EXPRESSIONS AND EQUATIONS TASKS: OVERVIEW

Resources:

Attached you will find a set of **15 tasks for Expressions and Equations**. The tasks are best used by partners or small groups. Since these tasks are more in-depth than the practice items, you can use them as the foundation of your lesson, especially if they address a standard you are working on that day.

The purpose of using tasks is to help you see how students solve problems, and understand their thought processes while they work. Being able to work with others to get productive discourse, to be able to explain their thinking to another student, and develop a solution, is the most effective way to assess student understanding. These tasks require students to do just that – think about an efficient strategy to solve the problem, show their work and justify their reasoning. This is the ultimate goal for what we want students to be able to do. Being able to gather evidence of student learning and misconceptions in the moment, will give you the flexibility to change your instruction to meet their needs. As the instructional decision-maker, you are able to adjust your methods for whole class or small groups to address student misconceptions and move them toward proficiency.

The goal is to have tasks that can be used as the lesson of the day along with or in lieu of the practice items. There are tasks for each day per week that represent the 5 domains in 8th grade. We would like for you to use these tasks along with the practice items for a 10 week period between the time you receive them and the end of January. If used daily, in accordance with our recommendations or tips, for student and teacher practice, the outcome will be an improvement in ACT ASPIRE test scores.

At the end of each task packet, you will find an answer key for your use. **Although answer keys are provided, students should explain their thinking during the discussion of the tasks.** Some tasks include several representations of the solution.

Recommendations or Tips:

When implementing the tasks with your students, please allow students read through the tasks before starting to see if they have any questions about vocabulary or what the task is asking them to do. Taking the time to do these things now, will help assure that the students are familiar with mathematical vocabulary and different question types before the actual test.

Providing Feedback to Students:

An important process for understanding student thinking, is to debrief and provide feedback. This can be done during the exploration phase of the sharing out process by asking effective questions similar to the ones below which will also help students verbalize their reasoning :

- What did the problem ask you to do?
- What information do you see in the problem?
- What did you do first to solve this problem?
- What did you do next?
- What strategy did you use to solve the problem? Why did you use that particular strategy?
- Is there another strategy that you could use to solve the problem?
- Who else started this same way?
- Who started a different way?

- What are some strategies that you heard today that you would like to try when solving a similar problem in the future?

Another option is to let the groups record their solution(s) to the task on chart paper. They can then share out with the whole class. With this option the students are able to present their thinking, justify their reasoning, and answer questions from the other students.

Answer Key:

The information above is intended to help teachers get at student understanding of the mathematical idea(s) in each problem. Also provided is an Answer Key for each set of tasks. The Answer Key provides more information on the expected student response for each task. While it is important for students to get the answer right, it is equally important for them to understand how their thinking leads or does not lead to a correct solution. Incorrect solutions set the stage for teachable moments!!!!

Name____ Date_______________________

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Illustrative Mathematics

Day 1 Task - Coupon versus discount

You have a coupon worth $18 off the purchase of a scientific calculator. At the same time the calculator is offered with a discount of 15%, but no further discounts may be applied. For what tag price on the calculator do you pay the same amount for each discount? Show your work and explain your answer.

Name____ Date_______________________

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Illustrative Mathematics

Day 2 Task - Sammy's Chipmunk and Squirrel Observations

For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide? Show your work.

Name____ Date_______________________

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Illustrative Mathematics

Day 3 Task - The Sign of Solutions

Without solving them, say whether these equations have a positive solution, a negative solution, a zero solution, or no solution. Explain your reasoning.

a. $3x = 5$
b. $5z + 7 = 3$
c. $7 - 5w = 3$
d. $4a = 9a$
e. $y = y + 1$

Name____ Date_______________________

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Day 4 Task - How Many Solutions?

Consider the equation $5x - 2y = 3$. If possible, find a second linear equation to create a system of equations that has:

a. Exactly 1 solution.
b. Exactly 2 solutions.
c. No solutions.
d. Infinitely many solutions.

Show your work and explain your answer for each case.

Bonus Question: In each case, how many such equations can you find?

Name____ Date_______________________

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Day 5 Task - Fixing the Furnace

Ivan's furnace has quit working during the coldest part of the year, and he is eager to get it fixed. He decides to call some mechanics and furnace specialists to see what it might cost him to have the furnace fixed. Since he is unsure of the parts he needs, decides to compare the costs based only on service fees and labor costs. Shown below are the price estimates for labor that were given to him by three different companies.

Each company has given the same time estimate for fixing the furnace.

