this work is warranted because in Module 6 students will be asked to add tenths and hundredths when working with decimal fractions and decimal notation.

At the start of Topic E, students use decomposition and visual models to add and subtract fractions less than 1 to or from whole numbers (e.g., 4 + 3/4= 4 3/4 and 4 – 3/4= (3 + 1) – 3/4). They use addition and multiplication to build fractions greater than 1 and represent them on the number line.

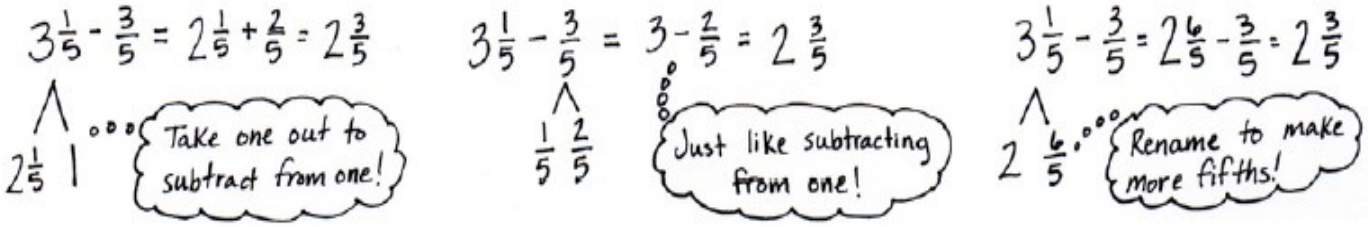


Students then use these visual models and decompositions to reason about the various forms in which a fraction greater than or equal to 1 may be presented: both as fractions and as mixed numbers. They practice converting between these forms and come to understand the usefulness of each form in different situations. Through this understanding, the common misconception that every improper fraction must be converted to a mixed number is avoided. Next, students compare fractions greater than 1, building on their rounding skills and using their understanding of benchmarks to reason about which of two fractions is greater. This activity continues to build understanding of the relationship between the numerator and denominator of a fraction. Students progress to finding and using like denominators or numerators to compare and order mixed numbers. They apply their skills of comparing numbers greater than 1 by solving word problems requiring the interpretation of data presented in line plots. Students use addition and subtraction strategies to solve the problems, as well as decomposition and modeling to compare numbers in the data sets.

In Topic F, students estimate sums and differences of mixed numbers, rounding before performing the actual operation to determine what a reasonable outcome will be. They go on to use decomposition to add and subtract mixed numbers. This work builds on their understanding of a mixed number being the sum of a whole number and a fraction.



Using unit form, students add and subtract like units first, ones and ones, fourths and fourths. Students use decomposition, shown with number bonds, in mixed number addition to make one from fractional units before finding the sum. When subtracting, students learn to decompose the minuend or the subtrahend when there are not enough fractional units to subtract from. Alternatively, students can rename the subtrahend, giving more units to the fractional units, which connects to whole number subtraction when renaming 9 tens 2 ones as 8 tens 12 ones.



In Topic G, students build on the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions. They use the associative property and their understanding of decomposition. Just as with whole numbers, the unit remains unchanged.

For example:

$$4 \times \frac{3}{5} = 4 \times \left(3 \times \frac{1}{5}\right) = (4 \times 3) \times \frac{1}{5} = \frac{4 \times 3}{5} = \frac{12}{5}$$

This understanding connects to students' work with place value and whole numbers. Students go on to explore the use of the distributive property to multiply a whole number by a mixed number. They recognize that they are multiplying each part of a mixed number by the whole number and use efficient strategies to do so. The topic closes with solving multiplicative comparison word problems involving fractions as well as problems involving the interpretation of data presented on a line plot.

$$5 \times 3\tfrac{3}{4} = 5 \times \left(3 + \tfrac{3}{4}\right)$$
$$= (5 \times 3) + \left(5 \times \tfrac{3}{4}\right)$$
$$= 15 + \tfrac{15}{4}$$
$$= 15 + 3\tfrac{3}{4}$$
$$= 18\tfrac{3}{4}$$

The final topic comprises an exploration lesson where students find the sum of all like denominators from 0/*n* to *n*/*n*. For example, they might find the sum of all fifths from 0/5 to 5/5. Students discover they can make pairs with a sum of 1 to add more efficiently, e.g., 0/5 + 5/5, 1/5 + 4/5, 2/5 + 3/5. They then extend this to similarly find sums of eighths, tenths, and twelfths, observing patterns when finding the sum of odd and even denominators.

**The sample questions/responses contained in this manual are straight from http://www.engageny.org/. They are provided to give some insight into the kinds of skills expected of students as the lesson is taught.

Terminology

New or Recently Introduced Terms

- Benchmark (standard or reference point by which something is measured)
- Common denominator (when two or more fractions have the same denominator)
- Denominator (bottom number in a fraction, represents how many equal portions make up 1 whole)
- Line plot (display of data on a number line, using an x or another mark to show frequency)
- Mixed number (number made up of a whole number and a fraction)
- Numerator (top number in a fraction, represents how many equal portions are being described)

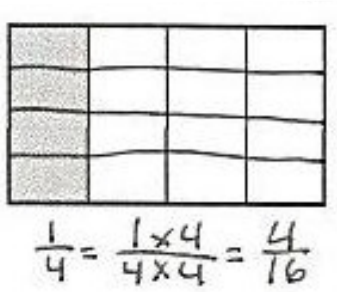
Familiar Terms and Symbols

- Compose (change a group of unit fractions with the same denominator to a single non-unit fraction or mixed number)
- Decompose (change a non-unit fraction or mixed number to the sum of its parts or unit fractions)
- Equivalent fractions (fractions that name the same size or amount)
- Fraction (e.g., 1/3, 2/3, 3/3, 4/3)
- Fraction greater than 1 (an improper fraction: a fraction with a numerator that is greater than the denominator)
- Fractional unit (e.g., half, third, fourth)
- Multiple (product of a given number and any other whole number)
- Non-unit fraction (fractions with numerators other than 1)
- Unit fraction (fractions with numerator 1)
- Unit interval (e.g., the interval from 0 to 1, measured by length)
- Whole (e.g., 2 halves, 3 thirds, 4 fourths)
- =, <, > (equal to, less than, greater than)

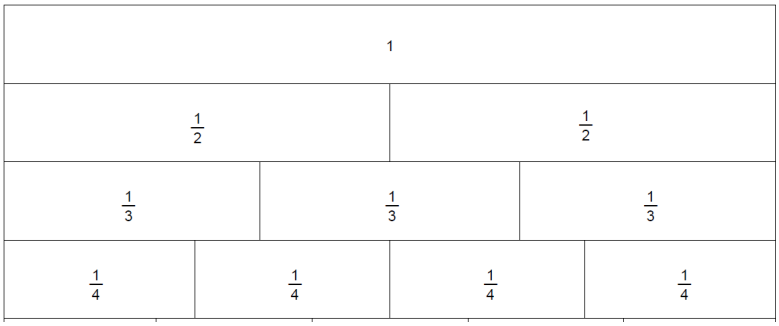
Suggested Tools and Representations

- Area model
- Fraction strips (made from paper, used to fold and model equivalent fractions)
- Line plot
- Number line
- Rulers
- Tape diagram

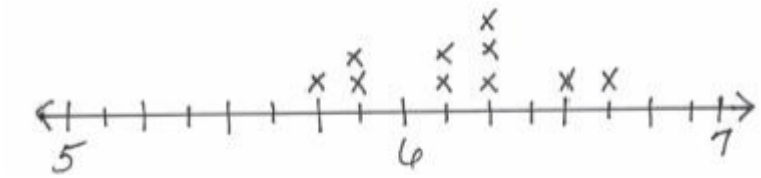
Area Model: This area model shows a visual representation of fraction equivalents, proving that 1/4 is equivalent to 4/16.



