# Mathematics



Grade 3 Module 7

# Grade 3 • Module 7

# **Geometry and Measurement Word Problems**

#### **OVERVIEW**

The final module of the year offers students intensive practice with word problems, as well as hands-on investigation experiences with geometry and perimeter.

Topic A begins with solving one- and two-step word problems based on a variety of topics studied throughout the year, using all four operations. The lessons emphasize modeling and reasoning to develop solution paths. They incorporate teacher facilitated problem solving, opportunities for students to independently make sense of problems and persevere in solving them, and time for students to share solutions and critique peer strategies.

Topic B introduces an exploration of geometry. Students build on Grade 2 ideas about polygons and their properties, specifically developing and expanding their knowledge of quadrilaterals. They explore

the attributes of quadrilaterals and classify examples into various categories, including recognizing the characteristics of polygons. Students draw polygons based on their attributes, producing sketches from descriptions like, "This shape has two long sides that are parallel, two short sides, and no right angles."



Students next use tangrams and tetrominoes (see examples to the right) to compose and decompose shapes. They reason about the relationships between shapes and between attributes. For example, students understand



that quadrilaterals can be decomposed into triangles, and recognize that the two smallest triangles in a tangram puzzle can be put together to form a parallelogram, a square, and a medium triangle.

Students tessellate to bridge geometry experience with the study of perimeter in Topic C. They first decompose a quadrilateral and then rearrange the parts. They use the new shape to tile. Students then define perimeter in two distinct ways: (1) as the boundary of a planar region and (2) as the length of the boundary curve. Students see varied examples from the tiles used to tessellate.



Cut on the line. Then slide the piece to the opposite side or rotate it to an adjacent side to make a new shape. As they learn about perimeter as an attribute of plane figures, students apply their knowledge to real world situations through problem solving. They measure side lengths of shapes in whole number units to determine perimeter and solve problems where side lengths are given. They use string and rulers to measure the length around circles of different sizes. This variation prompts students to think more flexibly about perimeter, and to understand that it can be the boundary of any shape and that its measurements are not limited to whole numbers. The topic ends with problems in which some measurements around the perimeter of a polygon are missing but can be determined by reasoning. Students consider the efficiency of their strategies and identify tools for solving; for example, they use multiplication as a tool when measurements are repeated.

Topic D utilizes the line plot, familiar from Module 6, to help students draw conclusions about perimeter and area measurements. Early in the topic, students find different possible perimeters or areas for rectangles based on information given about the rectangles. For example, using knowledge of factors from experience with multiplication, students determine the following:

- Different perimeters of rectangles comprised of a given number of unit squares.
- For example, given a rectangle composed of 24 unit squares, students find four possible perimeters: 50, 28, 22, and 20 length units.
- Different areas of rectangles comprised of unit squares with a given perimeter.
- For example, students use unit squares to build rectangles with a perimeter of 12 units and determine that they can do so using 5, 8, or 9 unit squares.

(Rectangles are formed with unit squares, and as a result they have whole number side lengths.)

Students then draw their rectangles on grid paper and reason about their findings, noticing, for example, that for rectangles of a given area, those with side lengths that are equal or almost equal (more square-like) have smaller perimeters than those whose side lengths are very different (a long and narrow shape). They use line plots to show the number of rectangles they were able to construct for each set of given information. The line plots are a tool that students use to help them reason and draw conclusions about their data.

As they move through the lessons in this topic, students notice and compare differences in the strategies for finding area when given a perimeter and for finding perimeter given an area. By the end of the topic they are able to conclude that there is no direct relationship between area and perimeter, meaning that if an area is given there is no way of knowing a shape's corresponding perimeter.

In Topic E, students solve problems involving area and perimeter. After an initial lesson problem solving with perimeter, students apply this knowledge to create a robot composed of rectangles. Given specific perimeter measurements, they reason about the different side lengths that may be produced. Students compare and analyze their work, discussing how different choices for side lengths can affect area while conforming to the criteria for perimeter. Students synthesize their learning in the final lessons through solving word problems involving area and perimeter using all four operations.

