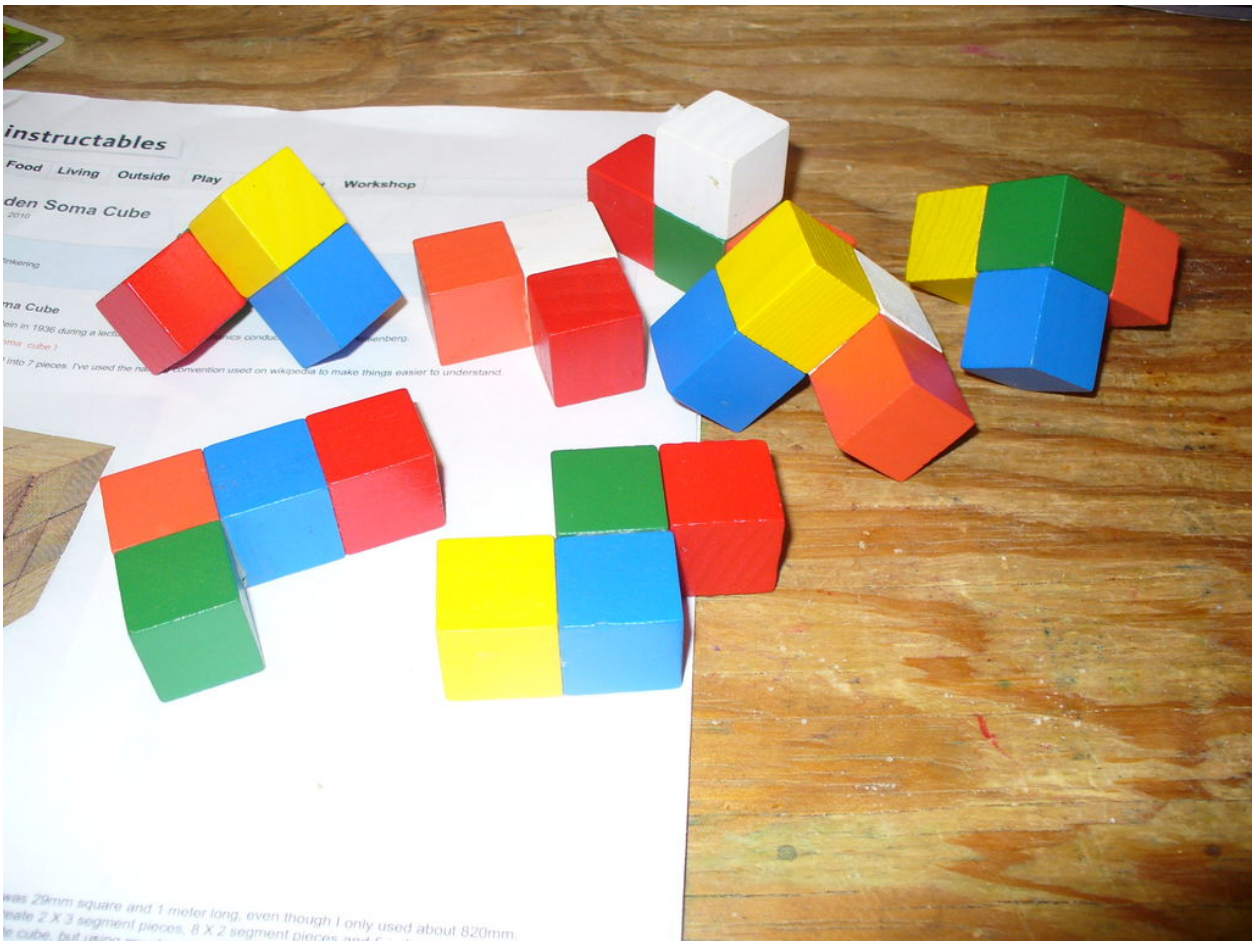
Mathematics Common Core State Standards:

Middle & High School Pathways to College, Careers, and Citizenship



A Report for the Piedmont Unified School District Mathematics Task Force

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Report Objective
This report summarizes findings from a review of research on student course-taking patterns and classroom grouping practices, particularly in 6th through 12th grade mathematics. It examines implications of student course-taking and classroom grouping for opportunities to learn mathematics and to prepare for college, careers, and citizenship.

Findings are intended to inform decision-making by the Math Taskforce of the Piedmont Unified School District about how best to support and challenge a diversity of students in their mathematics coursework throughout the middle and high school years. Considerations include opportunities for "accelerated" and "expanded" pathways consistent with California's Common Core State Standards (CCSS) for mathematics. In the Executive Summary, the review highlights key points to consider in the context of CCSS implementation, with an emphasis on the new possibilities and challenges presented by standards-based reforms.

Sources
The report draws on a review of articles identified by the Piedmont Unified School District, additional peer-reviewed articles found via JSTOR and ERIC, scholarly monographs, and a limited spectrum of other sources (white papers, policy reports, journalistic commentary) found via Google and consultations with colleagues. CCSS policy analyses, including implementation research and grey literature, are especially relevant to complementing the peer-reviewed literature.

This review is by no means exhaustive, given that the body of research pertaining to course-taking patterns, classroom grouping practices, and related topics is vast and multifaceted, encompassing decades of debate. However, this review highlights key research syntheses and representative themes, including contemporary debates.

More generally, the following book is full of suggestions and insights into deeper engagement with mathematics by everyone – including an appendix of "Recommended Math Puzzle Books":

Boaler, J. (2008) *What's Math Got To Do With It?: How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject*, Penguin Books.

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Executive Summary

"[O]pen approaches to learning not only give access to a depth of subject understanding but also encourage a personal and intellectual freedom that should be the right of all people in society."[1]
– Jo Boaler, Stanford mathematics education professor

Guiding Goals
Before delving into research findings on course-taking patterns, it may be valuable to note both the district's goal for CCSS course pathways, as well as the articulation of this goal with objectives outlined by the National Council of Teachers of Mathematics (NCTM).

Piedmont Unified School District:

> "The District has a range of learners, and this will continue to be true after the CCSS are fully implemented. The guiding consideration during the development of the pathways and other implementation is how the District can best support all learners, whether they are able to accelerate through the curriculum, follow a grade-level progression, or require more time to master the math concepts." ("Implementation of the Common Core Standards for Mathematics Curriculum," March 2014: 3)

National Council of Teachers of Mathematics:

> "In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed. NCTM challenges the assumption that mathematics is only for the select few. On the contrary, everyone needs to understand mathematics. All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence." (NCTM, *Principles and Standards for School Mathematics*, 2000)

New Possibilities in Common Core State Standards (CCSS) Context
This research review underscored that CCSS reforms present an opportunity to emphasize excellence and challenging, meaningful mathematics problem-solving for *all* students, both individually and in classroom learning communities. Multiple sources highlighted the potential of the CCSS to facilitate *deeper* mathematics learning and problem-solving by all students – with greater emphasis on conceptual understanding, modeling, applications, engagement with contexts of mathematical problems, and multiple approaches to problem formulation and problem-solving – through high school and into students' futures as professionals and citizens. The key findings below are oriented toward this context.

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[1] Boaler 2002: 254.

Key Findings:
Below is a summary of findings from the research review, highlighting core themes and considerations relevant to how the PUSD might best support a diversity of students in their math coursework through the middle and high school years, in the context of CCSS reforms.

Beyond "Excellence versus Equity"
As in deliberations over CCSS mathematics curricula, debates over course sequences and classroom grouping practices are often polarizing, sometimes deploying war rhetoric with a binary framing of "excellence versus equity." This can play out such that different students' challenges are seen in zero-sum relation – for example, the challenges of higher-performing students versus the challenges of lower-performing students. However, research does not validate such a zero-sum framing and offers examples of numerous strategies to support all students in their learning of challenging, meaningful mathematics en route to college (e.g. Burris 2008; Slavin 1991; Slavin 2010). This is especially true with the arrival of CCSS reforms, as schools move away from the assessments of the No Child Left Behind (NCLB) era (Phillips and Wong 2010). Such expectations are in keeping with those of other high-performing countries in mathematics, from Finland to Japan.

Importance of Elementary School Mathematics
While this report focuses on middle and high school math pathways, some research on elementary math education is included as well, highlighting how students' earlier math learning experiences lay the groundwork for advanced math courses and postsecondary pathways. Attending to the interdependencies of students' mathematics learning experiences across elementary, middle and high schools is crucial, particularly if the district adopts an integrated CCSS curriculum. An integrated curriculum could change the meanings of "acceleration" and approaches to Early Algebra, for example (Brizuela 2013).

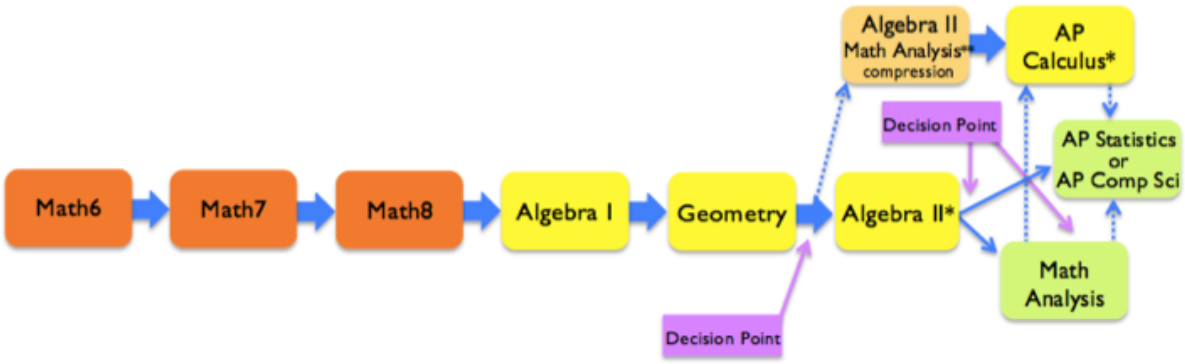
Middle School: Pivotal Years for Unfolding Futures
Research strongly indicates the pivotal importance of students' math course-taking patterns during their middle school years, as well as 9th grade, to their postsecondary educational outcomes (Wang and Goldschmidt 2003). Given the variable preparation students bring to middle school, and the pivotal importance of these years to postsecondary educational outcomes, it is crucial to plan for assessment and support of all students in achieving UC system and other college eligibility through their middle and secondary school math coursework (Choi and Shin 2004; Finkelstein et al. 2012). Possibilities for advanced and interdisciplinary coursework in Science, Technology, Engineering and Mathematics (STEM) fields should also be highlighted for all students.

Acceleration & Expansion: Multiple Pathways to College and STEM Fields
Based on this research review, there are significant potential learning benefits of accelerated mathematics courses for all students, including or especially in heterogeneous classrooms (e.g. Boaler 2008; Burris 2006). These benefits may even be greatest for students not identified as "high achieving" in middle school (e.g. Ma 2004; Ma 2005). However, it is crucial that students be adequately prepared for any accelerated course progression rather than being rushed to enroll. Especially in middle school, research indicates that schools should emphasize constructive assessments of students' mathematical proficiency, based on multiple forms of evidence of

proficiency and of areas for improvement (Faulkner et al. 2014). For all students, emphasis should be on depth of understanding and preparing for college-preparatory math sequences in high school. Schools should convey to students that this preparation is a process that may take multiple forms over the course of the middle and high school years – rather than conveying that students must enroll in a particular course in, for example, 7th or 8th grade. Though there are correlations between course enrollments and later academic outcomes, these correlations obscure the underlying pathways to learning and achievement for a variety of students.

One possible sequence of core math courses is this model middle and high school pathway, attuned to a range of issues surfaced by this research review:

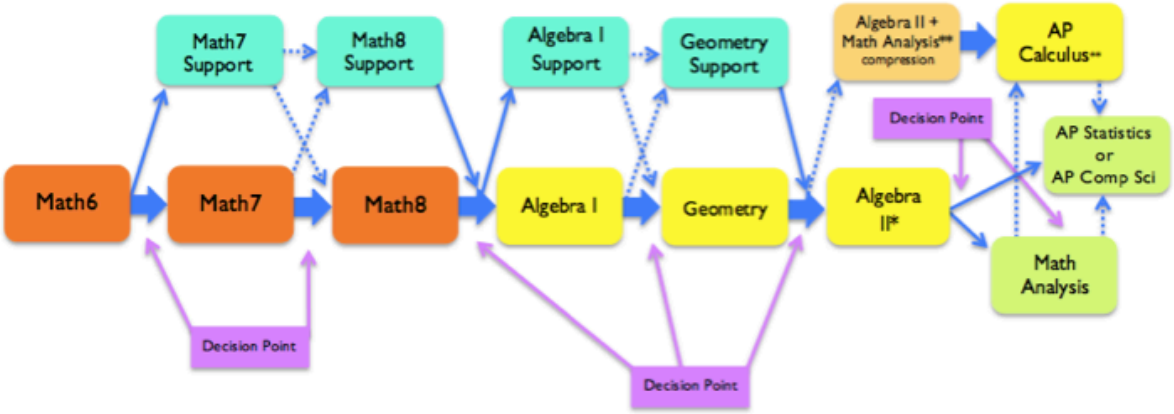
Figure 1.1: The Core Course Sequence



It is excerpted from "Oakland and San Francisco create course pathways through Common Core Mathematics," an April 2014 white paper by Phil Daro at the Strategic Education Research Partnership (SERP) Institute that includes fuller discussion of math pathway recommendations and alternatives (available at: https://hubs.serponline.org/app/short_links/flca/download).

For students who are not yet prepared for the CCSS and require additional support, or expanded course options, the white paper suggests this alternative pathway:

Figure 2A: Core Course Sequence with Support Course Options

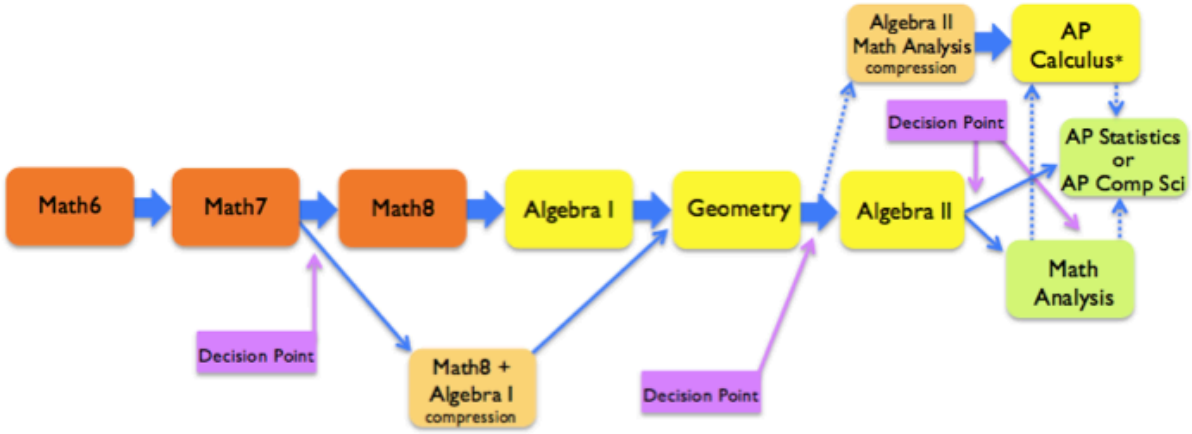


Reconceptualizing Expansion and Remediation

As discussed in the body of this review, expanded course options should not require students to simply repeat prior coursework in a remedial fashion (Balfanz et al. 2002; Hern 2012), an approach that is highly correlated with student failure. Rather that such remedial models, which "often break down complex skills and ways of thinking into discrete sub-skills, then deliver these skills up front to students in a linear, step-by-step curriculum" those calling for rethinking such approaches "argue instead for immersing students in challenging, authentic literacy and quantitative tasks and providing targeted reviews of foundational skills at the moment they are relevant to the higher-order work at hand." (Hern 2012: 64) These approaches align strongly with the CCSS reforms. They also blur conventional distinctions between acceleration and expansion; for example, the California Acceleration Project, directed by Hern, is devoted to developmental (or remedial) education: http://cap.3csn.org/.