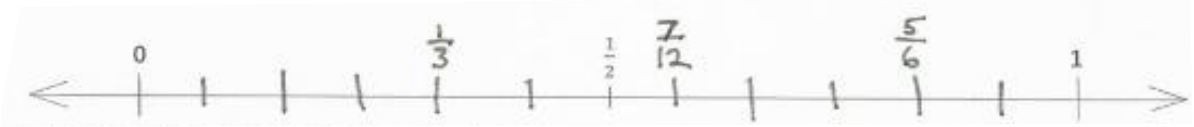
Fraction Strips: Fraction strips are tiles or strips that are proportionately sized to one whole so that students may physically make size comparisons and find equivalent amounts using different denominators.



Line Plot: Display of data on a number line, using an x or another mark to show frequency. For example, this line plot shows data that there is one item at 5 6/8, two at 5 7/8, etc…



Number Line: The number line is used to develop a deeper understanding of whole number units, fraction units, measurement units, decimals, and negative numbers. Throughout Grades K-5, the number line models measuring units.

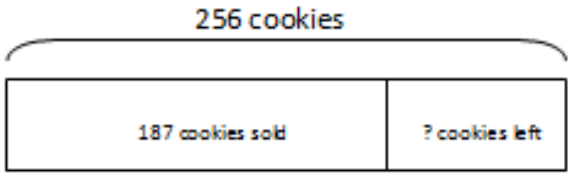


Tape Diagram: Tape diagrams, also called bar models, are pictorial representations of relationships between quantities used to solve word problems. At the heart of a tape diagram is the idea of *forming units*. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic.

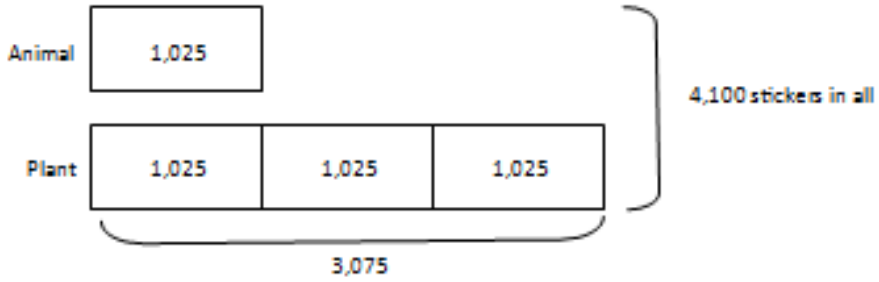
The tape diagram provides an essential bridge to algebra and is often called "pictorial algebra." There are two basic forms of the tape diagram model. The first form is sometimes called the part-whole model; it uses bar segments placed end-to-end (Grade 3 Example), while the second form, sometimes called the comparison model, uses two or more bars stacked in rows that are typically left justified (Grade 5 Example).

Grade 3 Example: *Sarah baked 256 cookies. She sold some of them. 187 were left. How many did she sell?* 256 – 187 = _____

256 –187 = 69 so Sarah sold 69 cookies.



Grade 5 Example: *Sam has 1,025 animal stickers. He has 3 times as many plant stickers as animal stickers. How many plant stickers does Sam have? How many stickers does Sam have altogether?*



1. He has 3,075 plant stickers.

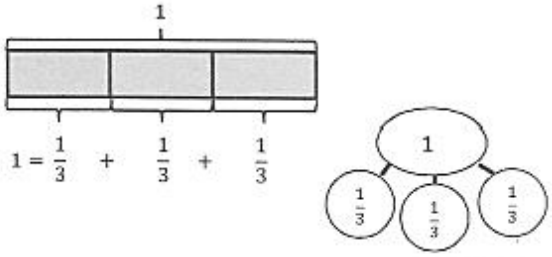
2. He has 4,100 stickers altogether.

Lesson 1

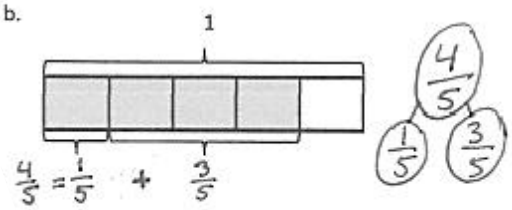
Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

Draw a number bond and write the number sentence to match each tape diagram. The first one is done for you.

a.

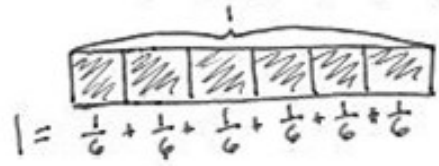


b.



Draw and label tape diagrams to model each decompositioin.

a. $1 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$



Lesson 2

Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

Step 1: Draw and shade a tape diagram of the given fraction.
Step 2: Record the decomposition of the fraction in 3 different ways using number sentences.

a. $\frac{7}{8}$

$\frac{7}{8} = \frac{1}{8} + \frac{1}{8} + \frac{2}{8} + \frac{3}{8}$

$\frac{7}{8} = \frac{5}{8} + \frac{2}{8}$

$\frac{7}{8} = \frac{4}{8} + \frac{3}{8}$

Lesson 3

Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

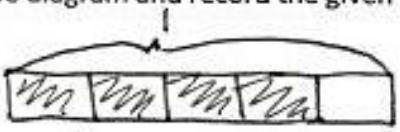
Write the following fractions greater than one as the sum of two products.

a.

$\frac{5}{3} = (3 \times \frac{1}{3}) + (2 \times \frac{1}{3})$

Draw a tape diagram and record the given fraction's decomposition into unit fractions as a multiplication sentence.

a. $\frac{4}{5}$



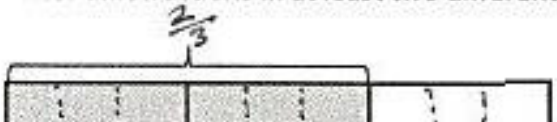
$\frac{4}{5} = 4 \times \frac{1}{5}$

Lesson 4

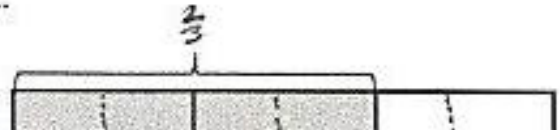
Objective: Decompose fractions into sums of smaller unit fractions using tape diagrams.

