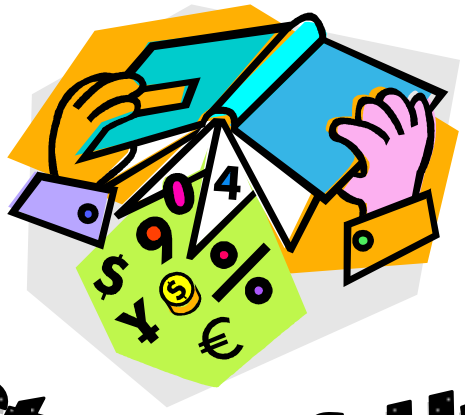
Mathematics:



"A Story of Units"

Parent Handbook

Grade 4
Module 4

Grade 4 • Module 4

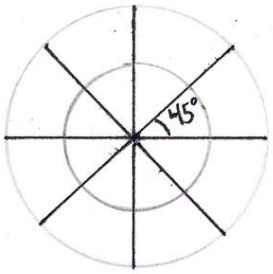
Angle Measure and Plane Figures

OVERVIEW

This module introduces points, lines, line segments, rays, and angles, as well as the relationships between them. Students construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. With angle measure playing a key role in their work throughout the module, students learn how to create and measure angles, as well as create and solve equations to find unknown angle measures. In these problems, where the unknown angle is represented by a letter, students explore both measuring the unknown angle with a protractor and reasoning through the solving of an equation. This connection between the measurement tool and the numerical work lays an important foundation for success with middle school geometry and algebra. Through decomposition and composition activities as well as an exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures based on them.

Topic A begins with students drawing points, lines, line segments, and rays and identifying these in various contexts and within familiar figures. Students recognize that two rays sharing a common endpoint form an angle. They create right angles through a paper folding activity, identify right angles in their environment, and see that one angle can be greater (obtuse) or less (acute) than a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines as they define, draw, and recognize intersecting, perpendicular, and parallel lines.

In Topic B, students explore the definition of degree measure, beginning with a circular protractor. By dividing the circumference of a circle into 360 equal parts, they recognize one part as representing 1 degree. Through exploration, students realize that although the size of a circle may change, an angle spans an arc representing a constant fraction of the circumference. By carefully distinguishing the attribute of degree measure from that of length measure, the common misconception that degrees are a measure of length is avoided. Armed with their understanding of the degree as a unit of measure, students use various protractors to measure angles to the nearest degree and sketch angles of a given measure. The idea that an angle measures the amount of "turning" in a particular direction is explored as students recognize familiar angles in varied contexts.



Topic C begins by decomposing 360 degrees using pattern blocks, allowing students to see that a group of angles meeting at a point with no spaces or overlaps add up to 360 degrees. With this new understanding, students now discover that the combined measure of two adjacent angles on a line is 180 degrees (supplementary angles), that the combined measure of two angles meeting to form a right angle is 90 degrees (complementary angles), and that vertically opposite angles have the same measure. These properties are then used to solve unknown angle problems.

An introduction to symmetry opens Topic D as students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry. Given one half of a line-symmetric figure and the line of symmetry, students draw the other half of the figure. This leads to their work with triangles. Students are introduced to the precise definition of a triangle and then classify triangles based on angle measure and side length. For isosceles triangles, a line of symmetry is identified, and a folding activity demonstrates that base angles are equal. Folding an

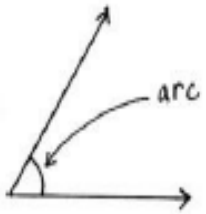
equilateral triangle highlights multiple lines of symmetry and establishes that all interior angles are equal. Students construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles).

Finally, students explore the definitions of familiar quadrilaterals and classify them based on their attributes, including angle measure and parallel and perpendicular lines. This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. The topic concludes with students using pattern blocks to compose and decompose compound figures based on a given set of classifying criteria.

