TRUMBULL PUBLIC SCHOOLS
Trumbull, Connecticut

UCONN Multivariable Calculus/Linear Algebra
Mathematics Department
Trumbull High School

2021

Curriculum Writing Team

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UCONN Multivariable Calculus/Linear Algebra
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The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in its employment.

CORE VALUES AND BELIEFS

The Trumbull School Community engages in an environment conducive to learning which believes that all students will **read** and **write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION

UCONN Multivariable Calculus/Linear Algebra is designed for the student who has successfully completed Advanced Placement / Early College Experience Calculus BC. The first semester is ECE Multivariable Calculus which is an extension of calculus in one variable to calculus with functions of several variables. In ECE Multivariable Calculus, students will become proficient with the differentiation and integration of functions involving multiple variables as well as applications in the real world such as volume and surface area. Students will be prepared to succeed on the Early College Experience Examination given by the University of Connecticut in January. The second semester is Linear Algebra which is a beautiful and important subject, rich in applications within mathematics and to many other disciplines. This is the first course to begin bridging the gap between concrete computations and abstract reasoning. Understanding the notions of vector spaces, linear (in)dependence, dimension, and linear transformations will help students make sense of matrix manipulations at a deeper level, clarifying the underlying structure.

PHILOSOPHY

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.
The successful mathematician will:

- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process
- Communicate effectively using mathematical terminology, both independently and collaboratively

- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011
Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourage students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.

Information, Media And Technology Skills

- Use real-world digital and other research tools to access, evaluate and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

- Work independently and collaboratively to solve problems and accomplish goals
- Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
- Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
- Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

- Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE GOALS

The Standards for Mathematical Practice describe varieties of expertise that all teachers of mathematics will develop in their students.

These practices rest on important "processes and proficiencies" that have long been valued in mathematics education.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry

software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

COURSE ENDURING UNDERSTANDINGS

Students will understand that we live in a three-dimensional world and understand that in this world, mathematicians need models that involve multivariables. Various functions will be explored and understood verbally, numerically, algebraically and visually. Students will be able to work with vector equations and operations, matrix equations and operations, subspaces. Students will understand how Linear Algebra relates to models in Economics and Engineering.

COURSE ESSENTIAL QUESTIONS

- How can the concepts of Multivariable Calculus and Linear Algebra help students understand the physical world around them?

- How can students use their previous knowledge of Single Variable Calculus to understand the concepts of Multivariable Calculus?
- How will the topics of Linear Algebra be applied to real life applications?

COURSE KNOWLEDGE & SKILLS

Students will understand . . .

- the applications of the derivative and the integral in Multivariable Calculus
- vector functions and tangent planes
- functions of two or more variables from verbal, numerical, visual and algebraic points of view
- the matrix equations and vector equations
- linear transformations

Students will be able to . . .

- perform vector operations and interpret the results geometrically
- use vector valued functions to analyze projectile motion
- find directional derivatives and gradients of functions
- find the volume of a solid region
- sketch a vector field, determine whether a vector field is conservative, find a potential function, find curl, and find divergence
- find solution sets of linear systems
- determine operations with matrices

COURSE SYLLABUS

Course Name

UCONN Multivariable Calculus/Linear Algebra

Level

Advanced Placement

Prerequisites

Completion of Advanced Placement / Early College Experience Calculus BC with a B or better and teacher recommendation.

Materials Required

Mymath lab

General Description of the Course

UCONN Multivariable Calculus/Linear Algebra follows the curriculum of the University of Connecticut's Math 2110Q (Multivariable Calculus) and 2210Q (Linear Algebra). The first semester of this course explores Multivariable Calculus. The topics include vectors, surfaces in

space in rectangular, cylindrical, and spherical coordinates, partial derivatives, directional derivatives, gradients, optimization, double integrals, line integrals, Green's, Divergence, and Stokes Theorems. The second semester is devoted to the study of Linear Algebra. The topics include systems of linear equations, matrices, vector spaces, determinants, eigenvalues, eigenvectors, and linear transformations. Students should not only be able to work through problems similar to ones seen in the homework, but should also have the ability to go beyond, presenting their knowledge in a clear and coherent manner as well.

Assured Assessments

Students will be evaluated by their performance on tests, quizzes, homework, problem sets, other formative and summative assessments, and midterm and final examinations including questions required by the University of Connecticut.

Core Text

Larson, Ron, and Bruce H. Edwards. *Calculus: AP Edition*. 9th ed. Boston: Brooks/Cole, 2010. Print.