The total length of each tape diagram represents 1 whole. Decompose the shaded fractions as the sum of smaller unit fractions in at least two different ways..

a.



$\frac{2}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$



$\frac{2}{3} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

Show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$ using a tape diagram and a multiplication sentence.

$\frac{1}{2} = 4 \times \frac{1}{8} = \frac{4}{8}$

Lesson 5

Objective: Decompose unit fractions using area models to show equivalence.

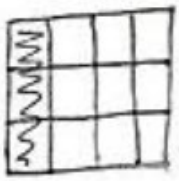
Draw area models to show the decompositions represented by the number sentences below. Represent the decomposition as a sum of unit fractions and as a multiplication sentence.

a. $\frac{1}{2}=\frac{3}{6}$

$\frac{1}{2}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$

$\frac{1}{2}=3\times\frac{1}{6}=\frac{3}{6}$

$\frac{1}{4}=\frac{3}{12}$



$\frac{1}{4}=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{3}{12}$

$\frac{1}{4}=3\times\frac{1}{12}=\frac{3}{12}$

Explain why $\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$ is the same as $\frac{1}{4}$.

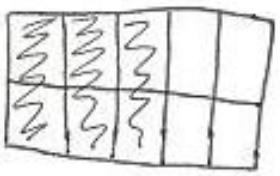
In the area model, the area of three twelfths $(\frac{1}{12}+\frac{1}{12}+\frac{1}{12})$ equals the area of $\frac{1}{4}$ of the model so the fractions are equal.

Lesson 6

Objective: Decompose fractions using area models to show equivalence.

Draw area models to show the decompositions represented by the number sentences below. Express each as a sum and product of unit fractions. Use parentheses to show the relationship between the number sentences.

a. $\frac{3}{5}=\frac{6}{10}$



$\frac{3}{5}=\frac{6}{10}$

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=(\frac{1}{10}+\frac{1}{10})+(\frac{1}{10}+\frac{1}{10})+(\frac{1}{10}+\frac{1}{10})=\frac{6}{10}$

$(\frac{1}{10}+\frac{1}{10})+(\frac{1}{10}+\frac{1}{10})+(\frac{1}{10}+\frac{1}{10})=(2\times\frac{1}{10})+(2\times\frac{1}{10})+(2\times\frac{1}{10})=\frac{6}{10}$

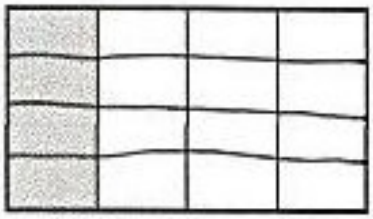
$\frac{3}{5}=6\times\frac{1}{10}=\frac{6}{10}$

Lesson 7

Objective: Use the area model and multiplication to show the equivalence of two fractions.

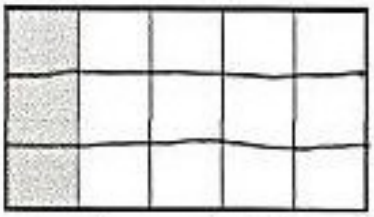
Decompose the shaded fractions into smaller units using the area models. Express the equivalent fractions in a number sentence using multiplication.

a.



$\frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$

b.



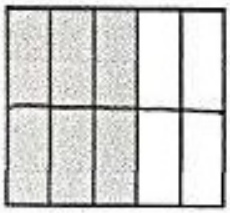
$\frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$

Lesson 8

Objective: Use the area model and multiplication to show the equivalence of two fractions.

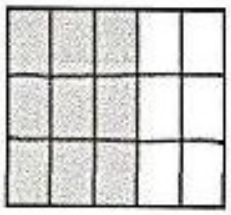
Decompose the shaded fractions into smaller units, as given below. Express the equivalent fractions in a number sentence using multiplication.

a. Decompose into tenths.



$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$

b. Decompose into fifteenths.



$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$

Use multiplication to rename each fraction below.

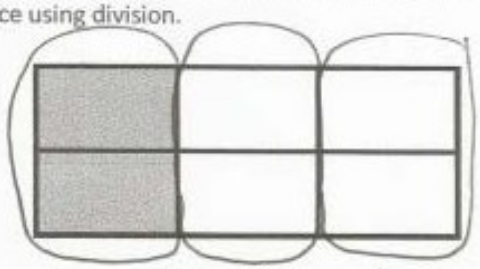
a. $\frac{3}{4}$ $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

Lesson 9

Objective: Use the area model and division to show the equivalence of two fractions.

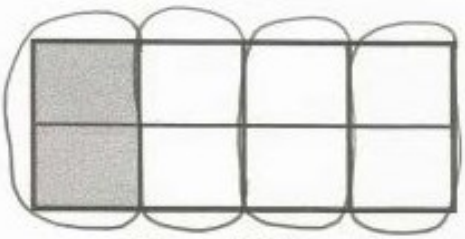
Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a.



$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

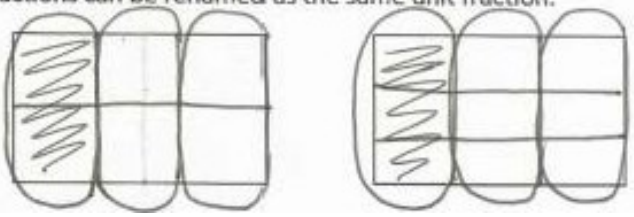
b.



$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$

a. In the first area model, show 2 sixths. In the second area model, show 3 ninths. Show how both fractions can be renamed as the same unit fraction.

$\frac{2}{6} = \frac{1}{3}$



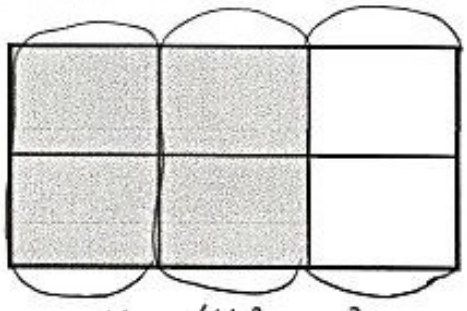
$\frac{3}{9} = \frac{1}{3}$

Lesson 10

Objective: Use the area model and division to show the equivalence of two fractions.

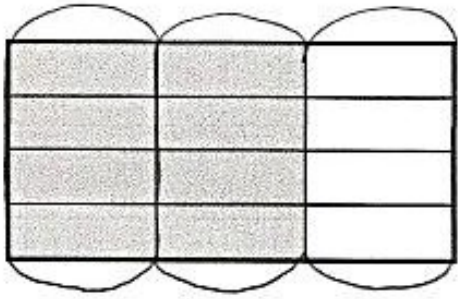
Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a.



