EXPRESSIONS AND EQUATIONS TASKS: OVERVIEW

Resources:

Attached you will find a set of **15 tasks for Expressions and Equations**. The tasks are best used by partners or small groups. Since these tasks are more in-depth than the practice items, you can use them as the foundation of your lesson, especially if they address a standard you are working on that day.

The purpose of using tasks is to help you see how students solve problems, and understand their thought processes while they work. Being able to work with others to get productive discourse, to be able to explain their thinking to another student, and develop a solution, is the most effective way to assess student understanding. These tasks require students to do just that – think about an efficient strategy to solve the problem, show their work and justify their reasoning. This is the ultimate goal for what we want students to be able to do. Being able to gather evidence of student learning and misconceptions in the moment, will give you the flexibility to change your instruction to meet their needs. As the instructional decision-maker, you are able to adjust your methods for whole class or small groups to address student misconceptions and move them toward proficiency.

The goal is to have tasks that can be used as the lesson of the day along with or in lieu of the practice items. There are tasks for each day per week that represent the 5 domains in 8^{th} grade. We would like for you to use these tasks along with the practice items for a 10 week period between the time you receive them and the end of January. If used daily, in accordance with our recommendations or tips, for student and teacher practice, the outcome will be an improvement in ACT ASPIRE test scores.

At the end of each task packet, you will find an answer key for your use. **Although answer keys are provided, students should explain their thinking during the discussion of the tasks.** Some tasks include several representations of the solution.

Recommendations or Tips:

When implementing the tasks with your students, please allow students read through the tasks before starting to see if they have any questions about vocabulary or what the task is asking them to do. Taking the time to do these things now, will help assure that the students are familiar with mathematical vocabulary and different question types before the actual test.

Providing Feedback to Students:

An important process for understanding student thinking, is to debrief and provide feedback. This can be done during the exploration phase of the sharing out process by asking effective questions similar to the ones below which will also help students verbalize their reasoning :

- What did the problem ask you to do?
- What information do you see in the problem?
- What did you do <u>first</u> to solve this problem?
- What did you do next?
- What strategy did you use to solve the problem? Why did you use that particular strategy?
- Is there another strategy that you could use to solve the problem?
- Who else started this same way?
- Who started a different way?

- What are some strategies that you heard today that you would like to try when solving a similar problem in the future?

Another option is to let the groups record their solution(s) to the task on chart paper. They can then share out with the whole class. With this option the students are able to present their thinking, justify their reasoning, and answer questions from the other students.

Answer Key:

The information above is intended to help teachers get at student understanding of the mathematical idea(s) in each problem. Also provided is an Answer Key for each set of tasks. The Answer Key provides more information on the expected student response for each task. While it is important for students to get the answer right, it is equally important for them to understand how their thinking leads or does not lead to a correct solution. Incorrect solutions set the stage for teachable moments!!!!

Name____ Date_______________________

Set 1 - Standard(s): 8.EE.1

Illustrative Mathematics

Day 1 Task - Raising to the zero and negative powers

In this problem *c* represents a positive number.
The quotient rule for exponents says that if *m* and *n* are positive integers with $m > n$, then

$$\frac{c^m}{c^n} = c^{m-n}.$$

After explaining to yourself why this is true, complete the following exploration of the quotient rule when $m \leq n$:

a. What expression does the quotient rule provide for $\frac{c^m}{c^n}$ when $m = n$?

b. If $m = n$, simplify $\frac{c^m}{c^n}$ without using the quotient rule.

c. What do parts *(a)* and *(b)* above suggest is a good definition for c^0?

d. What expression does the quotient rule provide for $\frac{c^0}{c^n}$?

e. What expression do we get for $\frac{c^0}{c^n}$ if we use the value for c^0 found in part *(c)*?

f. Using parts *(d)* and *(e)*, propose a definition for the expression c^{-n}.

Name____ Date_______________________

Set 1 - Standard(s): 8.EE.1, 8.EE.2

Illustrative Mathematics

Day 2 Task - Extending the Definitions of Exponents, Variation 1

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

a. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study				0	1	2	3	4
Population (thousands)				2				

b. If you know the size of the population at a certain time, how do you find the population one hour later?

c. Marco said he thought that they could use the equation $P = 2t + 2$ to find the population at time t. Seth said he thought that they could use the equation $P = 2 \cdot 2^t$. Decide whether either of these equations produces the correct populations for $t = 1, 2, 3, 4$.

d. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study? What about 3 hours before?

e. If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

f. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

g. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

h. Use the context to explain why it makes sense that $2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by $\frac{1}{2}$.