Lay, David C., Steven R. Lay, and Judi J.McDonald. *Linear Algebra and Its Applications.* 5th ed. Pearson 2016

Unit 1: Vectors and the Geometry of Space

Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Curriculum Guide.

- Write vectors, perform basic vector operations, and represent vectors graphically
- Plot points in a three-dimensional coordinate system and analyze vectors in space
- Find the dot product of two vectors
- Find the cross product of two vectors
- Find equations of lines and planes in space
- Write and recognize equations of cylindrical and quadric surfaces
- Use cylindrical and spherical coordinates to represent surfaces in space

Essential Questions

- What is the dot product and how can it be used to measure the work done by a force?
- What is a cross product and what are its applications in physics and engineering?
- How do the rectangular, cylindrical and spherical coordinate systems relate to each other in three-space?

Content (Scope and Sequence)

- Component form of a vector
- Vector operations
- Three-dimensional rectangular coordinate system

- Dot product and cross product
- Projection and Vector Components
- Area of parallelogram using cross product
- Volume of parallelepiped
- Distance between points, lines and planes
- Equations of cylindrical and quadric surfaces
- Cylindrical coordinate system
- Spherical coordinate system

Assured Experiences

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 3 weeks

Unit 2: Functions of Several Variables

Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Curriculum Guide.

- Various functions will be explored and understood verbally, numerically, algebraically and visually
- Students will investigate differentiation of functions of two independent variables; define and apply the gradient

Essential Questions

- What is a partial derivative and how is it interpreted?
- How is the chain rule applied when taking derivatives of functions of two variables?
- What is a directional derivative?
- What is a gradient vector and what meaning does it have?
- How does one calculate the minima and maxima values of a function of two variables?
- What are the applications of the maximum or minimum value of a function?

Content (Scope and Sequence)

- Sketch the graph of a function of two variables
- Sketch level curves for a function of two or three variables
- Find and use partial derivatives of a function of two or more variables
- Find higher-order partial derivatives of a function of two or three variables
- Extend the concept of differentiability to a function of two variables.

- Use the Chain Rules for functions of several variables
- Find partial derivatives implicitly
- Find and use directional derivatives of a function of two or more variables
- Find and use the gradient of a function in two or more variables
- Find equations of tangent planes and normal lines to surfaces
- Find the angle of inclination of a plane in space
- Find absolute and relative extrema of a function of two variables
- Use the Second Partials Test to find relative extrema of a function of two variables
- Solve optimization involving functions of several variables

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 4 weeks

Unit 3: Multiple Integration

Performance Standard

The following Performance Standard is TPS-created, and influenced by the Fairfield Public Schools Multivariable Curriculum Guide.

- Know how to use double and triple integrals to compute volumes, and surface areas
- Write and evaluate triple integrals in cylindrical and spherical coordinates

Essential Questions

- What is a double integral and how can it be used to find the surface area and volume of a solid?
- How does one write a double integral as an iterated integral?
- How does one use polar coordinates to simplify the integrations of solids over circular regions?
- What is a triple integral?

Content (Scope and Sequence)

- Evaluate an iterated integral
- Use an iterated integral to find the area of a plane region
- Use a double integral to represent the volume of a solid region
- Evaluate a double integral as an iterated integral
- Find the average value of a function over a region
- Write and evaluate double integrals in polar coordinates
- Use a double integral to find the area of a surface
- Use a triple integral to find the volume of a solid region
- Write and evaluate a triple integral in cylindrical and spherical coordinates

- Use a Jacobian to change variable in a double integral
- Understand the Method of Lagrange Multipliers
- Use Lagrange Multipliers to solve constrained optimization problems

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 4 weeks

Unit 4: Vector-Valued Functions

Performance Standards

The following Performance Standards are TPS-created, and influenced by the Fairfield Public Schools Multivariable Curriculum Guide.

- Know how to use vector-valued function to analyze projectile motion
- Apply to concepts of limits and continuity to vector-valued functions

Essential Questions

- What is a vector function?
- What is a derivative and integral of a vector function?
- What is a space curve and how do we measure its length and curvature?
- How can the ideas of a tangent and normal vectors and curvature be used in physics to study the motion of an object along a space curve?

