EXPRESSIONS AND EQUATIONS TASKS: OVERVIEW

Resources:

Attached you will find a set of **15 tasks for Expressions and Equations**. The tasks are best used by partners or small groups. Since these tasks are more in-depth than the practice items, you can use them as the foundation of your lesson, especially if they address a standard you are working on that day.

The purpose of using tasks is to help you see how students solve problems, and understand their thought processes while they work. Being able to work with others to get productive discourse, to be able to explain their thinking to another student, and develop a solution, is the most effective way to assess student understanding. These tasks require students to do just that – think about an efficient strategy to solve the problem, show their work and justify their reasoning. This is the ultimate goal for what we want students to be able to do. Being able to gather evidence of student learning and misconceptions in the moment, will give you the flexibility to change your instruction to meet their needs. As the instructional decision-maker, you are able to adjust your methods for whole class or small groups to address student misconceptions and move them toward proficiency.

The goal is to have tasks that can be used as the lesson of the day along with or in lieu of the practice items. There are tasks for each day per week that represent the 5 domains in 8^{th} grade. We would like for you to use these tasks along with the practice items for a 10 week period between the time you receive them and the end of January. If used daily, in accordance with our recommendations or tips, for student and teacher practice, the outcome will be an improvement in ACT ASPIRE test scores.

At the end of each task packet, you will find an answer key for your use. **Although answer keys are provided, students should explain their thinking during the discussion of the tasks.** Some tasks include several representations of the solution.

Recommendations or Tips:

When implementing the tasks with your students, please allow students read through the tasks before starting to see if they have any questions about vocabulary or what the task is asking them to do. Taking the time to do these things now, will help assure that the students are familiar with mathematical vocabulary and different question types before the actual test.

Providing Feedback to Students:

An important process for understanding student thinking, is to debrief and provide feedback. This can be done during the exploration phase of the sharing out process by asking effective questions similar to the ones below which will also help students verbalize their reasoning :

- What did the problem ask you to do?
- What information do you see in the problem?
- What did you do <u>first</u> to solve this problem?
- What did you do next?
- What strategy did you use to solve the problem? Why did you use that particular strategy?
- Is there another strategy that you could use to solve the problem?
- Who else started this same way?
- Who started a different way?

Name______________________________________ Date______________________

Set 2 - Standard(s): 7.EE.1, 7.EE.2, 8.EE.5, 8.EE.6

Day 1 Task

Malia is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets.

a. Write an expression that gives the number of tickets Malia has left in terms of *x*, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

b. 14 – 2x represents the number of tickets Malia has left after she has gone on *x* rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

14

-2

2x

c. 2 (7 – x) also represents the number of tickets Malia has left after she has gone on *x* rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

7

(7 – x)

2

Name____ Date_______________________

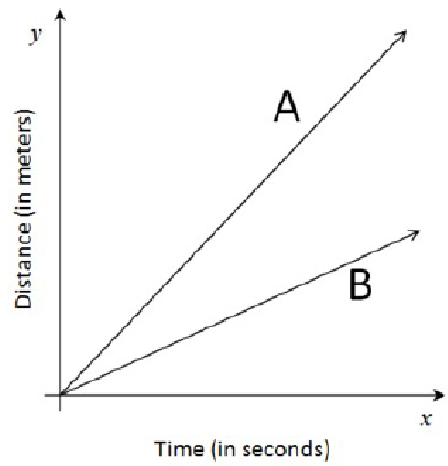
Set 2 - Standard(s): 8.EE.5, 8.EE.6

Illustrative Mathematics

https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/57

Day 2 Task - Comparing Speeds in Graphs and Equations

The graphs below show the distance two cars have traveled along the freeway over a period of several seconds. Car A is traveling 30 meters per second.