Topic F concludes the school year with a set of engaging lessons that briefly review the fundamental Grade 3 concepts of fractions, multiplication, and division. This topic comes after the End-of-Module Assessment. It begins with a pair of lessons on fractions, engaging students in analyzing and creating unusual representations of one-half such as those shown to the right. Students analyze and discuss these representations, using their knowledge of fractions to justify their constructions and critique the work of others to make adjustments as necessary. The final lessons in this topic are fluency based and engage students in games that provide practice to solidify their automaticity with Grade 3 skills. Using simple origami techniques they create booklets of

## **Terminology**

#### **New or Recently Introduced Terms**

- Attribute (any characteristic of a shape, including properties and other defining characteristics, e.g., straight sides, and non-defining characteristics, e.g., blue)
- Diagonal (e.g., the line drawn between opposite corners of a quadrilateral)

these games. The booklets go home and become resources for summer practice.

- Perimeter (boundary or length of the boundary of a two-dimensional shape)
- Property (e.g., having all sides equal in length)
- Regular polygon (polygon whose side lengths and interior angles are all equal)
- Tessellate (to tile a plane without gaps or overlaps)
- Tetrominoes (four squares arranged to form a shape so that every square shares at least one side with another square)

#### **Familiar Terms and Symbols**

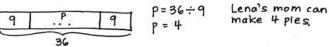
- Area (the measurement of two-dimensional space in a bounded region)
- Compose (put two or more objects or numbers together)
- Decompose (break an object or number into smaller parts)
- Heptagon (flat figure enclosed by seven straight sides and seven angles)

Hexagon (flat figure enclosed by six straight sides and six angles) Octagon (flat figure enclosed by eight straight sides and eight angles) Parallel (lines that do not intersect, even when extended in both directions) Parallelogram (a quadrilateral with both pairs of opposite sides parallel) Pentagon (flat figure enclosed by five straight sides and five angles) Polygon (a closed figure with three or more straight sides, e.g., triangle, quadrilateral, pentagon, hexagon) Quadrilaterals (a four-sided polygon, e.g., square, rhombus, rectangle, parallelogram, trapezoid) Rectangle (flat figure enclosed by four straight sides, having four right angles) Rhombus (flat figure enclosed by four straight sides of the same length) Right angle (e.g., a square corner) Square (rectangle with four sides of the same length) Tangram (special set of puzzle pieces with five triangles and two quadrilaterals that compose a square) Trapezoid (quadrilateral with at least one pair of parallel sides) Triangle (flat figure enclosed by three straight sides and three angles)

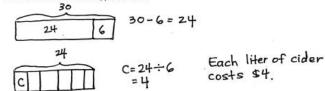
Objective: Solve word problems in varied contexts using a letter to represent the unknown.

Lena picked 17 apples and her brother picked 19. Lena's mom has a pie recipe that requires 9 apples. How many pies can Mom make with the apples that Lena and her brother picked?

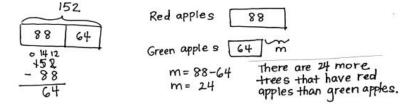




Lena's dad gives the cashier \$30 to pay for 6 liters of apple cider. The cashier gives him \$6 in change. How much does each liter of apple cider cost?



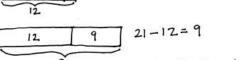
The apple orchard has 152 apple trees. There are 88 trees with red apples. The rest of the trees have green apples. How many more trees have red apples than green apples?



## Lesson 2

Objective: Solve word problems in varied contexts using a letter to represent the unknown.

Kami scored a total of 21 points during her basketball game. She made 6 two-point shots and the rest were three-point shots. How many three-point shots did Kami make?