Content (Scope and Sequence)

- Limits and continuity of vector valued functions
- Differentiate and integrate vector-valued functions
- Velocity and acceleration of vector-valued functions
- Unit tangent vectors
- Tangential and Normal components of acceleration
- Arc length of a space curve
- Curvature of a curve at a point on the curve

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 2 weeks

Unit 5: Vector Analysis

Performance Standards

The following Performance Standards are TPS-created.

- Know how to sketch a vector field and determine whether the field is conservative and find a potential function, find curl and divergence
- Use the Fundamental Theorem of Line Integrals
- Understand and apply the theorems of Green, Stokes and Gauss

Essential Questions

- What are some important properties of divergence and curl?
- How are double integrals and line integrals related?
- How does the Divergence Theorem relate triple integrals over a solid region and a surface integral over a surface?

Content (Scope and Sequence)

- Determine whether a vector field is conservative
- Find the curl and divergence of a vector field
- Understand and use the concept of a piecewise smooth curve
- Write and evaluate a line integral of a vector field
- Understand and use the Fundamental Theorem of Line Integrals
- Use Green's Theorem to evaluate a line integral
- Understand the definition of a parametric surface
- Find a normal vector and tangent vector plane to a parametric surface
- Find the area of a parametric surface
- Evaluate a surface integral as a double integral and for a parametric surface
- Understand and use the Divergence Theorem to calculate flux
- Understand and use Stoke's Theorem

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 4 weeks

Unit 6: Linear Equations in Linear Algebra

Performance Standards

The following Performance Standards are TPS-created, and influenced by the University of South Alabama by Professor Clontz.

- Systems as matrices: The student can translate back and forth between a system of linear equations and the corresponding augmented matrix.
- Row reduction: The student can put a matrix in reduced row echelon form.
- Systems of linear equations: The student can compute the solution set for a system of linear equations.

Essential Questions

- What are the methods we can use to solve system of linear equations?
- What is the relationship free variables and the solution of a homogeneous equation?

Content (Scope and Sequence)

- Solve systems of equations
- Find echelon form and reduced echelon form
- Solve the matrix equation A$\mathbf{x}$=$\mathbf{b}$
- Understand homogeneous equations
- Determine if vectors are linearly independent or dependent
- Describe and understand linearly transformations

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 4 weeks

Unit 7: Matrix Algebra

Performance Standards

The following Performance Standards are TPS-created, and influenced by the University of South Alabama by Professor Clontz.

- Matrix Multiplication: The student can multiply matrices.
- Invertible Matrices: The student can determine if a square matrix is invertible or not.
- Matrix inverses: The student can compute the inverse matrix of an invertible matrix.

Essential Questions

- What is the Invertible Matrix Theorem and how does it connect the concepts of Linear Algebra?
- How is the basis of a matrix relate to the span of a set?

Content (Scope and Sequence)

- Calculate matrix operations including multiplication, transpose, inverse, determinant
- Understand the Invertible Matrix Theorem
- Understand the three properties of subspaces
- Find bases for subspaces
- Find the dimension of subspaces
- Understand The Rank Theorem
- Understand The Basis Theorem

Assured Experiences (Projects)

- Problem Sets
- Homework
- Mid-Unit Quizzes
- Tests

Time Allocation

- Approximately 3 weeks

Unit 8: Determinants

Performance Standards

The following Performance Standards are TPS-created, and influenced by the University of South Alabama by Professor Clontz.

- Determinants: The student can compute the determinant of a 4 x 4 matrix.
- Row Operations: The student can describe how a row operation affects the determinant of a matrix, including composing two row operations.

Essential Questions

- What is criterion for a square matrix to be invertible?
- How does a linear transformation change the area of a figure?

Content (Scope and Sequence)

- Find the determinant using cofactor expansion
- Use row operations to calculate the determinant
- Use Cramer's Rule to find the solution of A**x**=**b**
- Understand the connection of matrices to area of parallelograms and volume of parallelepiped

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 3 weeks

Unit 9: Vector Spaces

Performance Standards

The following Performance Standards are TPS-created, and influenced by the University of South Alabama by Professor Clontz.

- Linear combinations: The student can determine if a Euclidean vector can be written as a linear combination of a given set of Euclidean.
- Spanning sets: The student can determine if a set of Euclidean vectors spans R^n.
- Subspaces: The student can determine if a subset of $R^n$is a subspace or not.
- Polynomial basis computation: The student can find a basis for the solution set of a homogeneous system of equations.

Essential Questions

- What is a vector space?
- How do other vector spaces resemble R^n?