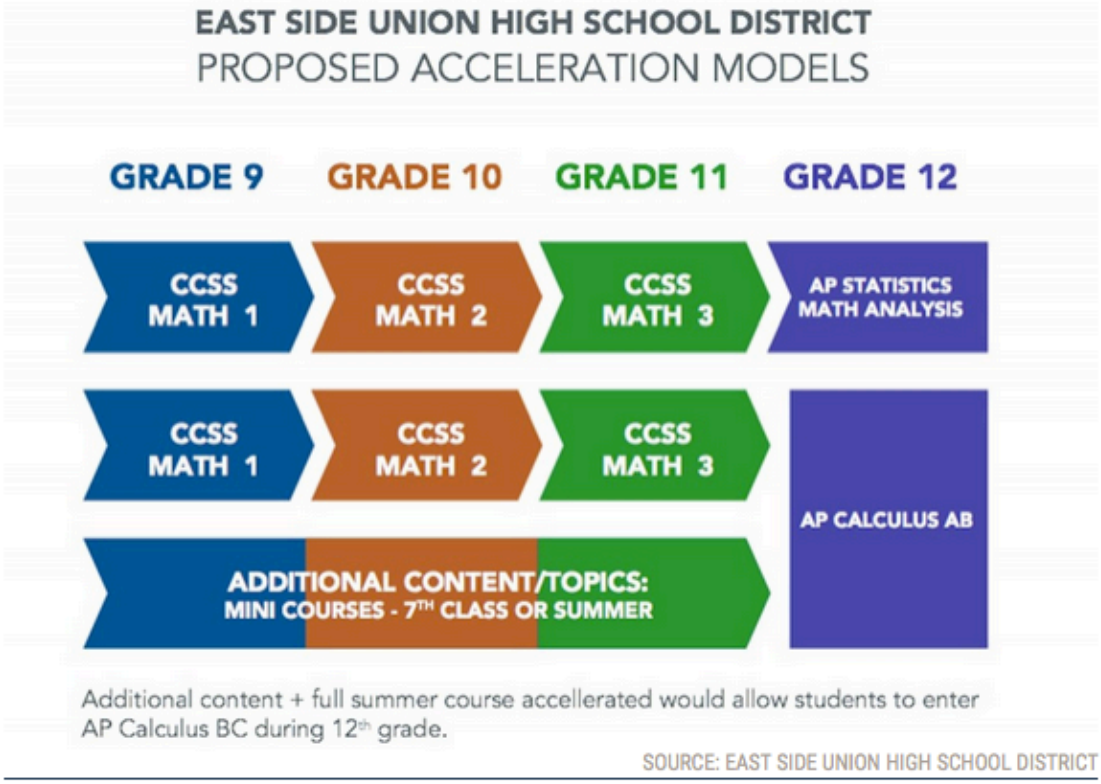
For students who are prepared for the CCSS and choose to enroll in accelerated courses earlier, there is this suggested pathway:

Figure 2B: Core Course Sequence with Challenge Course Options



Any initiatives on behalf of math acceleration should also ensure adequate support for learning for all students; acceleration should not be promoted as an end in itself. Again, fuller discussion of the above model pathways is available in the white paper at: https://hubs.serponline.org/app/short_links/flca/download. These pathways are also supported by the research in this review, particularly in their emphasis on the importance of all students establishing a solid, deep foundation of mathematical understanding during middle school.

Another interesting model pathway, for high school mathematics, is the following, from the East Side Union High School District (http://www.esuhsd.org/):



As reported in *EdSource*: http://edsource.org/2014/new-twist-to-old-debate-on-accelerated-math/66174#.VMKUTsaTCDo

This model is possible in an integrated curriculum through optional "plus standards," designed for prospective STEM majors and other interested students. Students bypass pre-Calculus through a progression of extra daily mini-courses. This pathway would facilitate students' preparation for advanced mathematics in high school as well as support learning in diverse, heterogeneous classrooms throughout core high school mathematics courses. It would allow students to wait until high school to make decisions about enrolling in advanced math courses, while emphasizing college preparation, mastery of the new CCSS, and depth of understanding for all students.

A later decision point – in students' sophomore year, for example – could also be worth considering. A 10th grade decision point might be especially useful if the district decides to offer new or alternative 12th grade course options – such as Statistics, Engineering Mathematics, or Mathematics for Computer Science – that students may not have been introduced to previously. Some districts have noted that such senior-year courses can be opportunities to mathematically reengage students who might otherwise not investigate fields in which they would thrive (http://www.edweek.org/ew/articles/2008/11/19/13seniors_ep.h28.html). A 10th grade acceleration path could be facilitated by compressing three years of material into two.

Further discussion and model pathways for CCSS middle and high school mathematics are in "California State Board of Education, Appendix A: Course Placement and Sequences," available at: http://www.cde.ca.gov/ci/ma/cf/documents/aug2013apxacourseplace.pdf (cf. models on p. 18-20).

Senior Year Mathematics: Rush to Calculus?
Students considering enrolling in Calculus for their senior year should be aware of the "Joint Position on Calculus" issued in 2012 by the Mathematical Association of America (MAA) and National Council of Teachers of Mathematics (NCTM), which speaks to the question: "How should secondary schools and colleges envision calculus as the course that sits astride the transition from secondary to postsecondary mathematics for most students heading into mathematically intensive careers?" It voices concerns about how the high school "rush to Calculus" is affecting students' mastery of the subject, especially in college. The statement reads:

> "Although calculus can play an important role in secondary school, the ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. The college curriculum should offer students an experience that is new and engaging, broadening their understanding of the world of mathematics while strengthening their mastery of tools that they will need if they choose to pursue a mathematically intensive discipline." (Available at: http://www.nctm.org/about/content.aspx?id=32351)

More background on the rationale for this statement can be found in the "Annotated Bibliography and Discussion of Sources" below, and at:
http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/MAA_NCTM_background.pdf

Transitions from High School to College for Prospective STEM Majors
In addition, members of both the MAA and NCTM have raised questions about the privileged position occupied by Calculus in both high school and college curricula, including or even especially its relevance to students interested in STEM fields. As former NCTM president J. Michael Shaughnessy put it, "Both high schools and colleges are operating under outdated assumptions" about mathematics curricula in the transition to college, articulating his concerns about the inadequacy of the K-12 mathematics curriculum in preparing students for the STEM challenges of the 21st century. He explains that in 2011 the MAA published *Curriculum Renewal Across the First Two Years* (CRAFTY) of college, "which examined the mathematical needs of many client disciplines, such as biology, chemistry, economics, engineering, physics, and others. CRAFTY advocates secondary mathematics that facilitates students' transition from high school to college by providing (1) a greater emphasis on modeling; (2) consideration of multivariate topics; (3) an emphasis on computational skills that are useful in other fields; and (4) a strong foundation in units, scaling, and dimensional analysis." (CRAFTY report available at: http://www.maa.org/sites/default/files/pdf/CUPM/crafty/introreport.pdf)

These issues are discussed further in the body of this research review and its sources. They suggest the value of students, parents and educators considering a wider array of mathematics course possibilities, particularly in 12th grade, rather than assuming that enrollment in AP Calculus is the only or best option for college preparation, including for STEM fields. Again, the CCSS provides an opportunity to rethink and revitalize both course pathways and key courses.

Acceleration and Classroom Grouping Practices

Research into course-taking patterns and acceleration also involves questions of classroom grouping practices, including long-standing debates about tracking and "ability grouping" in schools. This research review documented that students' course-taking patterns, especially during their middle and high school years, entail "path dependency" effects such that earlier courses and course sequences influence both students' academic attainments and their postsecondary trajectories (Schneider et al. 1998). Course-taking patterns have effects even controlling for prior achievement and performance (Wang and Goldschmidt 2003), with research indicating that schools with more rigid course sequences or "tracks" – especially if some are college preparatory and others are not – tend to contribute significantly to unequal student achievement across those course sequences.

Explanations often underscore the unequal opportunities to learn across contexts, with the most challenging curriculum and most experienced, effective teachers devoted to college preparatory course sequences (Gamoran 1992). In addition, researchers have documented negative affective outcomes that often accompany "low-track" courses, including students' experience of stigma. Some studies (in this review, principally those by economists) argue that more "homogeneous" classrooms can lead to improved, more efficient instruction for all students (Collins and Gan 2013). However, the studies cited are not longitudinal nor are they engaged with the wider body of educational and sociological research on these topics, which often is more attuned to classroom contexts and educational processes. The weight of the evidence underpinning this research indicates that tracking, especially in its more rigid forms, does not contribute to overall student achievement but can contribute to unequal educational outcomes among students (Gamoran 1992; Huang 2009; Lucas 1999; Oakes 2005; Slavin 1987; Slavin 1990).

That said, classroom grouping practices may take a variety of forms, and researchers have articulated a range of more nuanced positions depending on the particular contexts under consideration – including the specific subject matter and grade levels at issue. For example, Robert Slavin, who has conducted multiple "best evidence" research syntheses on these topics and is generally critical of ability grouping and tracking on that basis, has argued:

> "I am in favor of acceleration programs (especially in mathematics) for the gifted and I believe in differentiating instruction *within* heterogeneous classes to meet the needs of students above (and below) average in performance. But I see no evidence or logic to support separate enrichment programs for gifted students. Enrichment is appropriate for *all* students. I see little evidence at all for separate tracks for high achievers." (Slavin 1991: 70)

Schools also exhibit a range of classroom grouping practices across subjects and grade levels. Mathematics courses are most often tracked or grouped by students' assessed performance levels, even at schools in which heterogeneous grouping is otherwise the norm.

These more nuanced positions and specific contexts are most relevant to thinking through CCSS middle and high school mathematics pathways. Overall, this research suggests the potential educational value of accelerated course options in mathematics, especially in later grades, when

students are more mature and have laid a deep foundation of mathematical understanding and preparation. Again, research also indicates that accelerated course options could benefit and should be open to all students (Burris and Levin 2004; Burris and Garrity 2008), and may even especially benefit initially lower-achieving students (e.g. Ma 2004; Ma 2005). Given these contingencies, some researchers have emphasized the importance of flexibility in any course acceleration or classroom grouping arrangements (Choi and Shin 2004; Tieso 2003), for example: "Schools must make at least two sorts of investments to bring greater flexibility to their grouping systems: (1) they must reassess students' capabilities and take new information into account when making assignment decisions, and (2) they must enable students to make up curricular material they may have missed—for example, in tutorials during the school year or the summer—so that those who are ready to advance are not held back by lack of curriculum coverage." In addition, "Implementing more flexible grouping systems also means rotating teachers so that all students have opportunities to learn from the most effective teachers and to prevent the loss of morale that sometimes occurs for teachers who are assigned to low tracks year after year." (Gamoran 1992)

Beyond accelerated course pathways along these lines, the majority of research does not suggest that ability grouping contributes to improved academic or affective outcomes for students. Moreover, researchers have found that even in the absence of formal tracking, classroom-level student clustering can tend to reinforce existing inequalities among students (Lucas 1999).

The literature on these subjects will no doubt remain contested; it is vast and is marked by decades of debate over appropriate measures, effects, and contexts. In addition, focusing on classroom grouping practices can distract from other pivotal dimensions of classroom practice – such as curriculum, instructional strategies, and teacher professional development – demonstrated to affect student learning and the implications of any grouping practices.

Teacher Professional Development and CCSS Reforms

Many educators have pointed to the need for sustained teacher professional development to accompany CCSS reforms. Yet some researchers have noted that the preponderance of public attention to debating curricula has led to the neglect of different teaching practices and instructional strategies, even as adequate teacher preparation and sustained support are crucial to effectively teaching diverse students across new courses and pathways. Key strategies include, for example, explicitly addressing how to grapple with open-ended problems, thereby engaging in "metacognition." Researchers have documented the efficacy of such strategies, while noting that they are unevenly implemented across a range of classrooms. In the new CCSS context, such open-ended problems should not be limited only to certain students and classrooms. In order to truly implement the CCSS, teaching strategies emphasizing open-ended, multi-level problems should become the norm (Boaler 2002), along with accompanying professional development. This includes ongoing professional development for differentiated instruction, tailored to diverse student proficiency levels and learning styles, for educational reasons that include but are not limited to issues of equity.

Cooperative Learning and Collaborative Problem-Solving

Mathematics courses have, more often than other courses, tended to be singled out in U.S. schools for course sequence tracking and ability grouping, as mentioned above. This has meant

that classroom strategies oriented toward collaborative problem-solving among diverse students have been especially underutilized in math classrooms, despite the evidence base for their efficacy (Slavin 2010). Given that CCSS reforms emphasize the importance of complex problem-solving, as do 21st workplaces and civic life, there is a particular opportunity to better leverage collaborative learning strategies in heterogeneous math classrooms in the future. As Slavin (2010) writes:

> "Co-operative learning offers a proven, practical means of creating exciting social and engaging classroom environments to help students to master traditional skills and knowledge as well as develop the creative and interactive skills needed in today's economy and society. Co-operative learning itself is being reshaped for the 21st century, particularly in partnership with developments in technology." (10)

To this end, educational research that foregrounds in-class "task features" – such as complexity – alongside instructional practices may be especially relevant to developing models for effective learning and problem-solving among students in diverse, heterogeneous classrooms, including accelerated courses (Sears and Reagin 2013).

Such approaches dovetail with the CCSS Standards for Mathematical Practice, perhaps especially those that call on students to "Make sense of problems and persevere in solving them," "Construct viable arguments and critique the reasoning of others," "Model with mathematics," and "Look for and make use of structure." Moreover, researchers most familiar with the evidence base for cooperative learning, such as Slavin, emphasize its potential contributions across classrooms and students, including high-achieving students:

> "Educators of the gifted should be in the forefront of the cooperative learning movement, insisting on the use of forms of cooperative learning known to benefit gifted and other able students. If these methods also happen to be good for average and below average students, so much the better." (Slavin 1991: 71)

He specifies that the two key factors necessary for positive achievement outcomes are: "1) the presence of group goals (the learner groups are working towards a goal or to gain reward or recognition); and 2) individual accountability (the success of the group depends on the individual learning of every member)." (Slavin 2010) For example, one of the approaches that he highlights, which combines both features, is "Team Assisted Individualization" (TAI). This strategy combines group work with attention to individual pacing, in order to address students' diverse individual learning needs while leveraging the educational strengths of heterogeneous classrooms.

These issues are discussed further in the body of this research review and its sources, including links to professional development resources. They offer touchstones for mathematics education to prepare all students for the multifaceted challenges they will encounter along their paths to college, careers, and democratic citizenship.