- Company A charges \$35 per hour to its customers.
- Company B charges a \$20 service fee for coming out to the house and then \$25 per hour for each additional hour.
- Company C charges a \$45 service fee for coming out to the house and then \$20 per hour for each additional hour.
- For which time intervals should Ivan choose Company A, Company B, Company C? Support your decision with sound reasoning and representations. Consider including equations, tables, and graphs.

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Illustrative Mathematics

Day 1 Task - Coupon versus discount – KEY

Solution: Coupon versus discount

If p is the tag price in dollars then $p - 18$ is the price using the coupon while $0.85p$ is the price using the 15% discount. Then when the discounts are the same,

$$p - 18 = 0.85p$$
$$p - 0.85p = 18$$
$$0.15p = 18$$
$$P = 120.$$

Thus, the tag price is $120.

Solution: Coupon versus discount: solution by comparing the price reduction

The cost is the same when the reduction in price is the same. The coupon always gives an $18 reduction in price; if p is the tag price in dollars then the discount gives a reduction in price of 0.15p$. So the cost is the same when

$$18 = 0.15p$$
$$p = 120.$$

Thus, the tag price is $120.

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Day 2 Task - Sammy's Chipmunk and Squirrel Observations – KEY

Solution: 1 Table

We start by making a table with the number of holes dug by the chipmunk and squirrel and the number of acorns they have buried. We are looking for a common number of acorns and then need to study the number of holes.

Holes for Chipmunk	Chipmunk's acorns	Holes for Squirrel	Squirrel's acorns
1	3	1	4
2	6	2	8
3	9	3	12
4	12	4	16

Notice that the first common number of acorns we find is 12. The chipmunk hides 12 acorns in 4 holes while the squirrel hides 12 acorns in 3 holes. This is a difference of only one hole so if we want a difference of 4 holes we can repeat this scenario four times to get:

Holes for Chipmunk	Chipmunk's acorns	Holes for Squirrel	Squirrel's acorns
16	48	12	48

So the chipmunk and squirrel each buried 48 acorns. This is the only answer that works because with more acorns the difference in the number of holes dug goes up and with fewer acorns this difference becomes smaller.

Solution: 2 Using equations

We can introduce a variable h for the number of holes that the chipmunk has dug. Since the chipmunk hides 3 acorns in each hole this make a total of $3h$ acorns that the chipmunk hides. The squirrel, on the other hand, has dug 4 fewer holes than the chipmunk: this is represented by the expression $h - 4$. The squirrel hides 4 acorns in each hole so this means that squirrel has hidden $4 X (h – 4)$ acorns in total. We are given that the chipmunk and squirrel have hidden the same number of acorns so $3h = 4 X (h – 4)$.

Using the distributive property on the right hand side gives $3h = 4h – 16$.
Subtracting 3 from both sides and adding 16 to both sides gives $16 = h$.
So the squirrel and chipmunk have each made 16 wholes. Since the chipmunk hides 3 acorns in each hole this means that the chipmunk (and squirrel) have each hidden 48 acorns.

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Day 3 Task - The Sign of Solutions – KEY

Solution: Reasoning About Operations

a. 3x = 5 has a positive solution. Since 3 is positive, the second factor in the product 3x also has to be positive to produce the positive result 5.
b. 5z + 7 = 3 has a negative solution. This is because we are adding 7 to the number *5z*, and get a result of 3. The only way for that to happen is to have started with a negative number. So *5z* must be negative, and since 5 is positive, that is only true when *z* is negative.
c. 7 – 5w = 3 has a positive solution. We know this because we are starting with 7, a positive number, and we are ending at 3, which is a smaller number. To get there we are subtracting *5w*, which must also be positive in order for us to get from 7 to 3. Since *5w* is positive, it follows that *w* must be positive.
d. 4a = 9a has zero as its solution. 4 multiplied by a number can never equal 9 multiplied by the same number, unless that number is zero.
e. y = y + 1 has no solution. If it did, we would be saying that some number, *y*, is equal to the number that is one more than itself, which is impossible.

Solution: Solutions that are intended to be understandable to kids.

