GEOMETRY TASKS: OVERVIEW

Resources:

Attached you will find a set of **10 tasks for Geometry**. The tasks are best used by partners or small groups. Since these tasks are more in-depth than the practice items, you can use them as the foundation of your lesson, especially if they address a standard you are working on that day.

The purpose of using tasks is to help you see how students solve problems, and understand their thought processes while they work. Being able to work with others to get productive discourse, to be able to explain their thinking to another student, and develop a solution, is the most effective way to assess student understanding. These tasks require students to do just that – think about an efficient strategy to solve the problem, show their work and justify their reasoning. This is the ultimate goal for what we want students to be able to do. Being able to gather evidence of student learning and misconceptions in the moment, will give you the flexibility to change your instruction to meet their needs. As the instructional decision-maker, you are able to adjust your methods for whole class or small groups to address student misconceptions and move them toward proficiency.

The goal is to have tasks that can be used as the lesson of the day along with or in lieu of the practice items. There are tasks for each day per week that represent the 5 domains in 8^{th} grade. We would like for you to use these tasks along with the practice items for a 10 week period between the time you receive them and the end of January. If used daily, in accordance with our recommendations or tips, for student and teacher practice, the outcome will be an improvement in ACT ASPIRE test scores.

At the end of each task packet, you will find an answer key for your use. **Although answer keys are provided, students should explain their thinking during the discussion of the tasks.** Some tasks include several representations of the solution.

Recommendations or Tips:

When implementing the tasks with your students, please allow students read through the tasks before starting to see if they have any questions about vocabulary or what the task is asking them to do. Taking the time to do these things now, will help assure that the students are familiar with mathematical vocabulary and different question types before the actual test.

Providing Feedback to Students:

An important process for understanding student thinking, is to debrief and provide feedback. This can be done during the exploration phase of the sharing out process by asking effective questions similar to the ones below which will also help students verbalize their reasoning :

- What did the problem ask you to do?
- What information do you see in the problem?
- What did you do <u>first</u> to solve this problem?
- What did you do next?
- What strategy did you use to solve the problem? Why did you use that particular strategy?
- Is there another strategy that you could use to solve the problem?
- Who else started this same way?
- Who started a different way?

- What are some strategies that you heard today that you would like to try when solving a similar problem in the future?

Another option is to let the groups record their solution(s) to the task on chart paper. They can then share out with the whole class. With this option the students are able to present their thinking, justify their reasoning, and answer questions from the other students.

Answer Key:

The information above is intended to help teachers get at student understanding of the mathematical idea(s) in each problem. Also provided is an Answer Key for each set of tasks. The Answer Key provides more information on the expected student response for each task. While it is important for students to get the answer right, it is equally important for them to understand how their thinking leads or does not lead to a correct solution. Incorrect solutions set the stage for teachable moments!!!!

Name____ Date_______________________

Set 1 - Standard(s): 7.G.1, 7.G.4

Illustrative Mathematics

Day 1 Task - Circumference of a Circle

a. Draw circles with diameters as indicated below and measure their circumferences to complete the following table.

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1		
2		
3		
$\frac{1}{2}$		

b. The number π can be defined as the circumference of a circle with diameter 1 (unit). Using your knowledge about circles (that is, *without measuring*), complete the following table. Explain how you know the circumferences of the different circles.

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1	π	π
2		
3		
$\frac{1}{2}$		

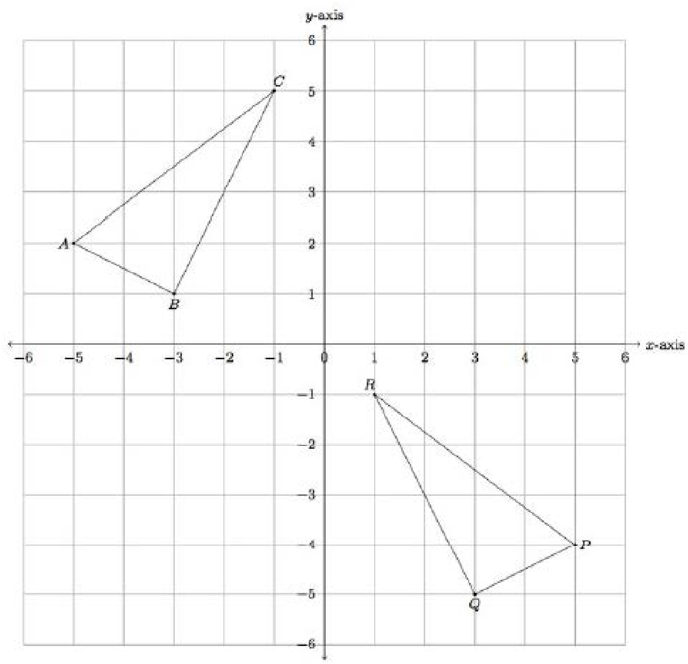
c. How does the information in the two tables compare? Explain.