* * * *

Annotated Bibliography and Discussion of Sources

1) Historical Perspectives: Numeracy and Course-Taking Patterns in U.S. Context

Cohen, P. (1999) *A Calculating People: The Spread of Numeracy in Early America*, Routledge.
This book presents a history of the post-Revolutionary U.S. that "traces the development of numeracy from its origins in the Enlightenment to its flowering in the first half of the nineteenth century" – including the first census, in Jamestown, and the role of statistics in social and political reform, to reason and to persuade. Numeracy is approached as "an ability with or knowledge of numbers," analogous to literacy, leading to new forms of quantitative argumentation and calculation.

Lucas, S. (1999) *Tracking Inequality: Stratification and Mobility in American High Schools*. New York: Teachers College Press.
This book draws on sophomore data from the High School and Beyond longitudinal survey, employing extensive quantitative analysis alongside qualitative, ethnographic data. It discusses how, prior to 1975, most students in U.S. schools were assigned to formal, rigid curricular programs, or "tracks," that led to different courses of study in high school that in turn mapped onto disparate postsecondary trajectories (e.g. college or vocational tracks). This system limited students' mobility across multiple dimensions, impeding their ability to move from one track to another or take courses outside their designated curriculum. This formal tracking system was criticized by many researchers and others for its perpetuation of broader social inequalities, particularly defined by race/ethnicity and class, such that by the 1980s tracking waned. In its place, a system of course-level differentiations emerged, e.g. between honors and regular algebra.

Lucas finds that these "detracking" changes resulted, in some ways, in greater opportunities for student mobility and opportunities to choose more challenging classes, helping students "transcend low achievement in one domain by seeking demanding instruction in other subjects." (131) In other words, he finds evidence that more flexible course taking patterns, some of which could be characterized as "acceleration," can benefit students, including students previously demonstrating lower achievement. However, Lucas also finds persistent effects of social class on student trajectories, including effects on student placements. For example, he finds that, even after controlling for students' aspirations, prior achievement, and race, among other controls, students from more advantaged socioeconomic backgrounds tend to secure better placements. In addition, he finds that middle class parents often play a key role in helping their children to secure superior placements – undermining the notion that honors and gifted programs are unproblematic meritocracies. Differential parent access to information, education, and social networks are key mechanisms of student stratification within even "detracked" schools, leading Lucas to argue that "disproportionate access to information seems to have played a major role in transforming a potentially egalitarian reform into an inegalitarian one." (132) These findings suggest that, even in the absence of formal tracking, classroom-level student clustering will tend to reinforce existing class inequalities among students. While the book does not focus on mathematics courses specifically, its findings are important to consider both in terms of their implications for students' academic outcomes as well as students' multifaceted readiness for college, career, and citizenship in the 21st century.

Stinson, D. (2004) "Mathematics as 'Gate-Keeper' (?): Three Theoretical Perspectives that Aim Toward Empowering All Children With a Key to the Gate," *The Mathematics Educator*, Vol. 14, No. 1.
This article asks, "How might mathematics educators ensure that gatekeeping mathematics becomes an inclusive instrument for empowerment rather than an exclusive instrument for stratification?" It begins with a historical discussion of the ways in which mathematics education has served as a gatekeeper in schools, going back further in time than most other research to observe, "the "gatekeeper" concept in mathematics can be traced back over 2,300 years ago to Plato's dialogue, *The Republic*…[in which] Plato argued that mathematics was 'virtually the first thing everyone has to learn...common to all arts, science, and forms of thought.' Although Plato believed that all students needed to learn arithmetic—'the trivial business of being able to identify one, two, and three'—he reserved advanced mathematics for those that would serve as philosopher guardians of the city." (9)

In the U.S., the nearly 20-fold increase in the student population between 1890 and 1940 led some to believe "that the overall intellectual capabilities of students had decreased." (10) In a 1926 survey, 18 of 48 high school mathematics teachers thought that fewer students should learn mathematics, in contrast with teachers of other

subjects who wanted more students to learn their subjects. "The question of who should be taught mathematics initially appeared in the debates of the 1920s," over who was prepared to learn algebra, and "led to an increase in grouping students according to their presumed mathematics ability. This 'ability' grouping often resulted in excluding female students, poor students, and students of color from the opportunity to enroll in advanced mathematics courses," (10) as discussed by a range of scholars. Then in the 1950s, in the context of the Cold War, mathematics education became a matter of national concern, and has remained so since. The article emphasizes that various mathematics education reform movements have not only been concerned with *what* mathematics should be taught, "but more importantly, who should be taught mathematics." (10) The article then presents three alternative perspectives on mathematics education, oriented toward empowering all children: the situated perspective, the culturally relevant perspective, and the critical perspective. Together, these perspectives emphasize the contextual, culturally embedded dimensions of mathematical thinking and problem-solving, including the ethical dimensions of different uses of mathematics.

2) Beyond "Excellence vs. Equity": Social Contexts of Academic Achievement & Assessment

Burris, C. and D. Garrity (2008) *Detracking for Excellence and Equity*, Alexandria, VA: Association for Supervision & Curriculum Development.
This book discusses the rationales, process, and outcomes of detracking and phasing out ability grouping in New York s suburban Rockville Centre School District. While it does not focus on mathematics specifically, it makes a case that detracking can lead to higher achievement by all students as well as to narrowed achievement gaps among students. To challenge the "equity versus excellence" dichotomy, it argues that all students must be offered opportunities to study rigorous curricula in heterogeneous classrooms. The book emphasizes the need to question many people's basic assumptions about intelligence, ability, and classroom instruction as part of implementing these processes. Such questioning may be a major challenge to their implementation.

Faulkner, V. et al. (2014) "Race and Teacher Evaluations as Predictors of Algebra Placement," *Journal for Research in Mathematics Education*, Vol. 45, No. 3 (May 2014).
This article examines longitudinal data from the Early Childhood Longitudinal Study–Kindergarten Class of 1998-1999 (ECLS-K) to analyze the impact of teacher evaluation of student performance, compared with student demonstrated performance, on the odds of being placed in 8th grade algebra. It finds that, even after controlling for mathematics performance, Black students have a lower probability of placement in algebra by 8th grade. It also finds that teacher evaluations had a larger and adverse role for Black students compared with their peers. The article highlights research on implicit bias and implicit associations, alongside critical race theory, to help explain these data. Specifically, teachers and others who may not consciously intend racial bias may nonetheless exhibit racial bias in practice. Thus the article calls for any student math placement decisions to be systematically monitored, to mitigate against such bias and "ensure high-achieving students are placed appropriately, regardless of racial background." One practical recommendation is the analysis of student math performance data after removing names and other identifiers that may bias decision-makers.

Fischer, C., M. Hout, M. S. Jankowski, S. R. Lucas, A. Swidler, and K. Voss. (1996) *Inequality by Design: Cracking the Bell Curve Myth*, Princeton, NJ: Princeton University Press.
This book presents a thoughtful critique of Richard Herrnstein and Charles Murray's bestselling *The Bell Curve* (1994), which argued that inherited differences in intelligence explain inequality. Here, the authors return to Herrnstein and Murray's data, presenting a re-analysis and alternative argument that "economic fortune depends more on social circumstances than on IQ, which is itself a product of society." The book proceeds to debunk the myth of innate racial inequality promoted by Herrnstein and Murray, presenting evidence that racial differences in academic achievement are consequences, not causes, of social inequality. Mechanisms for perpetuating these inequalities include different levels of family resources, including income and education, as well as public policies that shape the contours of people's lives in schools, workplaces and beyond, affecting whose individual abilities and efforts matter.

Phillips, V. and C. Wong (2010) "Tying Together the Common Core of Standards, Instruction, and Assessments," *The Phi Delta Kappan*, Vol. 91, No. 5.
This article, written by two staff of the Bill and Melinda Gates Foundation, discusses the multifaceted challenges of reforming K-12 school systems to prepare all high school graduates for college. These include the failures of the No

Child Left Behind Act to go beyond "prescriptive accountability" (38) to assessments that are useful as levers of good practices. It argues for the potential of new assessments associated with the CCSS to be useful as such levers: "The Common Core represents an opportunity to totally redesign assessment systems, using the standards and the college-ready goal as the guides." (39) It also calls for schools that are "high demand, high support," (42) to achieve the college-readiness goal for all students. Challenges include the problem of "going to scale," which "in this country also means influencing the vendors of textbooks and assessments," (42) actors that are not mentioned in most of the other studies in this research review.

3) Research on Course-Taking Patterns & Classroom Diversity

Classroom Grouping Practices: Research Syntheses

Gamoran, A. and M. Berends (1987) "The Effects of Stratification in Secondary Schools: Synthesis of Survey and Ethnographic Research," *Review of Educational research*, Vol. 57, No. 4.
This synthesis is valuable in discussing the contributions of both quantitative and qualitative research on tracking and classroom grouping practices to addressing questions of educational practice. It demonstrates rigor by considering both types of research from the vantage of the other – using "ethnographic research to interpret the findings or survey analyses" and using "survey studies to assess the causal implications and generalizability of ethnographic findings." (415) It calls for, "Longitudinal, quantitative research that is sensitive to the actual dimensions of stratification in schools, and to classroom conditions and processes that vary across levels of the academic hierarchy." (415) More substantively, the article discusses that the most consistent finding from existing survey research is that, "Students in academic tracks are more likely to plan on attending college and more likely to actually enroll, even with controls for plans and achievement prior to tracking." (418) This finding highlights the importance of course-taking patterns to students' college preparedness, as underscored by other studies in this research review. It also indicates the potential detrimental effects of disparate high school course tracks.

Gamoran, A. (1992) "Synthesis of Research: Is Ability Grouping Equitable?" *Educational Leadership*, Vol. 50, No. 2.
This article provides a helpful research overview of a wide range of studies of tracking and ability grouping, including engagement with other reviews such as those by Slavin (1987; 1990). Overall, the article argues, "Grouping and tracking do not increase overall achievements in schools, but they do promote inequity, research suggests. To reduce inequality, we should decrease the use of both practices, and, where ability grouping is retained, improve its use." (1) More specifically: "Schools must make at least two sorts of investments to bring greater flexibility to their grouping systems: (1) they must reassess students' capabilities and take new information into account when making assignment decisions, and (2) they must enable students to make up curricular material they may have missed—for example, in tutorials during the school year or the summer—so that those who are ready to advance are not held back by lack of curriculum coverage." In addition, "Implementing more flexible grouping systems also means rotating teachers so that all students have opportunities to learn from the most effective teachers and to prevent the loss of morale that sometimes occurs for teachers who are assigned to low tracks year after year." The article emphasizes the importance of teacher professional development and classroom instruction, including to facilitate challenging learning opportunities for a range of students: "The elimination of grouping must be accompanied by staff development opportunities for teachers to learn strategies for enhancing the learning of all students in classes that are more diverse than those to which they are accustomed." In addition, as noted above, the most experienced or effective teachers should not be themselves tracked to teach only certain classes and students, which can be a mechanism through which ability grouping can exacerbate inequalities.

Slavin, R. (1987) "Ability Grouping and Achievement in Elementary Schools: A Best-Evidence Synthesis." *Review of Educational Research* 57: 293–336.
This article presents a review of research on the effects of between- and within-class ability grouping on the achievement of elementary school students. Though relevant to general analyses of classroom grouping practices, its relevance to middle and high school is limited by both the primary grade levels of students and the different kinds of grouping that occur at different grade levels – with more within-class grouping during primary grade years. It excludes both studies of nongraded plans and "studies of special classes for the gifted and for low achievers," on the basis that these classes entail many other changes "that make them fundamentally different than comprehensive ability grouping plans." (297) Among other criteria, at least 3 experimental and 3 control teachers were involved in

all included studies. "Overall, evidence does not support assignment of students to self-contained classes according to ability (median effect size [ES] = .00), but grouping plans involving cross-grade assignment for selected subjects can increase student achievement." Regarding mathematics, the review finds, "Within-class ability grouping in mathematics is also found to be instructionally effective (median ES = +.34)." In addition, "Analysis of effects of alternative grouping methods suggests that ability grouping is maximally effective when done for only one or two subjects, with students remaining in heterogeneous classes most of the day; when it greatly reduces student heterogeneity in a specific skill; when group assignments are frequently reassessed; and when teachers vary the level and pace of instruction according to students' needs." Overall, these findings are in keeping with the research synthesis of Gamoran (1992), which pertains not only to elementary school students but middle and high school students. Both syntheses highlight the potential contribution of limited, flexible student grouping in mathematics classrooms.

Slavin, R (1990) "Achievement Effects of Ability Grouping in Secondary Schools: A Best-Evidence Synthesis." ***Review of Educational Research*** **60: 471–499.**
This article, combining meta-analytic and literature review assessment of existing research, presents a review of 29 studies that compared between-class ability grouping to heterogeneous placements, in order to assess the effects of ability grouping on secondary school students' achievement. These included 6 randomized experiments, 9 matched experiments, 14 correlational studies examining periods between one semester and 5 years. "Findings indicate that comprehensive between-class ability grouping plans, different forms of ability grouping, and ability grouping by subject (except in social studies) had no effect on student achievement. The finding of zero effects of grouping for all ability levels contradicts earlier conclusions that demonstrated benefits of ability grouping for high-level students and detriments for low-level students." Given this finding that these different types of ability grouping had no effect on student achievement, Slavin argues that, "policy decisions about ability grouping must be based on criteria other than effect on academic achievement," such as affective and social outcomes. In this context recommends "reduction of between class ability grouping practices and consideration of cooperative learning methods," discussed elsewhere in this report.

Classroom Grouping Practices in International Comparison

Boaler, J. (2008) "Chapter 5: Stuck in the Slow Lane: How American Grouping Systems Perpetuate Low Achievement," in ***What's Math Got To Do With It?: How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject*****, Penguin Books.**
This book features a chapter discussing the research on ability grouping, including in international comparison. It notes that, "Many parents support ability grouping because they want their high-attaining, motivated children to be working with similar children. This is completely understandable and makes perfect sense. But we know from several international studies that countries that reject ability grouping – countries as varied as Japan and Finland – are among the most successful in the world, whereas countries that employ ability grouping, such as America, are among the least successful." (106) In contrast to the U.S., it discusses how in Japan, "the main priority is to promote high achievement for all and teachers refrain from prejudging achievement, instead providing all students with complex problems that they can take to high levels." (107) Mixed ability grouping is correlated with higher academic achievement due to more opportunities to learn for all students. It also mitigates against "borderline casualties" of students assigned to lower classes in ways that impair their chances for longer-term success. Also, in contrast to the assumption that ability grouping leads to improved, more "efficient" teaching, the chapter discusses the value of differentiated instruction in heterogeneous classrooms in terms of improved, higher-quality instruction, to help students engage with the complexity of math problems, rather than only in terms of expediency, as discussed further below.

Huang, M. (2009) "Classroom Homogeneity and the Distribution of Student Math Performance: A Country-Level Fixed-Effects Analysis," ***Social Science Research*****, 38 (2009).**
This article seeks to answer the question: "Does homogeneous grouping of students by ability improve performance for all students alike, or does it affect different students differently?" (781) To answer this question, it compares the distributions of math performance for students between Grade 8 and Grade 4 across countries with variable between-classroom ability grouping, while controlling for country-level unobserved heterogeneity. It finds that homogeneous ability grouping has no significant impact on student performance, on average, relative to heterogeneous grouping. However, it finds that homogeneous ability grouping "does increase performance

inequality by benefiting the high achievers at the expense of the low achievers." (781) In its conclusions, the article also discusses ability grouping in ways synonymous with tracking, stating: "[T]he policy maker needs to consider the effects of classroom homogeneity on the distribution of student performance to make an informed decision about whether to implement a policy of tracking." (788) Here the author also makes explicit the assumption of "the theory of ability grouping, [that] the causal mechanisms through which track placements affect student performance are classroom homogeneity." (788) Problematizing this theory's assumption are both this article's findings and a wider body of interdisciplinary research emphasizing the contributions of heterogeneity to learning and innovation, including through cooperative learning strategies.