Name______________________________________ Date______________________

Set 1 - Standard(s): 8.EE.1, 8.EE.2, 8.EE.3, 8.EE.4

Day 3 task - Orders of Magnitude

It is said that the average person blinks about 1000 times an hour. This is an *order-of-magnitude* estimate, that is, it is an estimate given as a power of ten. Consider:

- 100 blinks per hour, which is about two blinks per minute.
- 10,000 blinks per hour, which is about three blinks per second.

Neither of these are reasonable estimates for the number of blinks a person makes in an hour. Make order-of-magnitude estimates for each of the following:

a. Your age in hours.
b. The number of breaths you take in a year.
c. The number of heart beats in a lifetime.
d. The number of basketballs that would fill your classroom.

Can you think of others questions like these?

Name_______________________________________ Date______________________

Set 1 - Standard(s): 7RP.3, 8.EE.3

Illustrative Mathematics

Day 4 Task- Choosing appropriate units

a. A computer has 128 gigabytes of memory. One gigabyte is $1 \text{ X } 10^9$ bytes. A floppy disk, used for storage by computers in the 1970's, holds about 80 kilobytes. There are 1000 bytes in a kilobyte. How many kilobytes of memory does a modern computer have? How many gigabytes of memory does a floppy disk have? Express your answers both as decimals and using scientific notation.
b. George told his teacher that he spent over 21,000 seconds working on his homework. Express this amount using scientific notation. What would be a more appropriate unit of time for George to use? Explain and convert to your new units.
c. A certain swimming pool contains about $3 \text{ X } 10^7$ teaspoons of water. Choose a more appropriate unit for reporting the volume of water in this swimming pool and convert from teaspoons to your chosen units.
d. A helium atom has a diameter of about 62 picometers. There are one trillion picometers in a meter. The diameter of the sun is about 1,400,000 km. Express the diameter of a helium atom and of the sun in meters using scientific notation. About many times larger is the diameter of the sun than the diameter of a helium atom?

Name_______________________________________ Date______________________

Set 1 - Standard(s): 8.EE.4

Illustrative Mathematics

Day 5 Task – Giantburgers

This headline appeared in a newspaper.

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about $8 \text{ X } 10^3$ Giantburger restaurants in America.
- Each restaurant serves on average $2.5 \text{ X } 10^3$ people every day.
- There are about $3 \text{ X } 10^8$ Americans.

Explain your reasons and show clearly how you figured it out.

Set 1 - Standard(s): 7.RP.2 8.EE.4

Day 1 Task - Raising to the zero and negative powers – KEY

The goal of this task is to use the quotient rule of exponents to help explain how to define the expressions c^k for $c > 0$ and $k \leq 0$. This important definition is motivated and explained by the law of exponents: adopting the definitions for the expressions c^0 and c^{-n} given in the task allows us to maintain the intuitive product and quotient rules known for all positive exponents (which this task assumes students are familiar with).

a. If we apply the quotient rule for exponents when $m = n$, we find

$$\begin{aligned}\frac{c^m}{c^n} &= \frac{c^m}{c^m}\\ &= c^{m-m}\\ &= c^0.\end{aligned}$$

b. Without the quotient rule, when $m = n$, we have $\frac{c^m}{c^m} = 1$.

c. Parts (a) and (b) give a compelling justification for choosing to define $c^0 = 1$ for any positive number c. Namely, on the one hand, $\frac{c^m}{c^m}$ is 1 by doing the explicit division. On the other hand, the extension of the quotient rule to the case $m = n$ gives $\frac{c^m}{c^m} = c^0$. If we want the quotient rule to continue hold in this case, we are *forced* to define $c^0 = 1$.

d. If we apply the quotient rule for exponents when $m = 0$, we find

$$\begin{aligned}\frac{c^0}{c^n} &= c^{0-n}\\ &= c^{-n}.\end{aligned}$$

We have a negative exponent in this case.

e. According to part (c), $c^0 = 1$. If we substitute this into the fraction $\frac{c^0}{c^n}$ we find that $\frac{c^0}{c^n} = \frac{1}{c^n}$.

f. Much like our definition for c^0, the previous two parts dictate that if we want the quotient rule to hold for the expression $\frac{c^0}{c^n}$, we are forced to define $c^{-n} = \frac{1}{c^n}$. That is, the quotient rule suggests we should define raising the number c to a negative power to be the reciprocal of the corresponding positive power.