$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

b.



$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Use division to rename each fraction given below. Draw a model if that helps you. See if you can use the largest common factor.

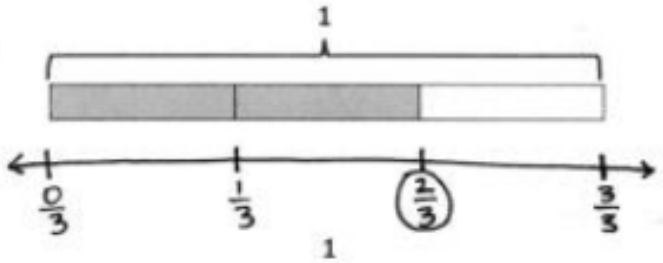
a. $\frac{4}{8}$ $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

Lesson 11

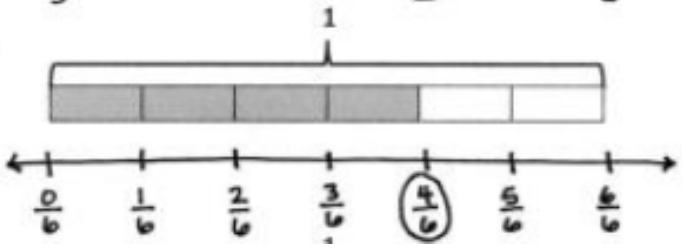
Objective: Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division.

Use each shaded tape diagram below as a ruler to draw a number line. Mark each number line with the unit fractions shown on the tape diagram, and circle the fraction that labels the point on the number line that also names the selected part of the tape diagram.

a.



b.

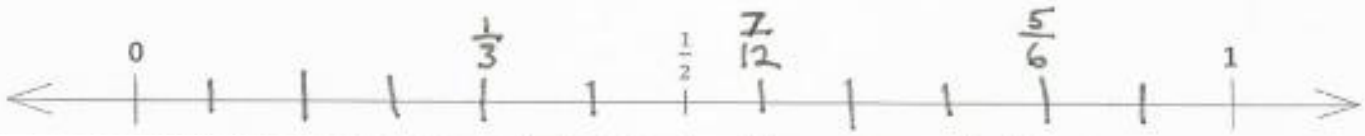


Lesson 12

Objective: Reason using benchmarks to compare two fractions on the number line.

a. Plot the following points on the number line without measuring.

i. $\frac{1}{3}$ ii. $\frac{5}{6}$ iii. $\frac{7}{12}$



b. Use the number line in (a) to compare the fractions by writing >, <, or = on the lines:

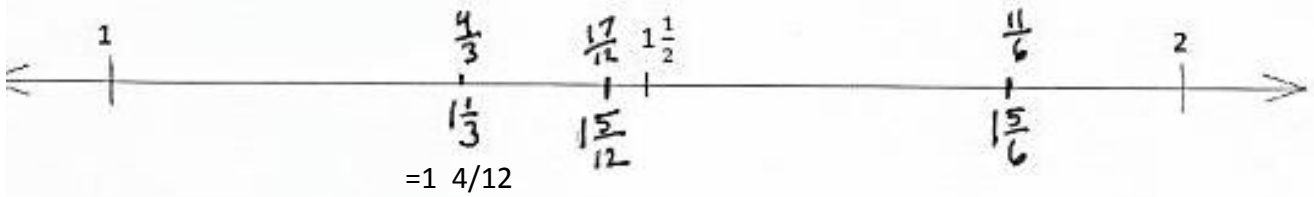
i. $\frac{7}{12} > \frac{1}{2}$ ii. $\frac{7}{12} < \frac{5}{6}$

Lesson 13

Objective: Reason using benchmarks to compare two fractions on the number line.

Place the following fractions on the number line given.

a. $\frac{4}{3}$ b. $\frac{11}{6}$ c. $\frac{17}{12}$



=1 4/12

Use the number line in Problem 1 to compare the fractions by writing >, <, or = on the lines:

a. $1\frac{5}{6} > 1\frac{3}{12}$ b. $1\frac{1}{3} < 1\frac{5}{12}$

Lesson 14

Objective: Find common units or number of units to compare two fractions.

Compare by reasoning about the following pairs of fractions with the same or related numerators. Use >, <, or =. Explain your thinking using words, pictures, or numbers. Problem 2(b) has been done for you.

a. $\frac{3}{5} < \frac{3}{4}$

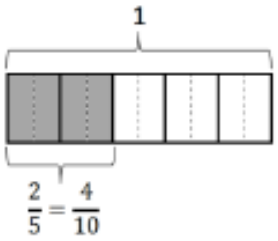
1 fifth < 1 fourth

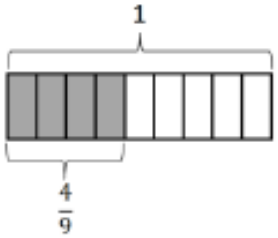
so $\frac{3}{5} < \frac{3}{4}$ → fourths are larger units than fifths.

b. $\frac{2}{5} < \frac{4}{9}$

because $\frac{2}{5} = \frac{4}{10}$

4 tenths is less than 4 ninths because tenths are smaller than ninths.

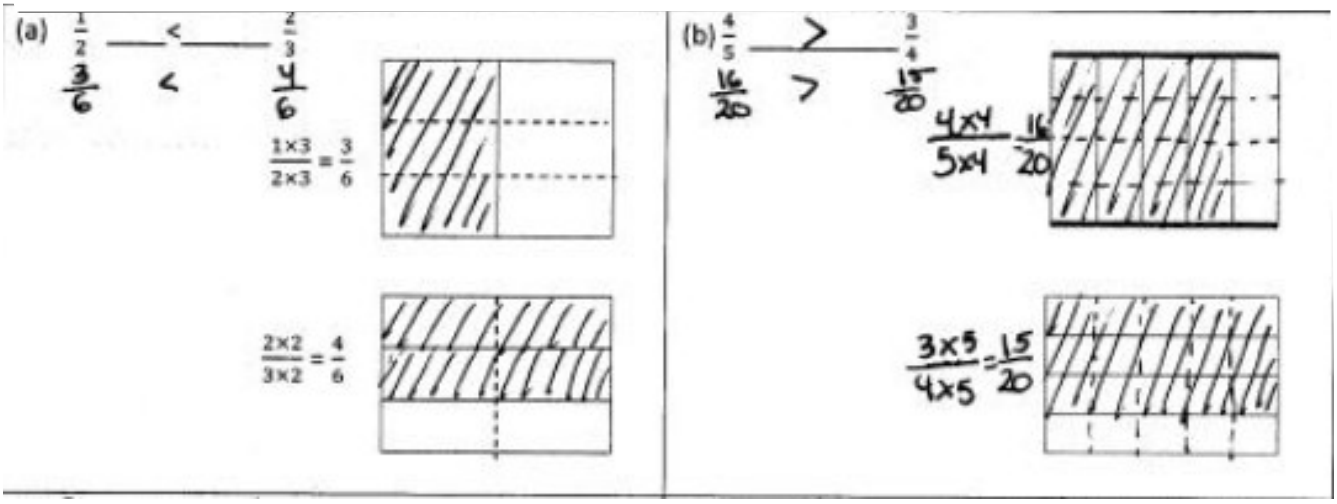




Lesson 15

Objective: Find common units or number of units to compare two fractions.