Contingencies of Acceleration & Classroom Grouping Practices: Multiple Possibilities

Brizuela, B. et al. (2013) "The Impact of Early Algebra: Results From A Longitudinal Intervention," *Journal of Research in Mathematics Education*, Vol. 2, No. 2.
The article provides evidence of the potential benefits of early algebra (EA) by examining the performance over time of an experimental group of 15 students on an algebra assessment, from 3rd to 5th grade, compared with the performance of a comparison student group from the same school. It finds that early access to algebra is associated with increased scores over time on assessment "items that involve inequalities and graphs," and that in comparisons of both student groups, "we find increased scores on items that involve variables, functional relations, intra-mathematical contexts, tables, and algebraic expressions." (209) Thus, "The study adds to a body of literature that has been arguing for EA as well as a need to thread algebra throughout the mathematics curriculum, starting in the earliest grades." This article is less relevant to considerations of accelerated math course-taking than to the potential merits of integrated over traditional math curricula, with an emphasis on the value of threading algebra instruction throughout the K-12 math curriculum from elementary school onward. It suggests the value of integrated math curricula, rather than traditional curricula with more discrete, linear sequences of math learning. While arithmetic is often demarcated from algebra –with the former considered "elementary school" math and the latter "middle school" or "high school" math – the authors "argue that a deep understanding of arithmetic requires mathematical generalizations and understandings of basic algebraic principles." (211-212) In questioning conventional K-12 math course-taking patterns, this article underscores the interconnectedness of students' math learning experiences over these years. Math course-taking and learning possibilities in students' middle and high school years will be shaped by their earlier math learning experiences, and course acceleration may be less relevant to students who have learned math, including algebra, in a more integrated way over time. One limitation of this study is the small sample size.

Burris, C., J. Heubert, and H. Levin. (2004) "Math Acceleration For All," *Improving Achievement in Math and Science*, Vol. 61, No. 5.
The article discusses the case of a New York state middle school, serving a diverse suburban community, which in 1995 ended tracked math classes and instead adopted an accelerated math program for all students, including heterogeneous grouping at the classroom level. It presents 6 years of longitudinal data on students' subsequent math course enrollments and levels of achievement. It finds both that more students took advanced math courses in middle and high school, and that students gained in their overall math achievement in these courses, including higher scores on AP calculus exams. "By every measure, students benefited from studying accelerated math in heterogeneously grouped classes," and that, "when compared with earlier cohorts, more students took math courses at higher levels. The high standards did not discourage even the initial low achievers." In addition, "The research documented a statistically significant increase in the percentages of all students who took math courses beyond Algebra 2 in high school. This benefit applied to every subgroup… The rates at which each group took precalculus and Advanced Placement calculus also increased." As for the distribution of high student achievement, the article finds that, "high achievers are doing better, and more students have become high achievers. As long as the curriculum is rigorous, heterogeneous math classes can benefit all students." The article also cites research on the significant long-term benefits of studying advanced math in high school, along with two key factors that inhibit many students from doing so: 1) tracking; and 2) attitudes often associated with tracking. (Also see further discussion of this case study in reference below: Burris, C., et al. 2006)

Burris, C., et al. (2006) "Accelerating Mathematics Achievement Using Heterogeneous Grouping," *American Educational Research Journal*, Vol. 43, No. 1.
This article provides a longer, more in-depth discussion of the case study above (Burris et al. 2004). Rather than starting from the assumption that classroom homogeneity contributes to improved student performance, as does the

theory of ability grouping critiqued by Huang (2009), this article presents findings from a longitudinal study investigating the potential contribution of classroom heterogeneity to improved student achievement in middle school mathematics in "a diverse suburban school district," (105) in the context of an accelerated curriculum. It deploys a quasi-experimental cohort design to find that the "probability of completion of advanced math courses increased significantly and markedly in all groups, including minority students, students of low socioeconomic status, and students at all initial achievement levels. Also, the performance of initial high achievers did not differ statistically in heterogeneous classes relative to previous homogeneous grouping, and rates of participation in advanced placement calculus and test scores improved." (105) These findings are significant in suggesting the potential contribution of heterogeneous grouping to all students' learning and in challenging the dichotomy of "equity versus excellence." That said, this study does not address more fine-grained comparisons of students across the spectrum of assessed performance, including students assessed in the top 3-5% on exams who may in other studies be considered in a separate gifted category, rather than in a broader category of high-achievers. These findings also highlight the importance of curriculum, among other factors, in intersection with grouping practices – a point also emphasized by Tieso (2003), though in the context of homogeneous grouping. Thus across these disparate studies, a challenging curriculum, appropriately tailored to diverse students, emerges as vital to learning. Heterogeneous grouping did not detract from these outcomes, including for initial high achievers.

Elkind, D. (2006) *The Hurried Child: Growing Up Too Fast, Too Soon (25th anniversary edition)*.
First published in 1981, this book by child psychologist David Elkind examines and critiques trends toward "hurrying" young people in multiple areas of life, including education, by increasing expectations that can lead to unhealthy levels of stress and anxiety. It sounds a cautionary note, emphasizing the importance of taking time to learn subjects deeply, including math.

George, P. and W. Grebing (1995) "Talent Development and Grouping in Middle Grades: Challenging the Brightest Without Sacrificing the Rest," *Middle School Journal*, Vol. 26, Nov. 4.
This article discusses successful strategies to educate all students mostly in heterogeneous classes, as well as via supplementary enrichment programs, at the Broomfield Heights Middle School in Colorado – including students identified as gifted and those needing additional support. Mathematics is the one subject for which there are ability-grouped courses. In this way the article highlights how math stands out as the subject most often tracked or organized by ability-grouped courses in U.S. schools. This arrangement is presented as largely due to parental advocacy for homogeneous math courses, rather than due to research findings or classroom experiences. The article is notable for its discussion of larger political economic contexts of debates about tracking and ability grouping in schools, including changing occupational structures with the decline of manufacturing. "The pressing need to secure a desirable future for one's children has convinced many parents that schooling is a zero sum game, and that there is not enough success to go around. Fear of the future has translated into a struggle to garner the best the school has to offer for one's child…We believe this drama is often being played out as a battle between advocates for gifted children and proponents of the middle school concept" (12-13) of success for all students. It calls for using a block schedule to enable limited, subject-based grouping in math, but otherwise urges decision-makers to "challenge all students," "raise test scores across the board," "explain [to parents] heterogeneous teams," "offer exploratory mini-courses," and "use appropriate acceleration," including to enable high school math coursework that supports the postsecondary goals of all students.

Lee, S-Y., P. Olszewski-Kubilius, and G. Peternel (2010) "The Efficacy of Academic Acceleration for Gifted Minority Students," *Gifted Child Quarterly*, Vol. 54, No. 3.
This article presents a study of Project EXCITE, a program offering math enrichment and accelerated learning opportunities "to help elementary to middle school aged gifted minority students prepare for advanced tracks in high school." (192) It is based on interviews with 30 students and 7 teachers in Grades 4 through 9, all of who are participants in Project EXCITE, as well as educational outcomes data on whether students were accelerated in math by at least one year at the start of Grade 9 and whether they were successful in these math courses. However, though Project EXCITE is centered around educational enrichment opportunities that "supplement rather than supplant the regular school activities" (192) – including afterschool, weekend, and summer classes – which the study finds to be successful in most cases in facilitating students' success in accelerated math in 9th grade, the article is not framed as a study of the efficacy of educational enrichment in leading to accelerated learning outcomes. Rather, it is framed as a study of "the efficacy of academic acceleration," problematically misconstruing the independent and the dependent variables. This study is among those demonstrating both the relevance of acceleration to help a wide range of

students achieve educational goals, as well as its interdependence with educational interventions, supports, and classroom practices to leverage this potential.

Ma, X. (2004) "A Longitudinal Assessment of Early Acceleration of Students in Mathematics on Growth in Mathematics Achievement," *Developmental Review 25 (2005)*.
This article uses data from the Longitudinal Study of American Youth to examine "the rate of growth in mathematics achievement of accelerated gifted, honors, and regular students" (104) over grades 7 through 12, in comparison with non-accelerated counterparts. Regarding whether early acceleration in math promotes growth in achievement: "The answer is mixed. Early acceleration of students in mathematics (when it takes the form of early access to formal abstract algebra) appears associated with (a) little advantage in achievement in mathematics for gifted students, (b) small advantage for honors students, but (c) large advantage for regular students." (121) More specifically, "Accelerated regular students grew faster in achievement than both accelerated gifted and honors students and gained remarkable final (Grade 12) status in achievement that is better than that of non-accelerated honors students and almost comparable with that of accelerated honors students." (125) In addition, "Accelerated regular students outperformed accelerated gifted and honors students on the equity front; that is, equity issues, especially gender, racial, and socioeconomic equities, are not a concern once regular students were accelerated, whereas there are serious concerns about racial gaps among honors students and both gender and racial gaps among gifted students once they were accelerated." (125) School-level educational conditions are noted as important to success in the early acceleration of students in mathematics. For example, suburban schools with "a high socio-economic background" were associated with enabling regular students to benefit from early acceleration. "[S]uccess in early acceleration requires two conditions—students have high achievement and attend schools having high achievement. These conditions appear to be particularly critical for accelerated regular students." (124)

Ma, X. (2005) "Early Acceleration of Students in Mathematics: Does it Promote Growth and Stability of Growth in Achievement Across Mathematical Areas?" *Contemporary Educational Psychology 30 (2005)*.
This article uses data from the Longitudinal Study of American Youth to examine the acceleration of students into formal algebra at the beginning of middle school to assess growth within and across different mathematical areas (basic skills, algebra, geometry, and quantitative literacy). Using multivariate analysis, it finds that low achieving accelerated students "grew faster than not only low achieving students who were not accelerated but also high achieving students who were not accelerated. The rates of growth of accelerated low achieving students were even comparable to those of accelerated high achieving students." (439) Moreover, these students demonstrated the same potential to take advantage of early acceleration regardless of individual, family, and school characteristics.

Mills, C., K. Ablard, and W. Gustin. (1994), "Academically Talented Students' Achievement in a Flexibly Paced Mathematics Program," *Journal for Research in Mathematics Education*, Vol. 25, No. 5.
This article examines, over a year-long period, the mathematics performance of 306 3rd through 6th grade students identified as mathematically talented in order to assess the potential contribution of flexibly paced instruction to math achievement. All participants had completed at least one year of a flexibly paced mathematics course at the Johns Hopkins Center for Talented Youth, and had scored in at least the top 3% on a nationally normed, grade-appropriate test, such as the California Achievement Test. The study was also "designed to demonstrate the wide range and exceptionally high levels of mathematical abilities found within a group of highly able students." (497) It finds that, "When compared to students several grade levels higher, these highly able students gained as much as 46 percentile points from pre- to post-testing." It argues that, "Restricting such students to a rigid instructional pace and a 'grade-appropriate' curriculum may place them at risk for declines in motivation and achievement." (495) In addition to critiquing "curricular restraints," it also problematizes the notion of "homogeneous groups" that may be associated with callss for ability grouping, arguing that, "grouping students for an accelerated class by general ability level alone, especially on a grade-appropriate test, is often not sufficient without further differentiation to accommodate such differences in more specific attributes." (497) Interestingly, this point is similar to those made by critics of the notion of "homogeneous groups," as well as of ability grouping, such as Boaler (2008). While these authors ultimately have different stances toward ability grouping, they share in common the implication that differentiated instruction is important in all classrooms to challenge and engage a diversity of students.

Slavin, R. (1991) "Are Cooperative Learning and 'Untracking' Harmful to the Gifted?," *Educational Leadership*, Vol. 48.
This article summarizes research on classroom grouping practices and cooperative learning strategies, with a particular eye to potential effects on high-achieving students, including students assessed in the top 5%, who may

typically be considered in a separate gifted category. The article asks, "Is ability grouping beneficial for high-ability students? My reviews of research on between-class ability grouping (tracking) found it was not…In almost every study I reviewed, the achievement differences between ability-grouped and heterogeneous placement were not statistically significant for high achievers. The possibility that the failure to find educationally meaningful effects could be due to ceiling effects on standardized tests is remote; standardized tests are certainly designed to adequately measure the achievement of the top 33 percent of students." (68-69) Based on this overview of research, the author ultimately presents his view "that schools must recognizes individual differences and allow students to reach their full potential, and they can do this by using flexible, within-class grouping strategies and other instructional techniques without turning to across-the-board between-class grouping…." (69) Moreover, he states that, "I am in favor of acceleration programs (especially in mathematics) for the gifted and I believe in differentiating instruction *within* heterogeneous classes to meet the needs of students above (and below) average in performance. But I see no evidence or logic to support separate enrichment programs for gifted students. Enrichment is appropriate for *all* students. I see little evidence at all for separate tracks for high achievers." (70) Thus Slavin's conclusions dovetail with those of Tieso (2003) in their common finding of the potential value of accelerated math classes. Though overall, Slavin emphasizes the value of heterogeneous classrooms with differentiated instruction and points to the detrimental effects of tracking if, for example, only some course-taking sequences prepare students for college.

Swiatek, M. and Benbow (1991) "A 10-Year Longitudinal Follow-up of Participants in a Fast-Paced Mathematics Course," *Journal for Research in Mathematics Education*, Vol. 22, No. 2.
This article presents data from a follow-up survey of the Study of Mathematically Precocious Youth (SMPY), which offered its first two fast-paced mathematics classes in 1972 and 1973 to match "the classwork to the ability level of the gifted participants." (139) Ten years later, it reports on the status of "students who attended SMPY's fast-paced mathematics program and a comparison group of qualified students who did not attend the class." (139) Surveys found a series of correlations between participation in the SMPY fast-paced course and educational outcomes, including positive correlations with attending prestigious undergraduate institutions and with attending graduate school. On the other hand, "Self-esteem ratings, although high for both groups, were found to be higher for students who qualified for the class but did not participate." (138) Attitudes toward math and science were equivalent. The fast-paced classes were also found to cause no harm. In discussing study limitations, the article also states, "Because of the self-selection of students to the participant and non-participant groups, cause and effect cannot be established. There may be alternate explanations of the overall differences between the treatment and comparison groups that have nothing to do with the fast-paced classes (e.g. in initial motivation). Also, the small sample size involved in this study made it difficult to establish statistically significant differences. Nevertheless, the current work suggests possible relationship that are worthy of further investigation." (148) One of the article's subsequent tentative conclusions is that, "The use of fast-paced classes to meet the needs of highly intellectually talented children is, therefore, justified on the basis of findings from this study." (148) More data and information on the SMPY are available at: https://my.vanderbilt.edu/smpy/; according to the Web site: "The aim of this research is to develop a better understanding of the unique needs of intellectually precocious youth and the determinants of the contrasting developmental trajectories they display over the lifespan."