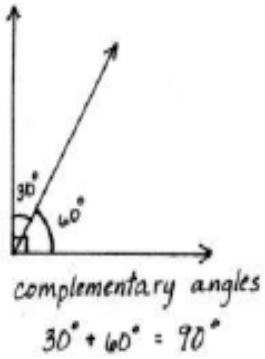
**The sample questions/responses contained in this manual are straight from http://www.engageny.org/. They are provided to give some insight into the kinds of skills expected of students as the lesson is taught.

Terminology

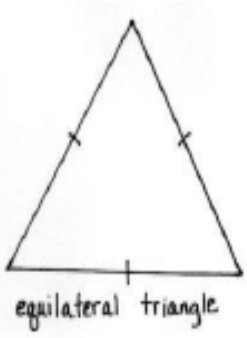
New or Recently Introduced Terms



- Acute angle (angle with a measure of less than 90 degrees)
- Acute triangle (triangle with all interior angles measuring less than 90 degrees)
- Adjacent angle (Two angles $\angle AOC$ and $\angle COB$, with a common side $\overrightarrow{OC}$, are *adjacent angles* if C is in the interior of $\angle AOB$.)
- Angle (union of two different rays sharing a common vertex)
- Arc (connected portion of a circle)
- Collinear (Three or more points are *collinear* if there is a line containing all of the points; otherwise, the points are *non-collinear*.)
- Complementary angles (two angles with a sum of 90 degrees)
- Degree measure of an angle (Subdivide the length around a circle into 360 arcs of equal length. A central angle for any of these arcs is called a *one-degree angle* and is said to have angle measure 1 degree.)



- Diagonal (A Line segment whose endpoints are two non-adjacent vertices of a straight sided shape– ie. Straight lines joining two opposite corners of a straight-sided shape.)
- Equilateral triangle (triangle with three equal sides)
- Figure (set of points in the plane)
- Interior of an angle (the convex[1] region defined by the angle)
- Intersecting lines (lines that contain at least one point in common)
- Isosceles triangle (triangle with at least two equal sides)
- Length of an arc (circular distance around the arc.)
- Line (straight path with no thickness that extends in both directions without end)
- Line of symmetry (line through a figure such that when the figure is folded along the line two halves are created that match up exactly)

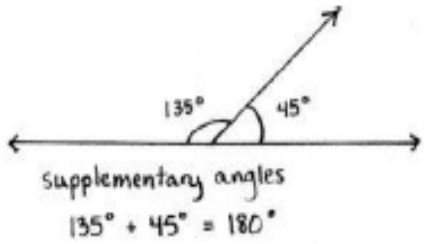


- Line segment (two points, A, B, together with the set of points on the line $\overleftrightarrow{AB}$ between A and B)
- Obtuse angle (angle with a measure greater than 90 degrees but less than 180 degrees)
- Obtuse triangle (triangle with an interior obtuse angle)
- Parallel (two lines in a plane that do not intersect)
- Perpendicular (Two lines are *perpendicular* if they intersect, and any of the angles formed between the lines is a 90° angle.)

- Point (precise location in the plane)
- Protractor (instrument used in measuring or sketching angles)

[1] In Grade 4, a picture will suffice. A precise definition of convexity will be given in Grade 10 geometry.

- Ray (The *ray* $\overrightarrow{OA}$ is the point O and the set of all points on the line $\overleftrightarrow{OA}$ that are on the same side of O as the point A.)
- Right angle (angle formed by perpendicular lines, measuring 90 degrees)
- Right triangle (triangle that contains one 90° degree angle)
- Scalene triangle (triangle with no sides or angles equal)
- Straight angle (angle that measures 180 degrees)
- Supplementary angles (two angles with a sum of 180 degrees)
- Triangle (A *triangle* consists of three non-collinear points and the three line segments between them. The three segments are called the *sides* of the triangle and the three points are called the *vertices*.)



- Vertex (a point, often used to refer to the point where two lines meet, such as in an angle or the corner of a triangle)
- Vertical angles (When two lines intersect, any two non-adjacent angles formed by those lines are called *vertical angles* or *vertically opposite angles*.)