Content (Scope and Sequence)

- Find subspaces of R^n and P^n.
- Find column spaces and null spaces
- Find bases for column and null spaces
- Find vectors using the coordinate vector
- Find change-of-coordinates
- Understand the relationship between the dimensions of the null space, column space, and the row space

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 3 weeks

Unit 10: Eigenvalues and Eigenvectors

Performance Standards

The following Performance Standards are TPS-created, and influenced by the University of South Alabama by Professor Clontz.

- The student can find the eigenvalues of a matrix.
- The student can find a basis for the eigenspace of a matrix associated with a given eigenvalue

Essential Questions

- What is the connection between eigenvalues and eigenvectors?
- How is a linear transformation related to a diagonal matrix?

Content (Scope and Sequence)

- Find eigenvalues of matrices
- Find eigenvectors of matrices
- Use the characteristic equation to find eigenvalues of a matrix
- Find the diagonalization of a matrix
- Compute linear transformations between two finite dimensional vector spaces

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 3 weeks

Unit 11: Orthogonality and Least Squares

Performance Standards

The following Performance Standards are TPS-created, and influenced by the University of South Alabama by Professor Clontz.

- The student can calculate orthogonal and orthonormal sets.
- The student can determine if a set of Euclidean vectors is a basis of R^n

Essential Questions

- How can orthogonality be used to find the closest point with in a subspace to a point outside of the subspace?
- What is the most efficient method to produce an orthogonal basis?

Content (Scope and Sequence)

- Calculate the inner product of two vectors
- Calculate the length of a vector
- Find the distance between two vectors
- Find an orthogonal basis for a subspace
- Find an orthonormal basis for a subspace
- Find an orthogonal projection
- Use the Gram-Schmidt process to produce an orthogonal basis

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
Tests

Time Allocation

Approximately 3 weeks

Culminating Activity

Midterm Exam

The midterm exam is worth 10% of the student's Trumbull High School course grade and 90% of the student's UCONN Calculus 2110Q grade.

Final Exam

The final exam is worth 10% of the student's Trumbull High School course grade.

COURSE CREDIT

One THS credit in Mathematics

One class period daily for a full year

PREREQUISITES

Advanced Placement / Early College Experience Calculus BC with a B or better and teacher recommendation.

TEXTS

Larson, Ron, and Bruce H. Edwards. *Calculus: AP Edition*. 9th ed. Boston: Brooks/Cole, 2010. Print.

Lay, David C., Steven R. Lay, and Judi J. McDonald. *Linear Algebra and Its Applications.* 5th ed. Pearson 2016

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

My Mathlab. Pearson

CURRENT REFERENCES

Fairfield Public Schools Multivariable Calculus

http://cdn.fairfieldschools.org/curriculum/math-2014/Multivariable%20Calculus%2061.pdf

TEACHER GUIDE

Multivariable Calculus

Unit 1: Vectors and the Geometry of Space

Sections:

11.2 Space Coordinates and Vectors in Space
11.1 Vectors in the Plane
11.6 Surfaces in Space
11.7 Cylindrical and Spherical Coordinates
11.3 The Dot product of Two Vectors
11.4 The Cross Product of Two Vectors in Space
11.5 Lines and Planes in Space

Unit 2: Functions of Several Variables
13.1 Introduction of Functions of Several Variables
13.3 Partial Derivatives
13.4 Differentials
13.5 Chain Rules for Functions of Several Variables
13.6 Directional Derivatives and Gradients
13.7 Tangent Lines and Normal Planes
13.8 Extrema of Functions of Two Variables
13.9 Applications of Extrema of Functions of Two Variables
13.10 Lagrange Multipliers

Unit 3: Multiple Integration
14.1 Iterated Integrals and Area in the Plane
14.2 Double Integrals and Volume
14.3 Change of Variables: Polar Coordinates
14.4 Center of Mass and Moments of Inertia
14.5 Surface Area
14.6 Triple Integrals and Applications
14.7 Triple Integrals in Cylindrical and Spherical Coordinates
14.8 Change of Variables: Jacobians

Unit 4: Vector-Valued Functions
12.1 Vector-Valued Functions
12.2 Differentiation and Integration of Vector-Valued Functions
12.4 Tangent Vectors and Normal Vectors
12.5 Arc Length and Curvature