Draw an area model for each pair of fractions, and use it to compare the two fractions by writing a >, <, or = symbol on the line. The first two have been partly done for you. Each rectangle represents one whole.

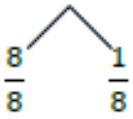


Lesson 16

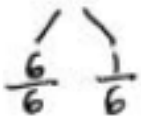
Objective: Use visual models to add and subtract two fractions with the same units.

Solve. Use a number bond to show how to convert the difference to a mixed number. Problem (a) has been completed for you.

a. $\frac{12}{8} - \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$



b. $\frac{12}{6} - \frac{5}{6} = \frac{7}{6} = 1\frac{1}{6}$



Solve. Write the sum in unit form.

a. 2 fourths + 1 fourth = 3 fourths

Solve.

a. $\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$

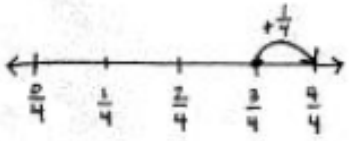
Lesson 17

Objective: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole.

Solve. Model each subtraction problem with a number line, and solve by both counting up and subtracting. Problem (a) has been solved for you.

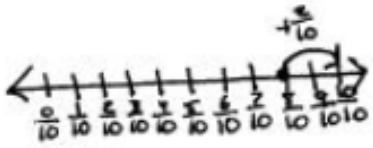
a. $1 - \frac{3}{4}$

$\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$



b. $1 - \frac{8}{10}$

$\frac{10}{10} - \frac{8}{10} = \frac{2}{10}$



Find the difference in two ways. Use number bonds to decompose the whole. Problem (a) has been completed for you.

a. $1\frac{2}{5} - \frac{4}{5}$

$\frac{5}{5}$ $\frac{2}{5}$

$\frac{5}{5} + \frac{2}{5} = \frac{7}{5}$

$\frac{7}{5} - \frac{4}{5} = \frac{3}{5}$

$\frac{5}{5} - \frac{4}{5} = \frac{1}{5}$

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

Lesson 18

Objective: Add and subtract more than two fractions.

Show one way to solve each problem. Express sums and differences as a mixed number when possible. Use number bonds when it helps you. Problem (a) is partially completed.

a. $\frac{2}{5} + \frac{3}{5} + \frac{1}{5}$

$= \frac{5}{5} + \frac{1}{5} = 1 + \frac{1}{5}$

$= 1\frac{1}{5}$

b. $\frac{3}{6} + \frac{1}{6} + \frac{3}{6}$

$\frac{6}{6}$

$= \frac{6}{6} + \frac{1}{6} = 1 + \frac{1}{6}$

$= 1\frac{1}{6}$

c. $\frac{5}{7} + \frac{7}{7} + \frac{2}{7}$

$\frac{7}{7}$

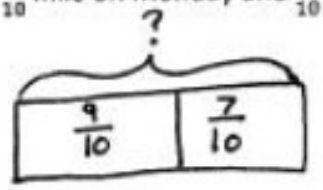
$= \frac{7}{7} + \frac{7}{7} = 1 + 1$

$= 2$

Lesson 19

Objective: Solve word problems involving addition and subtraction of fractions.

Sue ran $\frac{9}{10}$ mile on Monday and $\frac{7}{10}$ mile on Tuesday. How many miles did Sue run in the 2 days?



Sue ran $1\frac{6}{10}$ miles in the 2 days.

$\frac{9}{10} + \frac{7}{10} = \frac{16}{10} = 1\frac{6}{10}$

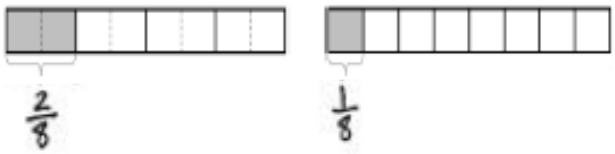
$\frac{16}{10}$ → $\frac{10}{10}$ $\frac{6}{10}$

Lesson 20

Objective: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, 12.

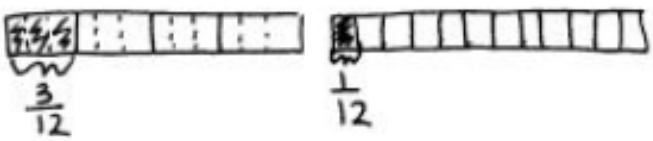
Use a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then write the complete number sentence. Part (a) is partially completed.

a. $\frac{1}{4} + \frac{1}{8}$



$\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$

b. $\frac{1}{4} + \frac{1}{12}$

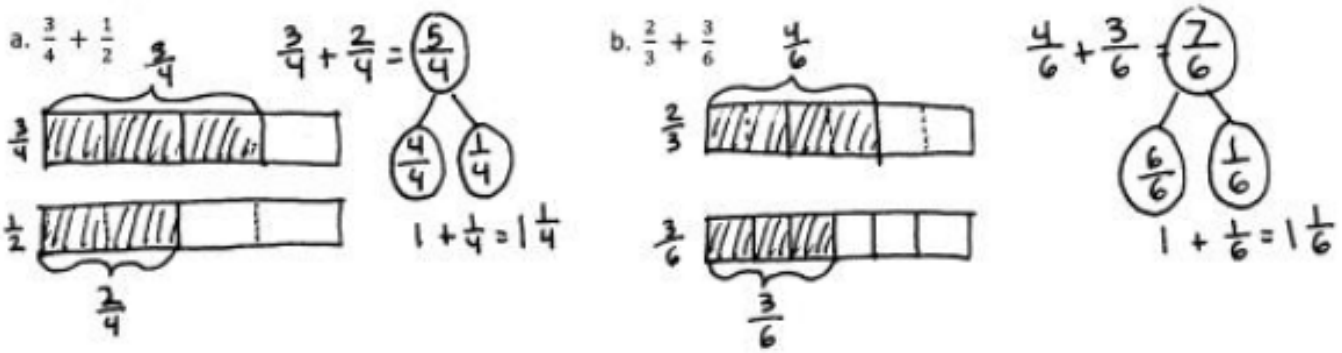


$\frac{3}{12} + \frac{1}{12} = \frac{4}{12}$

Lesson 21

Objective: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, 12.

Draw a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then write a complete number sentence. Use a number bond to write each sum as a mixed number.



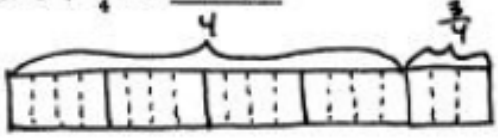
Lesson 22

Objective: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.