Familiar Terms and Symbols

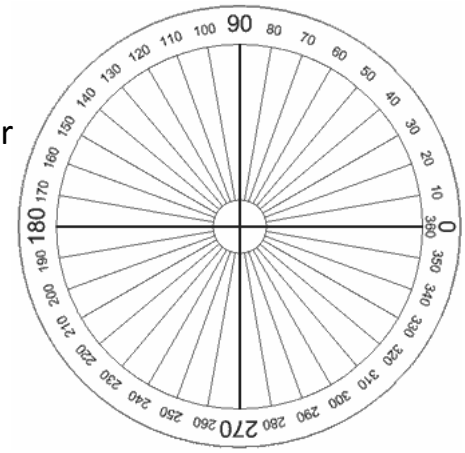
- Decompose (process of separating something into smaller components)
- Parallelogram (quadrilateral with two pairs of parallel sides)
- Polygon (closed two-dimensional figure with straight sides)
- Quadrilateral (polygon with four sides)
- Rectangle (quadrilateral with four right angles)
- Rhombus (quadrilateral with all sides of equal length)
- Square (rectangle with all sides of equal length)
- Sum (result of adding two or more numbers)
- Trapezoid (quadrilateral with exactly one pair of parallel sides)

Suggested Tools and Representations

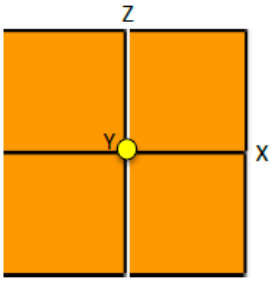
- Protractor, various diameters including a 360° and 180° protractor.
- Ruler, straightedge
- Set square
- Folded paper models
- Pattern blocks
- Rectangular and triangular grid paper

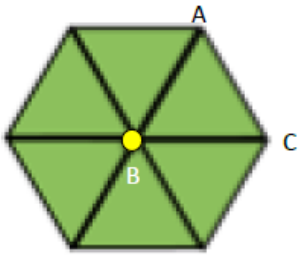
Protractors: Two types of protractors (360° and 180°) are used to measure angles.

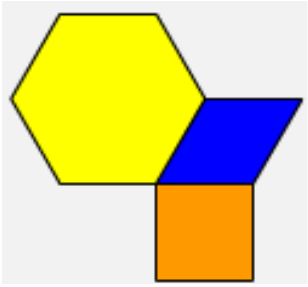
360° protractor



Pattern Blocks: These are sets of color-coded shapes that may be used for sorting or shape-recognition in the lower grades, but may be used to manipulate fractions, when compared to a whole amount/shape, or to manipulate angles that may either be composed to form a larger angle or decomposed to form smaller angles.



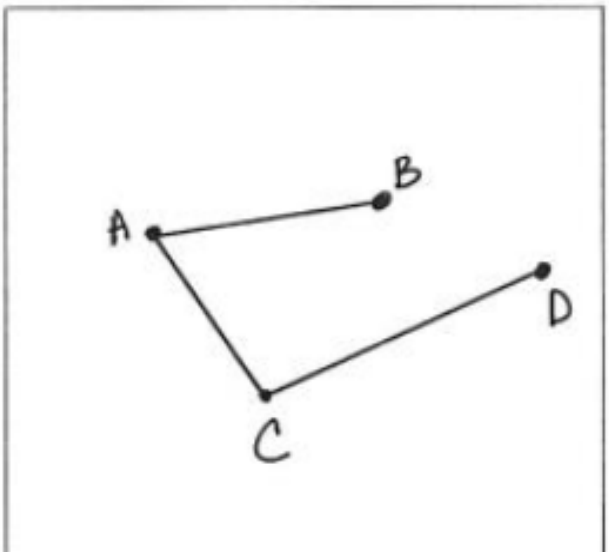




Lesson 1

Objective: Identify and draw points, lines, line segments, rays, and angles and recognize them in various contexts and familiar figures.