Tieso, C. (2003) "Ability Grouping is Not Just Tracking Anymore," *Roeper Review*, Vol. 26, No.1, 29-36.
This article argues that contemporary ability grouping should not be equated with tracking, stating that, "The present and future of ability grouping lies in the flexible use of grouping either between or within classrooms." (29) It discusses three types of grouping practices: 1) whole class instruction, 2) between-class grouping, and 3) within-class grouping. It characterizes the third type (within-class) as "flexible grouping," or a practice that groups students within the same class into smaller groups for specific activities and projects. The article summarizes: "Researchers are divided on the effects of such grouping arrangements, but most agree that some form of temporary ability grouping…when complemented by appropriate instruction, may have significant effects on student achievement." (32) It continues, "Grouping practices alone will have only small to moderate effects on achievement if they are not complemented with appropriately modified and differentiated curricula." (32) It also both cites Slavin's research in its discussion of the potential advantages of more flexible, nuanced approaches to grouping, and singles his research out for critique due to its supposed emphasis on "equity versus excellence," among other issues. These contradictions in representing Slavin's research (which he better represents himself, as cited below and above), including his multiple "best-evidence" syntheses, are not acknowledged. It is also dismissive of "research supporting the equity argument" as "largely qualitative and anecdoctal in nature" – ignoring (not citing) quantitative studies such as Lucas' *Tracking Inequality* (1999), discussed in this report, among other sources. In sum, while this article's

emphasis on the potential value of temporary, flexible, limited approaches to classroom grouping is important to consider and is reflected in the broader research literature, some of its larger claims are problematic.

Reconceptualizing Expansion and Remediation

Balfanz, R., J. McPartland, and A. Shaw (2002) "Reconceptualizing Extra Help for High School Students in a High Standards Era," Center for Social Organization of Schools, Johns Hopkins University. Paper prepared for the Office of Vocational and Adult Education, U.S. Department of Education.
This paper begins by noting that the CCSS reforms aim to "make intellectually demanding course work in high school the norm," and that "it is anticipated that insuring all students, regardless of background or income level, receive access to high quality instruction and rigorous coursework in high school will reap substantial social and economic benefits at both the individual and national levels." (1) It then acknowledges that these expectations place new demands on high schools, including providing adequate support for all students to succeed and "accelerate their learning and catch-up," (1) indicating the potential role of accelerated coursework for all students. In this context, the paper calls for reconceptualizing "extra help" not merely as remediation, but "organized learning opportunities to accelerate students' acquisition of intermediate level skills (e.g. fluency and vocabulary in reading and operating with rational numbers and integers in mathematics) and more advanced reading comprehension and mathematical reasoning strategies." (23) While the paper notes that the need for such support is greatest in high-poverty high schools, it emphasizes that, "if the goal is to have all high school students engage in an intellectually rigorous sequence of high school courses, most high schools in the United States will need to develop extra help programs and supports for significant numbers of their students." (23) To these ends, it offers the following recommendations: 1) time for substantial extra help needs to be found during the school day; 2) attempts to provide extra help need to avoid triggering the re-emergence of dead-end tracks and remediation; 3) the importance of teacher quality needs to be recognized and confronted; and 4) in low-performing high schools extra help needs to occur in the context of comprehensive whole school reform. (20-22)

Hern, K. (2012) "Acceleration Across California: Shorter Pathways in Developmental English and Math," *Change: The Magazine of Higher Learning*, Vol. 44, No. 3.
This article echoes arguments by Balfanz et al. (2002) about the need to rethink "remedial" education, presenting evidence of the benefits of accelerated learning opportunities for all students, including as a means for students to catch-up, meet, and excel beyond grade-level standards. Focusing on community college students, it presents acceleration as a means to help prevent the scenario in which "remedial course sequences have become the place where college dreams go to die." (60) It cites national studies that have found that the more semesters of remedial instruction students are required to take, the less likely they are to ever complete a college-level math or English course, as well as longer-term goals of degree completion. In other words, the article provides evidence of the multiple ends that different types of acceleration and expansion may serve, and the relevance of acceleration to a wide range of students. With regards to streamlined curricula, one key recommendation resonates strongly with CCSS reforms: "Current models of developmental education often break down complex skills and ways of thinking into discrete sub-skills, then deliver these skills up front to students in a linear, step-by-step curriculum. We argue instead for immersing students in challenging, authentic literacy and quantitative tasks and providing targeted reviews of foundational skills at the moment they are relevant to the higher-order work at hand." (64) Other recommendations emphasize the importance of "intentional support for affective issues," which can be the primary reason students are not successful in classes, as well as tailoring requirements to students' career objectives, which at the college level tend to be more clearly defined. The author is the director of the California Acceleration Project: http://cap.3csn.org/.

White, P., A., Gamoran, J. Smithson, A. Porter (1996) "Upgrading the High School Math Curriculum: Math Course-Taking Patterns in Seven High Schools in California and New York," *Educational Evaluation and Policy Analysis*, Vol. 18, No. 4.
This article examines course-taking patterns of students in seven high schools in California and New York that have attempted to enroll lower-level math students in more meaningful initial math courses – by both creating new transition math courses and eliminating the general math track. These changes were instituted in response to research findings that students enrolled in the general math track take less math overall and do not learn as much math as students in college-preparatory math courses. In this study, transcript data "indicate that the new transition math courses meet with partial success in providing a common curriculum to students with diverse math preparation," (285) with greater success in the CA schools studied (in San Francisco and San Diego) than those in

NY. CA's emphasis in its Math A program "on upgrading the lower-level math courses and providing teacher training specific to Math A" (305) is suggested as important to these successes.

Student Experiences of Classroom Grouping Practices

Boaler, J., D. William, and M. Brown. (2000) "Students' Experiences of Ability Grouping – Disaffection, Polarization, and the Construction of Failure," *British Educational Research Journal*, Vol. 26, Issue 5.
This article presents results from the first two years of a four-year longitudinal investigation into the ways that students' attitudes towards, and achievements in, mathematics are influenced by ability-grouping in six British schools, involving 943 students total. It draws on surveys, interviews, and classroom observations, over the course of students' transition from grade 8 to grade 9. During this period, students in four of the six schools transitioned from heterogeneous to homogeneous groups (or "sets," as they are referred to in the UK). The article finds that, "Ability-grouping was associated with curriculum polarisation. This was enacted through restriction of opportunity to learn for students in lower sets, and students in top sets being required to learn at a pace which was, for many students, incompatible with understanding. The same teachers employed a more restricted range of teaching approaches with 'homogeneous' groups than with mixed-ability groups, which impacted upon the students' experiences in profound and largely negative ways. Almost all of the students interviewed from 'setted groups' were unhappy with their placement." In other words, findings indicate overall negative outcomes from ability grouping, from a range of students' perspectives.

Oakes, J. (2005) *Keeping Track: How Schools Structure Inequality, 2nd edition*, New Haven: Yale University Press.
This book, first published in 1985, is a seminal text in debates over tracking in U.S. schools. "Tracking is the process whereby students are divided into categories so that they can be assigned in groups to various kinds of classes." (3) Through an examination of 13,719 teenagers at 25 junior and senior high schools, it finds that tracking, as a system for grouping students on the basis of assessed ability over time, both reflects and serves to perpetuate inequalities of race/ethnicity and social class in U.S. society. It argues that this is partly because, beyond subject matter content, part of what students learn from these educational experiences is that these inequalities, including their place in these hierarchies, are entirely natural and legitimate. It also discusses assumptions that often underlie support for tracking – including the belief that students learn best when grouped with others considered to be like them academically, as well as the assumption that student placement processes "accurately and fairly reflect past achievements and native abilities." (6) It notes that student placements tend to correlate with race/ethnicity and social class regardless of whether placements are based on test scores, teacher and counselor recommendations, or student and parent choices. The book also critiques the tendency of educational reformers to focus on attributed student characteristics rather than on "the content and processes of schooling itself for ways they might contribute to school failure." (xvi) Its analysis works to open up those processes for examination and critique, including by questioning the concept of "homogeneous ability grouping" – given that even in schools with ability grouping, there is considerable variance among students, whether in terms of their effort, learning styles, assessed achievement, or in other ways. Thus it argues for the value of differentiated instruction, tailored to diverse student proficiency levels and learning styles, for educational reasons that include but are not limited to issues of equity – a point echoed by Boaler (2008). Conversely, it argues that non-differentiated instruction, even if easier in some ways, should not be emphasized, given the social and educational costs to students in general.

Economics Literature on Peer Effects and Tracking

Collins, C. and L. Gan (2013) "Does Sorting Students Improve Scores?: An Analysis of Class Composition," National Bureau of Economic Research (NBER) Working Paper #18848.
This paper examines a student-level data set from the Dallas Independent School District to investigate how schools' measured variations in sorting students may affect student achievement. It finds that, "sorting homogeneously by previous performance significantly improves students' math and reading scores. This effect is present for students across the score distribution, suggesting that the net effect of sorting is beneficial for both high and low performing students." However, it looks only at test score data from 2004-2005, without attention to transcript data or to distinct grade-level progressions and outcomes across grades. Distinctions between elementary, middle, and high schools are not addressed. Thus, unlike more extensive and disaggregated longitudinal studies, it is not able to examine student course-taking patterns, tracking, and outcomes over time. This is a significant shortcoming in its assessment of

"tracking effects," which it assumes enable "teachers to direct their focus to a more narrow range of students," a description reflecting the assumption among other economists on this subject that tracking and student homogeneity lead to greater efficiency for teachers. It also asserts that there are "two potential effects that result from schools creating homogeneous classes—a 'tracking effect,'" per above, and "a peer effect, which causes a particular student's achievement to be influenced by the quality of peers in his classroom." However, a wider body of research by education and other scholars, which it does not engage, has found many and more complex "potential effects" from student grouping practices, including effects from biased placements and effects on student identity formation such as the internalization of social hierarchies that impinge on classroom grouping practices in ways that legitimize those inequalities. Relatedly, this article does not delve into the array of teaching practices, curricula, and other variables impinging on classroom learning and relations among both students and teachers. Data from larger "best-evidence" research syntheses, longitudinal quantitative studies, as well as qualitative studies focused on multi-causal pathways of student learning outcomes in specific contexts, better indicate some of these patterns.

Duflo, E., P. Dupas, and M. Kremer (2011) "Peer Effects, Teacher Incentives, and the Impact of Tracking: Evidence from a Randomized Evaluation in Kenya," *American Economic Review*, Vol. 101, No. 5.
This article examines data from first grade classrooms in Kenya, and does not look specifically at math, limiting its relevance to considerations of middle and high school math classes in the U.S. However, it is widely cited among other economists on the subject of school tracking and peer effects, including to generalize to quite different contexts (such as U.S. high schools) about the possible efficiency gains of tracking for all students. Most of this literature does not engage the extensive research by education scholars and sociologists on these topics, including Lucas (1999) and Slavin (1987, 1990), among many others. Nor does it address the finer-grained, contingent, contextual analyses of educational processes that many of these studies present, including attention to classroom dynamics and course content, which would enable questioning the meanings and mechanisms underlying ascribed "peer effects." In its findings, it starts from the premises, "To the extent that students benefit from high-achieving peers, tracking will help strong students and hurt weak ones," while on the other hand, "all students may benefit if tracking allows teachers to better tailor their instruction level." Also, "Lower-achieving pupils are particularly likely to benefit from tracking when teachers have incentives to teach to the top of the distribution," as they are in the case it presents. Here, however, another constraint on generalizing to the U.S. arises, as in Kenya most students drop out of school by the 8th grade, leading to a situation where "teachers have incentives to focus on the students at the top of the distribution" and "[these incentives] likely help maintain a culture in the educational system that is much more focused on the top of the distribution than in the United States." (1740-1741) Ultimately the article argues that, "While the direct effect of high-achieving peers is positive, tracking benefited lower-achieving pupils indirectly by allowing teachers to teach to their level." Among other issues, given that the article's argument and findings are based around a context of such differential teacher incentives to teach a range of students, they should not be construed as evidence for the general benefits of tracking in U.S. schools, particularly middle and high schools.

Imberman, S. et al. (2009) "Katrina's Children: Evidence on the Structure of Peer Effects From Hurricane Evacuees," National Bureau of Economic Research (NBER) Working Paper #15291.
A starting point of this paper's analysis is the aftermath of 2005 hurricanes Katrina and Rita, which forced many children and families to relocate, particularly throughout the Southeast. It notes that, "While schools quickly enrolled evacuees, receiving families worried about the impact of evacuees on non-evacuee students." It presents data from Houston and Louisiana and argues that, on average, elementary math test scores in Houston were "moderately reduced" due to the arrival of students displaced by hurricanes Katrina and Rita. However, while this paper frames these data in terms of a "natural experiment," the exceptional circumstances surrounding new students' arrival and schools' sudden need to accommodate them make generalizing problematic. Longitudinal data on student course-taking patterns and outcomes over time are also not addressed. In addition, though the article argues that, "student achievement improves with high ability and worsens with low ability peers," it also states, "We find that, on average, evacuees had little impact on native outcomes with the exception of a negative impact on math scores in elementary schools and attendance in middle/high schools in Houston." (36) It continues "exposure to undisciplined evacuees increased native absenteeism and disciplinary problems, supporting a 'bad apple' model in behavior," yet elaborates, "we see no evidence of this in terms of test scores, [though] we do find that having a few misbehaving evacuees in an elementary classroom appears to worsen behavior of native students both in terms of attendance and disciplinary infractions." (37) It does not delve into teaching practices and other classroom variables, nor address the broader education research literature. It also deploys models, such as a "bad apple" model of behavior, which much social science research would find simplistic and problematic. That said, it suggests the need for more skillful local interventions by school staff and others to facilitate more successful outcomes for all students, perhaps by drawing

on research by Burris (2008) and others. Research in this review by Schneider et al. (1998), on "curricular dislocations," is also relevant.

Journalistic Commentary on Classroom Grouping Practices

Garelick, B. (2013) "Let's Go Back to Grouping Students by Ability," *The Atlantic*, March 2013.
This article contends that U.S. students' math educations have suffered in recent decades due to watered down curricula tied to the waning of tracking and ability grouping, but that such ability grouping is now making a promising comeback. However, while it claims that, "Math classes are now more about math appreciation and being able to explain how a procedure works rather than the mastery of skills and procedures necessary to solve problems," it also seems to be an example of the ways that trends and research on tracking and ability grouping may be reported in the news media in ways that are overly simplistic or over-generalizing. For example, one of the article's own sources documents improvement in U.S. students' math proficiency in international assessments, despite its nostalgia about past math education. The article also simultaneously asserts an earlier trend toward, on the one hand, the elimination of ability grouping and tracking, and on the other hand, the rise and persistence of programs in "gifted and talented education," as well as honors tracks, apparently during this same period.

Petrilli, M. (2011) "All Together Now?: Educating High and Low Achievers in the Same Classroom," *Education Next*, Winter 2011.
This piece argues that the greatest challenge facing U.S. schools "isn't the budget crisis, or standardized testing, or 'teacher quality," but rather "the enormous variation in the academic level of students coming into any given classroom." (49) Though, the author also acknowledges that these different dimensions of education are mutually contingent in classroom practice – e.g. when he underscores how the success of differentiated instruction depends on adequate teacher professional development. Rather perplexingly, the article simultaneously presents a story of an elementary school in Takoma Park, MD in which differentiated instruction has been successfully implemented in a very diverse student community (with positive effects for students at all levels), and at the same time concludes that this approach "sure seems rickety" (55) – apparently due to its dependence on well-trained staff and adequate support.