Which equation from those shown below is the best choice for describing the distance traveled by car B after *x* seconds? Explain.

a. y = 85x
b. y = 60x
c. y = 30x
d. y = 15x

Name______________________________________ Date______________________

Set 2 - Standard(s): 8.EE.5, 8.EE.6

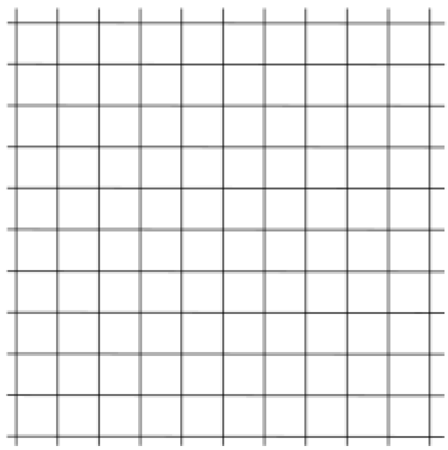
Day 3 Task - Coffee by the Pound

Lena paid $18.96 for 3 pounds of coffee.

a. What is the cost per pound for this coffee?

b. How many pounds of coffee could she buy for $1.00?

c. Draw a graph in the coordinate plane of the relationship between the number of pounds of coffee and the total cost.



d. In this situation, what is the meaning of the slope of the line you drew in part (c)?

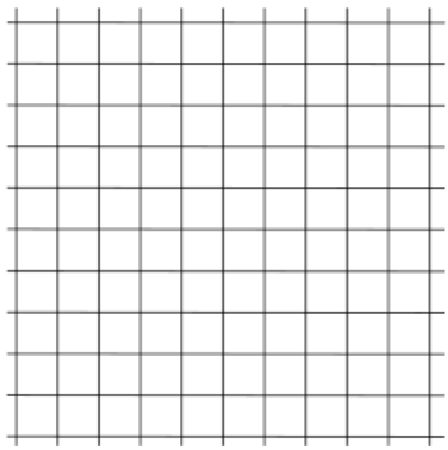
Name______________________________________ Date______________________

Set 2 - Standard(s): 8.EE.5, 8.EE.6

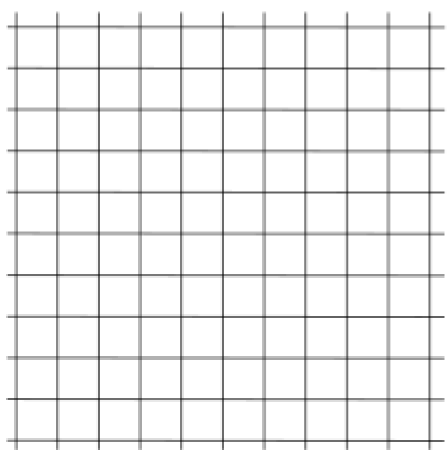
Day 4 Task - Stuffing Envelopes

Anna and Jason have summer jobs stuffing envelopes for two different companies. Anna earns \$14 for every 400 envelops she finishes. Jason earns \$9 for every 300 envelopes he finishes.

a. Draw graphs and write equations that show the earnings, *y* as functions of the number of envelopes stuffed, *n* for Anna and Jason.



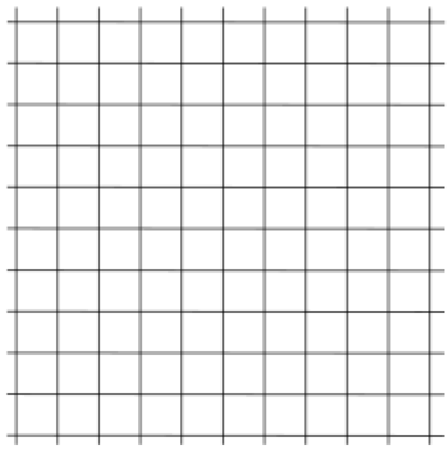
b. Who makes more from stuffing the same number of envelopes? How can you tell this from the graph?



Illustrative Mathematics

Day 4 Task

c. Suppose Anna has savings of $100 at the beginning of the summer and she saves all her earnings from her job. Graph her savings as a function of the number of envelopes she stuffed, *n*. How does this graph compare to her previous earnings graph? What is the meaning of the slope in each case?