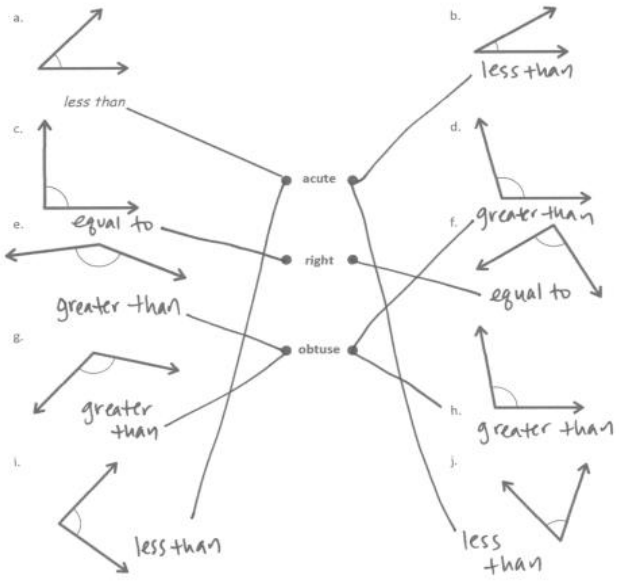
1. Use the following directions to draw a figure in the box to the right.
   a. Draw two points, A and B.
   b. Use a straightedge to draw $\overline{AB}$.
   c. Draw a new point that is not on $\overline{AB}$. Label it C.
   d. Draw segment $\overline{AC}$.
   e. Draw a point not on $\overline{AB}$ or $\overline{AC}$. Call it D.
   f. Construct line, $\overleftrightarrow{CD}$.
   g. Use the points you've already labeled to name one angle. $\angle BAC$



Lesson 2

Objective: Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles.

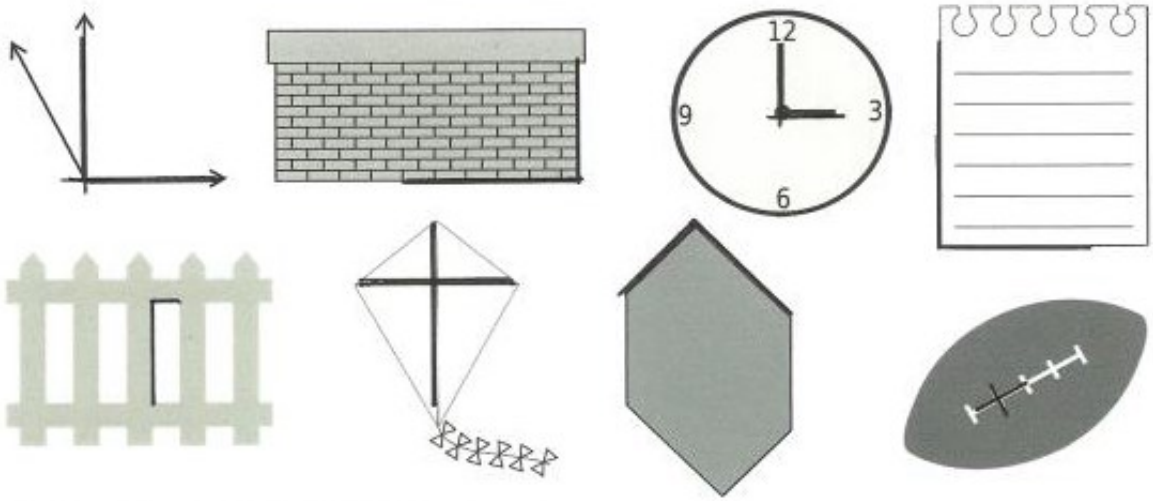
1. Use the right angle template that you made in class to determine if each of the following angles is greater than, less than, or equal to a right angle. Label each as *greater than*, *less than*, or *equal to*, and then connect each angle to the correct label of acute, right, or obtuse.
   The first one has been completed for you.



Lesson 3

Objective: Identify, define, and draw perpendicular lines.

1. On each object, trace at least one pair of lines that appear to be perpendicular.



2. How do you know if two lines are perpendicular?

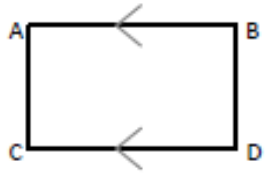
When two lines are perpendicular they make a right angle.