Name______________________________________ Date______________________

Set 1 - Standard(s): 8.G.2, 8.G.3

Day 2 Task - Triangle congruence with coordinates

Triangles *ABC* and *PQR* are shown below in the coordinate plane:



a. Show that $\triangle ABC$ is congruent to $\triangle PQR$ with a reflection followed by a translation.

b. If you reverse the order of your reflection and translation in part (a) does it still map $\triangle ABC$ to $\triangle PQR$?

c. Find a second way, different from your work in part (a), to map $\triangle ABC$ to $\triangle PQR$ using translations, rotations, and/or reflections.

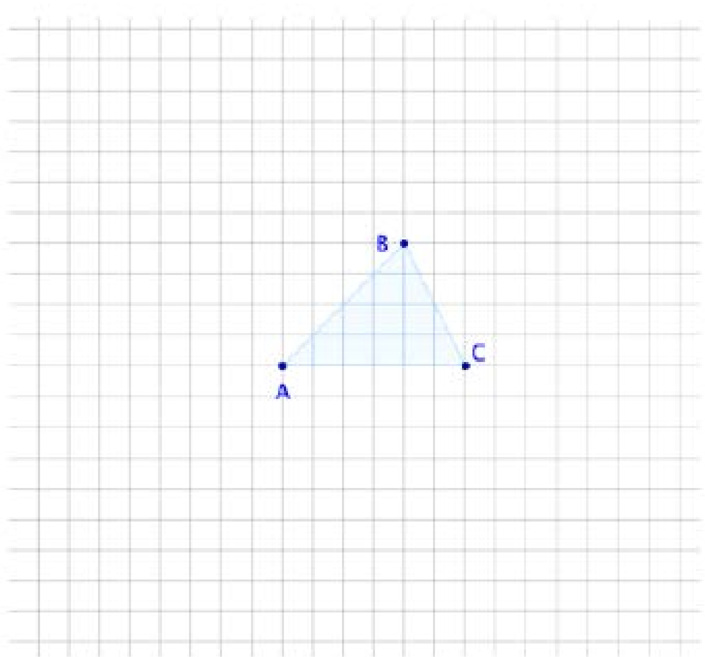
Name____ Date_______________________

Set 1 - Standard(s): 8.G.3

Illustrative Mathematics

Day 3 Task - Effects of Dilations on Length, Area, and Angles

Consider triangle *ABC*.



a. Draw a dilation of *ABC* with:

i. Center and scale factor 2.
ii. Center and scale factor 3.
iii. Center and scale factor $\frac{1}{2}$.

b. For each dilation, answer the following questions:

i. By what factor do the base and height of the triangle change? Explain.
ii. By what factor does the area of the triangle change? Explain.
iii. How do the angles of the scaled triangle compare to the original? Explain.

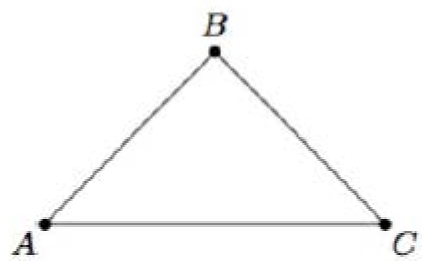
Name______________________________________ Date______________________

Set 1 - Standard(s): 8.G.4, 8.G.5

Illustrative Mathematics

Day 4 Task - Creating Similar Triangles

In triangle *ABC* below, $\angle B$ is a right angle and $|AB| = |BC|$:



Draw a line segment joining one of the vertices of $\triangle ABC$ to the opposite side so that it divides $\triangle ABC$ into two triangles which are both similar to $\triangle ABC$. Explain, using rigid motions and dilations, why the triangles are similar.

