Things to Know for the Algebra II Regents Exam

RADICALS

Simplifying Radicals: find two numbers that multiply to the number under the radical where one number must be a *perfect square.* (If there is a coefficient, it will multiply) *Ex)* $5\sqrt{32} = 5\sqrt{16}\sqrt{2} = 5 \cdot 4\sqrt{2} = 20\sqrt{2}$

Adding/Subtracting: Two radicals must have the SAME number under the radical. If so, add/subtract coefficients and leave common radical alone. *Ex)* $\sqrt{27} - 5\sqrt{12} = \sqrt{9}\sqrt{3} - 5\sqrt{4}\sqrt{3} = 3\sqrt{3} - 5 \cdot 2\sqrt{3} = -7\sqrt{3}$

Multiplying/Dividing: Any two radicals can mult/divide (do not have to be the same). Mult/divide the coefficients and mult/divide the radicands.

RULE: $\sqrt{x} \cdot \sqrt{x} = x$

Ex) Multiply and express result in simplest radical form:

$$9\sqrt{6} \cdot 7\sqrt{3} = 63\sqrt{18} = 63\sqrt{9}\sqrt{2} = 63 \cdot 3\sqrt{2} = 189\sqrt{2}$$

Ex) Divide: $\frac{6\sqrt{32}}{2\sqrt{2}} = 3\sqrt{16} = 3 \cdot 4 = 12$

Conjugates: to find the conjugate, change the MIDDLE sign. The product of a number and its conjugate will always be a rational number (no $\sqrt{\ }$).

Ex) conjugate of $2 + \sqrt{5}$ is $2 - \sqrt{5}$ and $(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$

Rationalizing Denominators: we can never leave a radical in the denominator of a fraction. To get rid of it…

If there is only one term… Multiply by the same radical on top & bottom

Ex) $\frac{4}{\sqrt{2}} = \frac{4 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

If there are two terms… Multiply by the Conjugate of the denominator

Equations with Radicals: Step 1: Get the radical alone. Step 2; Square BOTH sides to get rid of radical. Step 3: Solve for x and CHECK solution(s) in original eqn.

Ex) Solve: $\sqrt{x+7} - x = 1$

$$\sqrt{x+7} = x + 1$$
$$(\sqrt{x+7})^2 = (x+1)^2$$

FOIL

$$x + 7 = x^2 + 2x + 1$$
$$0 = x^2 + x - 6$$
$$0 = (x+3)(x-2)$$
$$\boldsymbol{\cancel{x = -3} \text{ or } x = 2}$$

<u>CHECK: x = -3</u>

$\sqrt{-3+7} - (-3) = 1$

$\sqrt{4} + 3 = 1$

$2 + 3 \neq 1$ <u>reject</u>!

<u>CHECK: x = 2</u>

$\sqrt{2+7} - 2 = 1$

$\sqrt{9} - 2 = 1$

$3 - 2 = 1$ ☑

FRACTIONS

Equations: Step 1: Find LCD for all fractions. (FACTOR them if necessary) Step 2: Multiply each fraction by what it is "missing". Step 3: Cancel out all common denominators. Step 4: Solve equation. Step 5: CHECK Solution(s) and REJECT any that make a fraction undefined.

Ex) $\frac{1}{x+1} + \frac{1}{x-1} = \frac{2}{x^2-1} \Rightarrow \frac{1}{x+1} + \frac{1}{x-1} = \frac{2}{(x+1)(x-1)}$

$\frac{1\ (\boldsymbol{x-1})}{(x+1)\ (\boldsymbol{x-1})} + \frac{1\ (\boldsymbol{x+1})}{(x-1)\ (\boldsymbol{x+1})} = \frac{2}{(x+1)(x-1)} \Rightarrow x - 1 + x + 1 = 2$

$2x = 2 \qquad x = 1$

<u>CHECK: x = 1</u>

makes $\frac{1}{x-1}$ undefined since it's $\frac{1}{0}$ $\therefore$ reject!

<u>Solution</u>: ∅ (empty set – there is NO solution)

Probability Formula: $\frac{\boldsymbol{number\ of\ favorable\ outcomes}}{\boldsymbol{total\ number\ of\ outcomes}}$

PROBABILITY

<u>Conditional Probability</u>: finds the probability of an event (E_2) occurs GIVEN that another event (E_1) already occurred…

$$P(E_2\ \ E_1) = \frac{P(E_2\ and\ E_1)}{P(E_1)}$$

Ex) The probability a student takes computers and Spanish is .15. The probability the student takes computers if 0.4. What is the probability t the student takes Spanish given that he/she takes computers?

Answer: $\frac{.15}{0.4}$ *= .375*

Ex) The table below has information regarding the percent of people (either with or without children in college) and how affordable they feel college is or is not)

	Too Expensive	Affordable	Too Cheap
Child in College	55%	4%	0%
Child not in College	30%	8%	3%

ge is too expensive

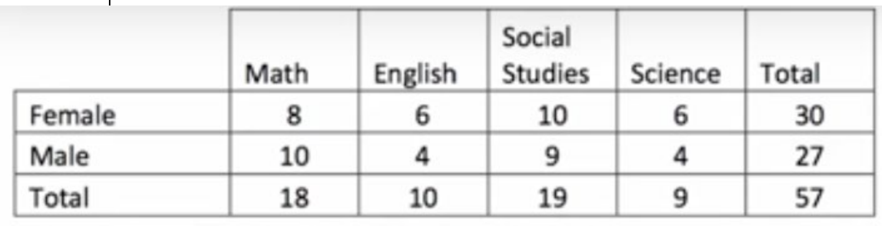
$$\frac{Number\ who\ have\ children\ in\ college\ AND\ think\ its\ expensive}{Total\ number\ having\ children\ in\ college} = \frac{.55}{.59}$$

<u>Independent Events:</u> when occurrence of 1 event does not change the occurrence of another.