Lesson 4

Objective: Identify, define, and draw parallel lines.

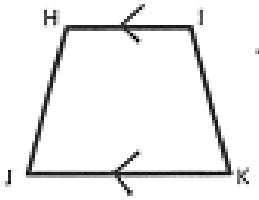
4. Determine which of the following figures have lines that are parallel by using a straightedge and the right angle template that you created. Circle the letter of the shapes that have at least one pair of parallel lines. Mark each pair of parallel lines with arrows and then identify the parallel lines with a statement modeled after the one in 4(a).

a.



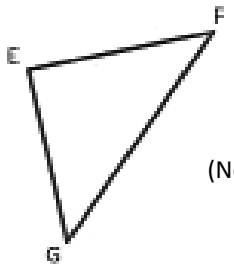
$\overline{AB} \parallel \overline{CD}$

b.



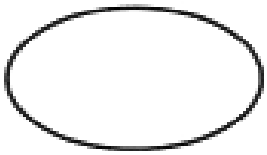
$\overline{HI} \parallel \overline{JK}$

c.



(No parallel lines)

d.



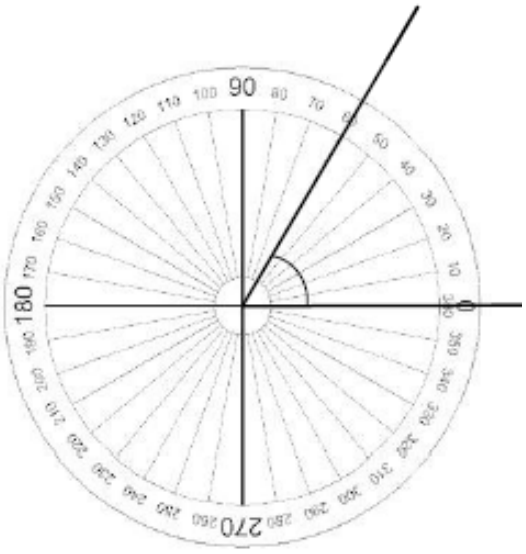
(No parallel lines)

Lesson 5

Objective: Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor.

1. Identify the measures of the following angles.

a.



This angle measures 60°.

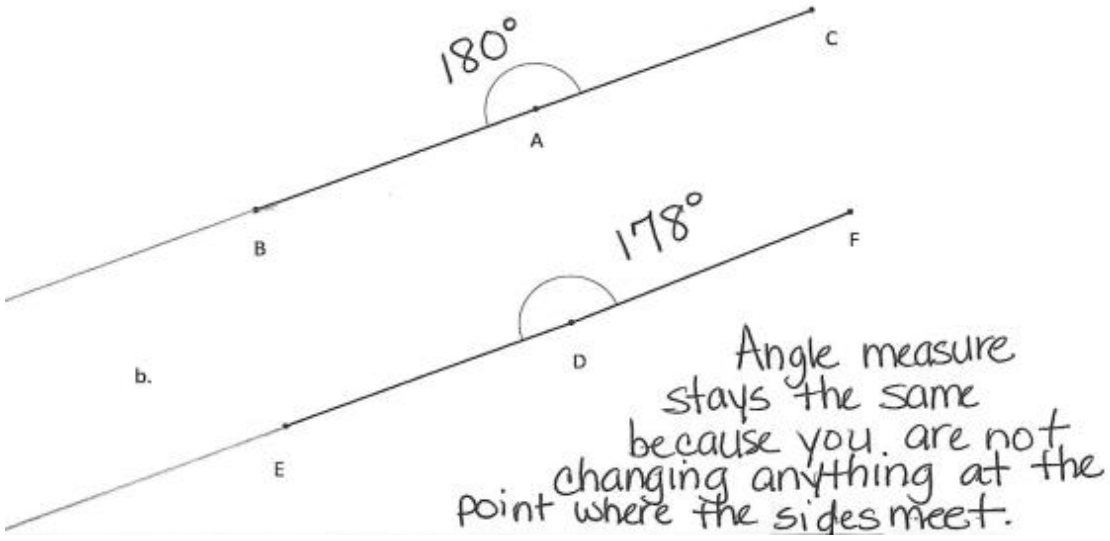
(This is a circular protractor.)