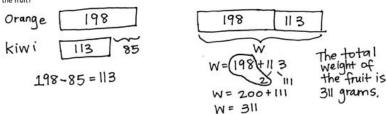


6 x 2 = 12

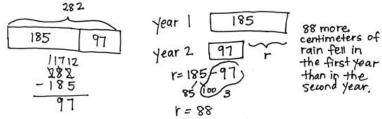
Kami made 3 three-point shots.

$$k=9\div3$$
 $k=3$ 

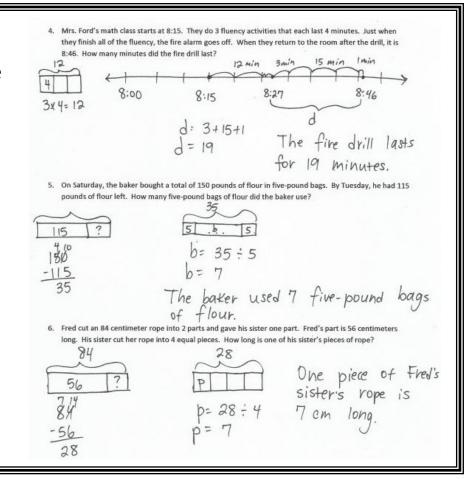
An orange weighs 198 grams. A kiwi weighs 85 grams less than the orange. What is the total weight of the fruit?



The total amount of rain that fell in New York City in two years was 282 centimeters. In the first year, 185 centimeters of rain fell. How many more centimeters of rain fell in the first year than in the second year?



Objective: Share and critique peer solution strategies to varied word problems.



## Lesson 4

Objective: Compare and classify quadrilaterals.

Closed shapes like these that have no gaps or overlaps between the straight sides are called **polygons**. Polygons with four straight sides are called **quadrilaterals**. Most quadrilaterals are made up of two triangles.

**Trapezoids** are quadrilaterals that have at least one set of **parallel** sides. Think of parallel sides like the two side lines of a capital *H*, or a slanted *H*, since not all parallel sides stand vertical.

Parallelograms these are four-sided polygons that have two sets of parallel sides.

**Squares** have four equal sides and four right angles.

Objective: Compare and classify other

They both have 8 sides, so they are both octagons. But Polygon M has all equal sides and Polygon X has sides that are equal.

polygons.

enny says. "Polygon N, Polygon R, and Polygon S are all regular quadrilaterals!" is she correct? Why or

No, Jenny is not correct. A regular quadrilateral has 4 equal sides and 4 equal angles. Only Polygon S is a regular quadrilateral.

"I have six equal sides and six equal angles. I have three sets of parallel lines. I have no right angles."

a. Write the letter and the name of the polygon described above.

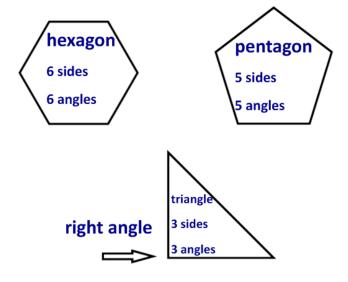
Polygon U is a regular hexagon.

b. Estimate to draw the same polygon, but with no equal sides.



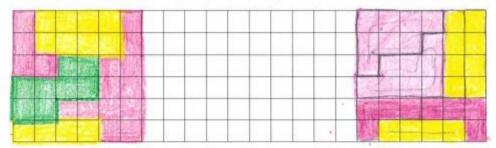
## Lesson 6

Objective: Draw polygons with specified attributes to solve problems.



Objective: Reason about composing and decomposing polygons using tetrominoes.

Use tetrominoes to create at least 2 squares each with an area of 36 square units. Then color the grid below to show how you created your squares. You may use the same tetromino more than once.