4) California Mathematics Course-Taking Patterns & Pathways to College

Overview of National Course-Taking Patterns in Math and Science

National Science Foundation (NSF), Science and Engineering Indicators 2014,
"Chapter 1: Elementary and Secondary Mathematics and Science Education"; available at:
http://www.nsf.gov/statistics/seind14/index.cfm/chapter-1/c1s2.htm
Includes discussion of Student Course-Taking in Mathematics and Science, comprising: 1) High School Graduation Requirements and Curriculum Standards; 2) Ninth Grade Mathematics and Science Course-Taking; and 3) Participation and Performance in the Advanced Placement Program.

Middle School: Pivotal Years for Unfolding Futures

Finkelstein, N. et al. (2012) "College Bound in Middle School and High School?: How Math Course Sequences Matter," *The Center for the Future of Teaching & Learning*, WestEd.
This report draws on transcript data for a representative cohort of approximately 24,000 California students throughout their middle and high school enrollments, from Grade 7 through Grade 12, to examine course-taking patterns and effects over time. It finds 7th grade to be pivotal: "Math performance in grade 7 is predictive of high-school math course-taking," (viii) and thus middle school years as crucial to laying a foundation for future math achievement and educational success, including college attendance. It also finds that, "While the majority of students who achieved at least Proficient on their math CSTs are those who took algebra 1 in grade 8, geometry in grade 9, and algebra 2 in grade 10, in general this accelerated pathway does not support students who are not proficient in math in grade 7." (viii) Again, grade 7 is a key year for assessing students' math proficiency and providing additional support for those who have fallen behind in elementary school. Such constructive assessment and support, rather than acceleration, should be emphasized for these students during the middle school years. Such

support might include course designed specifically to meet the needs of these students. Moreover, "[T]he large variation in students' grade-7 math performance suggests that more work must be done at the elementary level to prepare students for success in middle-grade math." (ix) The report calls on districts to improvement source-placement criteria and provide teacher professional development focusing on "strategies for teaching key algebraic concepts." (ix) It emphasizes that the implementation of the CCSS in an opportunity to open discussions about improved math instruction, course pacing, and course placement, including teachers' professional development needs.

Kurlaender, M., S. Reardon, and J. Jackson (2008) "Middle School Predicators of High School Achievement in Three California Districts," California Dropout Research Project (CDRP) Report #13 (CDRP: http://www.cdrp.ucsb.edu/).
This paper investigates predictors of high school graduation and success. It examines 7th grade cohorts from the San Francisco, Fresno, and Long Beach School Districts in terms of key middle school academic performance measures. It finds that standardized assessments, timing of algebra (specifically, completion of algebra by 8th grade), and course failures in middle school provide indication of students' high school academic success. The paper does not aim to identify causal mechanisms, but rather describe key associations relevant to policymakers. It suggests the importance of more students both enrolling in and successfully completing algebra by 8th grade – a goal contingent on multiple supplemental supports discusses in the broader research literature. In particular, high-quality instruction and extra support, as needed, may be crucial for many students.

Loveless, T. (2013) "Part III: Advanced Math in Eighth Grade," in report, "How Well Are American Students Learning?" 2013 Brown Center Report on American Education, Brown Center on Education Policy, Brookings Institution, Washington, DC.
This report examines the nationwide trend toward increased enrollments in Algebra in 8th grade, as well as student achievement outcomes on the National Assessment of Educational Progress (NAEP). It finds, "States boosting advanced math taking are no more likely to show NAEP gains than other states." (4) Further, "states that are most selective in math placements – not aggressively accelerating eighth graders into advanced courses – are more likely to show achievement gains in those courses." (4) Examining state enrollments and NAEP results only for 8th grade geometry, the report also finds that, "Geometry sits at the peak of the hierarchy of eighth-grade math courses, enrolling the nation's best math students (about 5%). Presumably, these are students who took algebra in seventh grade. Increases in eighth-grade geometry enrollments evidence no association with changes in mean achievement for the course," clarifying that, "analyses in the study are only correlational and cannot confirm or reject causality." (4) Here the report suggests that the push to enroll more students in 8th grade algebra seems to have resulted in more underprepared students being enrolled as well, leading to no average increase in NAEP scores; in contrast, the smaller percentage of students accelerated to enroll in 8th grade geometry appear to be better prepared, on average. One implication is that any initiatives emphasizing math acceleration should also ensure adequate support for learning for all students, and acceleration should not be promoted as an end in itself.

U.S. Department of Education (1997) "Mathematics Equals Opportunity," White Paper prepared for U.S. Secretary of Education Robert W. Riley. http://mathpl.us/docs/mathemat.pdf
This report is one touchstone of the observation that, for many U.S. students, algebra serves as a "gatekeeper" to postsecondary education and good jobs. It is based on data from the 1988 National Education Longitudinal Study (NELS: 88), with 88 samples of 24,599 eighth graders from 1,052 schools, and the 1992 follow-up study of 12,053 students. It begins, "In the United States today, mastering mathematics has become more important than ever. Students with a strong grasp of mathematics have an advantage in academics and in the job market. The 8th grade is a critical point in mathematics education." (5) It continues by noting how, "Achievement at that stage clears the way for students to take rigorous high school mathematics and science courses—keys to college entrance and success in the labor force. However, most 8th and 9th graders lag so far behind in their course taking that getting on the road to college is a long way off." (5) The report's findings include the key point that taking rigorous mathematics and science courses in high school appears to be especially important for low- income students, yet these students are less likely to take them. Also, students whose parents are involved in their school work are more likely to take challenging mathematics courses early. Lastly, the results of the Third International Mathematics and Science Study (TIMSS) reveal that the middle school mathematics curriculum may be a weak link in the U.S. education system.

National Center for Educational Statistics (1999) "Do Gatekeeper Courses Expand Education Options?"; available at: http://nces.ed.gov/pubs99/1999303.pdf

This report "examines the relationship between enrollment in algebra or foreign language as an eighth-grader in combination with high school course-taking patterns (math and foreign language) with applying to a four-year college/university." (1) It finds that: "Students who enrolled in algebra as eighth-graders were more likely to reach advanced math courses (e.g., algebra 3, trigonometry, or calculus, etc.) in high school than students who did not enroll in algebra as eighth-graders." (1) It also finds, "Students who enrolled in algebra as eighth-graders, and completed an advanced math course during high school, were more likely to apply to a four- year college than those eighth-grade students who did not enroll in algebra as eighth-graders, but who also completed an advanced math course during high school." (2) It concludes that while, "Not all students who take higher-level math or foreign language courses in high school apply to four-year colleges or universities during their senior year in high school," still, "students who do enroll in algebra or foreign language during eighth grade are more likely to pursue a four-year postsecondary education at the end of high school. This is true regardless of the level of math or foreign language attained by these students." (3) These findings substantiate the status of 8th grade algebra as a gatekeeper course and also help explain the push toward higher student enrollments in 8th grade algebra, a trend observed in California and beyond since this report was published.

Wang, J. and P. Goldschmidt (2003) "Importance of Middle School Mathematics on High School Students' Mathematics Achievement," *The Journal of Educational Research*, Vol. 97, No. 1.
This article examines "the consequences of middle school mathematics course-taking, a measure of opportunity to learn, disparity in students' high school mathematics achievement, and achievement growth." It uses longitudinal data over 4 years from an ethnically and linguistically diverse school district. "The results indicate that course-taking patterns, even when controlling for prior achievement, play a prominent role in identifying performance differences. The distribution of mathematics course taking among various subgroups not only differed in Grade 8 but also became increasingly inequitable by Grade 11." (3) These findings are generally consistent with the results of the research reviews summarized above, as well as social science literature on "path dependency" effects.

High School: Transitions to College and Careers

Choi, K. and E. Shin (2004) "What are the Chances of Getting into a UC School?: A Look at the Course-taking Patterns of High School Students For UC Admissions Eligibility," White Paper, Center for the Study of Evaluation National Center for Research on Evaluation, Standards, and Student Testing, UCLA.
This paper examines course-taking patterns of high school cohorts to investigate reasons for the disproportionately low rates of UC enrollments among traditionally underrepresented minority students. Based on district data, the study found that only 3.7% of African American and 4.4% of Hispanic students are eligible for UC admission by the end of high school. Results include the finding that the Algebra 1 course grade is associated with the highest probability of staying on-track for the students who attended and finished the 11th grade. In terms of general patterns of eligibility or falling "off-track" of eligibility, "most students fall off-track in the 9th grade and between grades (e.g., from Grade 9 to 10, and Grade 10 to 11)." (91)

Policy implications include: "to make more students to stay on-track, the school administrator must examine the time intervals during which a large proportion of students fall off-track because they do not take or complete the appropriate benchmark courses.…Finding the reasons that some students stop taking the required courses appropriate for a particular semester would be critical in developing an intervention program to help them get back on-track," as would assessing the adequacy of students' support systems. (94) Administrators also need to address the potential unintended consequences of automatic course enrollments, which may "be justified in terms of the student's performance history in math, but it closes the options through which the student increases the chances of going to a college such as a UC school. An alternative might be opening up the ninth-grade Algebra 1 class to everyone willing to do the work, but with the necessary assistance to complete the course." (95-96)

Finkelstein, N. and A. Fong (2008) "Course-taking Patterns and Preparation for Postsecondary Education in California's Public University Systems Among Minority Youth," a report (No. 035) of the Regional Educational Laboratory of WestEd; available at: http://ies.ed.gov/ncee/edlabs
This report highlights the importance of appropriate high school course course-taking, especially in mathematics, to facilitate college eligibility. For example, it finds that, "Many students appeared to be missing required college-preparatory courses, rendering them ineligible for the program even if they otherwise ranked in the top of their high school class." (i) Overall, it finds, "the high school program for college preparation begins in 9th grade and that making up missed preparatory courses and academic content is likely to be difficult for students who put off college-

preparatory work until later in their high school career." (i) In addition, regarding math preparation, it finds that "Completing one year of college-preparatory…mathematics in 9th grade is an enormous challenge for many students." (ii) Addressing these specific shortfalls in academic preparation would lead to significantly improved outcomes for these students, reducing the probability of high school subject requirements becoming a barrier to college eligibility and contingent careers.

Riegle-Crumb, C. (2006) "The Path Through Math: Course Sequences and Academic Performance at the Intersection of Race-Ethnicity and Gender," ***American Journal of Education*****, Vol. 113.**
This article examines national high school math course-taking patterns for students of different race-ethnicities and genders. It uses 1990s transcript data from the Adolescent Health and Academic Achievement data set, part of the National Longitudinal Study of Adolescent Health. It finds that African American and Latino males receive lower returns from taking Algebra I during their freshman year, compared with white males. Even when African American and Latino males begin in the same position and controlling for academic performance, the article finds that they reach lower levels of math course sequences in high school. Moreover, African American males receive less benefit for high math grades. Lower returns are not observed for minority female students, however. Thus, despite gender inequality in math, "the story here is not specifically one of feminine disadvantage. Instead, this article focuses much-needed attention on the particularly disadvantaged position of African American and Latino males in a core high school subject." (116) As it elaborates, "While African American and Latino students of both genders generally start high school in lower math courses compared with their white peers, for minority female students, this appears to be the primary hurdle to reaching comparable levels of math with white female students by the end of high school." (116) It calls for more attention to racial-ethnic inequality in math among male students. The article also argues that, "the patterns observed here may be partly a consequence of teachers' perceptions of minority students' lower academic abilities and motivation…as well as the content and form of the curriculum within math courses, which has shown to have little connection and relevance to minority students' cultures and experiences." (118) One shortcoming of this article is that it does not include discussion of data pertaining to Asian students, even though these data are included in its study and associated tables and figures. A finer-grained analysis within and among ethnic groups would be useful but is not addressed here.

Schneider, B., C. Swanson, and C. Riegle-Crumb (1998) "Opportunities for Learning: Course Sequences and Positional Advantages," ***Social Psychology of Education*****, Vo1. 2.**
This article examines data from the National Education Longitudinal Study of 1988–94 to examine the educational outcomes of course sequences as well as curricular dislocations within the context of school choice. It observes that, "Courses that are differentiated and sequentially organized, such as mathematics and science course sequences, create opportunities for positional advantages in a school's curricular structure. When students make a nonroutine change of schools – that is, transferring to a school outside of the regular attendance zone – they are at risk of changing their positional advantage." (25) The study finds that students' course sequences at 10th grade are the strongest predictors of 12th-grade mathematics and science course sequences. Moreover, students in higher mathematics and science sequences show greater achievement gains. Students in higher mathematics sequences are less likely to have behavior problems and are more likely to graduate from high school and attend college. Students who make a nonroutine change of school are more likely to be in lower course sequences than students who do not transfer. "Thus, course sequences in mathematics and science are tangible experiences with real consequences for students' lives during and beyond high school and are sensitive to transitions which disrupt the continuity of students' high school careers." (25)

Steinberg, A., C. Almeida, L. Allen, and S. Goldberger (2003) "Four Building Blocks for a System of Educational Opportunity: Developing Pathways To and Through College for Urban Youth," a report of the Jobs for the Future ***From the Margins to the Mainstream*** **program.**
This report highlights how economic and demographic changes over the previous two decades created a need to rethink existing educational policies to ensure that more young people acquire higher-level skills and credentials. It makes that argument especially because the fastest growing segments of our society comprise "the young people who have been least well-served by our secondary and postsecondary education and workforce development systems." (iii) Here it highlights that while 74% of young people graduate from high school and nearly three-quarters of these graduates enroll in postsecondary education, over half fail to complete a degree and one-third never make it to their sophomore year. It then discusses four types of institutions that "constitute the building blocks for a system of educational opportunity that includes vulnerable youth," including 1) "secondary/postsecondary blends"

to help students transition to college; 2) extended learning opportunities, including "creative use of time and resources outside the usual school building and usual school day to engage young people in intensive learning that is potentially credit-bearing (toward high school graduation)"; and 3) "education/employment blends," as in programs that combine academic and career-related studies. (iii-iv) These recommendations are notable in part for the role they highlight for earlier exposure to college-level learning for all students.

Transitions from High School to College for Prospective STEM Majors

Shaughnessy, J. M. (2011) "Endless Algebra – the Deadly Pathway from High School Mathematics to College Mathematics," *NCTM Summing Up*, February 2011; available at: http://www.nctm.org/about/content.aspx?id=28195
This piece, written by the president of the NCTM, highlights ways in which, "Both high schools and colleges are operating under outdated assumptions" about mathematics curricula in the transition to college. It articulates his concerns about the inadequacy of the K-12 mathematics curriculum in preparing students for the STEM challenges and fields of the 21st century. He explains that the MAA in 2011 published *Curriculum Renewal Across the First Two Years* (CRAFTY) of college, "which examined the mathematical needs of many client disciplines, such as biology, chemistry, economics, engineering, physics, and others. CRAFTY advocates secondary mathematics that facilitates students' transition from high school to college by providing (1) a greater emphasis on modeling; (2) consideration of multivariate topics; (3) an emphasis on computational skills that are useful in other fields; and (4) a strong foundation in units, scaling, and dimensional analysis." (CRAFTY report available at: http://www.maa.org/sites/default/files/pdf/CUPM/crafty/introreport.pdf)

As for implications for high schools, he presents an overview of a panel on "Transition from High School to College: Should There be an Alternate to Calculus?" at the 2011 Joint Mathematics Meetings conducted by the Mutual Concerns Committee of NCTM and the MAA. As he summarizes alternatives for high school math pathways:

> The NCTM/MAA Mutual Concerns panel presented four concrete, relevant, alternative mathematical transition paths for high schools and colleges to consider. One path emphasizes quantifying uncertainty and analyzing numerical trends. Its mathematical foci include data analysis, combinatorics, probability, and the use of data collection devices, interactive statistical software, and spreadsheet analyses of numerical trends. A second transition path concentrates entirely on the development of students' statistical thinking, beginning in high school and continuing into the first year of college. Statistical thinking involves understanding the need for data, the importance of data production, the omnipresence of variability, and decision making under uncertainty. This path differs both in purpose and approach from an AP statistics course. A third path recommends building a transition grounded in linear algebra. Linear algebra integrates algebra and geometry through powerful vector methods. It offers an arena in which students can work with important multivariable problems and provides students with general-purpose matrix methods that will serve them well in many fields, including mathematics, science, engineering, computer science, and economics. Finally, a fourth transition path incorporates a suggestion that an alternative to calculus can be found in calculus itself—but a vastly different calculus from the traditional calculus I. This path concentrates on multivariate applications of both calculus and statistics, because today's application problems rarely involve single-variable calculus or univariate statistics. We live in a multivariate world. Therefore, students' mathematics experience in preparation for their transition to college should emphasize multivariate functions, partial derivatives, multivariate data sets, and analyzing covariance.