Lesson 6

Objective: Use varied protractors to distinguish angle measure from length measurement.

3. Use a protractor to measure each angle. Extend the length of the lines if you need to. When you extend the lines, does the angle measure stay the same? Explain how you know.

a.

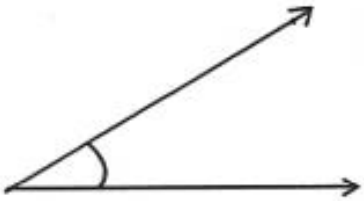


Lesson 7

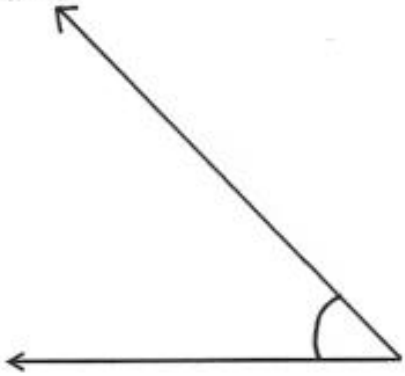
Objective: Measure and draw angles. Sketch given angle measures and verify with a protractor.

1. Construct angles that measure the given number of degrees. For a-d, use the ray shown as one of the rays of the angle with its endpoint as the vertex of the angle. Draw an arc to indicate the angle that was measured.

a. 30°



b. 45°



Lesson 8

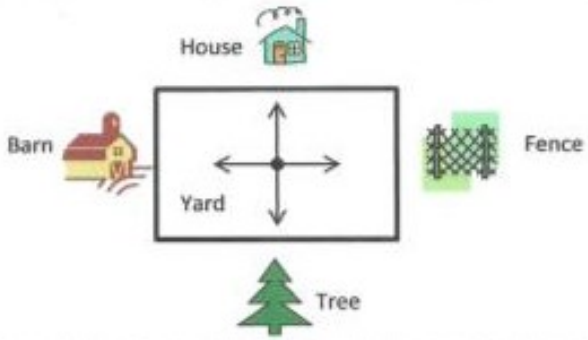
Objective: Identify and measure angles as turns and recognize them in various contexts.

1. Joe, Steve, and Bob stood in the middle of the yard and faced the house. Joe turned 90° to the right. Steve turned 180° to the right. Bob turned 270° to the right. To what was each boy now facing?

Joe fence

Steve tree

Bob barn



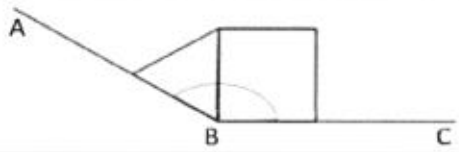
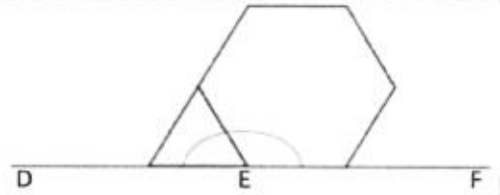
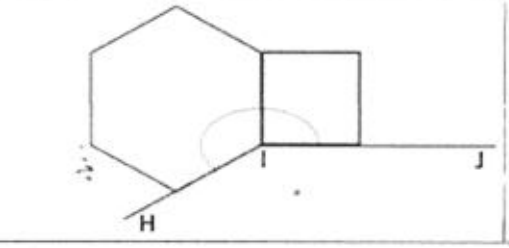
2. Monique looked at the clock at the beginning of class and at the end of class. How many degrees did the minute hand turn from the beginning of class until the end?

The minute hand moved 270 degrees.

Lesson 9

Objective: Decompose angles using pattern blocks.

