TRUMBULL PUBLIC SCHOOLS

Trumbull, Connecticut

Advanced Placement Statistics

Mathematics Department

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Curriculum Writing Team

Katie Laird	**Department Chair**
Brian Darrow, Jr.	**Teacher**

Jonathan S. Budd, Ph.D., Assistant Superintendent of Curriculum, Instruction, & Assessments

AP Statistics
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The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in any of its programs.

CORE VALUES AND BELIEFS

The Trumbull High School community engages in an environment conducive to learning which believes that all students will **read and write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION & PHILOSOPHY

Advanced Placement Statistics is a college-level course that is meant to be a challenging exploration of fundamental concepts in statistics while maintaining a relevance to life in modern society. While one of the major course goals is to prepare students for the AP Statistics College Board Examination, the course also aims to improve students' oral and written communication; critical thinking; scientific and quantitative reasoning; and interpretative skillset.

A Different Type of Math Class
In much of the mathematics students have done in their educational careers, the solutions to problems or answers to questions have always been absolute. Although there is tremendous creativity that can be applied to solving mathematics problems, the answer/solution is either deductively valid or deductively false. The answers to questions in statistics are far less concrete. To answer questions in statistics, we apply mathematics to analyze information in real (and often imperfect) situations in order to draw meaningful conclusions. In the real world, it is nearly impossible to have all the information about a situation, phenomena, or subject; often, we only have a limited collection of pieces of information (data) from which to draw conclusions and make decisions.

In statistics, we make educated "judgment calls," interpretations, and evidence-driven decisions. Statistical investigations are driven by the scientific method; therefore, all findings and limitations are reported. To support our conclusions, we formulate concise, detailed, and well-articulated arguments. Research findings need to be communicated (often to those who do not know anything about statistics) effectively through writing, oral presentation, or graphical representation. Research that is conducted using statistics must be carried out with integrity and research findings must be communicated objectively and dispassionately. For all of these reasons, investigation, interpretation, and communication are at the very heart of statistics. This course combines the quantitative and the qualitative. Statistics is as much about reading, writing, and critical thinking as it is about mathematical methods. This is because statistics is used to communicate research findings in nearly every academic discipline and research area – medicine, sociology, psychology, journalism, education, economics, and biology, to name a few. And it is for this reason that statistics is becoming the most relevant academic discipline to students' lives.

Statistics and the Future
The future is the analysis of data, and more specifically, big data. Statistical data analysis lies at the heart of advances from machine learning and artificial intelligence to technological

developments and medical breakthroughs. Not only are data scientists and statisticians the "rockstars of the industry" (with these professions consistently ranking as the best jobs in America in terms of demand, pay, lifestyle, and return on investment); their work is what is directly communicated to the public. Therefore, it is imperative that each citizen be an educated consumer of statistics. This means being able to separate fact from fiction, truth from speculation, and science from anecdotes in an age where evidence matters most. As H. G. Wells said, "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write."

As defined by the College Board, the purpose of AP Statistics is to introduce students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. Students are exposed to four broad conceptual themes:

(1) Selecting Statistical Methods
(2) Data Analysis
(3) Using Probability and Simulation
(4) Statistical Argumentation

Students who successfully complete the course and exam may receive credit, advanced placement, or both for a one-semester introductory college statistics course.

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.
The successful mathematician will:

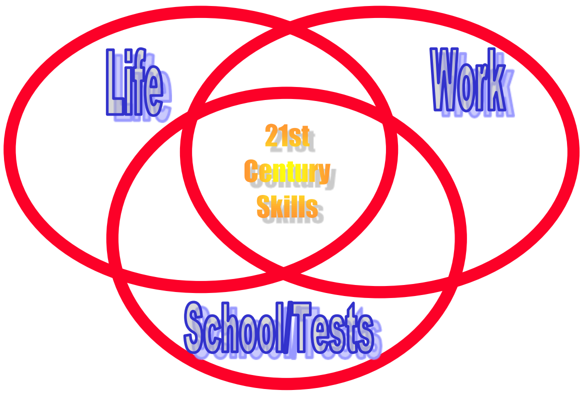
- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process
- Communicate effectively using mathematical terminology, both independently and collaboratively
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourge students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media and Technology Skills

1. Use real-world digital and other research tools to access, evaluate, and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE GOALS

The following Course Goals derive from the 2010 Connecticut Core Standards for Mathematical Practice, which describe varieties of expertise that all teachers of mathematics will develop in their students. These practices rest on important "processes and proficiencies" that have long been valued in mathematics education.

At the completion of this course, students will:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

The following Course Goals derive from the National Council of Teachers of Mathematics (NCTM) Mathematical Process Standards.

At the completion of this course, students will:

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Four overarching Course Goals for AP Statistics have been articulated by the College Board.

Selecting Statistical Methods: Students will select methods for collecting and/or analyzing data for statistical inference.

Data Analysis: Students will describe patterns, trends, associations, and relationships in data.

Using Probability and Simulation: Students will explore random phenomena.

Statistical Argumentation: Students will develop an explanation or justify a conclusion using evidence from data, definitions, or statistical inference.

Finally, the following Course Goals derive from the 2010 Connecticut Core Standards.

CCS.ELA-Literacy.RST.11-12.1	Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.
CCS.ELA-Literacy.RST.11-12.2	Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
CCS.ELA-Literacy.RST.11-12.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
CCS.ELA-Literacy.RST.11-12.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
CCS.ELA-Literacy.RST.11-12.5	Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.

CCS.ELA-Literacy.RST.11-12.6	Analyze the author’s purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
CCS.ELA-Literacy.RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
CCS.ELA-Literacy.RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
CCS.ELA-Literacy.RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
CCS.ELA-Literacy.RST.11-12.10	By the end of grade 12, read and comprehend science/technical texts in the grades 11-CCR text complexity band independently and proficiently.
CCS.ELA-Literacy.WHST.11-12.1	Write arguments focused on discipline-specific content.
CCS.ELA-Literacy.WHST.11-12.1.a	Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence.
CCS.ELA-Literacy.WHST.11-12.1.b	Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience’s knowledge level, concerns, values, and possible biases.
CCS.ELA-Literacy.WHST.11-12.1.c	Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.
CCS.ELA-Literacy.WHST.11-12.1.d	Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.
CCS.ELA-Literacy.WHST.11-12.1.e	Provide a concluding statement or section that follows from or supports the argument presented.

CCS.ELA-Literacy.WHST.11-12.2	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.
CCS.ELA-Literacy.WHST.11-12.2.a	Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.
CCS.ELA-Literacy.WHST.11-12.2.b	Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.
CCS.ELA-Literacy.WHST.11-12.2.c	Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify relationships among complex ideas and concepts.
CCS.ELA-Literacy.WHST.11-12.2.d	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.
CCS.ELA-Literacy.WHST.11-12.2.e	Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).
CCS.ELA-Literacy.WHST.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
CCS.ELA-Literacy.WHST.11-12.5	Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.
CCS.ELA-Literacy.WHST.11-12.6	Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
CCS.ELA-Literacy.WHST.11-12.7	Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize

	multiple sources on the subject, demonstrating understanding of the subject under investigation.
CCS.ELA-Literacy.WHST.11-12.8	Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
CCS.ELA-Literacy.WHST.11-12.9	Draw evidence from informational texts to support analysis, reflection, and research.
CCS.ELA-Literacy.WHST.11-12.10	Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

COURSE ENDURING UNDERSTANDINGS

Students will understand that . . .

- given that variation may be random or not, conclusions are uncertain.
- graphical representations and statistics allow us to identify and represent key features of data.
- the normal distribution can be used to represent some population distributions.
- regression models may allow us to predict responses to changes in an explanatory variable.
- the way we collect data influences what we can and cannot say about a population.
- well-designed experiments can establish evidence of causal relationships.
- simulation allows us to anticipate patterns in data.
- the likelihood of a random event can be quantified.
- probability distributions may be used to model variation in populations.
- probabilistic reasoning allows us to anticipate patterns in data.
- the normal distribution may be used to model variation.
- an interval of values should be used to estimate parameters, in order to account for uncertainty.
- significance testing allows us to make decisions about hypotheses within a particular context.
- probabilities of Type I and Type II errors influence inference.
- the *t*-distribution may be used to model variation.
- the chi-square distribution may be used to model variation.
- research literature has a particular style, including particular presentation of statistical information and particular research formatting, typically utilizing APA style guidelines.
- statistics has many practical uses, including its uses by other research disciplines to communicate research findings, and can be applied to many future career disciplines.
- statistical data management and analysis can be facilitated by Microsoft Excel.

- premier database systems and statistical software, such as Minitab, have significant benefits for the field of statistics.
- the study and application of statistics includes the creation of research questions, the design of data collection procedures to collect data conducive to statistical analysis, the determination of appropriate statistical tests for answering predetermined research questions, and the alignment of research questions, data collection procedures, data management procedures, statistical analyses, and interpretation of results into one cohesive structure.
- the communication of research findings in statistics can be done graphically, numerically, through language, and in a presentation format that includes fielding critical questions about research.

COURSE ESSENTIAL QUESTIONS

- What is statistics?
- What does it mean to do research?
- How do researchers use statistics?
- How can one harness the power of statistical analysis?
- How is data collection management/analysis related to statistics?
- How are ethics involved in statistical research?
- How does one communicate research findings in statistics?
- How does one summarize statistical research findings?
- What are the mathematical foundations of statistical procedures?
- How are mathematics and statistics related?
- How is statistics related to other research disciplines?
- Why is statistics important for the future?
- Where is statistics in our daily lives?
- How can we be skeptical of claims?
- How do we know if a research finding is important?
- When do we reject a research finding?
- How do we assess the validity of research findings?
- How can statistics be used in a dishonest way?
- Who is susceptible to dishonest statistics?
- How do we answer questions using statistics?
- How can we advance knowledge about individual phenomena using statistics?
- What are statistical tests?
- What is probability?
- What does it mean to anticipate phenomena?
- What is the role of probability in statistics?
- How do we choose which statistical procedures to use in which situations?
- How is the testing of claims different with proportions from with means? with categorical data?
- What are the mathematical foundations of statistical procedures?
- How do statisticians answer questions?

COURSE KNOWLEDGE & SKILLS

Students will know . . .

- that numbers may convey meaningful information, when placed in a context.
- that a variable is a characteristic that changes from one individual to another.
- that a categorical variable takes on values that are category names or group labels.
- that a quantitative variable is one that takes on numerical values for a measured or counted quantity.
- that a frequency table gives the number of cases falling into each category. A relative frequency table gives the proportion of cases falling into each category.
- that percentages, relative frequencies, and rates all provide the same information as proportions.
- that counts and relative frequencies of categorical data reveal information that can be used to justify claims about the data in context.
- that bar charts (or bar graphs) are used to display frequencies (counts) or relative frequencies (proportions) for categorical data.
- that the height or length of each bar in a bar graph corresponds to either the number or proportion of observations falling within each category.
- that there are many additional ways to represent frequencies (counts) or relative frequencies (proportions) for categorical data.
- that graphical representations of a categorical variable reveal information that can be used to justify claims about the data in context.
- that frequency tables, bar graphs, or other representations can be used to compare two or more data sets in terms of the same categorical variable.
- that a discrete variable can take on a countable number of values. The number of values may be finite or countably infinite, as with the counting numbers.
- that a continuous variable can take on infinitely many values, but those values cannot be counted. No matter how small the interval between two values of a continuous variable, it is always possible to determine another value between them.
- that in a histogram, the height of each bar shows the number or proportion of observations that fall within the interval corresponding to that bar. Altering the interval widths can change the appearance of the histogram.
- that in a stem and leaf plot, each data value is split into a “stem” (the first digit or digits) and a “leaf” (usually the last digit).
- that a dotplot represents each observation by a dot, with the position on the horizontal axis corresponding to the data value of that observation, with nearly identical values stacked on top of each other.
- that a cumulative graph represents the number or proportion of a data set less than or equal to a given number.
- that there are many additional ways to graphically represent distributions of quantitative data.
- that descriptions of the distribution of quantitative data include shape, center, and variability (spread), as well as any unusual features such as outliers, gaps, clusters, or multiple peaks.
- that outliers for one-variable data are data points that are unusually small or large relative to the rest of the data.
- that a distribution is skewed to the right (positive skew) if the right tail is longer than the left. A distribution is skewed to the left (negative skew) if the left tail is longer

than the right. A distribution is symmetric if the left half is the mirror image of the right half.