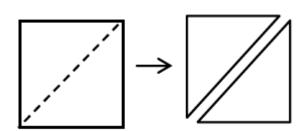


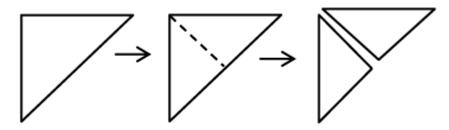
a. Write a number sentence to show the area of a square above as the sum of the areas of the tetrominoes you used to make the square.

b. Write a number sentence to show the area of a square above as the product of its side lengths.

## Lesson 8

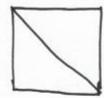
Objective: Create a tangram puzzle and observe relationships among the shapes.

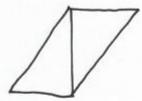


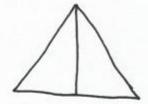


Objective:
Reason about
composing and
decomposing
polygons using
tangrams.

Use your two smallest triangles to create a square, a parallelogram, and a triangle. Show how you created them below.







Create your own shape on a separate sheet of paper using all seven pieces. Describe its attributes below. My Shape has 6 sides, which means it's a hexagon. It's not a regular hexagon because the sides aren't all equal. My Shape has I pair of parallel lines. My Shape doesn't have any right angles.

## Lesson 10

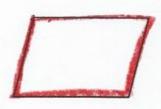
Objective:

Decompose quadrilaterals to understand perimeter as the boundary of a shape.

Outline the perimeter of the shapes below with a red crayon.



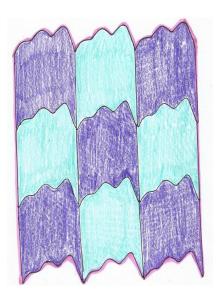




a. Explain how you know you outlined the perimeters of the shapes above.

I know I outlined the perimeters of the shapes because I colored the boundary of each shape and the boundary of a shape is the perimeter.

Objective: Tessellate to understand perimeter as the boundary of a shape.



- 1. Follow the directions below, using the shape you created yesterday.
- √a. Tessellate your shape on a blank piece of paper.
- √b. Color your tessellation to create a pattern.
- √c. Outline the perimeter of your tessellation with a highlighter.
- √d. Use a string to measure the perimeter of your tessellation.
- 2. Compare the perimeter of your tessellation to a partner's. Whose tessellation has a greater perimeter? How do you know?

My tessellation has a greater perimeter. I know because my partner and I compared our strings. The mark on my string that represents the perimeter was further down on the string than my partner's.

3. How could you increase the perimeter of your tessellation?

I could increase the perimeter of my tessellation by tessellating more shapes. If I tessellated another now of Shapes, that would increase the perimeter.

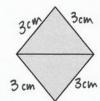
4. How would overlapping your shape when you tessellated change the perimeter of your tessellation?

If I had overlaps, the shapes wouldn't fit together. If I had overlaps with the Same number of tessellated shapes, the perimeter would decrease.

## Lesson 12

Objective: Measure side lengths in whole number units to determine the perimeter of polygons.

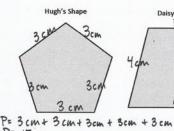
Carson draws 2 triangles to create the new shape shown below. Use a ruler to find the side lengths of Carson's shape in centimeters. Then find the perimeter.



P= 3cm+ 3cm + 3cm + 3cm = 12 cm

The perimeter of Carson's shape is 12 cm.

Hugh and Daisy draw the shapes shown below. Measure and label the side lengths in centimeters. Whose shape has a greater perimeter? How do you know?



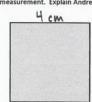
Daisy's Shape
3 cm
4 cm
4 cm

P= 3cm + 4cm + 4cm + 5cm = 16 cm

Daisy's shape has a greater perimeter. I measured the side lengths and added to find each perimeter.

16 is greater than 15.