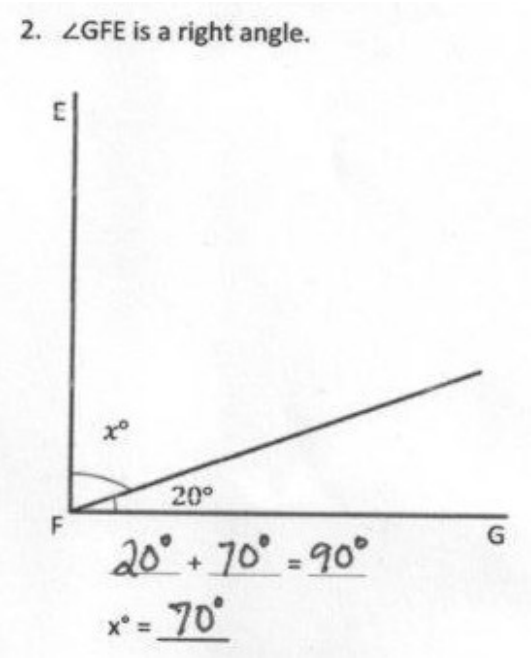
2. Find the measurements of the angles indicated by the arcs.

Pattern Blocks	Angle Measure	Addition Sentence
	150°	60° + 90° = 150°
	180°	60° + 120° = 180°
	210°	120° + 90° = 210°

Lesson 10

Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

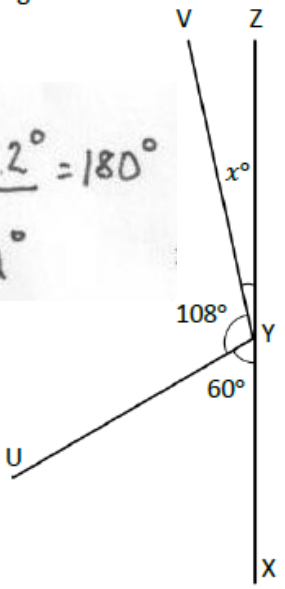
2. ∠GFE is a right angle.



20° + 70° = 90°

x° = 70°

6. Solve for the measurement of ∠ZYV.
∠XYZ is a straight angle.



108° + 60° + 12° = 180°

∠ZYV = 12°

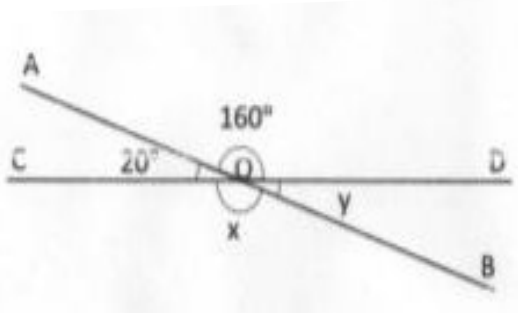
Lesson 11

Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

Write an equation and solve for the unknown angles numerically.

5. O is the intersection of $\overline{AB}$ and $\overline{CD}$.
   $\angle DOA$ is 160° and $\angle AOC$ is 20°

$x° = 160°$ $y° = 20°$



$\angle x + 20° = 180°$ $\angle y = 180° - 160°$

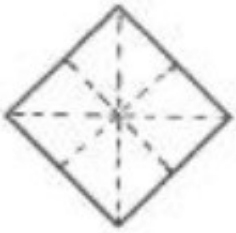
$\angle x = 160°$ $\angle y = 20°$

Lesson 12

Objective: Recognize lines of symmetry for given two-dimensional figures; identify line-symmetric figures and draw lines of symmetry.

2. Find and draw all lines of symmetry for the following figures. Write the number of lines of symmetry that you found in the blank underneath the shape.

a. 1



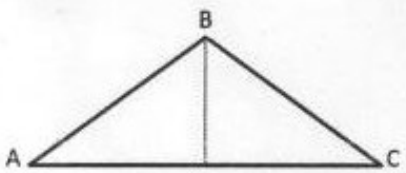
b. 4

c. 0

Lesson 13

Objective: Analyze and classify triangles based on side length, angle measure, or both.

2. ΔABC has one line of symmetry as shown. What does this tell you about the measures of $\angle A$ and $\angle C$?