Name______________________________________ Date______________________

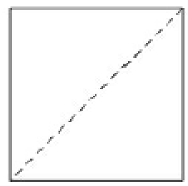
Set 1 - Standard(s): 8.G.1, G.CO.12

Illustrative Mathematics

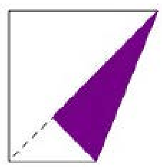
Day 5 Task - Origami Silver Rectangle

This task examines the mathematics behind an origami construction of a rectangle whose sides have the ratio ($\sqrt{2} : 1$). Such a rectangle is called a *silver rectangle*.

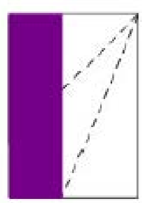
Beginning with a square piece of paper, first fold and unfold it leaving the diagonal crease as shown here:



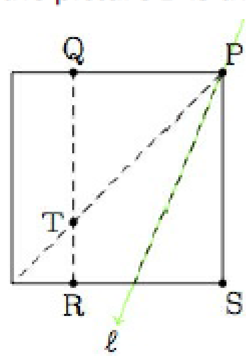
Next fold the bottom right corner up to the diagonal:



After unfolding then fold the left hand side of the rectangle over to the crease from the previous fold:



Here is a picture, after the last step has been unfolded, with all folds shown and some important points marked. In the picture T is the reflection of S about ℓ.



a. Suppose s is the side length of our square. Show that $|PT| = s$.

b. Show that $\triangle PQT$ is a 45-45-90 isosceles triangle.

c. Calculate $|PQ|$ and conclude that $PQRS$ is a silver rectangle.

Set 1 - Standard(s): 7.G.1, 7.G.4

Day 1 Task - Circumference of a Circle - KEY

a. Answers here will differ. For the circle with diameter one inch, for example, answers will likely vary between 3 inches and $3\frac{1}{4}$ inch and could be more (or less) depending on how careful the string is laid out around the circles. The table shows approximate answers:

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1	$3\frac{1}{8}$	$3\frac{1}{8}$
2	$6\frac{1}{4}$	$3\frac{1}{8}$
3	$9\frac{3}{8}$	$3\frac{1}{8}$
$\frac{1}{2}$	$1\frac{1}{2}$	3

b. To get a circle of diameter two inches from a circle of radius one inch we apply a scale factor of 2, which will double all lengths. The circumference of the circle is a measure of its "length" and so the circumference of a circle with diameter 2 inches will be twice the circumference of a circle with diameter 1 inch. In the same way, we apply a scale factor of 3 and $\frac{1}{2}$ to get the circles with diameter 3 inches and $\frac{1}{2}$ inch from the circle of radius 1 inch. The circumference of these circles will be scaled by the same factor as listed below in the table:

Diameter of Circle (inches)	Circumference of Circle (inches)	$\frac{\text{Circumference of Circle}}{\text{Diameter of Circle}}$
1	π	π
2	2π	π
3	3π	π
$\frac{1}{2}$	$\frac{1}{2}\pi$	π

c. The first table uses hand-drawn models of circles while the second table is for the abstract mathematical objects modeled by the drawings. This means that the diameters and circumferences in the second table are exact while they are only approximate for the first table. The first two columns of the second table is a ratio table while the first two columns of the first table may or may not be depending on how accurate the measurements are.

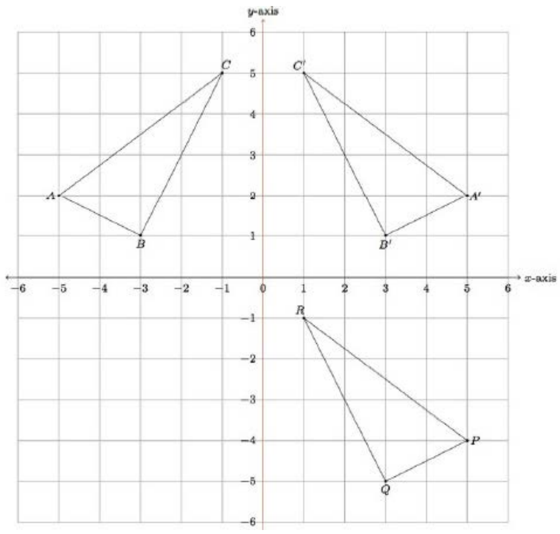
The number π is approximately equal to 3.1416 so an estimate of $3\frac{1}{8} = 3.25$ or 3 is reasonable using string and a ruler.