Name____ Date_______________________

Set 2 - Standard(s): 8.EE.5, 8.EE.6

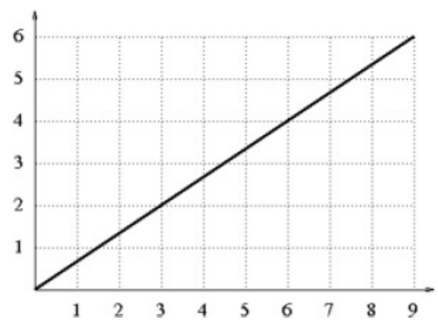
Illustrative Mathematics

Day 5 Task

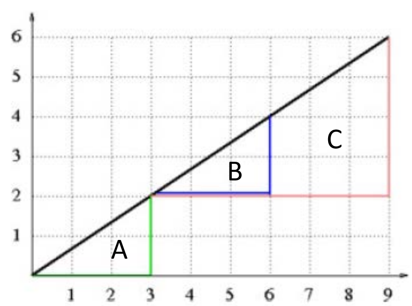
The slope between two points is calculated by finding the change in *y*-values and dividing by the change in x-values. For example, the slope between the points (7, -15) and (-8, 22) can be computed as follows:

- The difference in the y-values is $-15 - 22 = -37$.
- The difference in the x-values is $7 - (-8) = 15$.
- Dividing these two differences, we find that the slope is $-\frac{37}{15}$.

Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.



Eva finds the slope between the points (0, 0) and (3, 2). Carl finds the slope between the points (3, 2) and (6, 4). Maria finds the slope between the points (3, 2) and (9, 6). They have each drawn a triangle to help with their calculations (shown below).



i. Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the *x*- and *y*-values be interpreted geometrically in the pictures they have drawn?

ii. Consider any two points (x_1, y_1) and (x_2, y_2) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?

Set 2 - Standard(s): 7.RP.2 8.EE.4

Day 1 Task – KEY

a. Here are some possible expressions:

   $14 - 2x$
   $2(7 - x)$
   $14 - x - x$
   $10 - 2x + 4$
   $-2x + 14$

b. In the expression $14 - 2x$, the 14 represents the number of tickets Malia started with since the value of the expression is 14 when x = 0. The -2 represents the number of tickets she spends per ride. *2x* represents the number of tickets she has to subtract from her initial amount after riding *x* rides.

c. In the expression 2(7 – x), the represents the total number of rides Malia can go on. (7 – x) represents the number of rides she has left and the 2 represents the number of tickets required for each ride Malia has left.

Set 2 - Standard(s): 8.EE.5, 8.EE.6

Day 2 Task - Comparing Speeds in Graphs and Equations – KEY

Solution: Slope is speed

The graph that represents the distance of car A after seconds is steeper, so it must have a bigger slope. The slope can be interpreted as the unit rate; in this case it tells you the number of meters the car travels per second. Since the slope for car A is larger than the slope for car B, A is traveling faster. The only equation with a smaller slope (and thus a slower speed) is equation *d*. Thus, *y = 15x* is the best choice for the equation for car B of those that are given.

Solution: Add a scale to the graph, and plot the different possible answers

Since we know that car A travels 30 meters per second, we can place the point (1, 30) anywhere on the given graph. We can use this point to lay out a rough scale for the rest of the graph. Then we can plot points from each of the four equations given as possible answers, and see that answer *d* comes closest to the line for car B.