- that univariate graphs with one main peak are known as unimodal. Graphs with two prominent peaks are bimodal. A graph where each bar height is approximately the same (no prominent peaks) is approximately uniform.
- that a gap is a region of a distribution between two data values where there are no observed data.
- that clusters are concentrations of data usually separated by gaps.
- that descriptive statistics does not attribute properties of a data set to a larger population, but may provide the basis for conjectures for subsequent testing.
- that a statistic is a numerical summary of sample data.
- that the mean is the sum of all the data values divided by the number of values. For a sample, the mean is denoted by x-bar: $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, where x_i represents the i^{th} data point in the sample and *n* represents the number of data values in the sample.
- that the median of a data set is the middle value when data are ordered. When the number of data points is even, the median can take on any value between the two middle values. In AP Statistics, the most commonly used value for the median of a data set with an even number of values is the average of the two middle values.
- that the first quartile, Q1, is the median of the half of the ordered data set from the minimum to the position of the median. The third quartile, Q3, is the median of the half of the ordered data set from the position of the median to the maximum. Q1 and Q3 form the boundaries for the middle 50% of values in an ordered data set.
- that the p^{th} percentile is interpreted as the value that has *p*% of the data less than or equal to it.
- that three commonly used measures of variability (or spread) in a distribution are the range, interquartile range, and standard deviation.
- that the range is defined as the difference between the maximum data value and the minimum data value. The interquartile range (IQR) is defined as the difference between the third and first quartiles: Q3 − Q1. Both the range and the interquartile range are possible ways of measuring variability of the distribution of a quantitative variable.
- that standard deviation is a way to measure variability of the distribution of a quantitative variable. For a sample, the standard deviation is denoted by s: $s_x = \sqrt{\frac{1}{n-1}\sum(x_i - \overline{x})^2}$. The square of the sample standard deviation, s^2 , is called the sample variance.
- that changing units of measurement affects the values of the calculated statistics.
- that there are many methods for determining outliers. Two methods frequently used in this course are: i. An outlier is a value greater than *1.5 × IQR* above the third quartile or more than *1.5 × IQR* below the first quartile. ii. An outlier is a value located 2 or more standard deviations above, or below, the mean.
- that the mean, standard deviation, and range are considered nonresistant (or non-robust) because they are influenced by outliers. The median and IQR are considered resistant (or robust), because outliers do not greatly (if at all) affect their value.
- that, taken together, the minimum data value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum data value make up the five-number summary.
- that a boxplot is a graphical representation of the five-number summary (minimum, first quartile, median, third quartile, maximum). The box represents the middle 50%

of data, with a line at the median and the ends of the box corresponding to the quartiles. Lines (“whiskers”) extend from the quartiles to the most extreme point that is not an outlier, and outliers are indicated by their own symbol beyond this.

- that summary statistics of quantitative data, or of sets of quantitative data, can be used to justify claims about the data in context.
- that, if a distribution is relatively symmetric, then the mean and median are relatively close to one another. If a distribution is skewed right, then the mean is usually to the right of the median. If the distribution is skewed left, then the mean is usually to the left of the median.
- that any of the graphical representations, e.g., histograms, side-by-side boxplots, etc., can be used to compare two or more independent samples on center, variability, clusters, gaps, outliers, and other features.
- that any of the numerical summaries (e.g., mean, standard deviation, relative frequency, etc.) can be used to compare two or more independent samples.
- that a parameter is a numerical summary of a population.
- that some sets of data may be described as approximately normally distributed. A normal curve is mound-shaped and symmetric. The parameters of a normal distribution are the population mean, μ, and the population standard deviation, σ.
- that, for a normal distribution, approximately 68% of the observations are within 1 standard deviation of the mean, approximately 95% of observations are within 2 standard deviations of the mean, and approximately 99.7% of observations are within 3 standard deviations of the mean. This is called the empirical rule.
- that many variables can be modeled by a normal distribution.
- that a standardized score for a particular data value is calculated as (data value – mean)/(standard deviation), and measures the number of standard deviations a data value falls above or below the mean.
- that one example of a standardized score is a *z*-score, which is calculated as *z*-score= $\left(\frac{x_i-\mu}{\sigma}\right)$. A *z*-score measures how many standard deviations a data value is from the mean.
- that technology, such as a calculator, a standard normal table, or computer-generated output, can be used to find the proportion of data values located on a given interval of a normally distributed random variable.
- that, given the area of a region under the graph of the normal distribution curve, it is possible to use technology, such as a calculator, a standard normal table, or computer-generated output, to estimate parameters for some populations.
- that percentiles and *z*-scores may be used to compare relative positions of points within a data set or between data sets.
- that apparent patterns and associations in data may be random or not.
- that side-by-side bar graphs, segmented bar graphs, and mosaic plots are examples of bar graphs for one categorical variable, broken down by categories of another categorical variable.
- that graphical representations of two categorical variables can be used to compare distributions and/or determine if variables are associated.
- that a two-way table, also called a contingency table, is used to summarize two categorical variables. The entries in the cells can be frequency counts or relative frequencies.

- that a joint relative frequency is a cell frequency divided by the total for the entire table.
- that the marginal relative frequencies are the row and column totals in a two-way table divided by the total for the entire table.
- that a conditional relative frequency is a relative frequency for a specific part of the contingency table (e.g., cell frequencies in a row divided by the total for that row).
- that summary statistics for two categorical variables can be used to compare distributions and/or determine if variables are associated.
- that a bivariate quantitative data set consists of observations of two different quantitative variables made on individuals in a sample or population.
- that a scatterplot shows two numeric values for each observation, one corresponding to the value on the *x*-axis and one corresponding to the value on the *y*-axis.
- that an explanatory variable is a variable whose values are used to explain or predict corresponding values for the response variable.
- that a description of a scatter plot includes form, direction, strength, and unusual features.
- that the direction of the association shown in a scatterplot, if any, can be described as positive or negative.
- that a positive association means that as values of one variable increase, the values of the other variable tend to increase. A negative association means that as values of one variable increase, values of the other variable tend to decrease.
- that the form of the association shown in a scatterplot, if any, can be described as linear or non-linear to varying degrees.
- that the strength of the association is how closely the individual points follow a specific pattern, e.g., linear, and can be shown in a scatterplot. Strength can be described as strong, moderate, or weak.
- that unusual features of a scatter plot include clusters of points or points with relatively large discrepancies between the value of the response variable and a predicted value for the response variable.
- that the correlation, r, gives the direction and quantifies the strength of the linear association between two quantitative variables.
- that the correlation coefficient can be calculated by: $r = \frac{1}{n-1}\sum\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$.
  However, the most common way to determine *r* is by using technology.
- that a correlation coefficient close to 1 or −1 does not necessarily mean that a linear model is appropriate.
- that the correlation, r, is unit-free, and always between −1 and 1, inclusive. A value of $r = 0$ indicates is no linear association. A value of $r = 1$ or $r = -1$ indicates that there is a perfect linear association.
- that a perceived or real relationship between two variables does not mean that changes in one variable cause changes in the other. That is, correlation does not necessarily imply causation.
- that a simple linear regression model is an equation that uses an explanatory variable, *x*, to predict the response variable, *y*.
- that the predicted response value, denoted by $\hat{y}$, is calculated as $\hat{y} = a + bx$, where a is the *y*-intercept and *b* is the slope of the regression line, and *x* is the value of the explanatory variable.

- that extrapolation is predicting a response value using a value for the explanatory variable that is beyond the interval of *x*-values used to determine the regression line. The predicted value is less reliable as an estimate the further we extrapolate.
- that the residual is the difference between the actual value and the predicted value: residual $= y - \hat{y}$.
- that a residual plot is a plot of residuals versus explanatory variable values or predicted response values.
- that apparent randomness in a residual plot for a linear model is evidence of a linear form to the association between the variables.
- that residual plots can be used to investigate the appropriateness of a selected model.
- that the least-squares regression model minimizes the sum of the squares of the residuals and contains the point $(\overline{x}, \overline{y})$.
- that the slope, b, of the regression line can be calculated as $b = r\left(\frac{s_y}{s_x}\right)$, where r is the correlation between x and y, s_y is the sample standard deviation of the response variable, y, and s_x is the sample standard deviation of the explanatory variable, x.
- that, sometimes, the *y*-intercept of the line does not have a logical interpretation in context.
- that, in simple linear regression, r^2 is the square of the correlation, r. It is also called the coefficient of determination. r^2 is the proportion of variation in the response variable that is explained by the explanatory variable in the model.
- that the coefficients of the least-squares regression model are the estimated slope and *y*-intercept.
- that the slope is the amount that the predicted *y*-value changes for every unit increase in *x*.
- that the *y*-intercept value is the predicted value of the response variable when the explanatory variable is equal to 0. The formula for the *y*-intercept, a, is $= \overline{y} - b\overline{x}$.
- that an outlier in regression is a point that does not follow the general trend shown in the rest of the data and has a large residual when the Least Squares Regression Line (LSRL) is calculated.
- that a high-leverage point in regression has a substantially larger or smaller *x*-value than the other observations have.
- that an influential point in regression is any point that, if removed, changes the relationship substantially. Examples include much different slope, *y*-intercept, and/or correlation. Outliers and high leverage points are often influential.
- that transformations of variables, such as evaluating the natural logarithm of each value of the response variable or squaring each value of the explanatory variable, can be used to create transformed data sets, which may be more linear in form than the untransformed data.
- that increased randomness in residual plots after transformation of data and/or movement of r^2 to a value closer to 1 offers evidence that the least-squares regression line for the transformed data is a more appropriate model to use to predict responses to the explanatory variable than the regression line for the untransformed data.
- that methods for data collection that do not rely on chance result in untrustworthy conclusions.
- that a population consists of all items or subjects of interest.
- that a sample selected for study is a subset of the population.

- that, in an observational study, treatments are not imposed. Investigators examine data for a sample of individuals (retrospective) or follow a sample of individuals into the future collecting data (prospective) in order to investigate a topic of interest about the population. A sample survey is a type of observational study that collects data from a sample in an attempt to learn about the population from which the sample was taken.
- that, in an experiment, different conditions (treatments) are assigned to experimental units (participants or subjects).
- that it is only appropriate to make generalizations about a population based on samples that are randomly selected or otherwise representative of that population.
- that a sample is only generalizable to the population from which the sample was selected.
- that it is not possible to determine causal relationships between variables using data collected in an observational study.
- that, when an item from a population can be selected only once, this is called sampling without replacement. When an item from the population can be selected more than once, this is called sampling with replacement.
- that a simple random sample (SRS) is a sample in which every group of a given size has an equal chance of being chosen. This method is the basis for many types of sampling mechanisms. A few examples of mechanisms used to obtain SRSs include numbering individuals and using a random number generator to select which ones to include in the sample, ignoring repeats, using a table of random numbers, or drawing a card from a deck without replacement.
- that a stratified random sample involves the division of a population into separate groups, called strata, based on shared attributes or characteristics (homogeneous grouping). Within each stratum a simple random sample is selected, and the selected units are combined to form the sample.
- that a cluster sample involves the division of a population into smaller groups, called clusters. Ideally, there is heterogeneity within each cluster, and clusters are similar to one another in their composition. A simple random sample of clusters is selected from the population to form the sample of clusters. Data are collected from all observations in the selected clusters.
- that a systematic random sample is a method in which sample members from a population are selected according to a random starting point and a fixed, periodic interval.
- that a census selects all items/subjects in a population.
- that there are advantages and disadvantages for each sampling method depending upon the question that is to be answered and the population from which the sample will be drawn.
- that bias occurs when certain responses are systematically favored over others.
- that, when a sample is comprised entirely of volunteers or people who choose to participate, the sample will typically not be representative of the population (voluntary response bias).
- that, when part of the population has a reduced chance of being included in the sample, the sample will typically not be representative of the population (undercoverage bias).

- that individuals chosen for the sample for whom data cannot be obtained (or who refuse to respond) may differ from those for whom data can be obtained (nonresponse bias).
- that problems in the data gathering instrument or process result in response bias. Examples include questions that are confusing or leading (question wording bias) and self-reported responses.
- that non-random sampling methods (for example, samples chosen by convenience or voluntary response) introduce potential for bias because they do not use chance to select the individuals.
- that the experimental units are the individuals (which may be people or other objects of study) that are assigned treatments. When experimental units consist of people, they are sometimes referred to as participants or subjects.
- that an explanatory variable (or factor) in an experiment is a variable whose levels are manipulated intentionally. The levels or combination of levels of the explanatory variable(s) are called treatments.
- that a response variable in an experiment is an outcome from the experimental units that is measured after the treatments have been administered.
- that a confounding variable in an experiment is a variable that is related to the explanatory variable and influences the response variable and may create a false perception of association between the two.
- that a well-designed experiment should include the following: a. Comparisons of at least two treatment groups, one of which could be a control group. b. Random assignment/allocation of treatments to experimental units. c. Replication (more than one experimental unit in each treatment group). d. Control of potential confounding variables where appropriate.
- that, in a completely randomized design, treatments are assigned to experimental units completely at random. Random assignment tends to balance the effects of uncontrolled (confounding) variables so that differences in responses can be attributed to the treatments.
- that methods for randomly assigning treatments to experimental units in a completely randomized design include using a random number generator, a table of random values, drawing chips without replacement, etc.
- that, in a single-blind experiment, subjects do not know which treatment they are receiving, but members of the research team do, or vice versa.
- that, in a double-blind experiment neither the subjects nor the members of the research team who interact with them know which treatment a subject is receiving.
- that a control group is a collection of experimental units either not given a treatment of interest or given a treatment with an inactive substance (placebo) in order to determine if the treatment of interest has an effect.
- that the placebo effect occurs when experimental units have a response to a placebo.
- that, for randomized complete block designs, treatments are assigned completely at random within each block.
- that blocking ensures that at the beginning of the experiment the units within each block are similar to each other with respect to at least one blocking variable. A randomized block design helps to separate natural variability from differences due to the blocking variable.
- that a matched pairs design is a special case of a randomized block design. Using a blocking variable, subjects (whether they are people or not) are arranged in pairs

matched on relevant factors. Matched pairs may be formed naturally or by the experimenter. Every pair receives both treatments by randomly assigning one treatment to one member of the pair and subsequently assigning the remaining treatment to the second member of the pair. Alternately, each subject may get both treatments.