Set 1 - Standard(s): 8.G.2, 8.G.3

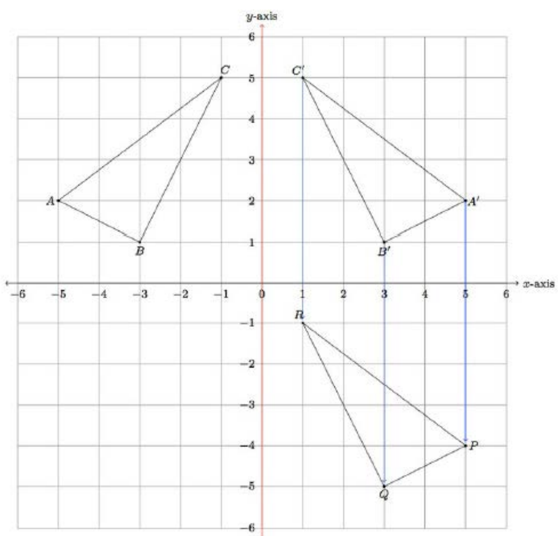
Illustrative Mathematics

Day 2 Task - Triangle congruence with coordinates - KEY

a. Below the y-axis is shaded red and triangle ABC is reflected over the y-axis. The image of this reflection is triangle $A'B'C'$. Reflecting about the y-axis leaves the y-coordinate of each point the same and switches the sign of the x-coordinate.



So, for example, $A = (-5, 2)$ so $A' = (5, 2)$. We can now see that translating triangle $A'B'C'$ down by 6 units puts it on top of triangle PQR:

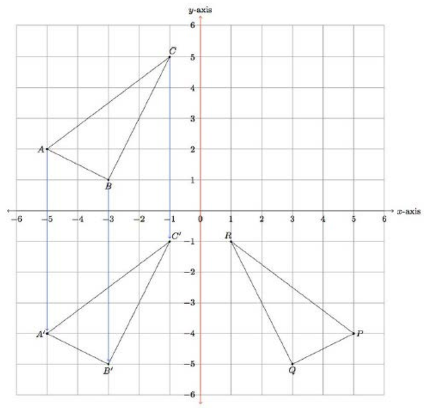


To find the coordinates after applying this translation, the x-coordinate stays the same and we subtract 6 from the y-coordinate of each point.

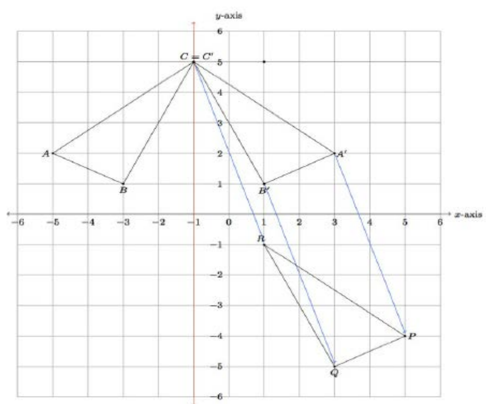
b. The answer here will depend on which reflection and translation have been chosen in part (a). For the reflection and translation chosen above, we reverse the order by first translating $\triangle ABC$ by 6 units downward and then reflecting over the y-axis. Below, the translated triangle is triangle $A'B'C'$ and its reflection over the y-axis is $\triangle PQR$:

Set 1 - Standard(s): 8.G.2, 8.G.3 (continued)

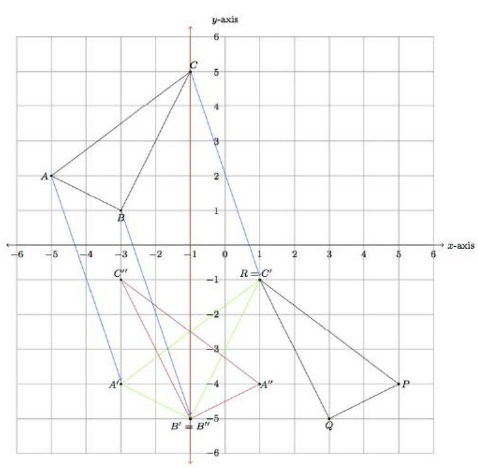
Day 2 Task - Triangle congruence with coordinates – KEY