If independent then: $P(A\ \ \ B) = P(A)$

If independent then: $P(\ A\ and\ B) = P(A) \cdot P(B)$

Ex) This table shows favorite subjects. Does it appear that choosing math as a favorite subject is <u>dependent</u> on gender?



	Math	English	Social Studies	Science	Total
Female	8	6	10	6	30
Male	10	4	9	4	27
Total	18	10	19	9	57

$P(math)$

$$\frac{8}{30} = \frac{18}{57}$$ No!

$$\therefore not\ independent, must\ be\ dependent$$

FUNCTIONS

To determine if a relation is a function…

Graphs: must pass the **"Vertical Line Test"** (no vertical line can ever intersect the graph more than once)

Points: All **x-values** must be **DIFFERENT** in a function.

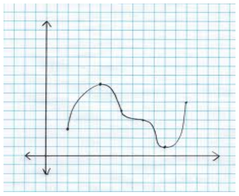
f(x) notation: If f(x) is given and we want to find "f(some number)", we substitute the number in place of x on the right side of the equation.

Ex) If $f(x) = 2x^2-3x + 1$*, then* $f(5) = 2(5)^2-3(5) + 1 = 36$

Domain: a list of the **x-values** used in the relation/function.

Range: a list of the **y-values** used in the relation/function.

Ex)



Domain: $2 \leq x \leq 13$

Range: $1 \leq y \leq 8$

Restricted Domains: many functions have a domain of "All x". Those that don't are…1) Fractions: whatever number(s) make the fraction UNDEFINED are NOT in the domain. *Ex)* Domain of $\frac{4x-1}{2x+3}$ is: $x \neq \frac{-3}{2}$

2) Square Roots: the radicand must be ≥ zero.

Ex) Domain of $\sqrt{2x+3}$ is: $x \geq \frac{-3}{2}$

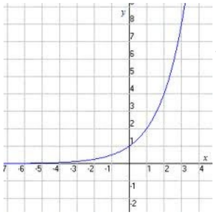
3) Square Root in Denominator: radicand must be > zero.

Ex) Domain of $\frac{4x-1}{\sqrt{2x+3}}$ is: $x > \frac{-3}{2}$

Inverse of a Function: notation: $f^{-1}(x)$. A function that has an inverse is called **one-to-one**. A graph must pass **BOTH** the Vertical and Horizontal Line Tests in order to be one-to-one.

Onto: when the range of the functionis all y's.

Ex)



This function is one-to-one but NOT onto.

(range is y > 0)

To find the inverse of a function:

Algebraically: Switch "x" and "y". (Remember that "f(x)" is the same as "y"). Then solve for "y". This new "y" is also called "$f^{-1}(x)$".

Ex) If $f(x) = \frac{2x-3}{7}$; find $f^{-1}(x)$.

$y = \frac{2x-3}{7}$

(switch) $x = \frac{2y-3}{7}$

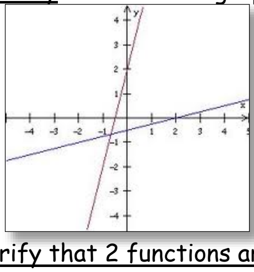
$7x = 2y - 3$

$7x + 3 = 2y$

$\frac{7x+3}{2} = y = f^{-1}(x)$

Graphically: Reflect the graph over the line **y = x**. All points (x, y) → (y, x)

Ex)



To Verify that 2 functions are inverses of one another: Show that

$$\boxed{(f \circ f^{-1})(x) = x}$$

QUADRATIC EQUATIONS

Quadratic Equation: an equation that has x^2 in it. To solve:

Step 1: Get one side equal to ZERO

Step 2: If possible, FACTOR (by GCF, DOTS, or TRInomial). If not, use the Quadratic Formula.

a = coefficient of x^2

b = coefficient of x

c = constant

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Every quadratic equation has two solutions (though not necessarily real). The graph of a quadratic equation is a parabola and the solutions represent its REAL ROOTS (the places it intersects the x-axis).

Nature (Type) of Roots: to determine the type of roots a quadratic eqn. has (without actually finding them), use the **Discriminant** = $b^2 - 4ac$

Discriminant $b^2 - 4ac$ is…	**Roots would be…**
NEGATIVE	Imaginary
ZERO	Equal (Real, Rational, & Equal)
POSITIVE PERFECT SQUARE	Rational & Unequal (Real, Rational, & Unequal)
POSITIVE but NOT a PERFECT SQUARE	Irrational (Real, Irrational, & Unequal)

Sum and Product of the Roots:

$$\boxed{\textbf{SUM} = \frac{-b}{a} \qquad \textbf{PRODUCT} = \frac{c}{a}}$$

To write a Quadratic Equation given its roots:

$$x^2 + \triangle x + \square = 0$$

Opposite sign of the Sum

Exact sign of the Product

Ex) Write a quadratic equation whose roots are: $4 - 3\sqrt{5}$ and $4 + 3\sqrt{5}$.

SUM: $(4 - 3\sqrt{5}) + (4 + 3\sqrt{5}) = 8$ (but use -8 in equation)

PRODUCT: $(4 - 3\sqrt{5})(4 + 3\sqrt{5}) = 16 - 9(5) = -29$ (actually use the -29)

Answer: $x^2 - 8x - 29 = 0$

Completing the Square: another way of solving a quadratic equation

Step 1: Divide by the number in front of x^2. Step 2: Keep all terms with x on the left and move constant to right side of =. Step 3: Set up a place holder ♡ on each side to fill in with the missing number $\left(\frac{b}{2}\right)^2$. Step 4: Rewrite the left side as $(x + \frac{b}{2})^2$. Step 5: Square root both sides – this will get rid of the "*squared*" on left side. **Include a " ± " on right side**. Step 6: Get x alone and simplify if possible.