- that there are advantages and disadvantages for each experimental design depending on the question of interest, the resources available, and the nature of the experimental units.
- that statistical inference attributes conclusions based on data to the distribution from which the data were collected.
- that random assignment of treatments to experimental units allows researchers to conclude that some observed changes are so large as to be unlikely to have occurred by chance. Such changes are said to be statistically significant.
- that statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect.
- that, if the experimental units used in an experiment are representative of some larger group of units, the results of an experiment can be generalized to the larger group. Random selection of experimental units gives a better chance that the units will be representative.
- that patterns in data do not necessarily mean that variation is not random.
- that a random process generates results that are determined by chance.
- that an outcome is the result of a trial of a random process.
- that an event is a collection of outcomes.
- that simulation is a way to model random events, such that simulated outcomes closely match real-world outcomes. All possible outcomes are associated with a value to be determined by chance.
- that the relative frequency of an outcome or event in simulated or empirical data can be used to estimate the probability of that outcome or event.
- that the law of large numbers states that simulated (empirical) probabilities tend to get closer to the true probability as the number of trials increases.
- that the sample space of a random process is the set of all possible non-overlapping outcomes.
- that, if all outcomes in the sample space are equally likely, then the probability an event E will occur is defined as the fraction: (number of outcomes in event E)/(total number of outcomes in sample space).
- that the probability of an event is a number between 0 and 1, inclusive.
- that the probability of the complement of an event E, E' or E , (i.e., not E) is equal to $1 - P(E)$.
- that probabilities of events in repeatable situations can be interpreted as the relative frequency with which the event will occur in the long run.
- that the probability that events A and B both will occur, sometimes called the joint probability, is the probability of the intersection of A and B, denoted $(A \cap B)$.
- that two events are mutually exclusive or disjoint if they cannot occur at the same time. So $P(A \cap B) = 0$.
- that the probability that event A will occur given that event B has occurred is called a conditional probability and denoted $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

- that the multiplication rule states that the probability that events A and B both will occur is equal to the probability that event A will occur multiplied by the probability that event B will occur, given that A has occurred. This is denoted $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A}) \cdot P(A|B)$.
- that events A and B are independent if, and only if, knowing whether event A has occurred (or will occur) does not change the probability that event B will occur.
- that if, and only if, events A and B are independent, then $P(A|B) = P(A), P(B|A) = P(B),$ and $P(A \cap B) = P(A) \cdot P(B)$.
- that the probability that event A or event B (or both) will occur is the probability of the union of A and B, denoted $P(A \cup B)$.
- that the addition rule states that the probability that event A or event B or both will occur is equal to the probability that event A will occur plus the probability that event B will occur minus the probability that both events A and B will occur. This is denoted $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- that the values of a random variable are the numerical outcomes of random behavior.
- that a discrete random variable is a variable that can only take a countable number of values. Each value has a probability associated with it. The sum of the probabilities over all of the possible values must be 1.
- that a probability distribution can be represented as a graph, table, or function showing the probabilities associated with values of a random variable.
- that a cumulative probability distribution can be represented as a table or function showing the probability of being less than or equal to each value of the random variable.
- that an interpretation of a probability distribution provides information about the shape, center, and spread of a population and allows one to make conclusions about the population of interest.
- that a numerical value measuring a characteristic of a population or the distribution of a random variable is known as a parameter, which is a single, fixed value.
- that the mean, or expected value, for a discrete random variable X is $\mu = \sum x_i \cdot P(x_i)$.
- that the standard deviation for a discrete random variable X is$\sigma_x = \sqrt{\sum(x_i - \mu_x)^2 \cdot P(x_i)}$.
- that parameters for a discrete random variable should be interpreted using appropriate units and within the context of a specific population.
- that, for random variables X and Y and real numbers a and b, the mean of $aX + bY$ + is $\mu_x + b\mu_y$.
- that two random variables are independent if knowing information about one of them does not change the probability distribution of the other.
- that, for independent random variables X and Y and real numbers a and b, the mean of $aX + bY$ + is $a\mu_x + b\mu_y$ and the variance of $aX + bY$ + is $a^2\sigma_x^2 + b^2\sigma_y^2$.
- that, for $Y = a + bX$, the probability distribution of the transformed random variable, Y, has the same shape as the probability distribution for X, so long as $a > 0$ and $b > 0$. The mean of Y is $\mu = a + b\mu_x$ The standard deviation of Y is $\sigma_y = |b|\sigma_x$.
- that a probability distribution can be constructed using the rules of probability or estimated with a simulation using random number generators.

- that a binomial random variable, X, counts the number of successes in n repeated independent trials, each trial having two possible outcomes (success or failure), with the probability of success p and the probability of failure $1 - p$.
- that the probability that a binomial random variable, X, has exactly x successes for n independent trials, when the probability of success is p, is calculated as $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0,1,2, \ldots, n$. This is the binomial probability function.
- that, if a random variable is binomial, its mean, μ_x, is np and its standard deviation, σ_x , is $\sqrt{np(1-p)}$.
- that probabilities and parameters for a binomial distribution should be interpreted using appropriate units and within the context of a specific population or situation.
- that, for a sequence of independent trials, a geometric random variable, X, gives the number of the trial on which the first success occurs. Each trial has two possible outcomes (success or failure) with the probability of success p and the probability of failure $1 - p$.
- that the probability that the first success for repeated independent trials with probability of success p occurs on trial x is calculated as $P(X = x) = (1-p)^{x-1} p, x = 1,2,3, \ldots$. This is the geometric probability function.
- that, if a random variable is geometric, its mean, μ_x, is $\frac{1}{p}$ and its standard deviation, σ_x is $\frac{\sqrt{(1-p)}}{p}$.
- that probabilities and parameters for a geometric distribution should be interpreted using appropriate units and within the context of a specific population or situation.
- that variation in statistics for samples taken from the same population may be random or not.
- that a continuous random variable is a variable that can take on any value within a specified domain. Every interval within the domain has a probability associated with it.
- that a continuous random variable with a normal distribution is commonly used to describe populations. The distribution of a normal random variable can be described by a normal, or "bell-shaped," curve.
- that the area under a normal curve over a given interval represents the probability that a particular value lies in that interval.
- that the boundaries of an interval associated with a given area in a normal distribution can be determined using z-scores or technology, such as a calculator, a standard normal table, or computer-generated output.
- that intervals associated with a given area in a normal distribution can be determined by assigning appropriate inequalities to the boundaries of the intervals:
  a. $P(X < x_a) = \frac{p}{100}$ means that the lowest $p\%$ of values lie to the left of x_a.
  b. $P(x_a < X < x_b) = \frac{p}{100}$ means that $p\%$ of values lie between x_a and x_b .
  c. $P(X > x_b) = \frac{p}{100}$ means that the highest $p\%$ of values lie to the right of x_b.
  d. To determine the most extreme $p\%$ of values requires dividing the area associated with p% into two equal areas on either extreme of the distribution: $P(X < x_a) = \frac{1}{2}\frac{p}{100}$ and $P(X > x_b) = \frac{1}{2}\frac{p}{100}$ means that half of the $p\%$ most extreme values lie to the left of x_a and half of the $p\%$ most extreme values lie to the right of x_b.
- that normal distributions are symmetrical and "bell-shaped." As a result, normal distributions can be used to approximate distributions with similar characteristics.

- that a sampling distribution of a statistic is the distribution of values for the statistic for all possible samples of a given size from a given population.
- that the central limit theorem (CLT) states that when the sample size is sufficiently large, a sampling distribution of the mean of a random variable will be approximately normally distributed.
- that the central limit theorem requires that the sample values are independent of each other and that n is sufficiently large.
- that a randomization distribution is a collection of statistics generated by simulation assuming known values for the parameters. For a randomized experiment, this means repeatedly randomly reallocating/reassigning the response values to treatment groups.
- that the sampling distribution of a statistic can be simulated by generating repeated random samples from a population.
- that, when estimating a population parameter, an estimator is unbiased if, on average, the value of the estimator is equal to the population parameter.
- that, when estimating a population parameter, an estimator exhibits variability that can be modeled using probability.
- that a sample statistic is a point estimator of the corresponding population parameter.
- that, for independent samples (sampling with replacement) of a categorical variable from a population with population proportion, p, the sampling distribution of the sample proportion, $\hat{p}$, has a mean, $\mu_{\hat{p}} = p$ and a standard deviation, $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
- that, if sampling without replacement, the standard deviation of the sample proportion is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.
- that, for a categorical variable, the sampling distribution of the sample proportion, $\hat{p}$, will have an approximate normal distribution, provided the sample size is large enough: $np \geq 10$ and $n(1-p) \geq 10$.
- that probabilities and parameters for a sampling distribution for a sample proportion should be interpreted using appropriate units and within the context of a specific population.
- that, for a categorical variable, when randomly sampling with replacement from two independent populations with population proportions p_1 and p_2, the sampling distribution of the difference in sample proportions $\widehat{p_1} - \widehat{p}_2$ has mean, $\mu_{\widehat{p_1}-\widehat{p_2}} = p_1 - p_2$ and standard deviation, $\sigma_{\widehat{p_1}-\widehat{p_2}} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$.
- that, if sampling without replacement, the standard deviation of the difference in sample proportions is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.
- that the sampling distribution of the difference in sample proportions $\widehat{p_1} - \widehat{p}_2$ will have an approximate normal distribution provided the sample sizes are large enough: $n_1p_1 \geq 10, n_1(1-p_1) \geq 10, n_2p_2 \geq 10, n_2(1-p_2) \geq 10$.
- that parameters for a sampling distribution for a difference of proportions should be interpreted using appropriate units and within the context of a specific populations.
- that, for a numerical variable, when random sampling with replacement from a population with mean μ and standard deviation, σ , the sampling distribution of the sample mean has mean $\mu_{\overline{x}} = \mu$ and standard deviation $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$.

- that, if sampling without replacement, the standard deviation of the sample mean is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.
- that, for a numerical variable, if the population distribution can be modeled with a normal distribution, the sampling distribution of the sample mean, $\overline{x}$, can be modeled with a normal distribution.
- that, for a numerical variable, if the population distribution cannot be modeled with a normal distribution, the sampling distribution of the sample mean, $\overline{x}$, can be modeled approximately by a normal distribution, provided the sample size is large enough, e.g., greater than or equal to 30.
- that probabilities and parameters for a sampling distribution for a sample mean should be interpreted using appropriate units and within the context of a specific population.
- that, for a numerical variable, when randomly sampling with replacement from two independent populations with population means μ_1 and $\mu_2$and population standard deviations σ_1 and σ_2, the sampling distribution of the difference in sample means $\overline{x}_1 - \overline{x}_2$ has mean $\mu_{(\overline{x}_1 - \overline{x}_2)} = \mu_1 - \mu_2$ and standard deviation $\sigma_{(\overline{x}_1 - \overline{x}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_1^2}{n_1}}$.
- that, if sampling without replacement, the standard deviation of the difference in sample means is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.
- that the sampling distribution of the difference in sample means $\overline{x}_1 - \overline{x}_2$ can be modeled with a normal distribution if the two population distributions can be modeled with a normal distribution.
- that the sampling distribution of the difference in sample means $\overline{x}_1 - \overline{x}_2$ can be modeled approximately by a normal distribution if the two population distributions cannot be modeled with a normal distribution but both sample sizes are greater than or equal to 30.
- that probabilities and parameters for a sampling distribution for a difference of sample means should be interpreted using appropriate units and within the context of a specific population.
- that variation in shapes of data distributions may be random or not.
- that the appropriate confidence interval procedure for a one-sample proportion for one categorical variable is a one sample *z*-interval for a proportion.
- that, in order to make assumptions necessary for inference on population proportions, means, and slopes, we must check for independence in data collection methods and for selection of the appropriate sampling distribution.
- that, in order to calculate a confidence interval to estimate a population proportion, p, we must check for independence and that the sampling distribution is approximately normal.
    a. To check for independence:
        i. Data should be collected using a random sample or a randomized experiment.
        ii. When sampling without replacement, check that $\leq 10\%N$, where N is the size of the population.
    b. To check that the sampling distribution of $\hat{p}$ is approximately normal (shape):
        i. For categorical variables, check that both the number of successes, $n\hat{p}$, and the number of failures, $n(1-\hat{p})$ are at least 10 so that the sample size is large enough to support an assumption of normality.