Below is a different reflection through the vertical line through vertex A, which can be followed by the translation indicated by the blue arrows to show the congruence of $\triangle ABC$ with $\triangle PQR$:



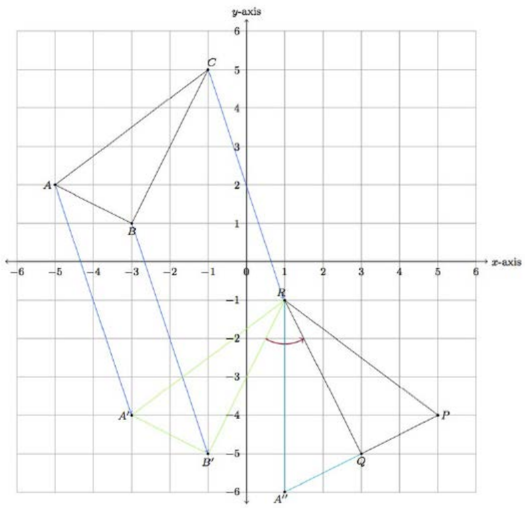
Unlike in the previous case, if we perform the translation first, giving the green triangle $A'B'C'$, and then the reflection, giving the purple triangle $A''B''C''$, this does not produce the triangle PQR. So in this case, performing the translation and reflection in a different order produces a different outcome.



c. One way to show the triangle congruence would be to align one vertex at a time. Graphically this is shown below:

Set 1 - Standard(s): 8.G.2, 8.G.3 (continued)

Day 2 Task - Triangle congruence with coordinates – KEY



First a translation is used to move C to R with the new triangle shown in green. If B' is the image of B under this translation, then a rotation, by the directed angle indicated in purple, moves B' to Q: the triangle after this transformation is indicated in blue, sharing one side with triangle PQR. If A'' is the image of A after the translation and rotation, then a reflection across $\overline{QR}$ moves A'' to P.

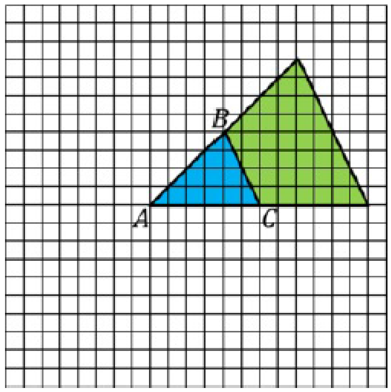
Set 1 - Standard(s): 8.G.3

Illustrative Mathematics

Day 3 Task - Effects of Dilations on Length, Area, and Angles – KEY

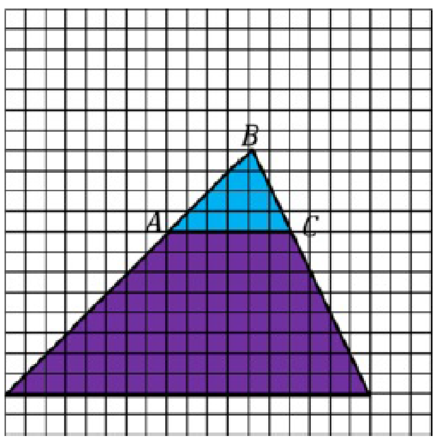
a. The three dilations are shown below along with explanations for the pictures:

i.



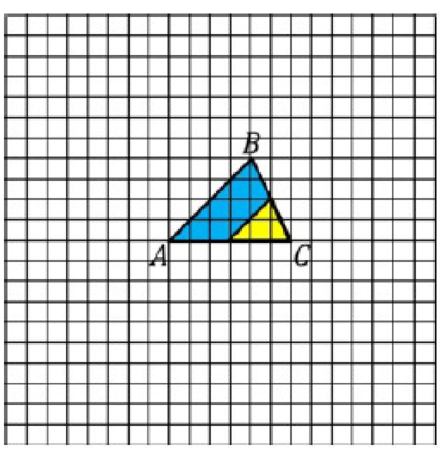
The dilation with center A and scale factor 2 doubles the length of segments $\overline{AB}$ and $\overline{AC}$. We can see this explicitly for $\overline{AC}$. For $\overline{AB}$, this segment goes over 6 units and up 4 so its image goes over 12 units and up 8 units.

ii.