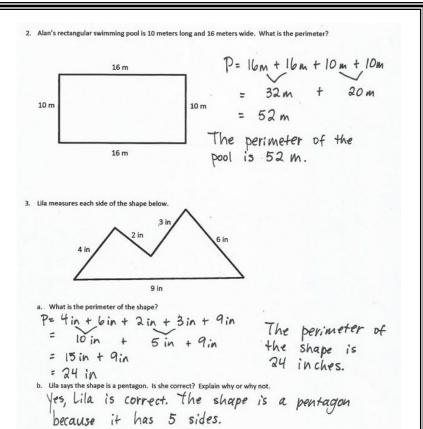
Andrea measures one side length of the square below and says she can find the perimeter with that
measurement. Explain Andrea's thinking. Then find the perimeter in centimeters.



Andrea can find the perimeter by measuring one side length of the square because squares have 4 equal sides. If she measures one side the rest of the sides are that length.

P= 4 cm + 4 cm + 4 cm + 4 cm is 16 cm.

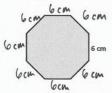
Objective: Explore perimeter as an attribute of plane figures and solve problems.



## Lesson 14

Objective: Determine the perimeter of regular polygons and rectangles when whole number measurements are missing.

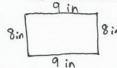
David draws a regular octagon and labels a side length as shown below. Find the perimeter of David's octagon.



8 x 6 = 48

The perimeter of David's octagon is 48 cm. I multiplied the number of sides times the side length.

4. Paige paints an 8 inch by 9 inch picture for her mom's birthday. What is the total length of wood that Paige needs to make a frame for the picture?



P= 9in + 9in + 8in + 8in = 18 in + 16 in Paige = 34 in of w

Mr. Spooner draws a regular hexagon on the board. One of the sides measures 4 centimeters. Giles and Xander find the perimeter. Their work is shown below. Whose work is correct? Explain your answer.

Giles' Work

Perimeter = 4 cm + 4 cm

Perimeter = 24 cm

Xander's Work

Perimeter = 6 × 4 cm

Perimeter = 24 cm

They are both right! A regular hexagon has 6 equal sides. Giles added 4 six times and Xander multiplied the number of sides by the length of each side. Both equations show the perimeter of the regular hexagon

Objective: Solve word problems to determine perimeter with given side lengths.

## Lesson 16

Objective: Use string to measure the perimeter of various circles to the nearest quarter inch.

2. Can you find the perimeter of the shape below using just your ruler? Explain your answer.

= 72 in



No I can't find the perimeter of this shape using just my ruler. The boundary of the shape has curved lines and I can't measure curved lines with just a ruler.

r= 72 in + 72 in +72 in

r = 144 in + 72 in

The troop

uses 216 inches of

ribbon.

Molly says the perimeter of the shape below is 6 <sup>1</sup>/<sub>4</sub> inches. Use your string to check her work. Do you agree with her? Why or why not?

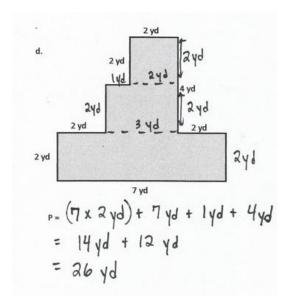


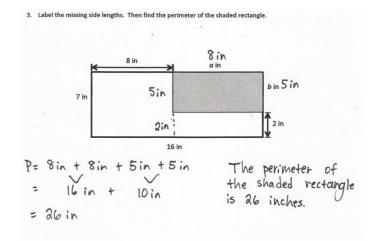
No, I don't agree with Molly. I used my string and ruler to find the perimeter and it's about 5 \( \frac{3}{4} \) inches.

4. Is the process you used to find the perimeter of a circular object an efficient method to find the perimeter of a rectangle? Why or why not?

No, I don't think this is an efficient method to find the perimeter of a rectangle. I can just use my ruler to measure the side lengths. Using the string adds an extra step and the answer is only an estimate.