- that, based on sample data, the standard error of a statistic is an estimate for the standard deviation for the statistic. The standard error of $\hat{p}$ is $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- that a margin of error gives how much a value of a sample statistic is likely to vary from the value of the corresponding population parameter.
- that, for categorical variables, the margin of error is the critical value (z*) times the standard error (SE) of the relevant statistic, which equals $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- that the formula for margin of error can be rearranged to solve for n, the minimum sample size needed to achieve a given margin of error. For this purpose, use a guess for $\hat{p}$ or use $\hat{p} = 0.5$ in order to find an upper bound for the sample size that will result in a given margin of error.
- that, in general, an interval estimate can be constructed as point estimate ± (margin of error). For a one-sample proportion, the interval estimate is $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- that critical values represent the boundaries encompassing the middle C% of the standard normal distribution, where C% is an approximate confidence level for a proportion.
- that confidence intervals for population proportions can be used to calculate interval estimates with specified units.
- that a confidence interval for a population proportion either contains the population proportion or it does not, because each interval is based on random sample data, which varies from sample to sample.
- that we are C% confident that the confidence interval for a population proportion captures the population proportion.
- that, in repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the population proportion.
- that interpreting a confidence interval for a one-sample proportion should include a reference to the sample taken and details about the population it represents.
- that a confidence interval for a population proportion provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- that, when all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases. For a population proportion, the width of the interval is proportional to $\frac{1}{\sqrt{n}}$.
- that, for a given sample, the width of the confidence interval for a population proportion increases as the confidence level increases.
- that the width of a confidence interval for a population proportion is exactly twice the margin of error.
- that the null hypothesis is the situation that is assumed to be correct unless evidence suggests otherwise, and the alternative hypothesis is the situation for which evidence is being collected.
- that, for hypotheses about parameters, the null hypothesis contains an equality reference (=, ≥, or ≤), while the alternative hypothesis contains a strict inequality (<, >, or ≠). The type of inequality in the alternative hypothesis is based on the question of interest. Alternative hypotheses with < or > are called one-sided, and alternative hypotheses with ≠ are called two-sided. Although the null hypothesis for a one-sided test may include an inequality symbol, it is still tested at the boundary of equality.

- that the null hypothesis for a population proportion is: $H_0: p = p_0$, where p_0 is the null hypothesized value for the population proportion.
- that a one-sided alternative hypothesis for a proportion is either $H_a: p < p_0$ or $H_a: p > p_0$. A two-sided alternate hypothesis is $H_a: p_1 \neq p_2$.
- that, for a one-sample z-test for a population proportion, the null hypothesis specifies a value for the population proportion, usually one indicating no difference or effect.
- that, for a single categorical variable, the appropriate testing method for a population proportion is a one-sample z-test for a population proportion.
- that, in order to make statistical inferences when testing a population proportion, we must check for independence and that the sampling distribution is approximately normal:
  a. To check for independence:
     i. Data should be collected using a random sample or a randomized experiment.
     ii. When sampling without replacement, check that $\leq 10\%N$.
  b. To check that the sampling distribution of pˆ is approximately normal (shape):
     i. Assuming that H_0 is true ($p = p_0$), verify that both the number of successes, np_0 and $n(1 - p_0)$, are at least 10 so that that the sample size is large enough to support an assumption of normality.
- that the distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or when a probability model is assumed to be true, a theoretical distribution (z).
- that, when using a z-test, the standardized test statistic can be written: test statistic=(sample statistic – null value of the parameter)/(standard deviation of the statistic). This is called a z-statistic for proportions.
- that the test statistic for a population proportion is: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.
- that a p-value is the probability of obtaining a test statistic as extreme or more extreme than the observed test statistic when the null hypothesis and probability model are assumed to be true. The significance level may be given or determined by the researcher.
- that the p-value is the proportion of values for the null distribution that are as extreme or more extreme than the observed value of the test statistic. This is:
  a. The proportion at or above the observed value of the test statistic, if the alternative is >.
  b. The proportion at or below the observed value of the test statistic, if the alternative is <.
  c. The proportion less than or equal to the negative of the absolute value of the test statistic plus the proportion greater than or equal to the absolute value of the test statistic, if the alternative is ≠.
- that an interpretation of the p-value of a significance test for a one-sample proportion should recognize that the p-value is computed by assuming that the probability model and null hypothesis are true, i.e., by assuming that the true population proportion is equal to the particular value stated in the null hypothesis.
- that the significance level, α, is the predetermined probability of rejecting the null hypothesis given that it is true.
- that a formal decision explicitly compares the p-value to the significance level, α. If the p-value $\leq \alpha$, reject the null hypothesis. If the p-value $> \alpha$, fail to reject the null hypothesis.

- that rejecting the null hypothesis means there is sufficient statistical evidence to support the alternative hypothesis. Failing to reject the null means there is insufficient statistical evidence to support the alternative hypothesis.
- that the conclusion about the alternative hypothesis must be stated in context.
- that a significance test can lead to rejecting or not rejecting the null hypothesis, but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.
- that small p-values indicate that the observed value of the test statistic would be unusual if the null hypothesis and probability model were true, and so provide evidence for the alternative. The lower the p-value, the more convincing the statistical evidence for the alternative hypothesis.
- that p-values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis and probability model were true, so do not provide convincing statistical evidence for the alternative hypothesis nor do they provide evidence that the null hypothesis is true.
- that a formal decision explicitly compares the p-value to the significance α. If the p-value $\leq \alpha$, then reject the null hypothesis, $H_0: p = p_0$. If the p-value $> \alpha$, then fail to reject the null hypothesis.
- that the results of a significance test for a population proportion can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
- that a Type I error occurs when the null hypothesis is true and is rejected (false positive).
- that a Type II error occurs when the null hypothesis is false and is not rejected (false negative).
- that the significance level, α, is the probability of making a Type I error, if the null hypothesis is true.
- that the power of a test is the probability that a test will correctly reject a false null hypothesis.
- that the probability of making a Type II error $= 1 - power$.
- that the probability of a Type II error decreases when any of the following occurs, provided the others do not change:
  i. Sample size(s) increases.
  ii. Significance level (α) of a test increases.
  iii. Standard error decreases.
  iv. True parameter value is farther from the null.
- that whether a Type I or a Type II error is more consequential depends upon the situation.
- that, since the significance level, α, is the probability of a Type I error, the consequences of a Type I error influence decisions about a significance level.
- that the appropriate confidence interval procedure for a two-sample comparison of proportions for one categorical variable is a two-sample z-interval for a difference between population proportions.
- that, in order to calculate confidence intervals to estimate a difference between proportions, we must check for independence and that the sampling distribution is approximately normal:
  a. To check for independence:

i. Data should be collected using random samples or a randomized experiment.
ii. When sampling without replacement, check that $n_1 \leq 10\%N_1$ and $n_2 \leq 10\%N_2$.

b. To check that sampling distribution of $\widehat{p_1}-\widehat{p_2}$ is approximately normal (shape).
   i. For categorical variables, check that $n_1p_1, n_1(1-p_1), n_2p_2, n_2(1-p_2)$ are all greater than or equal to some predetermined value, typically either 5 or 10.

- that, for a comparison of proportions, the interval estimate is $(\widehat{p_1}-\widehat{p_2}) \pm z^*\sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1}+\frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$.
- that confidence intervals for a difference in proportions can be used to calculate interval estimates with specified units.
- that, in repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference in population proportions.
- that interpreting a confidence interval for difference between population proportions should include a reference to the sample taken and details about the population it represents.
- that a confidence interval for difference in population proportions provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- that, for a two-sample test for a difference of two proportions, the null hypothesis specifies a value of 0 for the difference in population proportions, indicating no difference or effect.
- that the null hypothesis for a difference in proportions is: $H_0: p_1 = p_2$ or $H_a: p_1 - p_2 = 0$.
- that a one-sided alternative hypothesis for a difference in proportions is $H_a: p_1 < p_2$ or $H_a: p_1 > p_2$. A two-sided alternative hypothesis for a difference of proportions is $H_a: p_1 \neq p_2$.
- that, for a single categorical variable, the appropriate testing method for the difference of two population proportions is a two-sample *z*-test for a difference between two population proportions.
- that, in order to make statistical inferences when testing a difference between population proportions, we must check for independence and that the sampling distribution is approximately normal:
  a. To check for independence:
     i. Data should be collected using random samples or a randomized experiment.
     ii. When sampling without replacement, check that $n_1 \leq 10\%N_1$ and $n_2 \leq 10\%N_2$.
  b. To check that the sampling distribution of $\widehat{p_1}-\widehat{p_2}$ is approximately normal (shape):
     i. For the combined sample, define the combined (or pooled) proportion, $\widehat{p_c} = \frac{n_1\widehat{p_1}+n_2\widehat{p_2}}{n_1+n_1}$, check that $n_1\widehat{p_c}, n_1(1-\widehat{p_c}), n_2\widehat{p_c}, n_2(1-\widehat{p_c})$ are all greater than or equal to some predetermined value, typically either 5 or 10.
- that the test statistic for a difference in proportions is: $= \frac{(\hat{p}_1-\widehat{p_2})-0}{\sqrt{\hat{p}_c(1-\hat{p}_c)}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$, where $\widehat{p_c} = \frac{n_1\widehat{p_1}+n_2\widehat{p_2}}{n_1+n_1}$.
- that an interpretation of the *p*-value of a significance test for a difference of two population proportions should recognize that the *p*-value is computed by assuming

that the null hypothesis is true, i.e., by assuming that the true population proportions are equal to each other.

- that a formal decision explicitly compares the *p*-value to the significanceα. If the $p - value \leq \alpha$, then reject the null hypothesis, $H_0: p_1 = p_2$ or $H_a: p_1 - p_2 = 0$. If the $p - value > \alpha$, then fail to reject the null hypothesis.
- that the results of a significance test for a difference of two population proportions can serve as the statistical reasoning to support the answer to a research question about the two populations that were sampled.
- that random variation may result in errors in statistical inference.
- that, when *s* is used instead of σ to calculate a test statistic, the corresponding distribution, known as the *t*-distribution, varies from the normal distribution in shape, in that more of the area is allocated to the tails of the density curve than in a normal distribution.
- that, as the degrees of freedom increase, the area in the tails of a *t*-distribution decreases.
- that, because σ is typically not known for distributions of quantitative variables, the appropriate confidence interval procedure for estimating the population mean of one quantitative variable for one sample is a one-sample *t*-interval for a mean.
- that, for one quantitative variable, *X*, that is normally distributed, the distribution of $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ is a *t*-distribution with $n - 1$ degrees of freedom.
- that matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for confidence intervals proceeds as for a population mean.
- that, in order to calculate confidence intervals to estimate a population mean, we must check for independence and that the sampling distribution is approximately normal:
  a. To check for independence:
     i. Data should be collected using a random sample or a randomized experiment.
     ii. When sampling without replacement, check that $\leq 10\%N$, where N is the size of the population.
  b. To check that the sampling distribution of *x* is approximately normal (shape):
     i. If the observed distribution is skewed, *n* should be greater than 30.
     ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
- that the critical value $t*$ with $n - 1$ degrees of freedom can be found using a table or computer-generated output.
- that the standard error for a sample mean is given by $SE = \frac{s}{\sqrt{n}}$, where *s* is the sample standard deviation.
- that, for a one-sample *t*-interval for a mean, the margin of error is the critical value (*t**) times the standard error (*SE*), which equals $t * \left(\frac{s}{\sqrt{n}}\right)$.
- that the point estimate for a population mean is the sample mean, $\bar{x}$.
- that, for the population mean for one sample with unknown population standard deviation, the confidence interval is $\bar{x} \pm t * \left(\frac{s}{\sqrt{n}}\right)$.
- that a confidence interval for a population mean either contains the population mean or it does not, because each interval is based on data from a random sample, which varies from sample to sample.

- that we are C% confident that the confidence interval for a population mean captures the population mean.
- that an interpretation of a confidence interval for a population mean includes a reference to the sample taken and details about the population it represents.
- that a confidence interval for a population mean provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- that, when all other things remain the same, the width of a confidence interval for a population mean tends to decrease as the sample size increases.
- that, for a single mean, the width of the interval is proportional to $\frac{1}{n}$.
- that, for a given sample, the width of the confidence interval for a population mean increases as the confidence level increases.
- that the appropriate test for a population mean with unknown σ is a one-sample *t*-test for a population mean.
- that matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for significance testing proceeds as for a population mean.
- that the null hypothesis for a one-sample *t*-test for a population mean is $H_0: \mu = \mu_0$. Depending upon the situation, the alternative hypothesis is $H_a: \mu < \mu_0$, or $H_a: \mu > \mu_0$ or $H_a: \mu \neq \mu_0$.
- that, when finding the mean difference, μ_d , between values in a matched pair, it is important to define the order of subtraction.
- that, in order to make statistical inferences when testing a population mean, we must check for independence and that the sampling distribution is approximately normal:
  a. To check for independence:
     i. Data should be collected using a random sample or a randomized experiment.
     ii. When sampling without replacement, check that $\leq 10\%N$.
  b. To check that the sampling distribution of x is approximately normal (shape):
     i. If the observed distribution is skewed, n should be greater than 30.
     ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
- that, for a single quantitative variable when random sampling with replacement from a population that can be modeled with a normal distribution with mean μ and standard deviation σ, the sampling distribution of $t = \frac{\overline{x-\mu}}{\frac{s}{\sqrt{n}}}$ has a *t*-distribution with $n - 1$ degrees of freedom.
- that an interpretation of the *p*-value of a significance test for a population mean should recognize that the *p*-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population mean is equal to the particular value stated in the null hypothesis.
- that a formal decision explicitly compares the *p*-value to the significance α. If the *p*-value $\leq \alpha$, then reject the null hypothesis, $H_0: \mu = \mu_0$. If the *p*-value $> \alpha$, then fail to reject the null hypothesis.
- that the results of a significance test for a population mean can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
- that, considering a simple random sample from population 1 of size n_1 , mean μ_1 , and standard deviation σ_1 and a second simple random sample from population 2 of size n_2 , mean μ_2 , and standard deviation σ_2, if the distributions of populations 1 and

2 are normal or if both n_1 and n_2 are greater than 30, then the sampling distribution of the difference of means $\overline{x}_1 - \overline{x}_2$ is also normal. The mean for the sampling distribution of $\overline{x}_1 - \overline{x}_2$ is $\mu_1 - \mu_2$. The standard deviation of means $\overline{x}_1 - \overline{x}_2$ is $\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_1}}$.