Ex) Solve by completing the square: $2x^2 - 6x - 7 = 0$

$x^2 - 3x - \frac{7}{2} = 0$ *(Divide by 2)*

$x^2 - 3x = \frac{7}{2}$ *(Move constant)*

$x^2 - 3x + ♡ = \frac{7}{2} + ♡$ *(place holder)*

$x^2 - 3x + \frac{9}{4} = \frac{7}{2} + \frac{9}{4}$ $\left(\frac{-3}{2}\right)^2 = +\frac{9}{4}$

QUADRATIC

EQUATIONS

$\left(x-\frac{3}{2}\right)^2 = \frac{7}{2}+\frac{9}{4}$ *(left side is perfect square)*

$\left(x-\frac{3}{2}\right)^2 = \frac{23}{4}$ *(simplify)*

$\sqrt{\left(x-\frac{3}{2}\right)^2} = \pm\sqrt{\frac{23}{4}}$ *(square root- don't forget ± on right)*

$x - \frac{3}{2} = \pm\sqrt{\frac{23}{4}}$ (drop the squared on left)

$x = \frac{3}{2} \pm \sqrt{\frac{23}{4}} = \frac{3}{2} \pm \frac{\sqrt{23}}{2}$ or $\frac{3\pm\sqrt{23}}{2}$

EXPONENTS

Zero and Negative Exponents:

$$\boxed{\begin{aligned} x^0 &= 1 \\ x^{-n} &= \frac{1}{x^n} \\ \frac{1}{x^{-n}} &= x^n \end{aligned}} \quad \boxed{\begin{aligned} x^{\frac{p}{r}} &= \sqrt[r]{x^p} \\ &= (\sqrt[r]{x}\,)^p \end{aligned}}$$

Remember: a missing power is 1, but a missing root is 2

Notice the Difference: $-3^2 \neq (-3)^2$ because $-3^2 = -9$ yet $(-3)^2 = +9$

Equations with Exponents:

Exponent is a VARIABLE: Rewrite one or both bases so that they are the same number. (Usually the larger gets rewritten in terms of the smaller or both get written smaller.) *Note: if this is not possible, use LOGS!!* Then cancel out the like bases and solve the remaining equation (distribute if necessary). *Ex)* Solve: $8^x = 4^{x+3}$

$$(2^3)^x = (2^2)^{x+3}$$

$$(\cancel{2}^3)^x = (\cancel{2}^2)^{x+3} \quad \text{(distribute !)}$$

$$3x = 2x + 6 \quad \therefore \quad x = 6$$

Exponent is a NUMBER: Get the variable alone then to get rid of its exponent, raise both sides to the RECIPROCAL power.

Ex) Solve: $2x^{\frac{-2}{3}} + 7 = 25$

$$2x^{\frac{-2}{3}} = 18$$

$$x^{\frac{-2}{3}} = 9$$

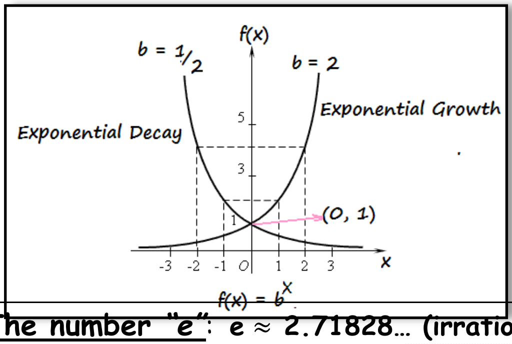
$$\left(x^{\frac{-2}{3}}\right)^{\frac{-3}{2}} = (9)^{\frac{-3}{2}} \longrightarrow \boxed{x = \frac{1}{27}}$$

EXPONENT GRAPHS

Graph of Exponential Functions: $y = b^x$ (b must be positive but ≠ 1)

If $0 < b < 1$ the graph represents DECAY (decreasing from left to right)

If $b > 1$ the graph represents GROWTH (increasing from left to right)

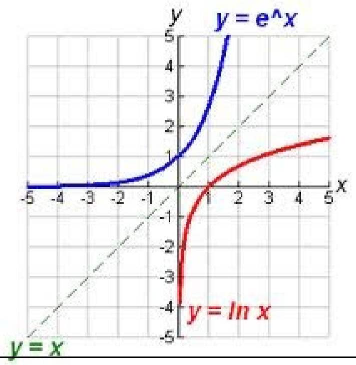


PROPERTIES of Exponential Graphs:

- ·Passes through (0, 1)
- ·Quadrants I and II only
- ·x – axis is an asymptote *(graph gets closer to but never touches x-axis)*

LOG GRAPHS

The number "e": **$e \approx 2.71828$... (irrational #)** "e" is the base of the natural logarithm "$\ln x$". That is... $\log_e x = \ln x$. **"e^x" and "ln x" are inverses of one another** (symmetric about the line y = x).



PROPERTIES of Log/Ln Graphs:

- Passes through (1, 0)
- Quadrants I and IV only
- y – axis is an asymptote *(graph gets closer to but never touches y-axis)*

e^x and ln x

Relationship between "e" and "ln": They are inverses of one another.

Ex) $e^{\ln 5} = 5$ (but be careful – there can't be anything in front of "ln")

Ex) $e^{2\ln 5} = e^{\ln 5^2} = 5^2 = 25$ (from Log Law III)

Ex) $\ln e^{4x-7} = 4x - 7$

$$e^{\ln x} = x$$
$$\ln e^x = x$$
$$\ln e = 1$$
$$\ln 1 = 0$$

LOGS

$\log_a b = c \longrightarrow a^c = b$ (*Note*: If no "a" is written, use 10)

$\ln x = y \longrightarrow e^y = x$ (*since the base is "e"*)

Ex) Find the value of: $\log_2 8$

This is the same as $\log_2 8 = x \longrightarrow 2^x = 8 \quad \therefore \quad x = 3$

Ex) Solve for x: $\log_x 3 = \frac{1}{3}$

$x^{\frac{1}{3}} = 3 \quad \therefore \quad \left(x^{\frac{1}{3}}\right)^3 = (3)^3 \therefore \quad x = 27$

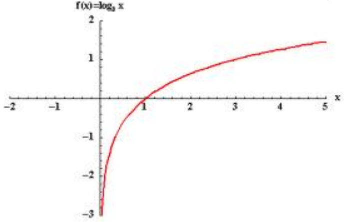
Logs on a Calculator:

log # $\longrightarrow$ **LOG** **#**

log$_a$ b $\longrightarrow$ **MATH** **logBASE**

ln # $\longrightarrow$ **LN** **#**

Domain of Log Functions: domain is x > 0 (this mean we can only find the "log" or "ln" of a positive number).