Draw a tape diagram to match each number sentence. Then complete the number sentence.

a. $3 + \frac{1}{3} = 3\frac{1}{3}$

b. $4 + \frac{3}{4} = 4\frac{3}{4}$

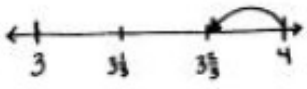


Solve using a number bond. Draw a number line to represent each number sentence. The first one has been done for you.

a. $4 - \frac{1}{3} = 3\frac{2}{3}$

$4 - \frac{1}{3} = 3\frac{2}{3}$

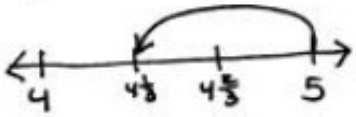
3 $\frac{2}{3}$



b. $5 - \frac{2}{3} = 4\frac{1}{3}$

$5 - \frac{2}{3} = 4\frac{1}{3}$

4 $\frac{2}{3}$



Lesson 23

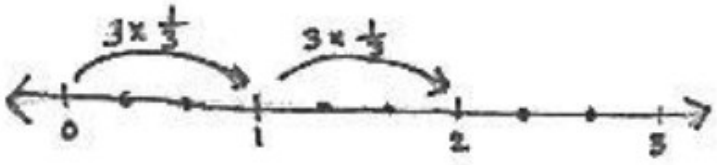
Objective: Add and multiply unit fractions to build fractions greater than 1 using visual models.

Use parentheses to show how to make ones in the following number sentence.

$$\left(\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}\right)+\left(\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}\right)+\left(\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}\right)=3$$

Multiply, as shown below. Draw a number line to support your answer.

a. $6 \times \frac{1}{3}$



$$6 \times \tfrac{1}{3} = 2 \times \tfrac{3}{3} = 2$$

Lesson 24

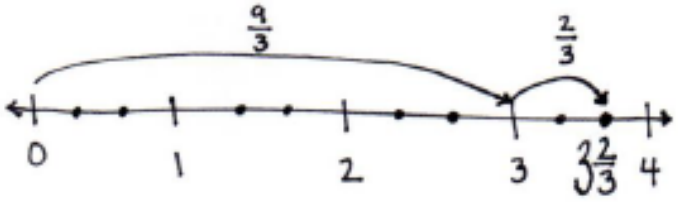
Objective: Decompose and compose fractions greater than 1 to express them in various forms.

Rename each fraction as a mixed number by decomposing it into two parts as shown below. Model the decomposition with a number line and a number bond.

a. $\frac{11}{3}$

$$\tfrac{11}{3} = \tfrac{9}{3}+\tfrac{2}{3} = 3+\tfrac{2}{3} = 3\tfrac{2}{3}$$

$\frac{9}{3}$ $\frac{2}{3}$



Lesson 25

Objective: Decompose and compose fractions greater than 1 to express them in various forms.

Convert each mixed number to a fraction greater than 1. Show your work as in the example.

(Note: $3 \times \frac{4}{4} = \frac{3 \times 4}{4}$)

$4\frac{1}{3}$

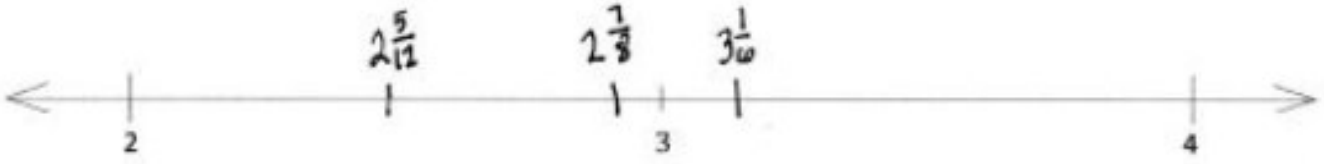
$4\frac{1}{3} = 4 + \frac{1}{3} = (4 \times \frac{3}{3}) + \frac{1}{3} = \frac{12}{3} + \frac{1}{3} = \frac{13}{3}$

Lesson 26

Objective: Compare fractions greater than 1 by reasoning using benchmark fractions.

a. Plot the following points on the number line without measuring.

i. $2\frac{7}{8}$ ii. $3\frac{1}{6}$ iii. $\frac{29}{12} = 2\frac{5}{12}$



b. Use the number line in (a) to compare the fractions by writing >, <, or =.

i. $\frac{29}{12}$ < $2\frac{7}{8}$ ii. $\frac{29}{12}$ < $3\frac{1}{6}$

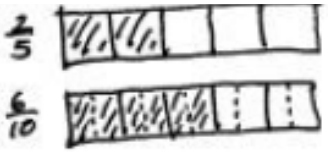
Lesson 27

Objective: Compare fractions greater than 1 by creating common numerators or denominators.

Draw a tape diagram to model each comparison. Use >, <, or = to compare.

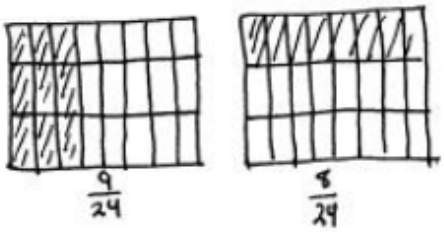
a. $3\frac{2}{3}$ < $3\frac{5}{6}$

b. $3\frac{2}{5}$ < $3\frac{6}{10}$



Use an area model to make like units. Then use >, <, or = to compare.

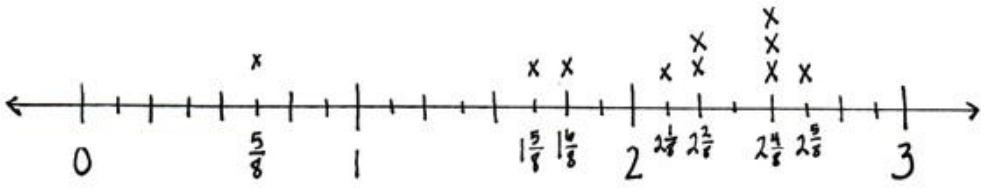
$2\frac{3}{8}$ > $2\frac{1}{3}$



Lesson 28

Objective: Solve word problems with line plots.

The chart to the right shows the distance 4th graders in Ms. Smith's class were able to run before stopping for a rest. Create a line plot to display the data in the table.



Student	Distance (in miles)
Joe	$2\frac{1}{2}$
Arianna	$1\frac{3}{4}$
Bobbi	$2\frac{1}{8}$
Morgan	$1\frac{5}{8}$
Jack	$2\frac{5}{8}$
Saisha	$2\frac{1}{4}$
Tyler	$2\frac{2}{4}$
Jenny	$\frac{5}{8}$
Anson	$2\frac{2}{8}$
Chandra	$2\frac{4}{8}$

Solve each problem.

a. Who ran a mile further than Jenny?