- that the appropriate confidence interval procedure for one quantitative variable for two independent samples is a two-sample *t*-interval for a difference between population means.
- that, in order to calculate confidence intervals to estimate a difference of population means, we must check for independence and that the sampling distribution is approximately normal:
  a. To check for independence:
     i. Data should be collected using two independent, random samples or a randomized experiment.
     ii. When sampling without replacement, check that $n_1 \leq 10\%N_1$ and $n_2 \leq 10\%N_2$.
  b. To check that the sampling distribution of $\overline{x}_1 - \overline{x}_2$ should be approximately normal (shape):
     i. If the observed distributions are skewed, both n_1 and n_2 should be greater than 30.
- that, for the difference of two sample means, the margin of error is the critical value (t*) times the standard error (*SE*) of the difference of two means.
- that the standard error for the difference in two sample means with sample standard deviations, s_1 and s_2, is $\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_1}}$.
- that the point estimate for the difference of two population means is the difference in sample means, $\overline{x}_1 - \overline{x}_2$.
- that, for a difference of two population means where the population standard deviations are not known, the confidence interval is $(\overline{x}_1 - \overline{x}_2) \pm t^* \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_1}}$ where $\pm t$ * are the critical values for the central C% of a *t*-distribution with appropriate degrees of freedom that can be found using technology.
- that, in repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference of population means.
- that an interpretation for a confidence interval for the difference of two population means should include a reference to the samples taken and details about the populations they represent.
- that a confidence interval for a difference of population means provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- that, when all other things remain the same, the width of the confidence interval for the difference of two means tends to decrease as the sample sizes increase.
- that, for a quantitative variable, the appropriate test for a difference of two population means is a two-sample *t*-test for a difference of two population means.
- that the null hypothesis for a two-sample *t*-test for a difference of two population means, μ_1 and μ_2, is $H_0: \mu_1 - \mu_2 = 0$ $H_0: \mu_1 = \mu_2$. The alternative hypothesis is $H_a: \mu_1 - \mu_2 < 0$ or $H_a: \mu_1 - \mu_2 > 0$ or $H_a: \mu_1 - \mu_2 \neq 0$ or $H_a: \mu_1 < \mu_2$ or $H_a: \mu_1 > \mu_2$ or $H_a: \mu_1 \neq \mu_2$.

- that, in order to make statistical inferences when testing a difference between population means, we must check for independence and that the sampling distribution is approximately normal:
  a. Individual observations should be independent:
     i. Data should be collected using simple random samples or a randomized experiment.
     ii. When sampling without replacement, check that $n_1 \leq 10\%N_1$ and $n_2 \leq 10\%N_2$.
  b. The sampling distribution of $\overline{x}_1 - \overline{x}_2$ should be approximately normal (shape).
     i. If the observed distribution is skewed, both n_1 and n_2 should be greater than 30.
     ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers. This should be checked for BOTH samples.
- that, for a single quantitative variable, data collected using independent random samples or a randomized experiment from two populations, each of which can be modeled with a normal distribution, the sampling distribution of $t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_1}}}$ is an approximate *t*-distribution with degrees of freedom that can be found using technology. The degrees of freedom fall between the smaller of $n_1 - 1$ and $n_2 - 1$ and $n_1 + n_2 - 2$.
- that an interpretation of the *p*-value of a significance test for a two-sample difference of population means should recognize that the *p*-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population means are equal to each other.
- that a formal decision explicitly compares the *p*-value to the significance α. If the *p*-value $\leq \alpha$, then reject the null hypothesis, $H_0: \mu_1 - \mu_2 = 0$ $H_0: \mu_1 = \mu_2$. If the the *p*-value $> \alpha$, then fail to reject the null hypothesis.
- that the results of a significance test for a two-sample test for a difference between two population means can serve as the statistical reasoning to support the answer to a research question about the populations that were sampled.
- that variation between what we find and what we expect to find may be random or not.
- that expected counts of categorical data are counts consistent with the null hypothesis. In general, an expected count is a sample size times a probability.
- that the chi-square statistic measures the distance between observed and expected counts relative to expected counts.
- that chi-square distributions have positive values and are skewed right. Within a family of density curves, the skew becomes less pronounced with increasing degrees of freedom.
- that, for a chi-square goodness-of-fit test, the null hypothesis specifies null proportions for each category, and the alternative hypothesis is that at least one of these proportions is not as specified in the null hypothesis.
- that, when considering a distribution of proportions for one categorical variable, the appropriate test is the chi-square test for goodness of fit.
- that expected counts for a chi-square goodness-of-fit test are (sample size) (null proportion).

- that, in order to make statistical inferences for a chi-square test for goodness of fit we must check the following:
  a. To check for independence:
     i. Data should be collected using a random sample or randomized experiment.
     ii. When sampling without replacement, check that $\leq 10\%N$.
  b. The chi-square test for goodness of fit becomes more accurate with more observations, so large counts should be used (shape).
     i. A conservative check for large counts is that all expected counts should be greater than 5.
- that the test statistic for the chi-square test for goodness of fit is $\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count}$ with $degrees\ of\ freedom = number\ of\ categories\ - 1$.
- that the distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or, when a probability model is assumed to be true, a theoretical distribution (chi-square).
- that the *p*-value for a chi-square test for goodness of fit for a number of degrees of freedom is found using the appropriate table or computer generated output.
- that an interpretation of the *p*-value for the chi-square test for goodness of fit is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
- that a decision to either reject or fail to reject the null hypothesis is based on comparison of the *p*-value to the significance level, α.
- that the results of a chi-square test for goodness of fit can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
- that the expected count in a particular cell of a two-way table of categorical data can be calculated using the formula: $expected\ count = \frac{(row\ total)(column\ total)}{table\ total}$.
- that the appropriate hypotheses for a chi-square test for homogeneity are: H_0: There is no difference in distributions of a categorical variable across populations or treatments. H_a: There is a difference in distributions of a categorical variable across populations or treatments.
- that the appropriate hypotheses for a chi-square test for independence are: H_0: There is no association between two categorical variables in a given population or the two categorical variables are independent. H_a: Two categorical variables in a population are associated or dependent.
- that, when, comparing distributions to determine whether proportions in each category for categorical data collected from different populations are the same, the appropriate test is the chi-square test for homogeneity.
- that, to determine whether row and column variables in a two-way table of categorical data might be associated in the population from which the data were sampled, the appropriate test is the chi-square test for independence.
- that, in order to make statistical inferences for a chi-square test for two-way tables (homogeneity or independence), we must verify the following:
  a. To check for independence:
     i. For a test for independence: Data should be collected using a simple random sample.

ii. For a test for homogeneity: Data should be collected using a stratified random sample or randomized experiment.
iii. When sampling without replacement, check that $\leq 10\%N$.
b. The chi-square tests for independence and homogeneity become more accurate with more observations, so large counts should be used (shape).
i. A conservative check for large counts is that all expected counts should be greater than 5.

- that the appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic: $\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count}$, with degrees of freedom equal to: $(number\ of\ rows - 1)(\ number\ of\ columns\ \ - 1)$.
- that the *p*-value for a chi-square test for independence or homogeneity for a number of degrees of freedom is found using the appropriate table or technology.
- that, for a test of independence or homogeneity for a two-way table, the *p*-value is the proportion of values in a chi-square distribution with appropriate degrees of freedom that are equal to or larger than the test statistic.
- that an interpretation of the *p*-value for the chi-square test for homogeneity or independence is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
- that a decision to either reject or fail to reject the null hypothesis for a chi-square test for homogeneity or independence is based on comparison of the *p*-value to the significance level, α.
- that the results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to a research question about the population that was sampled (independence) or the populations that were sampled (homogeneity).
- that variation in points' positions relative to a theoretical line may be random or non-random.
- that, considering a response variable, *y*, that is normally distributed with standard deviation, σ, the standard deviation σ can be estimated using the standard deviation of the residuals, $s = \sqrt{\frac{\sum(y_1 - \widehat{y_1})^2}{n-2}}$.
- that, for a simple random sample of *n* observations, let *b* represent the slope of a sample regression line. Then the mean of the sampling distribution of *b* equals the population mean slope: $\mu_b = \beta$. The standard deviation of the sampling distribution for β is $\sigma_b = \frac{\sigma}{\sigma_x \sqrt{n}}$ where $\sigma_x = \sqrt{\frac{\sum(x_i - \overline{x})}{n}}$.
- that the appropriate confidence interval for the slope of a regression model is a *t*-interval for the slope.
- that, in order to calculate a confidence interval to estimate the slope of a regression line, we must check the following:
  a. The true relationship between x and y is linear. Analysis of residuals may be used to verify linearity.
  b. The standard deviation for y, σ_y, does not vary with x. Analysis of residuals may be used to check for approximately equal standard deviations for all x.
  c. To check for independence:
  i. Data should be collected using a random sample or a randomized experiment.
  ii. When sampling without replacement, check that $\leq 10\%N$.

d. For a particular value of *x*, the responses (*y*-values) are approximately normally distributed. Analysis of graphical representations of residuals may be used to check for normality.
   i. If the observed distribution is skewed, n should be greater than 30. UNC

- that, for the slope of a regression line, the margin of error is the critical value (t^*) times the standard error (*SE*) of the slope.
- that the standard error for the slope of a regression line with sample standard deviation, s, is $= \frac{s}{s_x\sqrt{n-1}}$, where s is the estimate of σ and s_x is the sample standard deviation of the *x* values.
- that the point estimate for the slope of a regression model is the slope of the line of best fit, b.
- that, for the slope of a regression model, the interval estimate is $b \pm t^*(SE_b)$.
- that, in repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the slope of the regression model, i.e., the true slope of the population regression model.
- that an interpretation for a confidence interval for the slope of a regression line should include a reference to the sample taken and details about the population it represents.
- that a confidence interval for the slope of a regression model provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- that, when all other things remain the same, the width of the confidence interval for the slope of a regression model tends to decrease as the sample size increases.
- that the appropriate test for the slope of a regression model is a *t*-test for a slope.
- that the null hypothesis for a *t*-test for a slope is: $H_0: \beta = \beta_0$, where β_0, is the hypothesized value from the null hypothesis. The alternative hypothesis is $H_a: \beta < \beta_0$ or $H_a: \beta > \beta_0$ or $H_a: \beta \neq \beta_0$.
- that, in order to make statistical inferences when testing for the slope of a regression model, we must check the following:
  a. The true relationship between *x* and *y* is linear. Analysis of residuals may be used to verify linearity.
  b. The standard deviation for *y*, σ_y, does not vary with *x*. Analysis of residuals may be used to check for approximately equal standard deviations for all *x*.
  c. To check for independence:
     i. Data should be collected using a random sample or a randomized experiment.
     ii. When sampling without replacement, check that $n \leq 10\%N$.
  d. For a particular value of *x*, the responses (*y*-values) are approximately normally distributed. Analysis of graphical representations of residuals may be used to check for normality.
     i. If the observed distribution is skewed, n should be greater than 30.
     ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
- that the distribution of the slope of a regression model assuming all conditions are satisfied and the null hypothesis is true (null distribution) is a *t*-distribution.
- that, for simple linear regression when random sampling from a population for the response that can be modeled with a normal distribution for each value of the explanatory variable, the sampling distribution of $t = \frac{b-\beta}{SE_b}$ has a *t*-distribution with degrees of freedom equal to $n - 2$. When testing the slope in a simple linear regression model with one parameter, the slope, the test for the slope has $df = n$ –1.

- that an interpretation of the p-value of a significance test for the slope of a regression model should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population slope is equal to the particular value stated in the null hypothesis.
- that a formal decision explicitly compares the p-value to the significance α. If the p-value $\leq \alpha$, then reject the null hypothesis, $H_0: \beta = \beta_0$. If the p-value > α, then fail to reject the null hypothesis.
- that the results of a significance test for the slope of a regression model can serve as the statistical reasoning to support the answer to a research question about that sample.

Students will be able to . . .