Set 2 - Standard(s): 8.EE.5, 8.EE.6

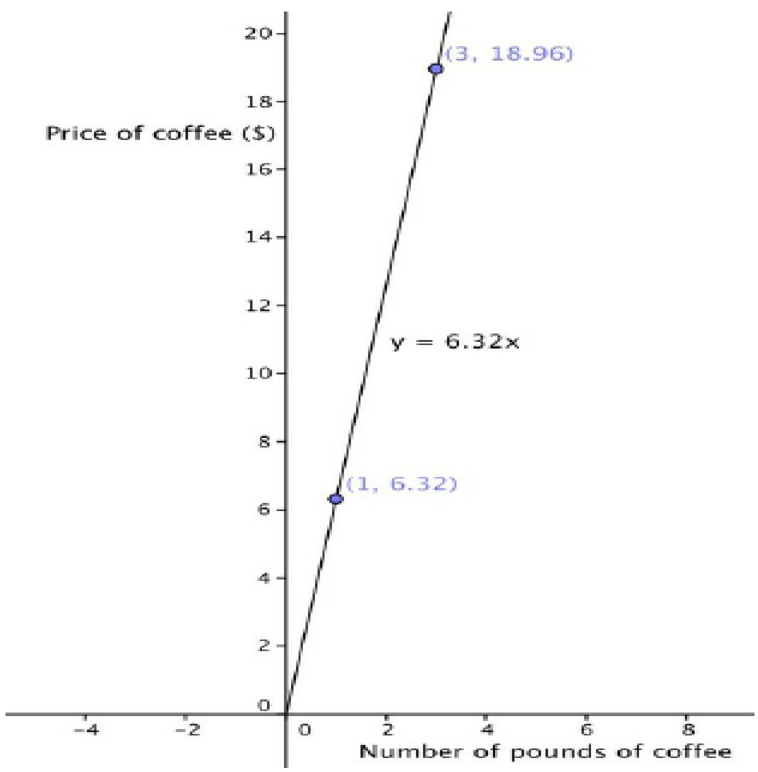
Day 3 Task - Coffee by the Pound – KEY

a. If you divide the cost for three pounds by three, you will get the cost per pound. Coffee costs $6.32 per pound.

b. If you divide the number of pounds by the cost for three pounds, you will get the amount of coffee one can purchase for $1.00. You can buy approximately 0.16 pounds of coffee for a dollar.

c. There are two possible graphs depending on what you choose x to represent and what you choose y to represent.

   If we let x indicate the number of pounds of coffee and let y indicate the total price, then the solver may produce a graph by drawing a line through the origin and the point (3, 18.96); see below.



   If we let *x* indicate the total price and let *y* indicate the number of pounds of coffee then the solver may produce a graph by drawing a line through the origin and the point (18.96, 3).

d. With the choice for *x* and *y* we made, the slope is the cost per pound of coffee, which is $6.32. If we had chosen the other order, the slope would have been the amount of coffee one could buy for a dollar, which is 0.16 pounds.

Set 2 - Standard(s): 8.EE.5, 8.EE.6

Illustrative Mathematics

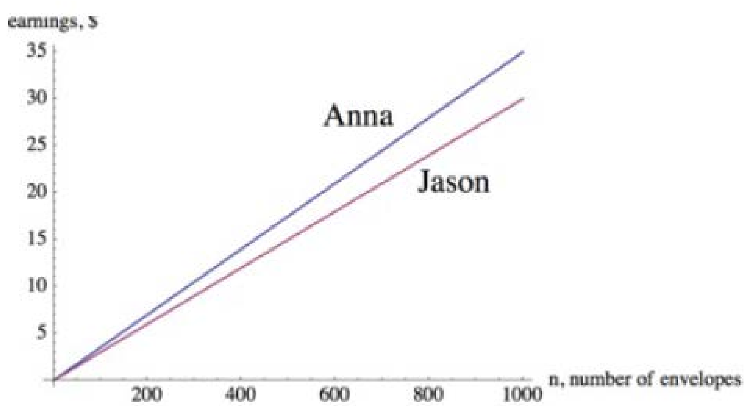
Day 4 Task - Stuffing Envelopes – KEY

a. The amount of money earned, *y*, and the number of envelopes stuffed, *n,* are proportional to each other. Since Anna earns $14 for 400 envelope, she makes $\frac{14}{400}$ = 0.035 dollars per envelope. Therefore, we have *y = 0.035n* for Anna's equation.

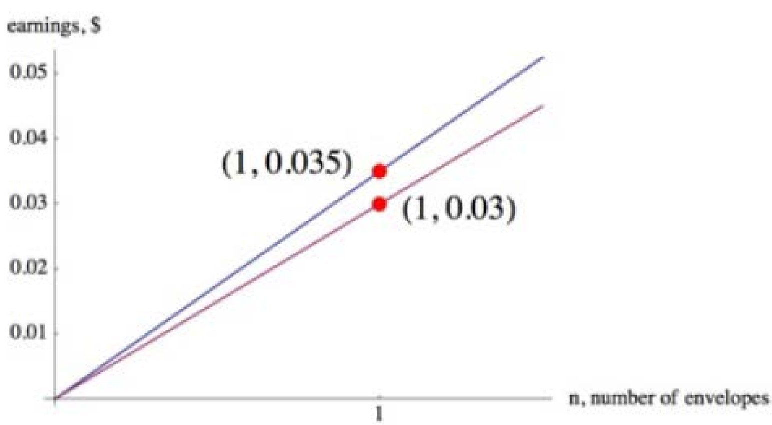
   Jason earns $9 for every 300 envelopes he stuffs, so he makes $\frac{9}{300}$ = 0.03 dollars per envelope. So we have *y= 0.03n* for Jason's equation.