The dilation with center B and scale factor 3 increases the length of $\overline{AB}$ and $\overline{AC}$ by a factor of 3. The point B does not move when we apply the dilation but A and C are mapped to points 3 times as far from B on the same line.

iii.



Set 1 - Standard(s): 8.G.3

Day 3 Task - Effects of Dilations on Length, Area, and Angles – KEY

The scale factor of $\frac{1}{2}$ makes a smaller triangle. The center of this dilation (also called a contraction in this case) is C and the vertices A and B are mapped to points half the distance from A on the same line segments.

b. i. When the scale factor of 2 is applied with center A the length of the base doubles from 6 units to 12 units. This is also true for the height which was 4 units for $\triangle ABC$ but is 8 units for the scaled triangle. Similarly, when the scale factor of 3 is applied with center B , the length of the base and the height increase by a scale factor of 3 and for the scale factor of $\frac{1}{2}$ with center C, the base and height of $\triangle ABC$ are likewise scaled by $\frac{1}{2}$.

ii. The area of a triangle is the base times the height. When a scale factor of 2 with center A is applied to $\triangle ABC$, the base and height each double so the area increases by a factor of 4: the area of $\triangle ABC$ is 12 square units while the area of the scaled version is 48 square units. Similarly, if a scale factor of 3 with center B is applied then the base and height increase by a factor of 3 and the area increased by a factor of 9. Finally, if a scale factor of 1/2 with center C is applied to $\triangle ABC$, the base and height are cut in half and so the area is multiplied by 1/4.

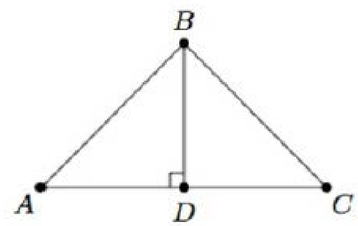
iii. The angle measures do not change when the triangle is scaled. For the first scaling, we can see that angle A is common to $\triangle ABC$ and its scaling with center A and scaling factor 2. Angle B is congruent to its scaled image as we can see by translating $\triangle ABC$ eight units to the right and 4 units up. Finally, angle C is congruent to its scaled image as we verify by translating $\triangle ABC$ 8 units to the right.

Set 1 - Standard(s): 8.G.4, 8.G.5

Illustrative Mathematics

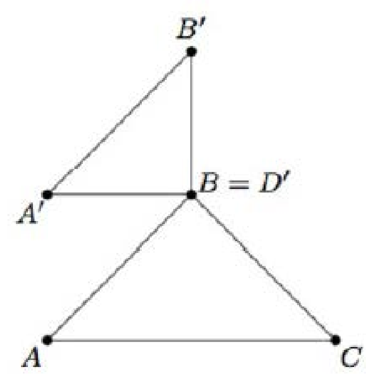
Day 4 Task - Creating Similar Triangles - KEY

Since $\triangle ABC$ is a right triangle, for our two smaller triangles to be similar to it, they will also need to be right triangles. This tells us which vertex our line segment must start from: If we start from vertex A, the line perpendicular to the opposite side is $\overleftrightarrow{AB}$ and this does not divide $\triangle ABC$ into two smaller triangles. Similar reasoning rules out vertex C. So our line must start from vertex B, and our line segment must be the one starting at B and perpendicular to $\overleftrightarrow{AC}$. Suppose then that D is the point on $\overline{AC}$ so that $\overleftrightarrow{BD}$ is perpendicular to $\overleftrightarrow{AC}$, shown below:



We will first show that $\triangle ADB$ is similar to $\triangle ABC$: the argument for $\triangle CDB$ is much the same. We can translate D to B and then rotate counterclockwise about B so that the right angle ADB matches up with the right angle CBA. We can then dilate the rotated triangle about B by a factor of $\frac{|BC|}{|DA|} = \frac{|BA|}{|DB|}$. A detailed explanation of each step in this argument is provided below. While the rigid motion part can be verified with patty paper, showing that $\frac{|BC|}{|DA|} = \frac{|BA|}{|DB|}$ to give us our scale factor, requires a more detailed argument.