Log Laws:
I. $\log(A \cdot B) = \log A + \log B$ (multiplication becomes addition)
II. $\log \frac{A}{B} = \log A - \log B$ (division becomes subtraction)
III. $\log A^c = c \log A$ (exponent becomes coefficient)

When do we use logs? WHENEVER WE ARE SOLVING FOR AN EXPONENT
Since we can take the "log" of both sides of an exponential equation and then apply Log Law III (above) to turn the exponent into a coefficient.

Ex) Solve for x: $5^x = 11$ *(these can't be rewritten with the same base)*

to nearest tenth

LOGS

log 5^x = log 11 Step 1: Take "log" of both sides

x log 5 = log 11 Step 2: Apply Log Law III

x = $\frac{\log 11}{\log 5}$ Step 3: Divide

x ≈ 1.9

Log Equations: Condense into a single logarithm (if necessary), then rewrite in exponential form and solve. CHECK in original equation. REJECT any value involving the log of a non-positive number *(see Domain of Log Functions pg. 7)*

Ex) $\log_3 x + \log_3(x-2) = 1$

$\log_3 x(x-2) = 1$

$3^1 = x(x-2)$

$3 = x^2 - 2x$

$0 = x^2 - 2x - 3$

$0 = (x+1)(x-3)$

$\cancel{x} = -1$ or $x = 3$

CHECK x = -1: can't have $\log_3(-1)$

CHECK x = 3: $\log_3 3 + \log_3 1 = 1$

1 + 0 = 1 ☑

POLYNOMIALS

Polynomial Equations: an equation with positive integral exponents of x.

Ex) y = $4x^5 - 3x^4 + x^3 - 7x - 9$ is an example of a polynomial function.

Zeros of a Function: the values of x where the graph intersects the x-axis. THE MAXIMUM NUMBER OF ZEROS A POLYNOMIAL HAS IS THE SAME AS THE HIGHEST EXPONENT IN ITS EQUATION.

Ex) y = $4x^5 - 3x^4 + x^3 - 7x - 9$ has a maximum of 5 zeros.

To find the zeros:

Algebraically: Set the function equal to zero and solve by factoring.

Ex) Find the zeros of y = $x^3 + x^2 - 12x$

$0 = x^3 + x^2 - 12x$

$0 = x(x^2 + x - 12)$

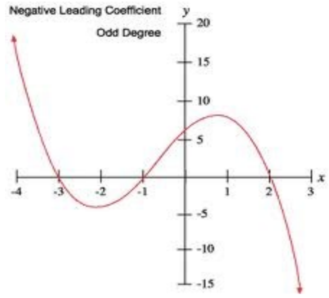
$0 = x(x + 4)(x - 3)$

$\boxed{x = 0 \text{ or } x = -4 \text{ or } x = 3}$

Graphically: Make sure equation it equal to zero then graph.

Ex) Solve graphically: $-x^3 - 2x^2 + 5x = -6$

Graph: y = $-x^3 - 2x^2 + 5x + 6$



COMPLEX NUMBERS

Definition of i: $i = \sqrt{-1}$

i on Calculator: MODE a+bi

Powers of i: we can leave i in an answer, but not with an exponent.

On Calculator, MATH ➡ NUM "iPart"

Additive Inverse: CHANGE BOTH SIGNS

Conjugate: CHANGE MIDDLE SIGN

Mult. Inverse: RECIPROCAL

Absolute Value: $|a + bi| = \sqrt{a^2 + b^2}$

Arithmetic: Adding/Subtracting: Combine real parts (w/o i). Combine imaginary parts (with i) *Ex)* $(-7 + 3i) - (-2 - 8i) = -7 + 3i + 2 + 8i = -5 + 11i$

Multiplying: Distribute and/or FOIL if necessary. Remember $\boxed{i \cdot i = i^2 = -1}$ *Ex)* $(-7 + 3i)(-2 - 8i) = 14 + 56i - 6i - 24i^2$

$= 14 + 50i + 24 = 38 + 50i$

CIRCLES

Circles: where r = radius

&
TRANSFORMATIONS

centered at origin: $x^2 + y^2 = r^2$

centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$

Notice: Change the signs of x and y to find center. If no number is written (as in x^2), then use zero. Also, notice that the number after the equal sign is the radius after squared.

Transformations: changes in an equation will move every point on a graph.

$\boldsymbol{f(x) + a}$: (outside change) moves the graph UP or DOWN

- $\boldsymbol{a}$ ***is positive***: graph moves UP
- $\boldsymbol{a}$ ***is negative***: graph moves DOWN

$\boldsymbol{f(x + a)}$: (inside change) moves the graph LEFT or RIGHT

- $\boldsymbol{a}$ ***is positive***: graph moves LEFT
- $\boldsymbol{a}$ ***is negative***: graph moves RIGHT

$\boldsymbol{f(-x)}$: (negative in front of x) graph reflects over Y-AXIS

$\boldsymbol{-f(x)}$: (negative in front of equation) graph reflects over X-AXIS

$\boldsymbol{a\,f(x)}$: (a coefficient in front of equation) graph STRETCHES or SHRINKS VERTICALLY. Multiply ally-coordinates by "a". **(Points on x-axis will therefore not move)**

TRIGONOMETRY

Trig. Values to MEMORIZE:

	30°	**45°**	**60°**
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

OR

Recognize these decimals:

$.707\ldots = \frac{\sqrt{2}}{2}$

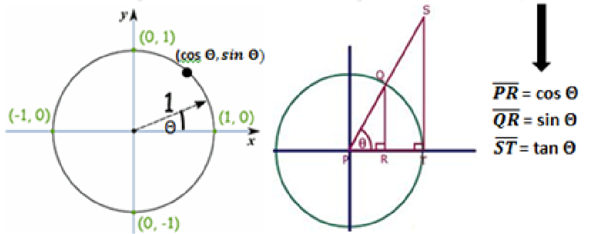
$.866\ldots = \frac{\sqrt{3}}{2}$

$.577\ldots = \frac{\sqrt{3}}{3}$

$1.732\ldots = \sqrt{3}$

Unit Circle: Each point on circle has coordinates **(cos ∢ , sin ∢)**

Since radius is length 1, the sides of the triangle are determined by ∢Θ



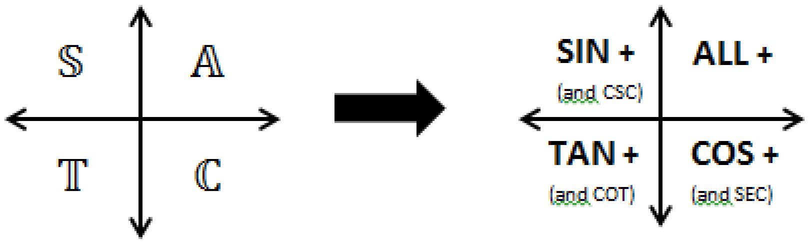
$\overline{PR}$ = cos Θ

$\overline{QR}$ = sin Θ

$\overline{ST}$ = tan Θ

TRIGONOMETRY

Quadrants: This is a way of remembering which trig. functions are POSITIVE each quadrant:



Reference Angles: the angle made with the x-axis. Any angle can be changed into its reference angle (positive and acute) AS LONG AS THE SIGN of the original answer is considered.

Ex) Find the exact value of: sec 135°.

$$\frac{1}{\cos 135} = \frac{1}{-\cos 45} = \frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$$

Radians: $\boxed{\pi^r = 180^\circ}$

To change radians into degrees:

Replace π with 180°

To change degrees into radians:

Multiply by $\frac{\pi}{180}$

ARC of a circle: to find the length of an arc of a circle…

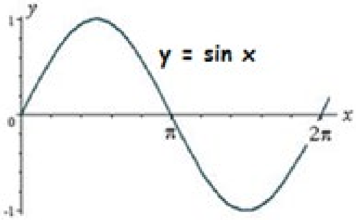
$$\boxed{A = RC}$$

A = Arc length

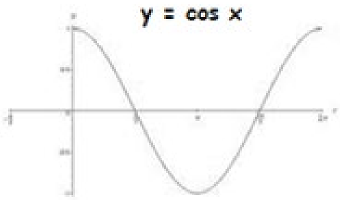
R = Radius

C = Central Angle* **(must be in radians)**

"SIN" wave:



"COS" wave:



General Equation for these sinusoidal graphs:

$$y = a \sin(bx + c) + d$$

$$y = a \cos(bx + c) + d$$

Amplitude: $|a|$ = the maximum height the graph reaches ABOVE and BELOW the midline "d"

Frequency: b = the number of full waves there will be from 0 to 2π

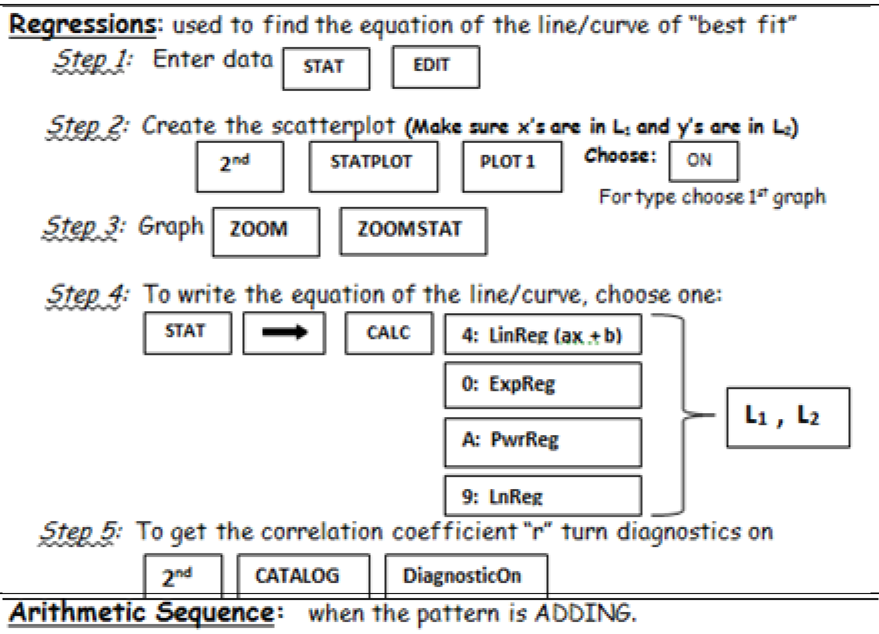
Period: $\frac{2\pi}{b}$ = the length of each full wave

Phase Shift: $\frac{-c}{b}$ = the horizontal shift (*Note: if no parentheses, graph starts at x = 0)*

Midline: d = the vertical shift

Increment: $\frac{Period}{4}$ = the scale on the x-axis

STATISTICS



Regressions: used to find the equation of the line/curve of "best fit"

Step 1: Enter data STAT EDIT

Step 2: Create the scatterplot **(Make sure x's are in L_1 and y's are in L_2)**

2nd STATPLOT PLOT 1 **Choose:** ON

For type choose 1st graph

Step 3: Graph ZOOM ZOOMSTAT

Step 4: To write the equation of the line/curve, choose one:

STAT ➡ CALC

4: LinReg (ax + b)