   Since Anna's equation has a larger unit rate, 0.035 dollars per envelope vs. 0.03 dollars per envelope for Jason, she has the higher paying job.

   The graphs of the equations are shown below.



b. We know that we can find the unit rate of proportional relationships by finding the point on the line with horizontal coordinate 1, as shown in the graph below.



   For every envelope they stuff, Anna makes half a cent more than Jason. Since Anna makes more money per envelope, her earnings increase faster than Jason's. Therefore, her earnings line is steeper than Jason's.

c. Anna still earns money at the same rate as before, but now her earnings are added to her savings of $100. The graph showing her total savings, including the money she earns, is still linear but it has a higher starting value. The new line is parallel to the previous earnings line but while the previous line went through the point (0, 0), the new line starts at the point (0, 100). This shows that when she starts working she already has $100 in savings.

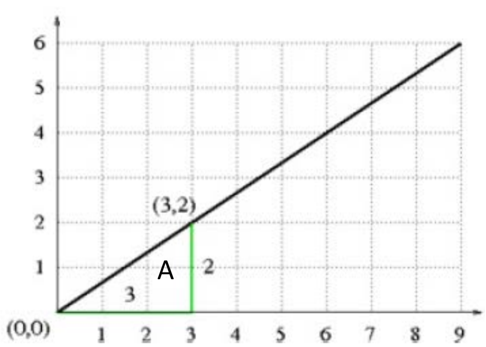
For her earnings graph, we see that she will make $0.035 for every envelope she stuffs, but for her savings, she will save an *additional* $0.035 for every *additional* envelope she stuffs.

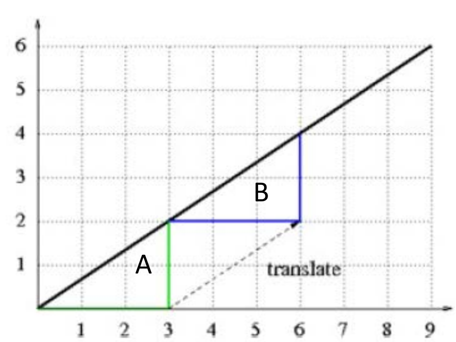
Set 2 - Standard(s): 8.EE.5, 8.EE.6

Illustrative Mathematics

Day 5 Task – KEY

a. Eva is using triangle A, since two of the vertices of the triangle are at (0, 0) and (3, 2). She will next find the length of each leg in the right triangle. The horizontal leg length is the difference between the *x*-coordinates, which is 3. The vertical leg length is the difference between the *y*-coordinates, which is 2. So the line rises by 2 units for every horizontal increase of 3 units. Therefore the slope is $\frac{2}{3}$.



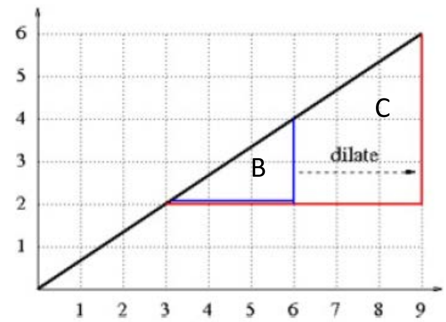


Carl is using triangle B, since two of the vertices of the triangle are at (3, 2) and (6, 4). He will next find the length of each leg in the right triangle. The horizontal leg length is the difference between the *x*-coordinates, which is 3. The vertical leg length is the difference between the *y*-coordinates, which is 2. So the line rises by 2 units for every horizontal increase of 3 units. Therefore the slope is $\frac{2}{3}$.

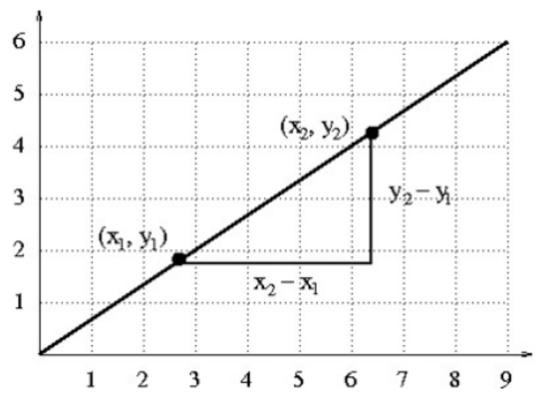
Maria is using triangle C, since two of the vertices of the triangle are at and (3, 2) and (9, 6). She will next find the length of each leg in the right triangle. The horizontal leg length is the difference between the *x*-coordinates, which is 6. The vertical leg length is the difference between the *y*-coordinates, which is 4. So the line rises by 4 units for every horizontal increase of 6 units. Therefore, the slope is $\frac{4}{6}$.