$\frac{5}{8} + 1 = 1\frac{5}{8}$ Morgan

b. Who ran a mile less than Jack?

$2\frac{5}{8} - 1 = 1\frac{5}{8}$ Morgan

Lesson 29

Objective: Estimate sums and differences using benchmark numbers.

Estimate each sum or difference to the nearest whole or half by rounding. Explain your estimate using words or a number line.

$\frac{16}{5} + \frac{11}{4} \approx 6$

$\frac{16}{5} = (3 \times \frac{3}{3}) + \frac{1}{5} = 3\frac{1}{5}$ $\frac{11}{4} = (2 \times \frac{4}{4}) + \frac{3}{4} = 2\frac{3}{4}$ $3+3=6$

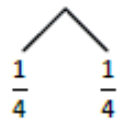
I rounded each number to the nearest whole and then found the sum of the rounded numbers.

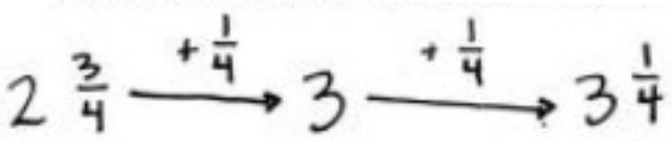
Lesson 30

Objective: Add a mixed number and a fraction.

Use a number bond and the arrow way to show how to make one. Solve.

a. $2\frac{3}{4} + \frac{2}{4}$





Solve.

a. $4\frac{2}{3} + \frac{2}{3}$

$4\frac{2}{3} + \frac{2}{3} = 4\frac{4}{3} = 5\frac{1}{3}$

$1\ \frac{1}{3}$

b. $3\frac{3}{5} + \frac{4}{5}$

$3\frac{3}{5} + \frac{4}{5} = 4 + \frac{2}{5} = 4\frac{2}{5}$

$\frac{2}{5}\ \frac{2}{5}$

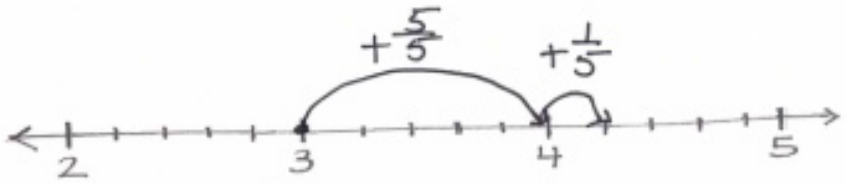
Lesson 31

Objective: Add mixed numbers.

Solve. Use a number line to show your work.

a. $2\frac{4}{5}+1\frac{2}{5}=3+\frac{6}{5}=4\frac{1}{5}$

$\frac{5}{5}$ $\frac{1}{5}$



Solve. Use the arrow way to show how you can "make one."

a. $2\frac{4}{6}+1\frac{5}{6}=3\frac{4}{6}+\frac{5}{6}=4\frac{3}{6}$

$\frac{2}{6}$ $\frac{3}{6}$

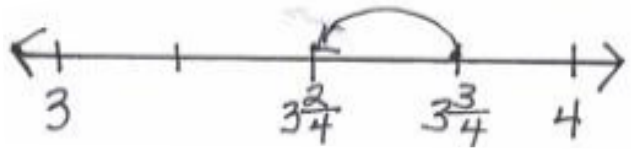
$3\frac{4}{6} \xrightarrow{+\frac{2}{6}} 4 \xrightarrow{+\frac{3}{6}} 4\frac{3}{6}$

Lesson 32

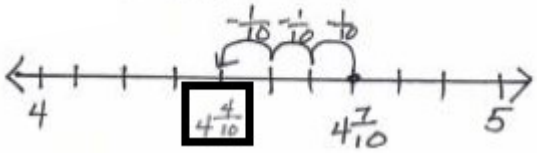
Objective: Subtract a fraction from a mixed number.

Subtract. Model with a number line or the arrow way.

a. $3\frac{3}{4}-\frac{1}{4}=3\frac{2}{4}$



b. $4\frac{7}{10}-\frac{3}{10}=4\frac{4}{10}$



Use decomposition to subtract the fractions. Model with a number line or the arrow way.

1. $5\frac{3}{5}-\frac{4}{5}=4\frac{4}{5}$

$\frac{3}{5}$ $\frac{1}{5}$

$5\frac{3}{5} \xrightarrow{-\frac{3}{5}} 5 \xrightarrow{-\frac{1}{5}} 4\frac{4}{5}$

2. $4\frac{1}{4}-\frac{2}{4}=3\frac{3}{4}$

$\frac{1}{4}$ $\frac{1}{4}$

$4\frac{1}{4} \xrightarrow{-\frac{1}{4}} 4 \xrightarrow{-\frac{1}{4}} 3\frac{3}{4}$

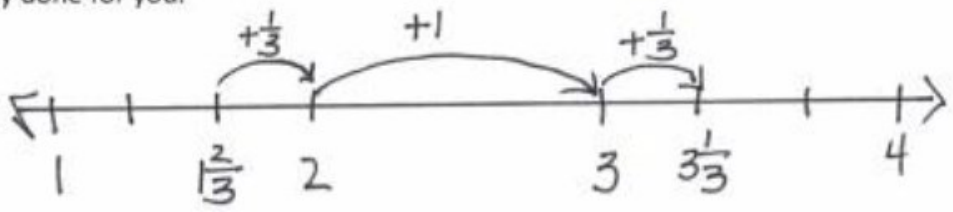
Lesson 33

Objective: Subtract a mixed number from a mixed number.

Write a related addition sentence. Subtract by counting on. Use a number line or the arrow way to help. The first one has been partially done for you.

a. $3\frac{1}{3} - 1\frac{2}{3} = 1\frac{2}{3}$

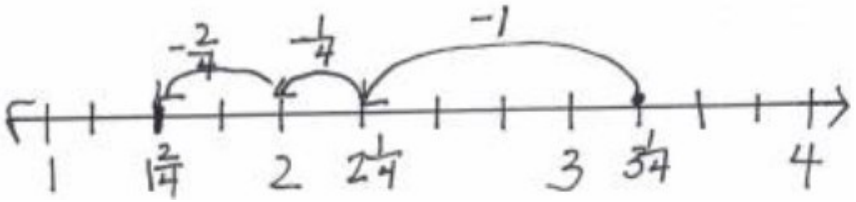
$1\frac{2}{3} + \_\_\_\_ = 3\frac{1}{3}$



Subtract, as shown in 2(a), by decomposing the fractional part of the number you are subtracting. Use a number line or the arrow way to help you.

a. $3\frac{1}{4} - 1\frac{3}{4} = 2\frac{1}{4} - \frac{3}{4} = 1\frac{2}{4}$

$\frac{1}{4}$ $\frac{2}{4}$



Lesson 34

Objective: Subtract mixed numbers.