0: ExpReg

A: PwrReg

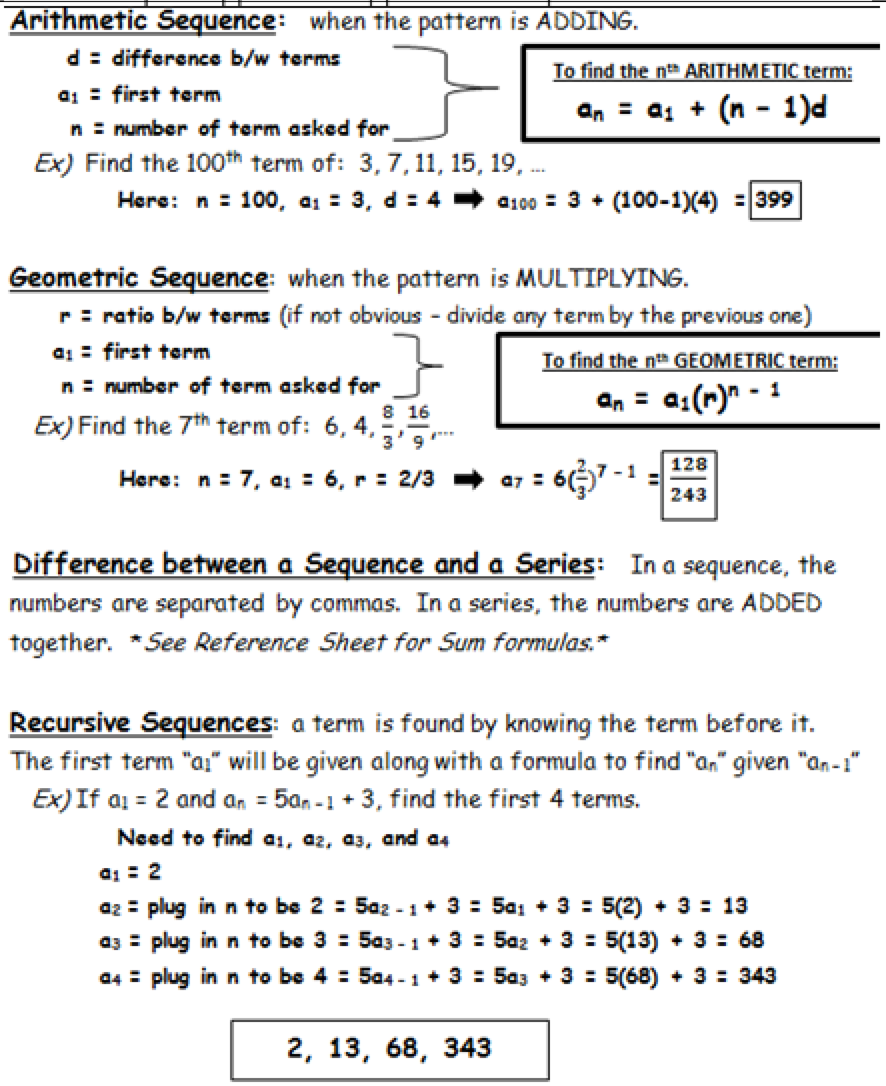
9: LnReg

L_1, L_2

Step 5: To get the correlation coefficient "r" turn diagnostics on

2nd CATALOG DiagnosticOn

SEQUENCES



Arithmetic Sequence: when the pattern is ADDING.

d = difference b/w terms

a_1 = first term

n = number of term asked for

To find the nth ARITHMETIC term:

$$a_n = a_1 + (n - 1)d$$

Ex) Find the 100th term of: 3, 7, 11, 15, 19, ...

Here: n = 100, a_1 = 3, d = 4 ➡ $a_{100} = 3 + (100-1)(4) = \boxed{399}$

Geometric Sequence: when the pattern is MULTIPLYING.

r = ratio b/w terms (if not obvious - divide any term by the previous one)

a_1 = first term

n = number of term asked for

To find the nth GEOMETRIC term:

$$a_n = a_1(r)^{n-1}$$

Ex) Find the 7th term of: $6, 4, \frac{8}{3}, \frac{16}{9}, \ldots$

Here: n = 7, a_1 = 6, r = 2/3 ➡ $a_7 = 6(\frac{2}{3})^{7-1} = \boxed{\frac{128}{243}}$

Difference between a Sequence and a Series: In a sequence, the numbers are separated by commas. In a series, the numbers are ADDED together. **See Reference Sheet for Sum formulas.**

Recursive Sequences: a term is found by knowing the term before it. The first term "a_1" will be given along with a formula to find "a_n" given "a_{n-1}"

Ex) If $a_1 = 2$ and $a_n = 5a_{n-1} + 3$, find the first 4 terms.

Need to find a_1, a_2, a_3, and a_4

$a_1 = 2$

a_2 = plug in n to be 2 = $5a_{2-1} + 3 = 5a_1 + 3 = 5(2) + 3 = 13$

a_3 = plug in n to be 3 = $5a_{3-1} + 3 = 5a_2 + 3 = 5(13) + 3 = 68$

a_4 = plug in n to be 4 = $5a_{4-1} + 3 = 5a_3 + 3 = 5(68) + 3 = 343$

$$\boxed{2,\ 13,\ 68,\ 343}$$

PARABOLAS

Equation of a Parabola: $y = ax^2 + bx + c$

To find the **vertex**: Complete the square (see page 5). Once in this form:

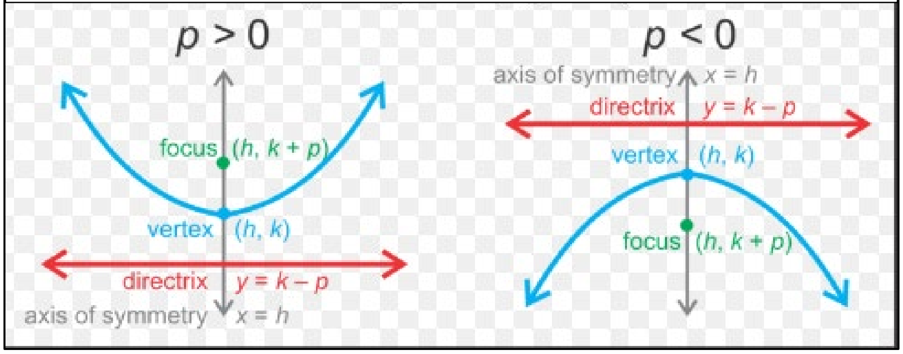
$$y = a(x-h)^2 + k$$

Vertex will be (+h, k)