b. To compute the slope between two points, we are computing the quotient of the lengths of the legs in a right triangle. We can see that triangles A and B (in the graph above) are congruent since we can translate triangle A along the line until it lines up with triangle B. Therefore, the quotient of the lengths of the legs must be the same.

Triangle C is not congruent to triangle B but it is similar to it. We can dilate triangle B by a factor of 2 to line it up with triangle C. The sides in similar triangles also have the same proportion. Therefore, the quotient of the lengths of the legs of the two triangles must be the same.



c. Parts (a) and (b) suggest the following picture:

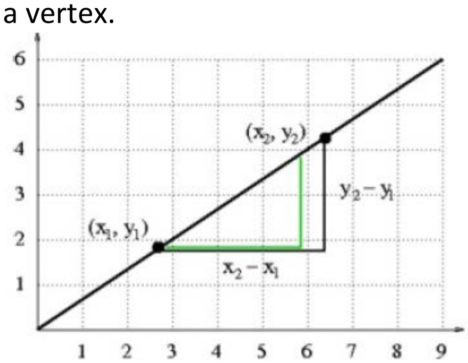


The lengths of the vertical leg of this triangle is $y_2 - y_1$, and the length of the horizontal leg is $x_2 - x_1$.

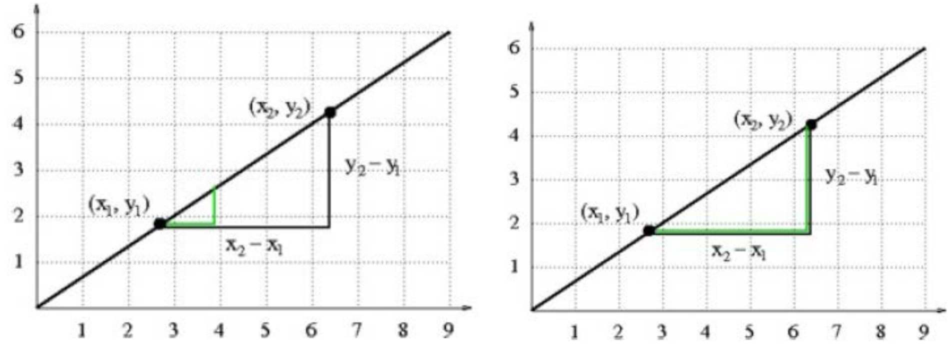
The slope between these two points is the quotient of these two lengths: $\frac{y_2 - y_1}{x_2 - x_1}$.

This slope should be the same as the slope obtained by Eva because her triangle A is similar to the triangle we drew above. To see this,

- First, translate Eva's triangle x_1 units to the right and y_1 units up. Now the two triangles share a vertex.



- Next, dilate Eva's triangle by a factor $\frac{1}{3}$ of (so the horizontal leg has a length of 1) and then by a factor of $x_2 - x_1$ (so the horizontal leg has a length of $x_2 - x_1$). Use the common vertex as the center of the dilation.



Because angles are preserved by translations and dilations, this shows that Eva's triangle and our triangle are similar and that the legs are proportional. So

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{3}$$

and the slope is the same no matter which two points on this line we choose to compute with.

- What are some strategies that you heard today that you would like to try when solving a similar problem in the future?

Another option is to let the groups record their solution(s) to the task on chart paper. They can then share out with the whole class. With this option the students are able to present their thinking, justify their reasoning, and answer questions from the other students.

Answer Key:

The information above is intended to help teachers get at student understanding of the mathematical idea(s) in each problem. Also provided is an Answer Key for each set of tasks. The Answer Key provides more information on the expected student response for each task. While it is important for students to get the answer right, it is equally important for them to understand how their thinking leads or does not lead to a correct solution. Incorrect solutions set the stage for teachable moments!!!!