Set 1 - Standard(s): 8.EE.1, 8.EE.2

Illustrative Mathematics

Day 2 Task - Extending the Definitions of Exponents, Variation 1– KEY

a. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2	4	8	16	32

b. You multiply it by 2, since it doubled.

c. The values predicted by Seth's equation agree exactly with those in the table above; Seth's equation works because it predicts a doubling of the population every hour. Marco's doesn't because it doesn't double the new population you have – instead it is doubling the time. Marco's equation predicts a linear growth of only two thousand bacteria per hour.

d. Since the population is multiplied by 2 every hour we would have to divide by 2 (which is the same as multiplying by $\frac{1}{2}$) to work backwards. The population 1 hour before the study started would be

$$\frac{1}{2} \cdot 2 = 1 \text{ thousand,}$$

and the population 3 hours before the study started would be

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = 0.25 \text{ thousand,}$$

so 250 bacteria.

e. Since the population is multiplied by 2 every hour we would have to divide by 2 (or multiply by $\frac{1}{2}$) to work backwards.

f. Time before the study started would be negative time; for example one hour before the study began was $t = -1$.

Hours into study	-3	-2	-1	0	1	2	3	4
Population (thousands)	$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$	$\frac{1}{2} \cdot 1 = 0.5$	1	2	4	8	16	32

g. Since one hour before the study started would be $t = -1$, we would simply plug this value into Seth's equation:

$$2 \cdot (2)^{-1} = 2 \cdot \left(\frac{1}{2}\right) = 1 \text{ thousand.}$$

Three hours before would be $t = -3$. Using the equation:

$$2 \cdot (2)^{-3} = \frac{2}{2^3} = 0.25 \text{ thousand,}$$

giving us the same answers as we got through reasoning.

h. Since the bacteria double every hour, we multiply the population by two for every hour we go forward in time. So if we want to know what the population will be 8 hours after the experiment started, we need to multiply the population at the start ($t = 0$) by 2 eight times. This explains why we raise 2 to the number of hours that have passed to find the new population; repeatedly doubling the population means we repeatedly multiply the population at $t = 0$ by 2.

In this context, negative time corresponds to time *before* the experiment started. To figure out what the population was before the experiment started we have to "undouble" (or multiply by $\frac{1}{2}$) for every hour we have to go back in time. So if we want to know what the population was 8 hours before the experiment started, we need to multiply the population at the start ($t = 0$) by $\frac{1}{2}$ eight times. The equation indicates that we should raise 2 to a power that corresponds to the number of hours we need to go back in time. For every hour we go back in time, we multiply by $\frac{1}{2}$. So it makes sense in this context that raising 2 to the -8 power (or any negative integer power) is the same thing as repeatedly multiplying $\frac{1}{2}$ 8 times (or the opposite of the power you raised 2 to). In other words, it makes sense in this context that

$$2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}.$$

Set 1 - Standard(s): 8.EE.1, 8.EE.2, 8.EE.3, 8.EE.4

Illustrative Mathematics

Day 3 Task - Orders of Magnitude – KEY

a. For this problem, the students must convert an age in years to an age in hours. For example, a 13-year-old student would calculate their age in hours in the following way:

$$13 \text{ years} \times 365 \frac{\text{days}}{\text{year}} \times 24 \frac{\text{hours}}{\text{day}} = 113,880 \text{ hours.}$$

Now we note that 113,880 hours is between 100,000, and 1,000,000 and is quite close to 100,000. To verify this, let's determine how many years 100,000 hours is:

$$100,000 \div 24 \div 365 \approx 11$$

How many years is 1,000,000 hours?

$$1,000,000 \div 24 \div 365 \approx 114$$

Which reaffirms that $100,000 = 10^5$ is the best order-of-magnitude estimate for the age in hours of a 13-year-old student.

b. The solution is presented in steps.

- Step 1: Approximate the number of breaths per minute for a person. Teachers can suggest students measure their own breathing rate. Teachers may also provide data. For an example, we use 12 breaths per minute.
- Step 2: Multiply the number of breaths per minute by 60 to find the number of breaths per hour. So:

$$12 \frac{\text{breaths}}{\text{minute}} \times 60 \frac{\text{minutes}}{\text{hour}} = 720 \frac{\text{breaths}}{\text{hour}}$$

- Step 3: Multiply by 24 to convert breaths per hour to breaths per day:

$$720 \frac{\text{breaths}}{\text{hour}} \times 24 \text{ hours} = 17,280 \frac{\text{breaths}}{\text{day}}$$