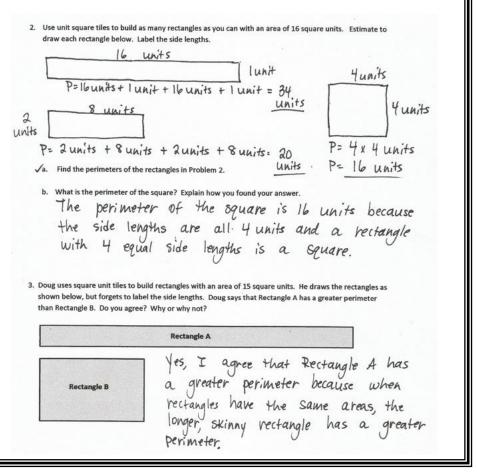
Objective: Use all four operations to solve problems involving perimeter and missing measurements.





## Lesson 18

Objective: Construct rectangles from a given number of unit squares and determine the perimeters.



Objective: Use a line plot to record the number of rectangles constructed from a given number of unit squares.

 Use unit square tiles to make rectangles for each given number of unit squares. Complete the charts to show how many rectangles you can make for each given number of unit squares. The is done for you. You might not use all the spaces in each chart.

Number of unit	squares = 12
Number of rectar	igles I made: 3
Width	Length
Width 1	Length 12
Width · · 1	000000000000000000000000000000000000000

Number of uni	t squares = 13
Number of rectan	gles I made: _
Width	Length

Number of uni	t squares = 14
Number of rectan	gles I made: 2
, Width	Length
, Width	Length

Number of uni	t squares = 15
Number of rectan	igles I made: 🙎
Width	Length
	_
	15

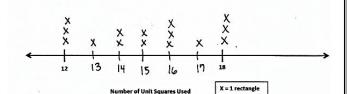
Number of rectar	t squares = 16 igles I made:
Width	Length
1	16
2	8

Number of recta	ngles I made:
Width	Length
1	17

	t squares = 1
Number of rectan	gles I made:
	-
Width	Length
1	18
2	0

2. Create a line plot with the data you collected in Problem 1.

Number of Rectangles Made With Unit Squares



3. Which numbers of unit squares produce 3 rectangles?

12, 16, and 18 unit squares all produce 3 rectangles.

4. Why do some numbers of unit squares, such as 13, only produce one rectangle?

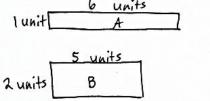
Some Numbers of Unit Squares only produce I rectangle because there is only one pair of factors that can multiply to make that humber. Like 13 only has I and 13.

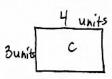
## Lesson 20

Objective: Construct rectangles with a given perimeter using unit squares and determine their areas.

2. Use your square unit tiles to build as many rectangles as you can with a perimeter of 14 units.

a. Estimate to draw your rectangles below. Label the side lengths of each rectangle.





b. Find the areas of all the rectangles in Problem 2(a).

Area of Reutangle A= 1 unit x 6 units = 6 sq units

Area of Rectangle B= 2 units x 5 units = 10 sq units

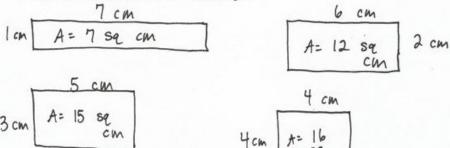
Atea of
Rectangle C
= 3 Units x 4 units
= 12 sq units

c. Given a rectangle's perimeter, what other information do you need to know about the rectangle to find its area?

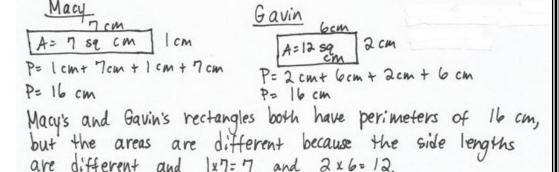
Given a rectangle's perimeter, you also need to know the width or length of the rectangle to find its area. For example, in the rectangles above, the perimeters are all 14 units, but the areas are different.

Objective: Construct rectangles with a given perimeter using unit squares and determine their areas.