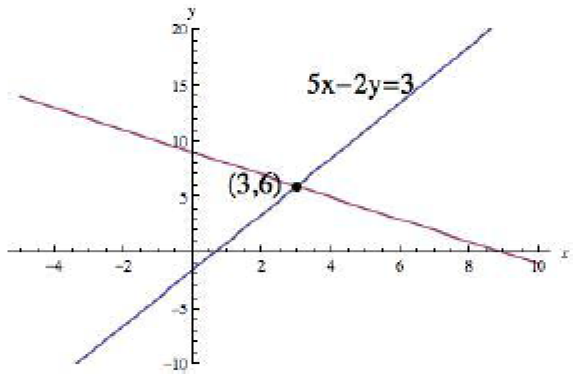
a. If 3*x* = 5, then *x* is positive. Five is the product of 3 and *x*. 5 is positive. For a product to be positive, both factors must have the same sign. 3 is positive, so *x* must be positive.
b. If 5*z* + 7 = 3, then *z* is negative. We are adding something to 7 and getting something smaller than 7. What we are adding is *5z*. This must be negative, so *z* must be negative.
c. If 7 − 5*w* = 3, then *w* must be positive. We are subtracting *5w* from 7 and getting something smaller than 7. So what we are subtracting must be positive. If is positive, then *w* is positive.
d. If 4*a* = 9*a* then *a* = 0. If a = 1, then the equation is saying 4 = 9, so 1 is not a solution. If a = 2, then the equation is saying 8 = 18, so 2 is not a solution. If a = 1/2, then the equation is saying 2 = 4.5, so 1/2 is not a solution. If a = -3, then the equation is saying -12 = -27, so – 3 is not a solution. Four things can never be equal to 9 things, since there are 5 more things in 9 things than in 4 things. But the exception is when the things are nothings. If you have 4 nothings and I have 9 nothings, then we both have nothing. Plenty of nothing is nothing. A bigger plenty of nothing is still nothing.
e. There is no number that stays the same when we add 1 to it. So there is no number *y* that makes y = y + 1 true.

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Illustrative Mathematics

Day 4 Task - How Many Solutions? - KEY

a. While it is possible to solve this problem purely algebraically, thinking about how the question relates to the graphs of the equations makes the problem much easier to solve. To have exactly one solution, we want the graph of 5x – 2y = 3 and the graph of the equation we come up with to intersect at exactly one point, as shown below.



To do this, we find a point on the line 5x – 2y = 3 and create another line that also contains that point but is not the same line. We can pick any value for *x* and find the corresponding value for *y*. Arbitrarily, let us consider x = 3. By substitution, we see that

$$5(3) - 2y = 3 \Longrightarrow$$
$$15 - 2y = 3 \Longrightarrow$$
$$y = 6$$

So (3, 6) is a point on the graph of 5x – 2y = 3. Now we find any values *a, b,* or *c* in the equation *ax + by = c* that "work" with x = 3 and y = 6 are not the same values as in the given equation. There are many possible ways to do this. For example, if we choose *a = 1* and *b = 1*, then *x + y = 3 + 6 = 9* so *c* must be 9. In other words, (3, 6) is a solution to the equation *x + y = 9*.

Taking the two equations together as a system of equations,

$$5x - 2y = 3$$
$$x + y = 9$$

we can verify algebraically that there is exactly one solution to this system, namely (3, 6).

b. To have exactly two solutions, we would want a second line that intersects the graph of 5x – 2y = 3 at exactly two points. However, this is not possible. Since two points determine one and only one line, we must conclude that if two lines intersect at two points, they must actually be the same line.

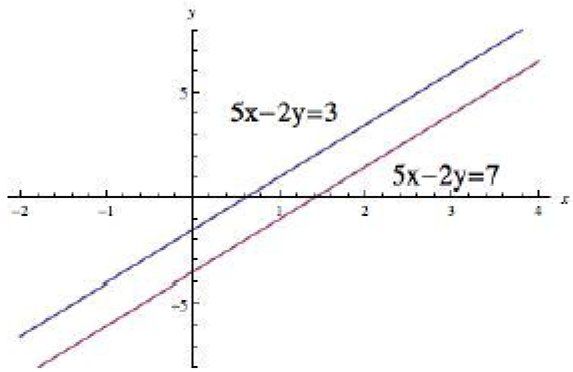
c. To have no solutions, we want our new line and the graph of 5x – 2y = 3 to not intersect anywhere, meaning that the two lines are parallel. Consider any equation of the form 5x – 2y = c where c ≠ 3, for example 5x – 2y = 7. Then our corresponding system of equations is

$$5x - 2y = 3$$

$5x - 2y = 7$

From this we can see that no matter what values of *x* and *y* we substitute into the two equations, $5x - 2y$ can never equal 3 and 7 simultaneously, meaning that no point (x, y) can be on both lines at the same time. So, this system of equations has no solutions.

Graphically we can think of two lines with the same slope but different *y*-intercepts.



d. To have infinitely many solutions, we want our equation and $5x - 2y = 3$ to intersect everywhere. In other words, they will be the same line. One way to denote this is to simply use the same equation, $5x - 2y = 3$, or just multiply both sides of the equation by a constant; let's say we multiply each term by 2. Then for the system of equations

$$5x - 2y = 3$$
$$10x - 4y = 6$$

any *x* and *y* pair that satisfies the first equation will satisfy the second, since taking two numbers that are equal and multiplying them both by 2 will result in two equal numbers. So this system has infinitely many solutions, as the equations both correspond to the same line and lines have infinitely many points.