Subtract.

a. $4\frac{1}{3} - \frac{2}{3} = 3\frac{4}{3} - \frac{2}{3} = 3\frac{2}{3}$

3 $\frac{4}{3}$

b. $5\frac{2}{4} - \frac{3}{4} = 4\frac{6}{4} - \frac{3}{4} = 4\frac{3}{4}$

4 $\frac{6}{4}$

Subtract the ones first.

a. $3\frac{1}{4} - 1\frac{3}{4} = 2\frac{1}{4} - \frac{3}{4} = 1\frac{2}{4}$

1 $\frac{5}{4}$

b. $4\frac{2}{5} - 1\frac{3}{5} = 3\frac{2}{5} - \frac{3}{5} = 2\frac{4}{5}$

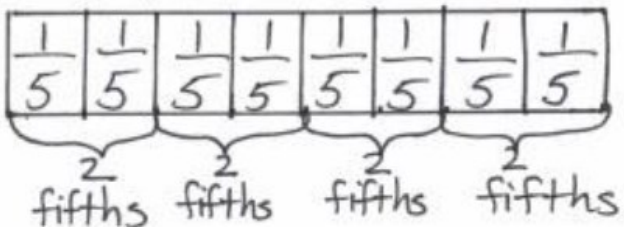
2 $\frac{7}{5}$

Lesson 35

Objective: Represent the multiplication of *n* times *a/b* as *(n x a)/b* using the associative property and visual models.

Draw and label a tape diagram to show the following are true.

a. 8 fifths = 4 × (2 fifths) = (4 × 2) fifths



Write the expression in unit form to solve.

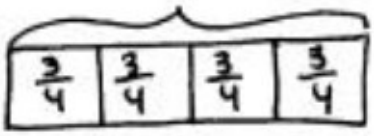
a. $7 \times \frac{2}{3} = \frac{14}{3}$

7 x 2 thirds = 14 thirds

Lesson 36

Objective: Represent the multiplication of *n* times *a/b* as *(n x a)/b* using the associative property and visual models.

Draw a tape diagram to represent $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$?



Write a multiplication expression equal to $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$.

$4 \times \frac{3}{4}$

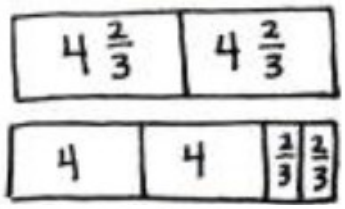
Rewrite each repeated addition problem as a multiplication problem and solve. Express the result as a mixed number. The first one has been started for you.

a. $\frac{7}{5}+\frac{7}{5}+\frac{7}{5}+\frac{7}{5}=4\times\frac{7}{5}=\frac{4\times7}{5}=\frac{28}{5}=5\frac{3}{5}$

Lesson 37

Objective: Find the product of a whole number and a mixed number using the distributive property.

Draw tape diagrams to show two ways to represent 2 units of $4\frac{2}{3}$.



Solve the following using the distributive property.

b. $2 \times 4\frac{2}{3} = 2(4+\frac{2}{3})$

$= (2\times4) + (2\times\frac{2}{3})$

$= 8 + \frac{4}{3}$

$= 8 + 1\frac{1}{3}$

$= 9\frac{1}{3}$

Lesson 38

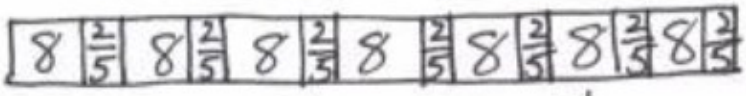
Objective: Find the product of a whole number and a mixed number using the distributive property.

Fill in the unknown factors.

a. $7 \times 3\frac{4}{5} = (\underline{7} \times 3) + (\underline{7} \times \frac{4}{5})$

Multiply. Use the distributive property.

a. $7 \times 8\frac{2}{5}$



$7 \times 8\frac{2}{5} = 56 + \frac{14}{5}$

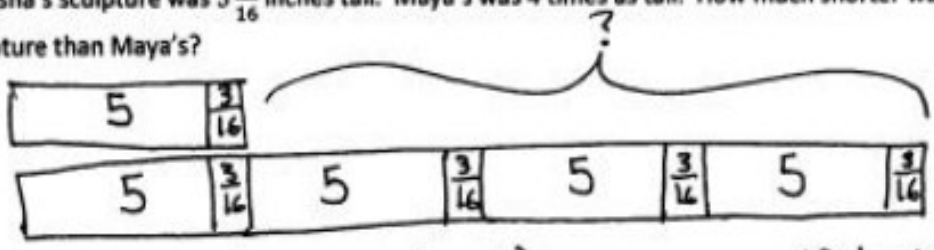
$= 56 + 2\frac{4}{5}$

$= 58\frac{4}{5}$

Lesson 39

Objective: Solve multiplicative comparison word problems involving fractions.

Natasha's sculpture was $5\frac{3}{16}$ inches tall. Maya's was 4 times as tall. How much shorter was Natasha's sculpture than Maya's?



$3 \times 5\frac{3}{16} = 3 \times (5 + \frac{3}{16})$

$= (3 \times 5) + (3 \times \frac{3}{16})$

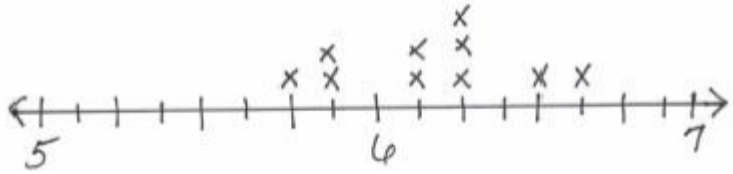
$= 15 + \frac{9}{16} = 15\frac{9}{16}$

Natasha's sculpture was $15\frac{9}{16}$ inches shorter than Maya's.

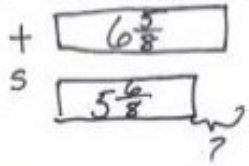
Lesson 40

Objective: Solve word problems involving the multiplication of a whole number and a fraction including those involving line plots.

The chart to the right shows the height of some football players. Use the data to create a line plot at the bottom of this page and to answer the questions below.



a. What is the difference in height of the tallest and shortest player?



$6\frac{5}{8} - 5\frac{6}{8} = 1\frac{5}{8} - \frac{6}{8} = \frac{13}{8} - \frac{6}{8} = \frac{7}{8}$

The difference in height between the tallest and the shortest player is $\frac{7}{8}$ ft.

Player	Height (in feet)
A	$6\frac{1}{4}$
B	$5\frac{7}{8}$
C	$6\frac{1}{2}$
D	$6\frac{1}{4}$
E	$6\frac{2}{8}$
F	$5\frac{7}{8}$
G	$6\frac{1}{8}$
H	$6\frac{5}{8}$
I	$5\frac{6}{8}$
J	$6\frac{1}{8}$

Lesson 41

Objective: Find and use a pattern to calculate the sum of all fractional parts between 0 and 1. Share and critique peer strategies.