To find the **Focus and Directrix** we first must find “p”. “p” is the distance between both the vertex and the focus OR the vertex and directrix (these are the same distances)

$$p = \frac{1}{4a}$$

To find Focus and Directrix once you know “p”:



Once p is found, add and subtract it to the y-value of the vertex (k) to find both the focus and directrix. Remember the focus is a point and the directrix is a line so…

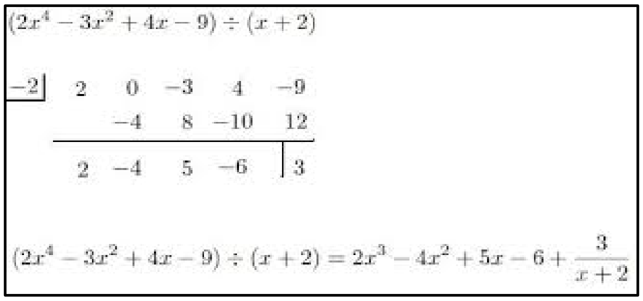
Focus: (h, k + p)
Directrix: y = k - p

Also the focus is always INSIDE the parabola while the directrix is on the OUTSIDE.

SYNTHETIC DIVISION

Whenever using synthetic division:

- Make sure polynomial is in standard form (from highest to lowest exponent). Fill in 0’s for any missing terms.
- Write the coefficients only of all terms in numerator
- On the left, write the root that corresponds to the term being divided by (the number that makes the denominator zero)
- Bring down the first term then multiply and put that product under the second term. Add. Continue this to end. Final number is remainder.
- To write answer, remember that the new polynomial will be one degree smaller. The remainder will become a fraction divided by the original denominator.



$(2x^4 - 3x^2 + 4x - 9) \div (x+2)$

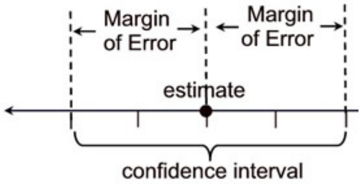
$$\begin{array}{r|rrrr|r} -2 & 2 & 0 & -3 & 4 & -9 \\ & & -4 & 8 & -10 & 12 \\ \hline & 2 & -4 & 5 & -6 & 3 \end{array}$$

$$(2x^4 - 3x^2 + 4x - 9) \div (x+2) = 2x^3 - 4x^2 + 5x - 6 + \frac{3}{x+2}$$

REMAINDER THEOREM	Another way of finding out a function's value for a particular value of x is to find the REMAINDER using synthetic division. **When we divide a polynomial f(x) by x-c, the remainder "r" is the same as f(c)** Ex) What is the remainder when $2x^2-5x-1$ is divided by -3 ? Notice, instead of doing synthetic division we can just find $f(3)$ $2(3)^2-5(3)-1=\boxed{2}$
FACTORING	In addition to the factoring techniques learned in Algebra 1 (Greatest Common Factor (GCF), Difference of Two Squares (DOTS) and Trinomial, you are expected to factor… 1. **Sum/Difference of CUBES**: $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$ Ex) Factor: $8x^6-1$ Since this is really $(2x^2)^3-(1)^3$ So a = $2x^2$ $\big\}$ $(2x^2-1)((2x^2)^2+(2x^2)(1)+1^2)$ b = 1 = $\boxed{(2x^2-1)(4x^4+2x^2+1)}$ 2. **Grouping**: used when there are 4 terms and, when grouped in pairs, each pair has a common factor. Step 1: Factor out the GCF of each pair. Step 2: What should be left in each of the parentheses should be the same. Factor this parenthesis out. Ex) Factor: $6ax+3bx-10a-5b$ $=3x(2a+b)-5(2a+b)$ $=\boxed{(2a+b)(3x-5)}$
SYSTEM OF 3 EQUATIONS	To solve for the 3 unknowns given 3 equations: 1. Pick any two pairs of equations from the system. 2. Eliminate the same variable from each pair using the Addition/Subtraction method. 3. Solve the system of the two new equations using the Addition/Subtraction method. Ex) $3x+2y+z=1$ $x-2y+2z=4$ $\underline{2x+4y+3z=9}$ Notice: If we add the first 2 equations, y will cancel out. So let's get y to cancel out from other two equations $4x+3z=5$ **2**$(x-2y+2z=4)$ ➡ $2x-4y+4z=8$ $\underline{2x+4y+3z=9}$ $4x+7z=17$ Solving these equations by subtracting, we get $-4z=-12$ $\boxed{z=3,\quad x=-1,\ y=\frac{1}{2}}$

MORE STATISTICS

Confidence Interval: a range of values used to estimate the TRUE value of a parameter.



PROPORTIONS:

Margin of Error: the distance from the estimate to one end of a confidence interval. The Margin of error is $\pm 2\sigma$. Since $\sigma = \sqrt{\frac{pq}{n}}$,

$$\boxed{\textbf{Margin of Error} = \pm 2\sqrt{\frac{pq}{n}}}$$

Where p = population proportion
q = 1 – p
n = sample size
σ = standard deviation (pop.)

MEANS:

Standard Error for a sample mean: $\frac{\sigma}{\sqrt{n}}$

Margin of Error for working with sample means: $2\frac{\sigma}{\sqrt{n}}$

Z-SCORES: a Standard score that represents the number of standard deviations a given value is from the mean.

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

where μ = population mean

To find the percentages (proportions, areas) that are associated with z-scores, use the TI-84 calculator. (Note: you can use either all original scores or their converted scores).