- identify the question to be answered or problem to be solved.
- identify key and relevant information to answer a question or solve a problem.
- describe an appropriate method for gathering and representing data.
- identify an appropriate inference method for confidence intervals.
- identify an appropriate inference method for significance tests.
- identify null and alternative hypotheses.
- describe data presented numerically or graphically.
- construct numerical or graphical representations of distributions.
- calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response.
- compare distributions or relative positions of points within a distribution.
- determine relative frequencies, proportions, or probabilities using simulation or calculations.
- determine parameters for probability distributions.
- describe probability distributions.
- construct a confidence interval, provided conditions for inference are met.
- calculate a test statistic and find a p-value, provided conditions for inference are met.
- make an appropriate claim or draw an appropriate conclusion.
- interpret statistical calculations and findings to assign meaning or assess a claim.
- verify that inference procedures apply in a given situation.
- justify a claim based on a confidence interval.
- justify a claim using a decision based on significance tests.
- identify the individuals and variables in a set of data.
- classify variables as categorical or quantitative.
- make and interpret bar graphs for categorical data.
- identify what makes some graphs of categorical data misleading.
- calculate the marginal and joint relative frequencies from a two-way table.
- calculate conditional relative frequencies from a two-way table.
- use bar graphs to compare distributions of categorical data.
- describe the nature of the association between two categorical variables.
- make and interpret dotplots, stemplots, and histograms of quantitative data.
- identify the shape of a distribution from a graph.
- describe the overall pattern (shape, center, and variability) of a distribution and identify any major departures from the pattern (outliers).
- compare distributions of quantitative data using dotplots, stemplots, and histograms.

- calculate measures of center (mean, median) for a distribution of quantitative data.
- calculate and interpret measures of variability (range, standard deviation, IQR) for a distribution of quantitative data.
- explain how outliers and skewness affect measures of center and variability.
- identify outliers using the 1.5XIQR rule.
- make and interpret boxplots for quantitative data.
- use boxplots and numerical summaries to compare distributions of quantitative data.
- find and interpret the percentile of an individual value within a distribution of data.
- estimate percentiles and individual values using a cumulative frequency graph.
- find and interpret the standardized score (*z*-score) of an individual value within a distribution of data.
- use a density curve to model distributions of quantitative data.
- identify the relative locations of the mean and median of a distribution from a density curve.
- use the 68-95-99.7 rule to estimate the proportion of values in a specified interval, or the value that corresponds to a given percentile in a Normal distribution.
- find the proportion of values in a specified interval in a Normal distribution using the *Z*-proportion table and/or technology.
- find the value that corresponds to a given percentile in a Normal distribution in a percentile using the *Z*-proportion table and/or technology.
- determine whether a distribution of data is approximately Normal from graphical and numerical evidence.
- distinguish between explanatory and response variables for quantitative data.
- make a scatterplot to display the relationship between two quantitative variables.
- describe the direction, form, and strength of a relationship displayed in a scatterplot and identify unusual features.
- understand the basic properties of correlation, including how a correlation is influenced by outliers
- interpret correlation.
- distinguish correlation from causation.
- make predictions using regression lines, keeping in mind the dangers of extrapolation.
- calculate and interpret a residual.
- interpret the slope and *y*-intercept of a least-squares regression line.
- determine the equation of a least-squares regression line using technology or computer output.
- construct and interpret residual plots to assess whether a regression model is appropriate.
- interpret the standard deviation of the residuals and *r*-squared and use these values to assess how well the least-squares regression line models the relationship between the two variables.
- describe how the slope, *y*-intercept, standard deviation of the residuals, and the *r*-squared are influenced by outliers.
- find the slope and *y*-intercept of the least-squares regression line from the means and standard deviations of *x* and *y* and their correlation.
- identify the population and sample in a statistical study.
- identify voluntary response sampling and convenience sampling and explain how these sampling methods can lead to bias.

- describe how to select a simple random sample with technology or a table of random digits.
- describe how to select a sample using stratified random sampling and cluster sampling, distinguish stratified random sampling from cluster sampling, and give an advantage of each method.
- explain how undercoverage, nonresponse, wording of questions, and other aspects of a sample survey can lead to bias.
- explain the concept of confounding and how it limits the ability to make cause-and-effect conclusions.
- distinguish between an observational study and an experiment, and identify the explanatory and response variables in each type of study.
- identify the experimental units and treatments in an experiment.
- describe the placebo effect and the purpose of blinding in an experiment.
- describe how to randomly assign treatments in an experiment using slips of paper, technology, or a table of random digits.
- explain the purpose of comparison, random assignment, control, and replication in an experiment.
- describe a completely randomized design for an experiment.
- describe a randomized block design and a matched pairs design for an experiment and explain the purpose of blocking in an experiment.
- explain the concept of sampling variability when making an inference about a population, and how sample size affects sampling variability.
- explain the meaning of "statistically significant" in the context of an experiment and use simulation to determine if the results of an experiment are statistically significant.
- identify when it is appropriate to make an inference about a population and when it is appropriate to make an inference about cause and effect.
- evaluate if a statistical study has been carried out in an ethical manner.
- interpret probability as a long-run relative frequency.
- use simulation to model chance behavior.
- give a probability model for a chance process with equally likely outcomes and use it to find the probability of an event.
- use basic probability rules, including the complement rule and the addition rule for mutually exclusive events.
- use a two-way table or Venn diagram to model a chance process and calculate probabilities involving two events.
- apply the general addition rule to calculate probabilities.
- calculate and interpret conditional probabilities.
- determine whether two events are independent.
- use the general multiplication rule to calculate probabilities.
- use a tree diagram to model a chance process involving a sequence of outcomes and to calculate probabilities.
- when appropriate, use the multiplication rule for independent events to calculate probabilities.
- use the probability distribution of a discrete random variable to calculate the probability of an event.
- make a histogram to display the probability distribution of a discrete random variable and describe its shape.

- calculate and interpret the mean (expected value) of a discrete random variable.
- use the probability distribution of a continuous random variable (uniform or Normal) to calculate the probability of an event.
- describe the effect of adding or subtracting a constant or multiplying or dividing by a constant on the probability distribution of a random variable.
- calculate the mean and standard deviation of the sum or difference of random variables.
- find the probabilities involving the sum or difference of independent Normal random variables.
- determine whether the conditions for a binomial setting are met.
- calculate and interpret probabilities involving binomial distributions.
- calculate the mean and standard deviation of a binomial random variable, and interpret these values in context.
- when appropriate, use the Normal approximation to the binomial distribution to calculate probabilities.
- find probabilities involving geometric random variables.
- distinguish between a parameter and a statistic.
- create a sampling distribution using all possible samples from a small population.
- use the sampling distribution of a statistic to evaluate a claim about a parameter.
- distinguish among the distribution of a population, the distribution of a sample, and the sampling distribution of a statistic.
- determine if a statistic is an unbiased estimator of a population parameter.
- describe the relationship between sample size and the variability of a statistic.
- calculate the mean and standard deviation of the sampling distribution of a sample proportion ($\hat{p}$) and interpret the standard deviation.
- determine if the sampling distribution of $\hat{p}$ is approximately Normal.
- if appropriate, use a Normal distribution to calculate the probabilities involving $\hat{p}$.
- calculate the mean and standard deviation of the sampling distribution of a sample mean ($\overline{x}$) and interpret the standard deviation.
- determine if, and explain how, the sampling distribution of $\overline{x}$ is affected by the shape of the population distribution and the sample size.
- if appropriate, use a Normal distribution to calculate probabilities involving $\overline{x}$.
- identify an appropriate point estimator and calculate the value of a point estimate.
- interpret a confidence interval in context.
- determine the point estimate and margin of error from a confidence interval.
- use a confidence interval to make a decision about the value of a parameter.
- interpret a confidence interval in context.
- describe how the sample size and confidence level affect the margin of error.
- explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval for calculating C% confidence interval for a population mean using a table or technology.
- state and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.
- determine the critical value for calculating a C% confidence interval for a population proportion using a table and/or technology.
- construct and interpret a confidence interval for a population proportion.

- determine the sample size required to obtain a C% confidence interval for a population proportion with a specified margin of error.
- determine the critical value for calculating C% confidence interval for a population mean using a table and/or technology.
- state and check the Random, 10%, and Normal/Large Sample conditions for constructing a confidence interval for a population mean.
- construct and interpret a confidence interval for a population mean.
- determine the sample size required to obtain a C% confidence interval for a population mean with a specified margin of error.
- state appropriate hypotheses for a significance test about a population parameter.
- interpret a *p*-value in context.
- make an appropriate conclusion for a significance test.
- interpret a Type I and Type II error in context, and give a consequence of each error in a given setting.
- state and check the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion.
- calculate the standardized test statistic and *p*-value for a test about a population.
- perform a significance test about a population proportion.
- state and check the Random, 10%, and Normal/Large Sample conditions for performing a significance test about a population mean.
- calculate the standardized test statistic and *p*-value for a test about a population mean.
- perform a significance test about a population mean.
- use a confidence interval to make a conclusion for a two-sided test about a population parameter.
- interpret the power of a significance test and describe what factors affect a power of a test.
- describe the shape, center, and variability of the sampling distribution for a difference of two sample proportions.
- determine whether the conditions are met for doing inference about a difference between two proportions.
- construct and interpret a confidence interval for a difference between two proportions.
- calculate the standardized test statistic and *p*-value for a test about a difference between two proportions.
- perform a significance test about a difference between two proportions.
- describe the shape, center, and variability of the sampling distribution of the difference between two sample means.
- determine whether the conditions are met for doing inference about a difference between two means.
- construct and interpret a confidence interval for a difference between two means.
- calculate the standardized test statistic and *p*-value for a test about a difference between two means.
- perform a significance test for the difference between two means.
- analyze the distribution of differences in a paired data set using graphs and summary statistics.
- construct and interpret a confidence interval for a mean difference.
- perform a significance test about a mean difference.

- determine when it is appropriate to use paired *t* procedures versus two-sample *t* procedures.
- state appropriate hypotheses and compute the expected counts and chi-square statistic for a Chi-Square test for goodness of fit.
- calculate the chi-square statistic, degrees of freedom, and *p*-value for a Chi-Square test for goodness of fit.
- state and check the Random, 10%, and Large Counts conditions for performing a chi-square test for goodness of fit.
- perform a chi-square test for goodness of fit.
- conduct a follow-up analysis when the results of a chi-square test are statistically significant.
- state appropriate hypotheses and compute the expected counts and chi-square test statistic for a chi-square test based on data in a two-way table.
- state and check the Random, 10%, and Large Counts conditions for a chi-square test based on data in a two-way table.
- perform a chi-square test for homogeneity.
- perform a chi-square test for independence.
- choose the appropriate chi-square test in a given setting.
- check the conditions for performing inference about the slope (beta-1) of the population (true) regression line.
- interpret the values of the *y*-intercept (b_0), slope (b_1), *s*, and *SE* (of b_1) in context, and determine these values from computer output.
- construct and interpret a confidence interval for the slope (beta-1) of the population (true) regression line.
- use transformations involving powers and roots to find a power model that describes the relationship between two variables, and use the model to make predictions.
- use transformations involving logarithms to find a power model that describes the relationship between two quantitative variables, and use the model to make predictions.
- determine which of several transformations is better in producing linear data.
- use Microsoft Excel effectively to manage data.
- use Microsoft Excel effectively to analyze data.
- use Minitab effectively to manage data.
- use Minitab effectively to analyze data.
- conduct original research.
- find and interpret academic literature.
- summarize academic literature.
- read dense academic literature.
- write reviews of relevant literature.
- write concise introductions.
- write cohesive reviews of literature.
- write detailed methodology sections.
- write objective, dispassionate results sections.
- write objective, dispassionate limitations sections.
- write easy-to-read and insightful discussion sections.
- write an academic paper that is well organized.
- develop a set of research questions.

- devise a data collection procedure.
- select appropriate statistical analyses.
- summarize, report, and interpret findings from analyses.
- communicate research findings to those unfamiliar with the statistical discipline.
- communicate research findings in oral presentation and in writing.
- work effectively with others.

COURSE SYLLABUS

Course Name

Advanced Placement Statistics

Level

Advanced Placement

Prerequisites

Grade of B or better in ACP Algebra II or successful completion of H Algebra II.

Materials Required

TI-84 Plus graphing calculator

General Description of the Course

Advanced Placement Statistics is a class for any student who wants to learn how to make sense of the world around him/her through the use of authentic data. Students will be introduced to the major concepts and tools used in collecting, analyzing, and drawing conclusions from data. Students will be exposed to four broad conceptual themes: (a) Exploring data: Describing patterns and departures from patterns; (b) Sampling and Experimentation: Planning and conducting a study; (c) Anticipating Patterns: Exploring random phenomena using probability and simulation; (d) Statistical Inference: Estimating population parameters and testing hypotheses. The content of this course requires students to use high-level problem solving skills to analyze, describe, and make conclusions about data. AP Statistics is an excellent option for all students meeting the prerequisites, regardless of their intended college major.

Assured Assessments

Formative Assessments:

Formative assessments can include, but are not limited to:

- Classroom discourse (Units 1, 2, 3)
- Self-reflections of progress (Units 1, 2, 3)
- Checked homework (Units 1, 2, 3)
- Checkpoint quizzes (Units 1, 2, 3)
- Quizlets (small homework quizzes) (Units 1, 2, 3)
- Group quizzes (Units 1, 2, 3)
- Teacher's observation of progress (Units 1, 2, 3)
- Reading and writing assignments developing the ability to engage with scientific literature (Units 1, 2, 3)

Summative Assessments:

- Multiple-choice and free-response quizzes on each textbook chapter (Units 1, 2, 3)
- Summative test on each textbook chapter (Units 1, 2, 3)
- Original research projects (Units 1, 2, 3)

Core Text

- Starnes, Daren E., and Josh Tabor. *The Practice of Statistics*. 6th ed. New York: Bedford, Freeman, & Worth, 2018. Print.