These possibilities are discussed specifically in the context of CCSS reforms:

> The Common Core State Standards provide us with an opportunity to rethink the sequence of school mathematics, as well as a challenge to provide exciting new pathways and transitions from high school to college mathematics. We need to offer students alternative pathways as they make their transition from secondary school and into colleges. The mathematics paths that we provide

> for our students need to prepare them for existing fields that are changing rapidly, as well as for emerging fields—and for fields that don't yet exist. In my view, the current deadly sequence of ever-repetitive and out-of- touch experiences in algebra—the sequence intended to lead students to a single variable calculus course—will not accomplish this goal. It is time that we replace the eternal algebra transition from high school to college with some viable and exciting 21st century mathematics alternatives.

Rosenstein, J. and A. Ahluwalia (2012) "Putting Brakes on the Rush to AP Calculus," Working Paper, Rutgers University and Brookdale (NJ) Community College.; available at: http://dimacs.rutgers.edu/~joer/AP-Calculus.pdf

The authors are professors of mathematics at Rutgers University and at Brookdale (NJ) Community College. This paper presents their findings on Rutgers students' transitions from high school to college calculus courses as well as data from a follow-up study on students' reasons for enrolling in AP Calculus in high school, including an array of motivations characterized as "intrinsic" and "extrinsic." Ultimately, it finds that, "there is no evidence that encouraging more students to take AP Calculus will expand the STEM pipeline." (13) It argues that, "The problem is not that there are too few students in the STEM pipeline, but that the pipeline is too leaky." (16) Here the paper proposes "strategies for encouraging students to consider STEM careers," including: early interventions such as STEM-related field trips; conversations with STEM professionals; teacher discussion of real-life applications of course material, especially in AP Calculus; and developing courses "that students see as relevant to their education, their careers, and their lives; that students get the assistance that they need to succeed in mathematics; and, that they are not pushed ahead at a pace that will cause them to avoid mathematics in the future." (16-17)

Rosenblum, I. and R. Kazis (2014) "Middle-Skill STEM State Policy Framework," a report of the Jobs for the Future Postsecondary State Policy initiatives and Achieving the Dream, Inc.

This report focuses on student preparation for so-called "middle-skill" STEM jobs – defined as those requiring less than a B.A. degree level of skill but often postsecondary certificates or associate degrees. These include jobs in manufacturing, health care, and construction, and often oriented toward implementation rather than invention. The report highlights five policy recommendation to advance education and workforce development in the "middle-skill STEM labor market," as an opportunity for greater "efficiency and equity," including by helping students move from high school to college most effectively. These recommendations include: 1) **"**Create pathways to careers: Ensure that STEM programs meet employer needs"; 2) "Open doors to STEM: Improve math preparation and developmental education to boost student success"; 3) "Focus on student completion: Create new models that lead to degree attainment"; 4) "Make informed decisions: Improve data collection and data use to enhance transparency, accountability, effectiveness and equity"; and 5) "Provide incentives for success to both students and community colleges: Encourage innovation and reward better outcomes for STEM students and the STEM workforce." (7-22) In particular, the report argues, "Realizing the potential of community colleges to be the primary source of well-prepared middle-skill STEM workers can go a long way toward strengthening regional economies and the employers that depend upon this segment of the workforce. It can also be a boon to low- income, minority and first-generation students who are disproportionately served in our nation's Associate's degree-granting institutions and who are seeking the stability that a quality job can provide." (2) Here, accelerated high school math pathways again have a potential role to play in helping students prepare for and complete college, including community college options leading to well-paid jobs.

Stone, J. R., C. Alfeld, and D. Pearson (2008) "Rigor and Relevance: Enhancing High School Students' Math Skills through Career and Technical Education," *American Educational Research Journal*, Vol. 45, No. 3 (Sept. 2008).

This study examines a classroom intervention to integrate more mathematics into a high school occupational curriculum – including programs in agriculture, auto technology, business and marketing, health, and information technology. It found that "After 1 year of the math-enhanced CTE lessons, students in the experimental classrooms performed equally on technical skills and significantly better than control students on two standardized tests of math ability (TerraNova and ACCUPLACER)." Notably, the intervention analyzed here includes both curricular and pedagogic dimensions, along with intensive teacher professional development.

Senior Year Mathematics: Rush to Calculus?

MAA/NCTM (2012) "Joint Position on Calculus," Mathematical Association of America (MAA) and National Council of Teachers of Mathematics (NCTM).
This statement on calculus speaks to the question: "How should secondary schools and colleges envision calculus as the course that sits astride the transition from secondary to postsecondary mathematics for most students heading into mathematically intensive careers?" More background on this question and the rationale for this MAA/NCTM joint statement can be found in the document below.

The core of the MAA/NCTM joint statement states: "Although calculus can play an important role in secondary school, the ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. The college curriculum should offer students an experience that is new and engaging, broadening their understanding of the world of mathematics while strengthening their mastery of tools that they will need if they choose to pursue a mathematically intensive discipline." (Available at: http://www.nctm.org/about/content.aspx?id=32351)

MAA/NCTM (2012) "Background to the MAA/NCTM Statement on Calculus," available at: http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/MAA_NCTM_background.pdf
This document provides background on the rationale for the MAA/NCTM statement cited above, discussing reasons that center on students' transitions from high school to college, especially students enrolling in high school Calculus and demonstrating interest in STEM college majors and STEM careers. As the background document elaborates, the MAA/NCTM statement was formulated for reasons that included:

1) Concerns "that Calculus instruction was acting as a filter, keeping many students out of careers in Science or Engineering," given the "low rate of completion among those who enter college with the intention of pursuing a career in the STEM (Science, Technology, Engineering, or Mathematics) fields" (1) and the significant contribution of college Calculus classes to this loss;
2) Despite growing high school enrollments in Calculus, evidence that "for many students their experience of Calculus in secondary school did not give them the preparation they needed to succeed in college" (1);
3) Moreover, understanding of other fields of high school mathematics may suffer, as increasing numbers of students are being accelerated to enroll in Calculus in 12th grade, with many "short-changing their preparation in and knowledge of algebra, geometry, trigonometry, and other precalculus topics so that calculus will appear on their secondary school transcript" (3);
4) In college, curricular repetition and wasting students' time may also be a problem – ironically, as an unintended consequence of earlier acceleration – since, even when students are prepared for college Calculus, "Today's reality for most students headed into STEM careers is a double dose of introductory calculus, once on each side of the high school to college divide" (3);
5) Nor is Calculus necessarily the best preparation for college mathematics, including for STEM majors, as recognized by the Committee on the Undergraduate Program in Mathematics (CUPM), which as early as 1987 "recommended analytical geometry, discrete mathematics, and matrix algebra as alternatives to calculus for those who would study college-level mathematics in 12th grade" (3);
6) For those interested in STEM majors, Calculus has also become an unnecessary "bottleneck," as a prerequisite for the pursuit of other math and science courses, such that, "We also need to encourage alternatives to calculus for the first-year of college. There is no reason for discrete mathematics or linear algebra or a course in transformational geometry to require calculus as a prerequisite" (3);
7) In sum, the document emphasizes that, "There is no question of the central importance of calculus…But what students need to know about calculus, when they need to know it, and how they should learn it are questions that still need to be actively engaged…[I]t is essential that MAA and NCTM work together to address these questions and to re-envision the role of calculus in secondary and post-secondary mathematics." (4)

Rosenstein, J. and A. Ahluwalia (2012) "Putting Brakes on the Rush to AP Calculus," Working Paper, Rutgers University and Brookdale (NJ) Community College.; available at: http://dimacs.rutgers.edu/~joer/AP-Calculus.pdf
This paper, also mentioned in the section above, presents results from two separate but related studies of Rutgers student cohorts, investigating their math course-taking patterns and transitions from high school. The paper begins by pointing out that the first author, a mathematics professor, did not himself take calculus while in high school – indicating shifting generational expectations and assumptions about course-taking. AP calculus was created to

provide "the very best math students with 'advanced placement,'" but over the past 25 years, "has been transformed into a policy of accelerating the curriculum, including pushing Algebra 1 into 8th grade," (1) while the number of high school students taking AP calculus increases by over 7% annually. Yet an analysis of Rutgers students' high school and college transcripts indicates that few (5.4%) achieve and maintain advanced placement in Calculus in their first year of college. "This data suggests that only 1 of 10 students currently taking AP Calculus actually takes advantage of the 'advanced placement' that taking AP Calculus makes possible. One might draw the conclusion that a district or school that encourages 10 times that many students into AP Calculus is misleading its students and distorting its curriculum." (3) In addition, the article argues that based on the data, "it is reasonably clear that the preponderance of the students who took AP Calculus but did not receive advanced placement…would have been better served by a non-AP Calculus course." (5)

Shaughnessy, J. M. (2011) "Endless Algebra – the Deadly Pathway from High School Mathematics to College Mathematics," *NCTM Summing Up*, February 2011; available at:
http://www.nctm.org/about/content.aspx?id=28195
This piece, also referenced above and written by the president of the NCTM, articulates his concerns about the inadequacy of the K-12 mathematics curriculum in preparing students for the STEM challenges of the 21st century. It questions current high school math course-taking patterns and whether they adequately prepare students for college – not only in terms of math "levels" but in terms of content, especially in the context of STEM majors and careers. He is particularly skeptical about the place and role of high school courses in algebra and calculus, asking: "(1) Is the 'layer cake' of algebra-dominated mathematics that pervades our U.S. secondary schools still relevant? (2) Is calculus the be-all and end-all goal for the preparation of students for a successful transition to college? My answer is, I think not." He states that these concerns are long-standing, "For more than 20 years national organizations and prominent leaders in mathematics education, including NCTM, have warned that our national rush to calculus is misguided and not even an appropriate path for many students."

Hacker, A. (2012) "Is Algebra Necessary?"; available at:
http://www.nytimes.com/2012/07/29/opinion/sunday/is-algebra-necessary.html?pagewanted=all
This op-ed argues that high school mathematics, especially algebra and calculus courses, are unnecessarily and irrationally serving as impediments to many students' college and career goals, as well as to their potential engagement with quantitative and mathematical reasoning as adults. It contends that this situation is due in part to the disconnect between math learned in school and the math actually needed in an array of jobs, where on-the-job training is often most appropriate. It mentions that, "Toyota, for example, recently chose to locate a plant in a remote Mississippi county, even though its schools are far from stellar. It works with a nearby community college, which has tailored classes in 'machine tool mathematics.'" The author continues, "That sort of collaboration has long undergirded German apprenticeship programs. I fully concur that high-tech knowledge is needed to sustain an advanced industrial economy. But we're deluding ourselves if we believe the solution is largely academic." The article sometimes paints math with a problematically large brush, with statements like, "The toll mathematics takes begins early," before presenting an array of high school and college dropout statistics correlated with math courses. Still, it raises many important questions and provocations.

Cavanagh, S. (2008) "Senior Year Inviting More Math Choices," *Education Week*; available at:
http://www.edweek.org/ew/articles/2008/11/19/13seniors_ep.h28.html
This article discusses options for alternative high school math courses, especially in 12th grade in lieu of precalculus or calculus. It mentions that considering alternatives is relevant among a diversity of students, "Across the country, interest in introducing new senior-year math options is rising as more states require four years of the subject for graduation and schools explore alternatives for both struggling and high-achieving students." As for what schools more generally are coming up with, "The most popular emerging courses include statistics and discrete math…as well as classes in quantitative reasoning, math modeling, and math in business and finance." Other alternatives mentioned include "Engineering Math." These alternative courses are discussed not only as potentially more relevant to preparing for college and careers, but also for their potential to mathematically reengage students who may have bogged down in algebra or other earlier courses and not appreciate the wider world of math possibilities, including STEM careers, in which they might thrive. The article also mentions alternative 12th grade math courses being developed at UT-Austin: http://www.utdanacenter.org/k12mathbenchmarks/resources/capstone.php/

Fensterwald, J. (2014) "New Twist to Old Debate on Acclerated Math," *EdSource*, August 12, 2014; available at: http://edsource.org/2014/new-twist-to-old-debate-on-accelerated-math/66174

This article highlights how California school districts are grappling to figure out the most appropriate middle and high school math pathways, in the context of CCSS reforms. Beyond the article's discussion of student placements and preparation, it also documents the push mentioned by Rosenstein et al. for students to enroll in Calculus in high school, including Calculus BC for students intending to major in STEM fields at top colleges. It does not mention schools, parents, or students being aware of the debates among mathematics educators and others about the value and relevance of high school AP Calculus. Yet these debates, which are not limited to questions of placement, raise important questions about students' successful transitions to college and their preparation for an array of STEM fields, interdisciplinary collaborations, and careers.

5) 21st Century Learning Environments: Math Classes & Collaborative Problem-Solving

Teaching and CCSS Reforms

Boaler, J. (2002) "Learning from Teaching: Exploring the Relationship Between 'Reform' Curriculum and Equity," *Journal for Research in Mathematics Education*, Vol. 33, No. 4.
This article examines the conditions under which reform-oriented math curricula may also promote equity, based on two studies in which middle and high school teachers using reform math curricula achieved a reduction in linguistic, ethnic, and class inequalities in their schools. It emphasizes that the preponderance of public attention to debating different curricula has meant a neglect of different teaching practices and instructional strategies, including teacher professional development needs, which are nonetheless central to student learning outcomes. In addition, it argues that teachers' decisions about the degree of structure versus open-endedness to present to students should not correlate with students' mathematical level or social class. "As long as we hold conceptual understanding as a goal for students, then it is imperative that such a goal is held for *all* students. Awareness that students of low SES or achievement encounter difficulties interpreting open work, must be accompanied by a drive to understand the students' experiences better and provide action to make the teaching of open ended approaches more equitable." (255) The teachers in this study "all spent time sharing understanding of the learning practices that students needed for their work on open-ended mathematics problems." (255) In other words, teachers explicitly addressed how to grapple with open-ended problems, which some researchers would refer to as engaging in "metacognition." The article emphasizes that such critical, independent thinking should not be reserved only for some students. "[O]pen approaches to learning not only give access to a depth of subject understanding but also encourage a personal and intellectual freedom that should be the right of all people in society." (254) It addition, well-prepared teachers are vital to student success, as are teachers' commitment both to reform curricula (and e.g. open-ended problems) and equity.