Unit 5: Vector Analysis
15.1 Vector Fields
15.3 Conservative Vector Fields and Independence
15.2 Line Integrals
15.4 Green's Theorem
15.5 Parametric Surfaces
15.6 Surface Integrals
15.7 Divergence Theorem
15.8 Stoke's Theorem

LINEAR ALGEBRA

Unit 6: Linear Equations in Linear Algebra
1.1 Systems of Linear Equations

1.2 Row Reduction and Echelon Form
1.3 Vector Equations
1.4 The Matrix Equation A**x=b**
1.5 Solution Sets of Linear Systems
1.7 Linear Independence
1.8 Introduction to Linear Transformations
1.9 The Matrix of a Linear Transformation

Unit 7: Matrix Algebra
2.1 Matrix Operations
2.2 The Inverse of a Matrix
2.3 Characterizations of Invertible Matrices

Unit 8: Determinants
3.1 Introduction to Determinants
3.2 Properties of Determinants
3.3 Cramer's Rule, Volume, and Linear Transformations

Unit 9:Vector Spaces
4.1 Vector Spaces and Subspaces
4.2 Null Spaces, Column Spaces, and Linear Transformations
4.3 Linearly Independent Sets; Bases
4.4 Coordinate Systems
4.5 The Dimension of a Vector Space
4.6 Rank
4.7 Change of Basis

Unit 10: Eigenvalues and Eigenvectors
5.1 Eigenvalues and Eigenvectors
5.2 The Characteristic Equation
5.3 Diagonalization
5.4 Eigenvectors and Linear Transformations

Unit 11: Orthogonality and Least Squares and quadratic Forms
6.1 Inner product, Length, Orthogonality
6.2 Orthogonal Sets
6.3 Orthogonal Projections
6.4 The Gram-Schmidt Process

ASSURED STUDENT PERFORMANCE RUBRICS

- Trumbull High School School-Wide Writing Rubric
- Trumbull High School School-Wide Problem-Solving Rubric
- Trumbull High School School-Wide Independent Learning and Thinking Rubric

Rubric 2: Write Effectively

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	• Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task	• Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task	• Establishes a purpose • Demonstrates an awareness of audience and task	• Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_____	• Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions	• Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions	• Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions	• Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_____	• Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples	• Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples	• May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples	• Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_____	• Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning	• Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors. Most errors do not detract from meaning	• Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning	• Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Rubric 3: Problem Solving through Critical Thinking

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	Student demonstrates clear understanding of the problem and the complexities of the task	Student demonstrates sufficient understanding of the problem and most of the complexities of the task	Student demonstrates some understanding of the problem but requires assistance to complete the task	Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_____	Student gathers compelling information from multiple sources including digital, print, and interpersonal	Student gathers sufficient information from multiple sources including digital, print, and interpersonal	Student gathers some information from few sources including digital, print, and interpersonal	Student gathers limited or no information
Reasoning and Strategies X_____	Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	Student demonstrates some critical thinking skills to develop a plan integrating some strategies	Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_____	Solution shows deep understanding of the problem and its components. Solution shows extensive use of 21st Century Technology Skills.	Solution shows sufficient understanding of the problem and its components. Solution shows sufficient use of 21st Century Technology Skills.	Solution shows some understanding of the problem and its components. Solution shows some use of 21st Century Technology Skills.	Solution shows limited or no understanding of the problem and its components. Solution shows limited or no use of 21st Century Technology Skills.

Rubric 5: Independent Learners and Thinkers

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work.	Student demonstrates initiative by generating appropriate questions, creating original projects/work.	Student demonstrates some initiative by generating questions, creating appropriate projects/work.	Student demonstrates limited or no initiative by generating few questions and creating projects/work.
Independent Research & Development X_____	Student is analytical, insightful, and works independently to reach a solution.	Student is analytical, and works productively to reach a solution.	Student reaches a solution with direction.	Student is unable to reach a solution without consistent assistance.
Presentation of Finished Product X_____	Presentation shows compelling evidence of an independent learner and thinker. Solution shows deep understanding of the problem and its components. Solution shows extensive and appropriate application of 21st Century Skills.	Presentation shows clear evidence of an independent learner and thinker. Solution shows adequate understanding of the problem and its components. Solution shows adequate application of 21st Century Skills.	Presentation shows some evidence of an independent learner and thinker. Solution shows some understanding of the problem and its components. Solution shows some application of 21st Century Skills.	Presentation shows limited or no evidence of an independent learner and thinker. Solution shows limited or no understanding of the problem. Solution shows limited or no application of 21st Century Skills.