- Step 4: Multiply by 365 to convert breaths per day to breaths per year. So:

$$17,280 \frac{\text{breaths}}{\text{day}} \times 365 \text{ days} = 6,307,200 \frac{\text{breaths}}{\text{year}}$$

- Step 5: Determine the order of magnitude. According to our example, the order of magnitude is 10^7. Again, students should explore why 10^6 or 10^8 are not reasonable estimates. Working backwards, 10^6 corresponds to about 2 breaths per minute, which is well below a healthy range, whereas 10^8 yields a breathing rate of about 190 breaths per minute, which is well above a healthy range.

c. Students can use the fact that there is a direct correlation between breaths and heart rate. Once this relationship is determined, they can use the calculations from part b to determine the solution. For instance, a healthy human heart beats approximately 6 times per breath. Students can multiply the number of breaths per year and the number of heartbeats per breath to find the number of heartbeats per year. For our example:

$$6,307,200\frac{\text{breaths}}{\text{year}} \times 6\frac{\text{heartbeats}}{\text{breath}} = 37,843,200\frac{\text{heartbeats}}{\text{year}}$$

Finally, to determine the number of heart beats per lifetime, they can multiply by the number of years in a lifetime. For our example, we place the lifespan at 80 years:

$$37,843,200\frac{\text{heartbeats}}{\text{year}} \times 80 \text{ years} = 3,027,456,000\frac{\text{heartbeats}}{\text{lifetime}}$$

This is between 10^9 and 10^{10}, but is closer to 10^9. Again, students should explore why 10^8 or 10^{10} are not reasonable estimates. Working backwards, 10^8 corresponds to about 2.4 heartbeats per minute (well below a healthy range), whereas 10^{10} corresponds to about 240 heartbeats per minute (well above a healthy range).

d. We need to use a measurement system that allows us to easily compare the size of a basketball to the size of a room. In this case, we will measure both the room and the basketball in terms of feet. First let us consider the size of a basketball. We can determine the circumference by simply using a tape measure. We can then use the circumference formula to find the diameter. We should find that the diameter of a standard basketball is approximately 9 in, or 0.75 feet. We can estimate that a basketball will fit into a cube that measures 0.75 feet × 0.75 feet × 0.75 feet. First, assume that the basketball almost fills the cube, as if we were stacking the balls directly on top of each other in a rectangular array. The basketball fits into a cube with volume of about 0.75^3 ft^3, or 0.421875 ft^3.

Now, let's consider the size of a classroom. Students can use whatever is at their disposal to measure their own classroom, such as yardsticks or measuring tape. As an example, we will use a room that measures 20 feet by 20 feet by 10 feet, since this is a reasonable estimate of an average classroom. This example gives us a classroom with a volume of 4000 ft^3 (using the volume formula mentioned above). To compute about how many basketballs fit in the room, we will divide the volume of the room by the volume of the cube that contains the basketballs:

$$4000 \text{ ft}^3 \div 0.75^3 \text{ ft}^3 \approx 9481$$

so about 9500 basketballs will fit in the room.

So as an order of magnitude estimate, we find that we can fit about 10,000 basketballs in the room. We might ask ourselves what would happen if we packed the basketballs more snugly together. We could imagine that there is no gap between the balls and divide by the actual volume of a basketball with a diameter of $\frac{3}{4}$ feet. The radius is half of that, or $\frac{3}{8}$, and the volume is

$$\frac{4}{3}\pi\frac{3}{8}^3 \approx 0.22$$

Again, the volume of the room is 4,000 cubic feet, so we can divide to find the number of basketballs:

$$4000 \text{ ft}^3 \div 0.22 \text{ ft}^3 \approx 18,108$$

So with this estimate, about 18,000 basketballs will fit in the room. This still seems closer to 10,000 than 100,000.

Let's check: How much space would 10^3 basketballs take up? Imagine they are loosely packed (and so taking up a lot of space); each one takes up the space of a containing cube which has a volume of about 0.42 cubic feet. The total volume would be about 420 cubic feet. Even if the ceiling in the classroom is only 8 feet tall, that would leave only 52.5 square feet for the area, which would be a very tiny classroom--a square with 7 foot sides, for example.

How about 10^5 basketballs? Imagine 10^5 tightly packed without any gaps between them, so each one takes up the space of a containing cube which has a volume of about 0.22 cubic feet. The total volume would be about 22,000 cubic feet. If the ceiling in the classroom is only 10 feet tall, the area of the classroom would be 2200 square feet, which is more like the size of a gym. It is possible to have a classroom that large, but not likely, and any real packing of the basketballs would take up more space anyway.