- On your centimeter grid paper, shade and label as many rectangles as you can with a perimeter of 16 centimeters.
  - a. Sketch the rectangles below and label the side lengths.

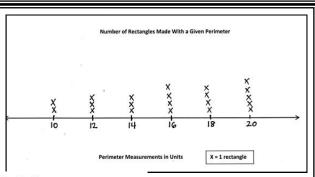


Macy and Gavin both draw rectangles with perimeters of 16 centimeters. Use words and pictures to
explain how it's possible for Macy's and Gavin's rectangles to have the same perimeters, but different
areas.



## Lesson 22

Objective: Use a line plot to record the number of rectangles constructed in Lessons 20 and 21.



CM

Why are all of the perimeter measurements even? Do all rectangles have an even perimeter?

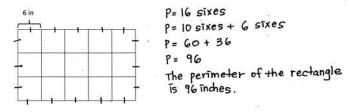
The perimeters are even because we made rectangles with whole number side lengths and when you double the sum of whole numbers, you get an even number. All rectangles don't have an even perimeter because we used string to show you can have an odd perimeter.

Sumi uses unit square tiles to build 3 rectangles that have an area of 32 square units. Does knowing this help her find the number of rectangles she can build for a perimeter of 32 units? Why or why not?

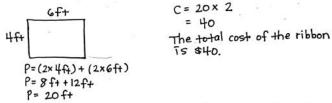
No, knowing the number of rectangles she built with on area of 32 sq units does not help her find the number of rectangles she can build for a perimeter of 32 units. If you halve 32 units to get 16 units, there will be more than 3 pairs of numbers that add to 16.

Objective: Solve a variety of word problems with perimeter.

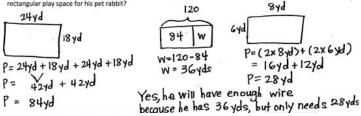
4. Raj uses 6-inch square tiles to make a rectangle, as shown below. What is the perimeter of the rectangle in inches?



5. Mischa makes a 4 foot by 6 foot rectangular banner. She puts ribbon around the outside edges. The ribbon costs \$2 per foot. What is the total cost of the ribbon?



6. Colton buys a roll of wire fencing that is 120 yards long. He uses it to fence in his 18 yard by 24 yard rectangular garden. Will Colton have enough wire fencing left over to fence in a 6 yard by 8 yard rectangular play space for his pet rabbit?



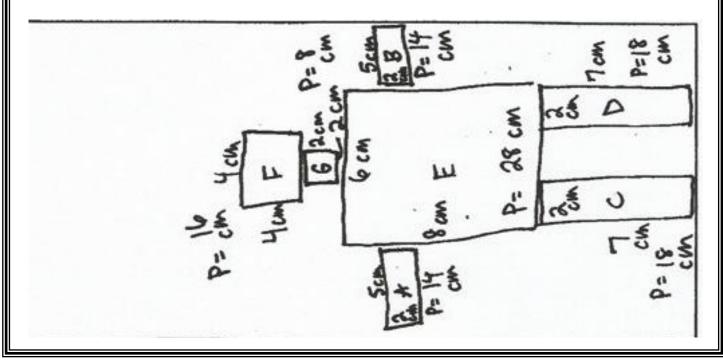
Lesson 24

Objective: Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced.

Use the given perimeters in the chart below to choose the widths and lengths of your robot's rectangular body parts. Write the widths and lengths in the chart below. Use the blank rows if you want to add extra rectangular body parts to your robot.

etter	Body Part	Perimeter	Width and Length
Α	arm	14 cm	
В	arm	14 cm	2 <sub>cm by</sub> 5 <sub>cm</sub>
С	leg	18 cm	
D	leg	18 cm	cm by7cm
E	body	Double the perimeter of one arm = 28 cm	6cm by8cm

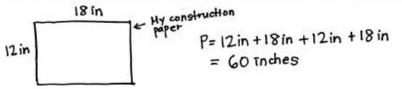
Objective: Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced.