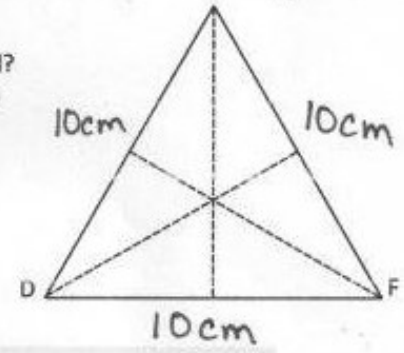


They must be the same! The sides of symmetrical shapes have to match when you fold it! That is what the line of symmetry means. The angles match.

3. ΔDEF has three lines of symmetry as shown.

a) How can the lines of symmetry help you figure out which angles are equal?

No matter which line you fold it on, the opposite angles would be the same, so they all have to be the same.



b) ΔDEF has a perimeter of 30 cm. Label the side lengths.

30cm ÷ 3 = 10cm

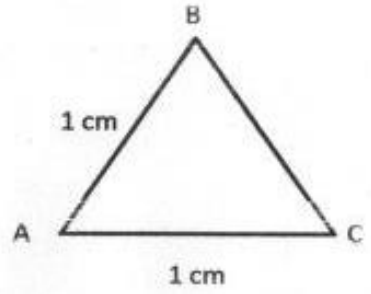
Lesson 14

Objective: Define and construct triangles from given criteria. Explore symmetry in triangles.

Are the following statements true or false? Explain using pictures or words.

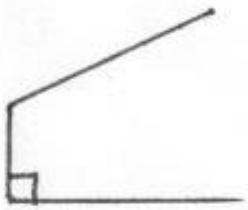
3. If ΔABC is an equilateral triangle, $\overline{BC}$ must be 2 cm. True or False?

False. "Equilateral" means all the sides are the same, so $\overline{BC}$ has to be 1cm.



4. A triangle cannot have one obtuse angle and one right angle. True or False?

True! If you have a right angle and an obtuse angle, there is no way the sides could connect to make the third corner.

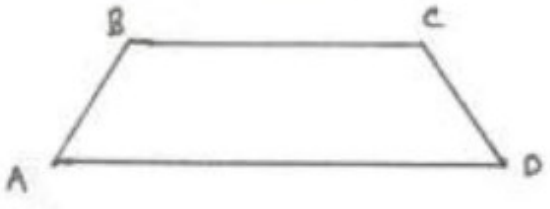


Lesson 15

Objective: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size.

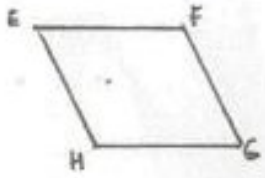
Construct the figures with given attributes. Name the shape you created. Be as specific as possible. Use extra blank paper as needed.

1. Construct quadrilaterals with at least one set of parallel sides.



ABCD is a trapezoid. It has 4 sides and 1 set of parallel lines.

2. Construct a quadrilateral with two sets of parallel sides.



EFGH is a parallelogram and a trapezoid, too. It has 4 sides and 2 sets of parallel lines.

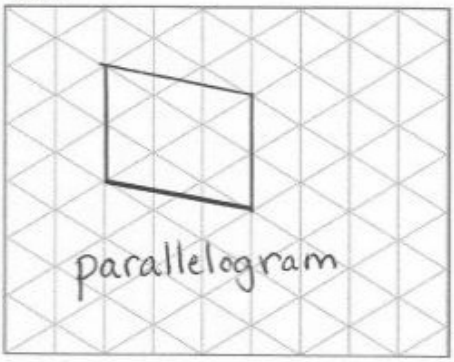
Lesson 16

Objective: Reason about attributes to construct quadrilaterals on square or triangular grid paper.

2. On the grid paper, draw at least one quadrilateral to fit the description. Use the given segment as one segment of the quadrilateral. Name the figure you drew using one of the terms below.

Parallelogram	trapezoid	rectangle
Square		rhombus

a. A quadrilateral that has two sets of parallel sides.



b. A quadrilateral that has four right angles.