Examples of others questions like these:

- The number of hours they spend in school each year
- The number of hours they spend doing homework per day, per week, per month, per year, per elementary school, and/or per lifetime
- The number of hours they spend sleeping in their lifetime as compared to the number of hours they spend awake/studying/eating/exercising in their lifetime

Set 1 - Standard(s): 7RP.3, 8.EE.3

Illustrative Mathematics

Day 4 Task - Choosing appropriate units – KEY

a. If one gigabyte is 1×10^9 bytes then 128 gigabytes is 128×10^9 bytes. Rewriting this number in appropriate scientific notation we find

$$128 \times 10^9 = 1.28 \times 100 \times 10^9$$
$$= 1.28 \times 10^{11}$$

for the computer storage. Since there are 1000 bytes in a kilobyte we can find the number of kilobytes in the computer memory by dividing the number of bytes by 1000 or 1×10^3. So the computer has 1.28×10^8 kilobytes of memory. This is 128,000,000 or 128 million kilobytes.

The floppy disk has 80 kilobytes of storage and there are 1000 bytes in a kilobyte so this is 80,000 or 8×10^4 bytes of storage. There are 1×10^9 bytes in a gigabyte so the floppy disk storage is

$$\frac{8 \times 10^4 \text{ bytes}}{1 \times 10^9 \text{ bytes per gigabyte}}.$$

This is $8 \times 10^{4-9} = 8 \times 10^{-5}$ gigabytes. There are 0.00008 gigabytes of storage on a floppy disk.

b. We have

$$21{,}000 = 2.1 \times 10{,}000$$
$$= 2.1 \times 10^4$$

so it took George 2.1×10^4 seconds to do his homework. This is not very helpful for giving an intuition of how long George worked on the homework. A more appropriate unit would probably be hours. To check, there are 60 minutes per hour and 60 seconds per minute so there are 60 × 60 = 3600 seconds per hour. This means that George worked on the homework for 21,000 ÷ 3600 hours. This is a little less than 6 hours.

c. For a swimming pool, cubic meters or yards would be an appropriate unit. In metric units, there are 100 centimeters in a meter and so $100^3 = 1 \times 10^6$ cubic centimeters in a cubic meter. One teaspoon of water takes up about 4.9 cubic centimeters of space so 3×10^7 teaspoons would take up about $3 \times 4.9 \times 10^7$ cubic centimeters (or about 1.5×10^8 cubic centimeters). So, converting from cubic centimeters to cubic meters, our water takes up about 1.5×10^2 (or 150) cubic meters of space. This is reasonable as a swimming pool 2 meters deep, 5 meters wide, and 15 meters long would have this volume of water.

d. One trillion is 1,000,000,000,000 or, in scientific notation, 1×10^{12}. So one trillionth is 1×10^{-12}. One picometer is 1×10^{-12} meters and the diameter of a helium atom is about 62×10^{-12} or 6.2×10^{-11} meters. The diameter of the sun is about 1,400,000 km and there are 1000 meters in a kilometer. So the diameter of the sun is about 1,400,000,000 meters or 1.4×10^9 meters. To compare these two diameters, we compute the quotient $1.4 \times 10^9 \div 6.2 \times 10^{-11} \approx 0.23 \times 10^{20}$ or, writing this in proper scientific notation, 2.3×10^{19}. The diameter of the sun is more than 1×10^{19} as large as a helium atom, a fantastically large number!

Set 1 - Standard(s): 8.EE.4

Illustrative Mathematics

Day 5 Task – Giantburgers – KEY

If there are about 8×10^3 Giantburger restaurants in America and each restaurant serves about 2.5×10^3 people every day, then about

$$8 \times 10^3 \cdot 2.5 \times 10^3 = 20 \times 10^6 = 2 \times 10^7$$

people eat at a Giantburger restaurant every day.

Since there are about 3×10^8 Americans, the percent of Americans who eat at a Giantburger restaurant every day can be computed by dividing the number of restaurant patrons by the total number of people:

$$2 \times 10^7 \div 3 \times 10^8 = \frac{2}{3} \times 10^{-1}$$

Since

$$\frac{2}{3} \times 10^{-1} = \frac{2}{3} \times \frac{1}{10} = \frac{2}{30} = \frac{1}{15} = 0.06\overline{6},$$

our estimate is that $6\frac{2}{3}\%$ of Americans eat a Giantburger restaurant every day, which is reasonably close to the claim in the newspaper.