2nd DISTR 2: NormalCDF

(lower bound (left), upper bound (right), mean, standard deviation)

(Note: If you use z-scores, then mean = 0 and standard dev = 1)

Ex) Given a normal distribution of values for which the mean is 70 and the standard deviation is 4.5. Find the probability that a value is between 65 and 80.

Normalcdf(65, 80, 70, 4.5) = .8536055925
OR Normalcdf(-1.111, 2.222, 0, 1) = .853574137

$\boxed{= 85.36\%}$

Ex) A normally distributed population of test scores has a mean of 80 and a standard deviation of 5.2. Find the percentage of scores that lies below 73.

Normalcdf(-9999, 73, 80, 5.2) = .0891265138 $\boxed{= 8.91\%}$

EXPONENTIAL GROWTH

<u>Continuous</u> Growth/Decay:

$$A = Pe^{rt}$$

where P = initial amount (principal)

r = <u>continuous</u> rate of interest (as a decimal)
t = time (in years)
A = ending amount

<u>Compound Interest</u>:

$$A = P(1 + \frac{r}{n})^{nt}$$

where P = initial amount (principal)

r = interest rate (as a decimal)
n = # of compoundings in a year *
t = time (in years)
A = ending amount

*The value of n when compounding:

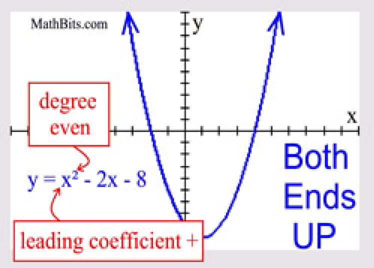
• annual (yearly)	n= 1
• semi-annual	n = 2
• quarterly	n = 4
• monthly	n = 12
• daily	n = 365

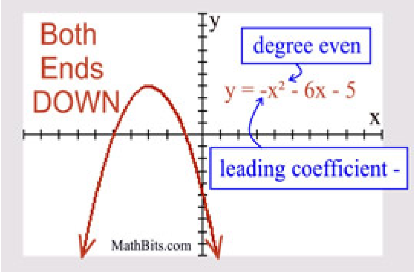
END BEHAVIOR OF POLYNOMIALS

By looking at the degree of the polynomial (its Highest exponent) as well as the Leading Coefficient, we can tell what the “ends” of the graph would look like.

If the DEGREE is EVEN and…

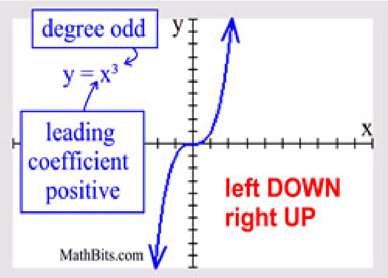
Leading coefficient is POSITIVE then ends would be UP-UP
Leading coefficient is NEGATIVE then ends would be DOWN-DOWN

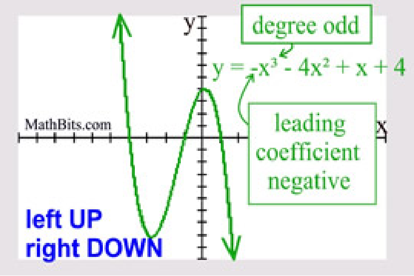




If the DEGREE is ODD and…

Leading Coefficient is POSITIVE then ends would be DOWN-UP
Leading Coefficient is NEGATIVE then ends would be UP-DOWN





REGRESSIONS

When points are given, we can find the equation of a function that may closely resemble the data. The reason an equation is important is that we can use it to find the value of one of the variables if given the value of the other.

The types of Regression covered in Algebra 1 were Linear, Quadratic, Power, and Exponential Regressions. The additional types in Algebra II are Logarithmic and Sinusoidal Regressions.

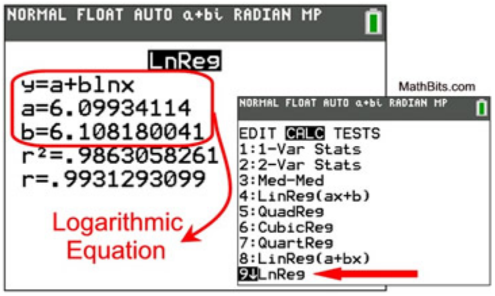
To find the Regression Equation:

Step 1: Enter Data using STAT then EDIT (see page 10 for more details)

Step 2: Choose STAT then CALC then choose the type of regression

<u>For Logarithmic</u>: Choose 9: LnReg

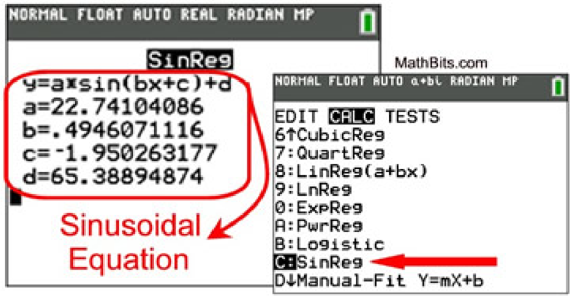
<u>For Sinusoidal:</u> Choose C: SinReg



```
NORMAL FLOAT AUTO a+bi RADIAN MP
LnReg
y=a+blnx
a=6.09934114
b=6.108180041
r²=.9863058261
r=.9931293099
```

Logarithmic Equation

MathBits.com



```
NORMAL FLOAT AUTO REAL RADIAN MP
SinReg
y=a*sin(bx+c)+d
a=22.74104086
b=.4946071116
c=⁻1.950263177
d=65.38894874
```

Sinusoidal Equation

MathBits.com

What do the numbers in the “SinReg” equation stand for?

y = asin(bx + c) + d

| a | = amplitude

b = frequency

2π / b = period

d = vertical shift

c / b = horizontal shift (right if c < 0 and left if c > 0)

NOTE: If your calculator is not giving you the “Correlation Coefficient” (which is denoted “r”), then go to MODE and then Page 2 of MODE and Choose:

STATDIAGNOSTICS: <u>ON</u>

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