(Bonus) In parts (a), (c), and (d), there are infinitely many equations that can be found.
In part (a), the point/solution (3, 6) was arbitrary, and we could have picked any point, on the given line and drawn a line through the point at any slope. In part (c), we could have chosen any constant (other than 3) to take the place of the 3, and it would have still resulted in a system with no solutions. In part (d), we could have multiplied the terms of the given equation by any constant (besides 0) and it would have described the same line, though the equation would look different, resulting in infinitely many solutions to the corresponding system of equations. However, the graphs of the different equations in (d) would all look identical.

Set 3 - Standard(s): 7.EE.3, 7.EE.4, 8.EE.7, 8.EE.8

Illustrative Mathematics

Day 5 Task - Fixing the Furnace – KEY

Let x be the number of hours it takes to fix the furnace, and y the cost in dollars of fixing the furnace. Company A's cost can be modeled with the equation

$$y = 35x,$$

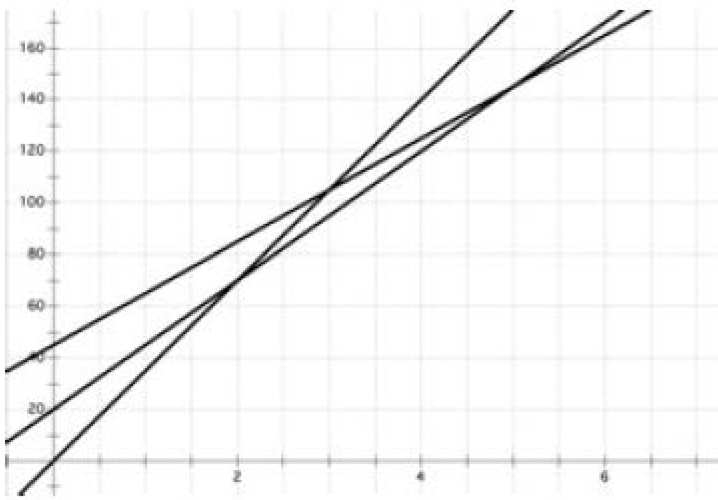
company B's with the equation

$$y = 25x + 20,$$

and company C's with the equation

$$y = 20x + 45.$$

Graphing the equations helps students visualize the solution.



Looking at the graph, it appears that the graph that represents the cost for company A is below the other two graphs for any amount of time less that 2 hours, where the graph intersects the line that represents the cost of company B. The graph for company B is below the other two up until 5 hours, where it intersects the graph that represents the cost for company C. In other words, company A is least expensive for 0 to 2 hours, company B is least expensive for 2 to 5 hours, and company C is least expensive for more than five hours.

Students should get in the habit of checking the coordinates of the intersection points algebraically, since it is often the case that the coordinates of the intersection points are not whole numbers (and therefore not easy to read from the graph). This also reinforces the relationship between the algebraic representation of the solution and the graphical representation of the solution.

To find the solution algebraically, consider each pair of equations as a system. Let x be the number of hours it takes to repair the furnace and y be the cost of the repair (without parts).

To find the number of hours for which company A and company B cost the same, consider $y = 35x$ and $y = 25x + 20$. Substituting for y, we get

$$35x = 25x + 20.$$

The solution to this equation gives the number of hours for which company A and company B cost the same. Solving this equation, we find that the cost of company A and company B is the same for 2 hours of labor. The cost is $70.

To find the number of hours for which company A and company C cost the same, consider $y = 35x$ and $y = 20x + 45$. Substituting for y, we get

$$35x = 20x + 45.$$

Solving this equation, we find that the cost of company A and company C is the same for 3 hours labor, for a total cost of $105.

To find the number of hours for which company B and company C cost the same, consider $y = 25x + 20$ and $y = 20x + 45$. Substituting for y, we get

$$25x + 20 = 20x + 45.$$

Solving this equation, we find that the cost of company B and company C is the same for 5 hours labor, for a total cost of $145.

Some additional substitution of values shows that company C is $15 more expensive than A and B at 2 hours. Company B is $10 less than A and C at 3 hours. And company A is $30 more expensive at 5 hours than companies B and C.

As before, we found that company A is the least expensive up to a time of 2 hours, at which point company A and B are the same cost. From 2 hours to 5 hours, company B is the least expensive, and at 5 hours company B and C both cost $145. For more than 5 hours, Company C will be the least expensive.