McPartland, J. and B. Schneider (1996) "Opportunities to Learn and Student Diversity: Prospects and Pitfalls of a Common Core Curriculum," *Sociology of Education*, Vol 69, Special Issue on Sociology and Educational Policy.
This article analyses the potentials and pitfalls of the proposed common core curriculum, with its demanding content for all students, through an "opportunities to learn" framework. It provides an overview of research findings that students will learn more when offered a demanding curriculum. At the same time, it acknowledges that adopting higher curricular standards without proper supports may undermine their implementation, for example if high rates of failure by students result. To hedge against such possibilities, the article recommends "enhanced resources and flexible scheduling that allow teachers to give extra help and more time to the most needy students, reformulations of the criteria for academic success that recognize both individual progress and the achievement of absolute levels of proficiency, and restructuring the roles and relationships of teachers toward achievement outcomes that emphasize support for students, along with the enforcement of standards." (78)

Holmes, J. E. (1970) "Enrichment or Acceleration?" *The Mathematics Teacher*, Vol. 63, No. 6.
This piece by a college mathematics instructor, though rather dated, embodies a skeptical stance toward acceleration not on the basis of equity issues per se but rather out of concern for students' depth of mathematical learning. It therefore calls for greater enrichment of curricula and instruction, not only acceleration, in ways highly conducive to the CCSS reforms and those interested in acceleration. "A competent secondary school mathematics teacher will encounter no difficulty in enriching a course if he tries to introduce, at a simplified or intuitive level, topics that he knows will be studied at a later time." (471) As he continues, "During the second year of algebra, for example, it is possible to introduce parabolic interpolation at an extremely low level. This could lead to better understanding of

Newton's formula for approximating roots of a polynomial equation, which is customarily introduced about the twelfth year, and eventually to LaGrange's formula, which is usually not encountered until a course in differential equations is taken." (471) Such approaches undoubtedly require additional teacher professional development and sustained support, indicating the ways in which teacher preparation for CCSS reforms and more in-depth, differentiated instruction may go hand in hand.

Debating Differentiated Instruction

Boaler, J. (2008) "Chapter 5: Stuck in the Slow Lane: How American Grouping Systems Perpetuate Low Achievement," in *What's Math Got To Do With It?: How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject*, Penguin Books.
This book features a chapter discussing research on ability grouping, including issues of differentiated instruction. In explaining why "mixed ability grouping is repeatedly found to be associated with higher academic achievement," it discusses the value of differentiated instruction in heterogeneous classrooms in terms of improved, higher-quality instruction, rather than only in terms of expediency. This contrasts with the assumption, made by economists studying tracking and peer effects, that ability grouping leads to improved, more "efficient" teaching. As the chapter explains, "When students are placed into a tracked group, high or low, assumptions are made about their potential achievement. Teachers tend to pitch their teaching to students in the middle of the group, and they teach a particular level of content, assuming that all students are more or less the same…In a mixed-ability group the teacher has to *open* the work, making it suitable for students working at different levels and different speeds. Instead of prejudging the attainment of students and delivering work at a *particular* level, the teacher has to provide work that is multileveled and that enables students to work at the highest levels they can reach." (113) The chapter underscores that open, multilevel problems are one vital condition for mixed ability courses to work. The second condition is for students to be taught to work respectfully with each other. (119) (Also cited above, under "Classroom Grouping Practices in International Comparison.")

Hertberg-Davis, H. (2009) "Myth 7: Differentiation in the Regular Classroom Is Equivalent to Gifted Programs and Is Sufficient," *Gifted Child Quarterly*, Vol. 53, No. 4.
This commentary on differentiated instruction makes the case indicated by its title, namely that, "it does not seem that we are yet at a place where differentiation within the regular classroom is a particularly effective method of challenging our most able learners." (252) The reasons cited include: "lack of sustained teacher training in the specific philosophy and methods of differentiation, underlying beliefs prevalent in our school culture that gifted students do fine without any adaptations to curriculum, lack of general education teacher training in the needs and nature of gifted students, and the difficulty of differentiating instruction without a great depth of content knowledge." (252) The author also notes detrimental effects of "teaching to the test" – for example, for high-stakes exams associated with No Child Left Behind. Though many educators have highlighted how these effects are more generally detrimental to students. The author underscores the importance of sustained teacher professional development to facilitate differentiated instruction, though she remains skeptical that adequate levels will prove practical.

Sears, D. and J. M. Reagin (2013) "Individual versus Collaborative Problem Solving: Divergent Outcomes Depending on Task Complexity," *Instructional Science*, Vol. 41, Issue 6.
This article emphasizes task features and instructional practices, more than classroom homogeneity or heterogeneity, in relation to learning and problem-solving outcomes. It is a study of student problem-solving in accelerated and traditional 7th grade math classrooms, and is concerned with the features characterizing "naturally productive collaborative tasks," as distinguished from external supports for productive collaboration. It notes that research has variously shown "demonstrability" or "complexity" to be the "primary task feature" associated with productive collaboration – with demonstrability meaning "tasks that permit all group members to readily acknowledge a correct solution when one has been found" and complexity deemed a means of increasing motivation and opportunities for all group members to participate. "This study examined the problem-solving performance of 110 seventh grade students on a demonstrable mathematical task, including 69 in three traditional math classrooms (for whom the task was complex) and 41 in two accelerated math classrooms (for whom the task was not complex). Students were further assigned to one of four conditions split by two factors: grouping (individual versus dyad) and number of problems (one or two). For the accelerated math classes, individuals performed significantly better than dyads. For the traditional math classes, dyads performed significantly better than individuals and exceeded the truth-wins criterion (a theoretical maximum indicating how individuals would perform if they shared knowledge perfectly)."

The paper ultimately proposes a "complex-demonstrable task framework" for "characterizing naturally productive collaborative tasks," arguing that complex, demonstrable tasks best facilitate group collaborative problem-solving, and that collaborative problem-solving can best grapple with such tasks. In terms of math classroom practices, these findings suggest that cooperative learning may be most effective in taking on complex problem-solving, with complexity also facilitating such collaboration. Moreover, such learning is possible in traditional, non-accelerated classrooms. Again, this analysis suggests the value of considering task features and instructional practices as core variables in relation to learning and problem-solving outcomes.

Cooperative Learning & Collaborative Problem-Solving

Slavin, R. (2010) "Chapter 7: Co-operative Learning: What Makes Groupwork Work?" in *The Nature of Learning: Using Research to Inspire Practice*.
This chapter presents an overview of the research on cooperative learning methods in K-12 classrooms, particularly studies of cooperative learning compared with control groups being taught with more traditional methods. It finds that cooperative learning methods lead to overwhelmingly positive affective outcomes, and that achievement outcomes depend on two key factors: 1) the presence of group goals (the learner groups are working towards a goal or to gain reward or recognition); and 2) individual accountability (the success of the group depends on the individual learning of every member). The chapter emphasizes the robust evidence base of positive outcomes for cooperative learning methods, though cooperative learning "remains at the edge of school policy" and is often poorly implemented due to lack of understanding of the two factors above. The chapter also discusses how cooperative learning incorporates flexibly-paced, differentiated instruction that emphasizes the importance of improved student achievement relative to students' earlier levels of achievement, rather than encouraging comparisons with other students' achievement. Here the chapter (and underlying research) considers a broader range of affective or "psychosocial" outcomes than are usually considered in, e.g., literature that focuses primarily on students' social adjustment in the context of grade-level acceleration.

The chapter concludes with discussion of the synergies between cooperative learning and 21st century work and learning environments, including the importance of students' active engagement with tasks and with each other. As the author puts it, "Co-operative learning offers a proven, practical means of creating exciting social and engaging classroom environments to help students to master traditional skills and knowledge as well as develop the creative and interactive skills needed in today's economy and society. Co-operative learning itself is being reshaped for the 21st century, particularly in partnership with developments in technology." (10) It lists the following cooperative learning professional development resources:

Success for All Foundation: www.successforall.org
Peer-Assisted Learning Strategies: www.peerassistedlearningstrategies.net
Kagan Publishing and Professional Development: www.kaganonline.com

Slavin, R. (1991) "Are Cooperative Learning and 'Untracking' Harmful to the Gifted?," *Educational Leadership*, Vol. 48.
This article, also cited above under "Contingencies of Acceleration and Classroom Grouping Practices," provides an overview of research on classroom grouping practices and cooperative learning strategies, with a particular eye to potential effects on high-achieving students, including students assessed in the top 5%, who may typically be considered in a separate gifted category. The article makes a case for the value of cooperative learning strategies for all students, elaborating that, "with the use of effective cooperative learning programs, especially those that differentiate instruction within the class, high achievers are likely to benefit in achievement, even the very top-achieving 5%. Educators of the gifted should be in the forefront of the cooperative learning movement, insisting on the use of forms of cooperative learning known to benefit gifted and other able students. If these methods also happen to be good for average and below average students, so much the better." (70-71)

Strober, M. (2010) *Interdisciplinary Conversations: Challenging Habits of Thought*, Stanford University Press.
Though focused on post-secondary education, this book may be useful in fostering dialogue among teachers, administrators, and others about interdisciplinarity, "cognitive diversity," and collaborative problem-solving.

6) Math Pathways & Preparation for College, Career, and Citizenship

Future California Word Problems…

Given that California school districts are grappling with the implementation of the CCSS in mathematics, with a spectrum of priorities and desired outcomes, what words problems might current or future students write to capture the key relations, variables, quantities, proportions, and other dimensions relevant to assessing districts' math education "success"? 25 years from now? 50? 100? 1000? Another duration? How might the word problems vary depending on the spatial scale, location, or number of districts considered – e.g. in assessing a single district, or all the districts in the Bay Area, or the state of California, or some other scale?

An extension of The Sneakers Problem:

"In the Sneakers Problem, the students encounter the notion of multiple factors that could be used in developing a rating system for purchasing sneakers and the notion that not all factors are equally important to all people. Students were asked 'What factors are important to you in buying a pair of sneakers?' This generated a list of factors where not all factors were equally important to the students; the students then worked in small groups to determine how to order these factors in deciding which pair of sneakers to purchase. The students naturally produced different lists. The teacher then posed the problem that the sneaker manufacturer needed a single set of factors that represents the view of the whole class; in other words, the group rankings needed to be aggregated into a single class ranking. As we report below, the context of this problem, beginning with the point of view of the manufacturer, provided the teacher with the opportunity to use perspective-taking to support the sense-making efforts of her students." (2-359)

From:
"Perspective-Taking in Middle School Mathematical Modelling"
http://files.eric.ed.gov/fulltext/ED500952.pdf

On word problems and mathematics learning in everyday life:

DeFranco, T. and F. Curcio (1997) "A Division Problem With a Remainder Embedded Across Two Contexts: Children's Solutions in Restrictive Versus Real-World Settings," *Focus on Learning Problems in Mathematics*, Vol. 19, No. 2.

Lave, J. (1992) "Word Problems: A Microcosm of Theories of Learning," in *Context and Cognition: Ways of Learning and Knowing*, P. Light and G. Butterworth, Eds., New York: Harvester Wheatsheaf.

Appendix: Questions About Basic Assumptions and Larger Goals

The questions below arose organically in the course of this research review, in part to facilitate thinking through the basic assumptions of the vast, multifaceted research literature addressing the topics of this report. Since these questions are often not directly raised by research sources, nor the primary focus of specific research findings, it is often left to readers to ask them. And, since different readers will tend to have different answers to these questions, they will tend to read research differently as well – if not at the level of individual articles, then at the level of the larger research literatures, including how they would demarcate literature "relevant" to understanding human development and mathematics learning.

These questions may also be useful in reflecting on the larger goals, priorities, and values served by different educational practices. These larger goals and values are often left implicit in research literature focused mainly on "what works" in classrooms. ("Works" toward what larger ends? Why those ends? Trade-offs among various ends and values? Possible syntheses of approaches and ends served?)

Questions about basic assumptions and larger goals:

- What is "intelligence"? "Ability"? "Learning"? "Problem-solving"? What are their underpinnings? What are their relations to school-based instruction and assessments?
- What are the underpinnings of various students' higher or lower achievements in mathematics? What are the implications for district-, school- and classroom-level educational policies and practices?
- What is the meant by "homogeneous grouping"? "Heterogeneous grouping"? What are the specific reasons that one might be preferable to the other, in terms of various outcomes? Are there trade-offs among various outcomes? Why? If there are trade-offs, which are preferable, and why? Possible syntheses of approaches and ends served?
- What is meant by "acceleration"? "Expansion"? "Flexible-pacing"? "Depth of understanding"? "Complexity"? "Rigor"? What are their primary purposes or end goals? Why are these purposes and end goals important? For whom? How do they figure into different kinds of problem-solving?
- What are the relationships between students' middle and high school mathematics achievements and their later preparation for college? For career? For democratic citizenship? (E.g.: Gate-keeping relationships? Applying school math knowledge in these contexts? Complementing or supplementing other kinds of math learning in these contexts? Complementing or supplementing other kinds of problem-solving in these contexts? Mathematics as an end in itself or recreational pursuit?)
- Are middle and high school mathematics preparing students adequately in each of these areas (college, career, citizenship)? Why or why not? If not, what are the most important shortcomings? Why are these most important?
- If there are significant inequalities among students, in how they are prepared for each of these areas of adult life, what are the consequences – for a range of individual students, and for society as a whole?
- Which students are faring most and least well in middle and high school mathematics, at present? Why? (Again, what are the underpinnings of various students' higher or lower achievements in mathematics?) What are the consequences?

- What are the relationships among these students and their respective challenges? Are they mutually exclusive challenges? Mutually exclusive groups of students?
- What are various students learning from and about each other, in their daily math classrooms and over time, as they navigate through the school system and the world beyond? What aren't they learning from and about each other?
- How will various answers to these questions inform different responses to contemporary educational challenges?
- What other questions are crucial?
- How might various responses lay the groundwork for the challenges faced by the next generation of students, educators, parents, researchers, and society as a whole?

Research Touchstones:
Two touchstones relevant to these questions – one examining childhood development generally and the other focused explicitly on mathematics education – are the following National Research Council reports (both freely and fully available online):

Shonkoff, J. and D. Philips, eds. (2000) *From Neurons to Neighborhoods: The Science of Early Childhood Development*. National Academy Press: http://www.nap.edu/openbook.php?record_id=9824

Kilpatrick, J., J. Swafford, and B. Findell, eds. (2001) *Adding It Up: Helping Children Learn Mathematics*. National Academy Press: http://www.nap.edu/openbook.php?record_id=9822
This book, a review of research literature on pre-K through 8th grade mathematics learning, also includes the observation:

> "Many educational questions, however, cannot be answered by research. Choices about the mathematics curriculum and the methods used to bring about that curriculum depend in part on what society wants educated adults to know and be able to do. Research can inform these decisions—for example, by demonstrating what knowledge, skills, and abilities employees need in the workplace. But ideas about what children need to know also depend on value judgments based on previous experience and convictions, and these judgments often fall outside the domain of research." (3)

Both books represent syntheses of large, interdisciplinary bodies of research. The latter also references the former, which is co-edited by the director of the Center for the Developing Child at Harvard University, Dr. Jack Shonkoff: http://developingchild.harvard.edu/.

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