We can move vertex D to match up with vertex B by translating along segment $\overline{DB}$. The effect of this is pictured below, with the translated image of $\triangle ADB$ being denoted $\triangle A'D'B'$:

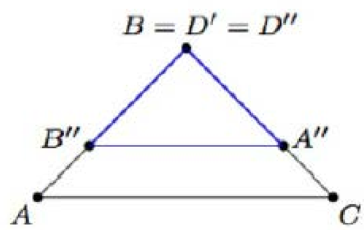


Next we apply a rotation, about B, through angle $A'BC$. We denote the image of $\triangle A'D'B'$ under the rotation as $\triangle A''D''B''$. The rotation will send $\overrightarrow{D'A'}$ to $\overrightarrow{BC}$ as shown below:

Set 1 - Standard(s): 8.G.4, 8.G.5

Illustrative Mathematics

Day 4 Task - Creating Similar Triangles – KEY



We know that B'' lies on $\overrightarrow{BA}$ because $\angle B''D''A''$ has the same measure as $\angle ABC$ as both are right angles.

We now apply a dilation with center B which maps A'' to C and B'' to A, which finishes the argument.

(Here is a formal argument that the dilation works as intended: As long as $\frac{|BC|}{|D''A''|} = \frac{|BA|}{|D''B''|}$, we can take this as our scale factor for the dilation. We know $|BA| = |BC|$ since $\triangle ABC$ is isosceles. We know that $m(\angle A''B''D'') = m(\angle ABD)$ and $m(\angle B''A''D'') = m(\angle BAD)$ since rigid motions do not change angle measures. But $m(\angle BAD) = 45$ since this is the same as $\angle A$ in $\triangle ABC$. This means that $m(\angle ABD)$ is also 45 because $m(\angle ADB) = 90$ and the sum of the angles in $\triangle ADB$ is 180 degrees. This means that $|DA| = |DB|$ and so $|D''A''| = |D''B''|$. Since we have already seen that $|BC| = |BA|$ we can conclude that $\frac{|BC|}{|D''A''|} = \frac{|BA|}{|D''B''|}$ and this is the desired scale factor to complete the similarity between $\triangle ADB$ and $\triangle CBA$.)

The similarity between $\triangle CDB$ and $\triangle ABC$ can be shown as above. Alternatively, $\triangle ABC$ is a right isosceles triangle with line of symmetry $\overleftrightarrow{BD}$. So we can reflect $\triangle CDB$ about $\overleftrightarrow{BD}$, mapping it to $\triangle ADB$. Then we can follow the identical steps outlined above.

Set 1 - Standard(s): 8.G.1, G.CO.12

Illustrative Mathematics

Day 5 Task - Origami Silver Rectangle – KEY

a. Since reflection about ℓ maps P to itself and maps S to T this means that segments $\overline{PS}$ and $\overline{PT}$ are interchanged by reflection about ℓ. Hence

$$|PS| = |PT|$$

because reflections preserve lengths of segments. We are given that $|PS| = s$ since it is one side of the square so $|PT| = s$.

b. We know that $m(\angle QPT) = 45$ because the diagonal of a square bisects the 90 degree angles. Alternatively, reflection over the diagonal $\overleftrightarrow{PT}$ is a symmetry of the square: this reflection interchanges angles QPT and SPT and so these must each measure 45 degrees. Angle TQP is a right angle because $\overline{QR}$ is a crease resulting from a horizontal fold of the paper. Since

$$m(\angle QTP) + m(\angle QPT) + m(\angle TQP) = 180$$

this means that $m(\angle QTP) = 45$. Thus $\triangle PQT$ is a 45-45-90 triangle: it is isosceles because $m(\angle QTP) = m(\angle QPT)$.

c. To find $|PQ|$ we use the fact that $\triangle PQT$ is a right isosceles triangle. Thus we know that $|TQ| = |PQ|$ and, from the Pythagorean Theorem,

$$|PQ|^2 + |TQ|^2 = |PT|^2.$$

Substituting $|PQ|$ for $|TQ|$ this is equivalent to

$$2|PQ|^2 = |PT|^2.$$

From part (a) this means that

$$|PQ|^2 = \frac{s^2}{2}.$$

So

$$|PQ| = \frac{s}{\sqrt{2}}.$$

We can now check that $PQRS$ is a silver rectangle. Our calculations show that $|PS| : |PQ|$ is $s : \frac{s}{\sqrt{2}}$. But the ratio $s : \frac{s}{\sqrt{2}}$ is equivalent to the ratio $\sqrt{2} : 1$ as we can see using a scaling factor of $\frac{\sqrt{2}}{s}$.