UNIT 1
Summarization and Collection of Data

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2019 College Board Curriculum Framework for Advanced Placement Statistics.

VAR-1.A	Identify questions to be answered, based on variation in one-variable data.
VAR-1.B	Identify variables in a set of data.
VAR-1.C	Classify types of variables.
UNC-1.A	Represent categorical data using frequency or relative frequency tables.
UNC-1.B	Describe categorical data represented in frequency or relative tables.
UNC-1.C	Represent categorical data graphically.
UNC-1.D	Describe categorical data represented graphically.
UNC-1.E	Compare multiple sets of categorical data.
UNC-1.F	Classify types of quantitative variables.
UNC-1.G	Represent quantitative data graphically.
UNC-1.H	Describe the characteristics of quantitative data distributions.
UNC-1.I	Calculate measures of center and position for quantitative data.
UNC-1.J	Calculate measures of variability for quantitative data.
UNC-1.K	Explain the selection of a particular measure of center and/or variability for describing a set of quantitative data.
UNC-1.L	Represent summary statistics for quantitative data graphically.
UNC-1.M	Describe summary statistics of quantitative data represented graphically.
UNC-1.N	Compare graphical representations for multiple sets of quantitative data.
UNC-1.O	Compare summary statistics for multiple sets of quantitative data.
VAR-2.A	Compare a data distribution to the normal distribution model.

VAR-2.B	Determine proportions and percentiles from a normal distribution.
VAR-2.C	Compare measures of relative position in data sets.
VAR-1.D	Identify questions to be answered about possible relationships in data.
UNC-1.P	Compare numerical and graphical representations for two categorical variables.
UNC-1.Q	Calculate statistics for two categorical variables.
UNC-1.R	Compare statistics for two categorical variables.
UNC-1.S	Represent bivariate quantitative data using scatterplots.
DAT-1.A	Describe the characteristics of a scatter plot.
DAT-1.B	Determine the correlation for a linear relationship.
DAT-1.C	Interpret the correlation for a linear relationship.
DAT-1.D	Calculate a predicted response value using a linear regression model.
DAT-1.E	Represent differences between measured and predicted responses using residual plots.
DAT-1.F	Describe the form of association of bivariate data using residual plots.
DAT-1.G	Estimate parameters for the least-squares regression line model.
DAT-1.H	Interpret coefficients for the least-squares regression line model.
DAT-1.I	Identify influential points in regression.
DAT-1.J	Calculate a predicted response using a least-squares regression line for a transformed data set.
VAR-1.E	Identify questions to be answered about data collection methods.
DAT-2.A	Identify the type of a study.
DAT-2.B	Identify appropriate generalizations and determinations based on observational studies.
DAT-2.C	Identify a sampling method, given a description of a study.
DAT-2.D	Explain why a particular sampling method is or is not appropriate for a given situation.
DAT-2.E	Identify potential sources of bias in sampling methods.

VAR-3.A	Identify the components of an experiment.
VAR-3.B	Describe elements of a well-designed experiment.
VAR-3.C	Compare experimental designs and methods.
VAR-3.D	Explain why a particular experimental design is appropriate.
VAR-3.E	Interpret the results of a well-designed experiment.
VAR-1.F	Identify questions suggested by patterns in data.

The following Unit Goals align with the 2010 Connecticut Core Standards.

CCS.ELA-Literacy.RST.11-12.1	Cite specific textual evidence to support analysis of academic research articles that relate to the topic under analysis, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.
CCS.ELA-Literacy.RST.11-12.2	Determine the central ideas or conclusions of formal and informal sources; summarize complex concepts, processes, or information presented in a text by accurately paraphrasing them in a concise, understandable manner that is free of plagiarism of any kind.
CCS.ELA-Literacy.RST.11-12.5	Analyze how academic research articles are structured, including information or ideas that are organized into categories or hierarchies.
CCS.ELA-Literacy.RST.11-12.6	Analyze the author's purpose in providing the claims of an academic research article, and whether or not the methodology and findings validate those claims.
CCS.ELA-Literacy.WHST.11-12.1	Write an argument based on information from a range of sources contributing to an overall position.
CCS.ELA-Literacy.WHST.11-12.1.d	Establish and maintain a formal style (particularly based on APA style guidelines) and objective tone while attending to the norms and conventions of the discipline of statistics.
CCS.ELA-Literacy.WHST.11-12.7	Conduct a short research project focused on data collection and summarization.
CCS.ELA-Literacy.WHST.11-12.8	Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively

	to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation to write a formal review of literature for a short research project.
CCS.ELA-Literacy.WHST.11-12.9	Effectively write a review of literature by drawing evidence from informational texts to support analysis, reflection, and research.

Unit Essential Questions

- What does it mean to do research?
- How do we summarize data?
- How do we represent data numerically, graphically, and with language?
- How do we effectively communicate patterns in data to an audience who has limited knowledge of statistics?
- How can we be creative in the way we represent patterns in data?
- What constitutes a pattern in data?
- What does it mean to deviate from patterns in data?
- How do we analyze univariate data?
- How do we analyze bivariate data?
- What role do units play in data summarization?
- How do we compare two sets of data?
- How do we measure the strength of patterns and relationships in data?
- When is a pattern in data noteworthy?
- How do researchers collect data?
- What is bias?
- How does bias influence the collection of data?
- How does bias influence the results of a research study?
- Are there methods of data collection that are inherently biased?
- What are research ethics?
- Why have Institutional Review Boards and other ethical conduct committees been established?
- Why is it important to protect those who are being studied?
- What are the ethics involved in data collection?
- Which types of data collection are acceptable and which types are not?
- Why is the methodology section of a research study so important?
- What are the various types of studies?
- What are the key differences between an observational study and an experiment?
- When would using a census versus a sample be more appropriate?
- Why is randomization so important in sampling?
- What is the placebo effect?
- What does it take to establish causation?

- What is the difference between correlation and causation?
- When are the findings of a study important?
- When should we generalize the findings of a study to a larger population?
- What does it mean to express findings in an objective and dispassionate manner?
- What is the style of academic research?
- How do we effectively read academic research?
- How do we summarize academic research?
- How do we write reviews of literature?
- How do we write research proposals?
- How do we conform to an academic style, such as APA?
- How do we find appropriate research articles?
- Which sources are trustworthy and which ones are not?

Scope and Sequence

- Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot)
  - Center and spread
  - Clusters and gaps
  - Outliers and other unusual features
  - Shape
- Summarizing distributions of univariate data
  - Measuring center: median, mean
  - Measuring spread: range, interquartile range, standard deviation
  - Measuring position: quartiles, percentiles, standardized scores (z-scores)
  - Using boxplots
  - The effect of changing units on summary measures
- Comparing distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots)
  - Comparing center and spread: within-group variation, between-group variation
  - Comparing clusters and gaps
  - Comparing outliers and other unusual features
  - Comparing shapes
- Exploring bivariate data
  - Analyzing patterns in scatterplots
  - Correlation and linearity
  - Least-squares regression line
  - Residual plots, outliers, and influential points
- Exploring categorical data
  - Frequency tables and bar charts
  - Marginal and joint frequencies for two-way tables
  - Conditional relative frequencies and association
  - Comparing distributions using bar charts

- Overview of methods of data collection
  - Census
  - Sample survey
  - Experiment
  - Observational study
- Planning and conducting surveys
  - Characteristics of a well-designed and well-conducted survey
  - Populations, samples, and random selection
  - Sources of bias in sampling and surveys
  - Sampling methods, including simple random sampling, stratified random sampling, and cluster sampling
- Planning and conducting experiments
  - Characteristics of a well-designed and well-conducted experiment
  - Treatments, control groups, experimental units, random assignments, and replication
  - Sources of bias and confounding, including placebo effect and blinding
  - Completely randomized design
  - Randomized block design, including matched pairs design
- Generalizability of results and types of conclusions that can be drawn from observational studies, experiments, and surveys

Assured Assessments

Formative Assessments:

- Students will participate in classroom discourse, self-reflections of progress, checked homework, checkpoint quizzes, quizlets (small homework quizzes), group quizzes, and the teacher's observation of progress.
- Students will also be given reading and writing assignments aimed at developing their ability to engage with scientific literature. They will write their first reviews of literature during this unit. Each of these assessments will count as either a homework or a quiz grade.

Summative Assessments:

- Students will take quizzes throughout the unit, with multiple-choice and free-response quizzes given on each of the four textbook chapters.
- Students will also take a summative test at the end of each textbook chapter to assess content knowledge.
- Students will also work individually or in a group (of up to five students) on an original research project. This unit's research project will focus on writing an effective review of literature, collecting data in a well-defined and ethical manner, and effectively summarizing the collected data numerically, graphically, and through language. The research project write-up will be in the style of an academic research paper, and will count as a test grade. They will be required to write and submit a project proposal consisting of a review of literature, proposed research questions, and a methodological plan; this assignment will count as a quiz grade.

Resources

Core

- Starnes, Daren E., and Josh Tabor. *The Practice of Statistics*. 6th ed. New York: Bedford, Freeman, & Worth, 2018. Print. Chps. 1-4.
- Sternstein, Martin. *Barron's AP Statistics*. 10th ed. New York: Barron's, 2019. Print.
- Minitab software
- TI-84 emulator software

Supplemental

- American Statistical Association. *This is Statistics*. https://thisisstatistics.org/. Web.
- Huff, Darrell. *How to Lie with Statistics*. New York: Norton, 1954. Print.
- Keshav, S. "How to Read a Paper." *ACM SIGCOMM Computer Communication Review* 37.3 (July 2007): 83-84. http://ccr.sigcomm.org/online/files/p83-keshavA.pdf. Web.
- Klingenberg, Bernhard. "Statistics: The Art & Science of Learning from Data." http://www.artofstat.com/. 2017. Web.
- LOCUS Project. "LOCUS: Levels of Conceptual Understanding in Statistics." https://locus.statisticseducation.org/. 2019. Web.
- Molesky, Jason. "StatsMonkey." http://www.apstatsmonkey.com/StatsMonkey/Statsmonkey.html. 2012. Web.
- National Science Foundation. "STATS4STEM." https://www.stats4stem.org/. Web.
- O'Neil, Cathy. *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*. New York: Penguin, 2016. Print.
- Purugganan, Mary, and Jan Hewitt. "How to Read a Scientific Article." *Cain Project for Engineering and Professional Communication*. 2004. http://www.owlnet.rice.edu/~cainproj/courses/HowToReadSciArticle.pdf. Web.
- "Rossman/Chance Applet Collection." http://www.rossmanchance.com/applets/. Web.
- "Stapplets." https://www.stapplet.com/. Web.
- "StatCrunch." https://www.statcrunch.com/. Web.
- "StatKey." http://www.lock5stat.com/StatKey/. Web.
- TheStatsMedic. "Stats Medic." https://www.statsmedic.com/. 2019. Web.
- Subramanyam, R.V. "Art of Reading a Journal Article: Methodically and Effectively." *Journal of Oral and Maxillofacial Pathology* 17.1 (January – April 2013): 65-70. https://www.researchgate.net/publication/236612069_Art_of_reading_a_journal_article_Methodically_and_effectively. Web.

Time Allotment

- Approximately 7 weeks

UNIT 2
Probability, Estimation, and the Mathematical Foundations of Inference

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2019 College Board Curriculum Framework for Advanced Placement Statistics.

UNC-2.A	Estimate probabilities using simulation.
VAR-4.A	Calculate probabilities for events and their complements.
VAR-4.B	Interpret probabilities for events.
VAR-4.C	Explain why two events are (or are not) mutually exclusive.
VAR-4.D	Calculate conditional probabilities.
VAR-4.E	Calculate probabilities for independent events and for the union of two events.
VAR-5.A	Represent the probability distribution for a discrete random variable.
VAR-5.B	Interpret a probability distribution.
VAR-5.C	Calculate parameters for a discrete random variable.
VAR-5.D	Interpret parameters for a discrete random variable.
VAR-5.E	Calculate parameters for linear combinations of random variables.
VAR-5.F	Describe the effects of linear transformations of parameters of random variables.
UNC-3.A	Estimate probabilities of binomial random variables using data from a simulation.
UNC-3.B	Calculate probabilities for a binomial distribution.
UNC-3.C	Calculate parameters for a binomial distribution.
UNC-3.D	Interpret probabilities and parameters for a binomial distribution.
UNC-3.E	Calculate probabilities for geometric random variables.
UNC-3.F	Calculate parameters of a geometric distribution.
UNC-3.G	Interpret probabilities and parameters for a geometric distribution.