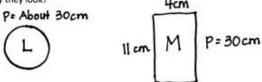
## Lesson 26

Objective: Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced.

2. Measure and calculate the perimeter of your construction paper in inches. Show your work below.

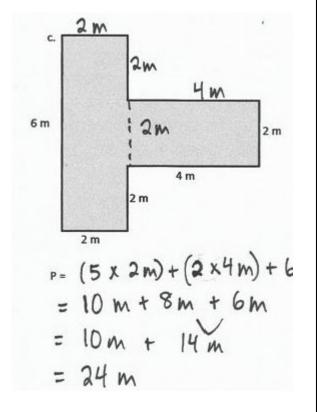


3. Sketch and label 2 shapes with the same perimeter from the robot's environment. What do you notice about the way they look?



Shapes L and M both have the same perimeter, but are 2 different shapes. Shape L is a circle and Shape M is a rectangle.

Objective: Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced.

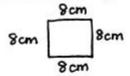


## Lesson 28

Objective: Solve a variety of word problems involving area and perimeter using all four operations.

Elijah draws a square that has side lengths of 8 centimeters.

a. Estimate to draw Elijah's square and label the side lengths.



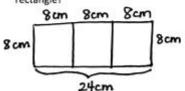
b. What is the area of Elijah's square?

8cm × 8cm = 64 sq cm The area of Elijah's square is 64 sq cm.

c. What is the perimeter of Elijah's square?

8cm + 8cm + 8cm + 8cm or 4 × 8cm = 32cm.
The perimeter of Elijah's square is 32cm.

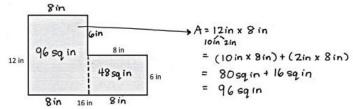
d. Elijah connects 3 of these squares to make one long rectangle. What is the perimeter of this rectangle?



P=24cm+8cm+24cm+8cm =64cm The perimeter of this rectangle is 64cm.

Objective: Solve a variety of word problems involving area and perimeter using all four operations.

Kyle puts 2 rectangles together to make the "L" shaped figure below. He measures some of the side lengths and records them as shown.



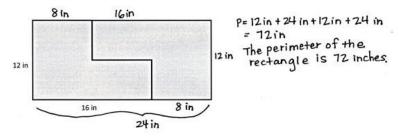
- a. Find the perimeter of Kyle's shape.

  P= 12 in + 8 in + 6 in + 8 in + 6 in + 16 = 56 in

  Zoin

  Zoin

  The perimeter of Kyle's shape is 56 in.
- b. Find the area of Kyle's shape. A = 96 sq in + 48 sq in 4'44 = 100 sq in + 44 sq in = 144 sq inThe area of Kyle's shape is 144 sq in
- c. Kyle makes 2 copies of the "L" shaped figure to create the rectangle shown below. Find the perimeter of the rectangle.



#### Lesson 30

Objective: Share and critique peer strategies for problem solving.

- Drew a picture of a rectangle and labeled Side lengths. · Added to find perimeter. Strategy/strategies Used a tape diagram to find total of 32 perimeters. · Used all steps in RDW. · Realized that 52+48=100, so the perimeter Things my classmate did well: īs 2 x 100 = 200. · Drew and labeled a tape diagram to show thinking for the last step. · Use a letter to show the unknown in the last step. Suggestions for improvement: · Include units in all steps. Thinking about numbers, like 52+48=100, Strategies I would like to try based on so I can use mental math, or do less my classmate's Work.
- 48 yds

  A jogging path around the outside edges of a rectangular playground measures 48 yards by 52 yards.

  Maya runs 3½ laps on the jogging path. What is the total number of yards Maya runs?

  P of jogging path = 48 yds + 52 yds + 48 yds + 52 yds

  = 200 yds

  Half of 200 yds = 100 yds

  3½ laps = (3×200 yds) + 100 yds

  = 600 + 100

  = 700

  May a runs 700 yards