VAR-1.G	Identify questions suggested by variation in statistics for samples collected from the same population.
VAR-6.A	Calculate the probability that a particular value lies in a given interval of a normal distribution.
VAR-6.B	Determine the interval associated with a given area in a normal distribution.
VAR-6.C	Determine the appropriateness of using the normal distribution to approximate probabilities for unknown distributions.
UNC-3.H	Estimate sampling distributions using simulation.
UNC-3.I	Explain why an estimator is or is not unbiased.
UNC-3.J	Calculate estimates for a population parameter.
UNC-3.K	Determine parameters of a sampling distribution for sample proportions.
UNC-3.L	Determine whether a sampling distribution for a sample proportion can be described as approximately normal.
UNC-3.M	Interpret probabilities and parameters for a sampling distribution for a sample proportion.
UNC-3.N	Determine parameters of a sampling distribution for a difference in sample proportions.
UNC-3.O	Determine whether a sampling distribution for a difference of sample proportions can be described as approximately normal.
UNC-3.P	Interpret probabilities and parameters for a sampling distribution for a difference in proportions.
UNC-3.Q	Determine parameters for a sampling distribution for sample means.
UNC-3.R	Determine whether a sampling distribution of a sample mean can be described as approximately normal.
UNC-3.S	Interpret probabilities and parameters for a sampling distribution for a sample mean.
UNC-3.T	Determine parameters of a sampling distribution for a difference in sample means.
UNC-3.U	Determine whether a sampling distribution of a difference in sample means can be described as approximately normal.
UNC-3.V	Interpret probabilities and parameters for a sampling distribution for a difference in sample means.

VAR-1.H	Identify questions suggested by variation in the shapes of distributions of samples taken from the same population.
UNC-4.A	Identify an appropriate confidence interval procedure for a population proportion.
UNC-4.B	Verify the conditions for calculating confidence intervals for a population proportion.
UNC-4.C	Determine the margin of error for a given sample size and an estimate for the sample size that will result in a given margin of error for a population proportion.
UNC-4.D	Calculate an appropriate confidence interval for a population proportion.
UNC-4.E	Calculate an interval estimate based on a confidence interval for a population proportion.
UNC-4.F	Interpret a confidence interval for a population proportion.
UNC-4.G	Justify a claim based on a confidence interval for a population proportion.
UNC-4.H	Identify the relationships between sample size, width of a confidence interval, confidence level, and margin of error for a population proportion.

The following Unit Goals align with the 2010 Connecticut Core Standards.

CCS.ELA-Literacy.RST.11-12.3	Follow precisely a complex multistep procedure when conducting a research study.
CCS.ELA-Literacy.WHST.11-12.1	Write arguments focused on the results of statistical procedures conducted on original data.
CCS.ELA-Literacy.WHST.11-12.1.a	Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence.
CCS.ELA-Literacy.WHST.11-12.1.b	Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases.
CCS.ELA-Literacy.WHST.11-12.1.c	Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.

CCS.ELA-Literacy.WHST.11-12.1.d	Establish and maintain a formal style (particularly based on APA style guidelines) and objective tone while attending to the norms and conventions of statistics in the writing of a final research project.
CCS.ELA-Literacy.WHST.11-12.1.e	Provide a concluding discussion section that highlights "what the researchers see in the results."
CCS.ELA-Literacy.WHST.11-12.2	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.
CCS.ELA-Literacy.WHST.11-12.2.a	Introduce the research topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole.
CCS.ELA-Literacy.WHST.11-12.2.b	Develop the research topic thoroughly by selecting the most significant and relevant findings from the study.
CCS.ELA-Literacy.WHST.11-12.2.c	Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify relationships among complex ideas and concepts, with each section anticipating the next.
CCS.ELA-Literacy.WHST.11-12.2.d	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.
CCS.ELA-Literacy.WHST.11-12.5	Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience; revise reviews of literature, research proposals, etc., to meet the needs of the final product.

Unit Essential Questions

- What is probability?
- What does it mean to anticipate patterns?
- How do we anticipate patterns in data?
- What mathematical definitions, concepts, etc., underlie probability?
- What are the properties of probability?
- How can we represent variation and expectation in terms of probability?
- How does probability underlie statistical procedures?
- How does probability play a role in data collection?
- How does probability play a role in data analysis?

- How does probability play a role in data management?
- What is the binomial distribution?
- What are binomial coefficients?
- How are binomial coefficients related to the binomial distribution?
- Why is the study of the binomial distribution so important to mathematics and statistics?
- What is the geometric distribution?
- How do the binomial and geometric distributions relate?
- What is a sampling distribution?
- What are the mathematical foundations of sampling distributions?
- Why is the study of sampling distributions so foundational to statistical procedures?
- How does the abstract nature of a sampling distribution relate to statistical procedures in practice?
- What is the normal distribution?
- Why is the normal distribution so important to the study of statistics?
- What are the mathematical origins of the normal distribution?
- Why does the normal distribution underlie so many statistical procedures?
- How can the properties of the normal distribution be leveraged to make the most of statistical procedures?
- How do the sampling distributions for means and proportions differ?
- What is the Central Limit Theorem and to what situations does it apply?
- Why is the Central Limit Theorem so important to the study of statistics?
- What are the mathematical foundations of the Central Limit Theorem?
- What is the Chi-Square distribution?
- What is the *z*-distribution?
- What is the normal distribution?
- What are standardized measures?
- How do the Chi-Square, *z*-, and *t*-distributions relate?
- How is probability related to sampling distributions?
- What is estimation?
- Why is point estimation so important to the study of statistics?
- Why do most statistical procedures consist of point estimation?
- Why is it of interest to researchers to estimate parameters?
- What is the relationship between the estimation of a parameter and sample statistics?
- How do probability and sampling distributions relate to the estimation of population parameters?
- What is a confidence interval?
- How do we test claims based on confidence intervals?
- What are confidence levels, and how do they relate to the overall procedure of constructing a confidence interval?
- How does error relate to estimation?
- How does bias and error relate to the construction of confidence intervals?

- How do confidence intervals for proportions differ from those of means?
- How do confidence intervals for differences in parameters differ from those of single-point estimates?
- What is the difference between an estimate and an estimator?
- What makes an estimator unbiased?

Scope and Sequence

- Probability
  - Interpreting probability, including long-run relative frequency interpretation
  - "Law of Large Numbers" concept
  - Addition rule, multiplication rule, conditional probability, and independence
  - Discrete random variables and their probability distributions, including binomial and geometric
  - Simulation of random behavior and probability distributions
  - Mean (expected value) and standard deviation of a random variable, and linear transformation of a random variable
- Combining independent random variables
  - Notion of independence versus dependence
  - Mean and standard deviation for sums and differences of independent random variables
- The normal distribution
  - Properties of the normal distribution
  - Using tables of the normal distribution
  - The normal distribution as a model for measurements
- Sampling distributions
  - Sampling distribution of a sample proportion
  - Sampling distribution of a sample mean
  - Central Limit Theorem
  - Sampling distribution of a difference between two independent sample proportions
  - Sampling distribution of a difference between two independent sample means
  - Simulation of sample distributions
  - *t*-distribution
  - Chi-square distribution
- Estimation (point estimators and confidence intervals)
  - Estimating population parameters and margins of error
  - Properties of point estimators, including unbiasedness and variability
  - Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals
  - Large sample confidence interval for a proportion
  - Large sample confidence interval for a difference between two proportions
  - Confidence interval for a mean
  - Confidence interval for a difference between two means (unpaired and paired)

- Confidence interval for the slope of a least-squares regression line

Assured Assessments

Formative Assessments:

- Students will participate in classroom discourse, self-reflections of progress, checked homework, checkpoint quizzes, quizlets (small homework quizzes), group quizzes, and the teacher's observation of progress.
- Students will also be given reading and writing assignments aimed at developing their ability to engage with scientific literature. Each of these assessments will count as either a homework or a quiz grade.

Summative Assessments:

- Students will take quizzes throughout the unit, with multiple-choice and free-response quizzes given on each of the four textbook chapters.
- Students will also take a summative test at the end of each textbook chapter to assess content knowledge.
- Students will also work individually or in a group (of up to five students) on an original research project. This unit's research project will focus on developing a set of research questions centered on the estimation of a population parameter, devising a data collection procedure to estimate the parameter, and effectively writing the findings in an academic manner. The research project write-up will be in the style of an academic research paper, and will count as a test grade. They will be required to write and submit a project proposal consisting of a review of literature, proposed research questions, and a methodological plan; this assignment will count as a quiz grade.

Resources

Core

- Starnes, Daren E., and Josh Tabor. *The Practice of Statistics*. 6th ed. New York: Bedford, Freeman, & Worth, 2018. Print. Chps. 5-8.
- Sternstein, Martin. *Barron's AP Statistics*. 10th ed. New York: Barron's, 2019. Print.
- Minitab software
- TI-84 emulator software

Supplemental

- American Statistical Association. *This is Statistics*. https://thisisstatistics.org/. Web.
- Huff, Darrell. *How to Lie with Statistics*. New York: Norton, 1954. Print.
- Keshav, S. "How to Read a Paper." *ACM SIGCOMM Computer Communication Review* 37.3 (July 2007): 83-84. http://ccr.sigcomm.org/online/files/p83-keshavA.pdf. Web.
- Klingenberg, Bernhard. "Statistics: The Art & Science of Learning from Data." http://www.artofstat.com/. 2017. Web.
- LOCUS Project. "LOCUS: Levels of Conceptual Understanding in Statistics." https://locus.statisticseducation.org/. 2019. Web.
- Molesky, Jason. "StatsMonkey." http://www.apstatsmonkey.com/StatsMonkey/Statsmonkey.html. 2012. Web.
- National Science Foundation. "STATS4STEM." https://www.stats4stem.org/. Web.

- O'Neil, Cathy. *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*. New York: Penguin, 2016. Print.
- Purugganan, Mary, and Jan Hewitt. "How to Read a Scientific Article." *Cain Project for Engineering and Professional Communication*. 2004. http://www.owlnet.rice.edu/~cainproj/courses/HowToReadSciArticle.pdf. Web.
- "Rossman/Chance Applet Collection." http://www.rossmanchance.com/applets/. Web.
- "Stapplets." https://www.stapplet.com/. Web.
- "StatCrunch." https://www.statcrunch.com/. Web.
- "StatKey." http://www.lock5stat.com/StatKey/. Web.
- TheStatsMedic. "Stats Medic." https://www.statsmedic.com/. 2019. Web.
- Subramanyam, R.V. "Art of Reading a Journal Article: Methodically and Effectively." *Journal of Oral and Maxillofacial Pathology* 17.1 (January – April 2013): 65-70. https://www.researchgate.net/publication/236612069_Art_of_reading_a_journal_article_Methodically_and_effectively. Web.

Time Allotment

- Approximately 10 weeks

UNIT 3
Statistical Inference

Unit Goals

At the completion of this unit, students will:

The following Unit Goals align with the 2019 College Board Curriculum Framework for Advanced Placement Statistics.

VAR-6.D	Identify the null and alternative hypotheses for a population proportion.
VAR-6.E	Identify an appropriate testing method for a population proportion.
VAR-6.F	Verify the conditions for making statistical inferences when testing a population proportion.
VAR-6.G	Calculate an appropriate test statistic and *p*-value for a population proportion.
DAT-3.A	Interpret the *p*-value of a significance test for a population proportion.
DAT-3.B	Justify a claim about the population based on the results of a significance test for a population proportion.
UNC-5.A	Identify Type I and Type II errors.
UNC-5.B	Calculate the probability of a Type I and Type II errors.
UNC-5.C	Identify factors that affect the probability of errors in significance testing.
UNC-5.D	Interpret Type I and Type II errors.
UNC-4.I	Identify an appropriate confidence interval procedure for a comparison of population proportions.
UNC-4.J	Verify the conditions for calculating confidence intervals for a difference between population proportions.
UNC-4.K	Calculate an appropriate confidence interval for a comparison of population proportions.
UNC-4.L	Calculate an interval estimate based on a confidence interval for a difference of proportions.
UNC-4.M	Interpret a confidence interval for a difference of proportions.
UNC-4.N	Justify a claim based on a confidence interval for a difference of proportions.

VAR-6.H Identify the null and alternative hypotheses for a difference of two population proportions.

VAR-6.I Identify an appropriate testing method for the difference of two population proportions.

VAR-6.J Verify the conditions for making statistical inferences when testing a difference of two population proportions.

VAR-6.K Calculate an appropriate test statistic for the difference of two population proportions.

DAT-3.C Interpret the *p*-value of a significance test for a difference of population proportions.

DAT-3.D Justify a claim about the population based on the results of a significance test for a difference of population proportions.

VAR-1.I Identify questions suggested by probabilities of errors in statistical inference.

VAR-7.A Describe *t*-distributions.

UNC-4.O Identify an appropriate confidence interval procedure for a population mean, including the mean difference between values in matched pairs.

UNC-4.P Verify the conditions for calculating confidence intervals for a population mean, including the mean difference between values in matched pairs.

UNC-4.Q Determine the margin of error for a given sample size for a one-sample *t*-interval.

UNC-4.R Calculate an appropriate confidence interval for a population mean, including the mean difference between values in matched pairs.

UNC-4.S Interpret a confidence interval for a population mean, including the mean difference between values in matched pairs.

UNC-4.T Justify a claim based on a confidence interval for a population mean, including the mean difference between values in matched pairs.

UNC-4.U Identify the relationships between sample size, width of a confidence interval, confidence level, and margin of error for a population mean.

VAR-7.B Identify an appropriate testing method for a population mean with unknown σ, including the mean difference between values in matched pairs.

VAR-7.C Identify the null and alternative hypotheses for a population mean with unknown σ, including the mean difference between values in matched pairs.

VAR-7.D	Verify the conditions for the test for a population mean, including the mean difference between values in matched pairs.
VAR-7.E	Calculate an appropriate test statistic for a population mean, including the mean difference between values in matched pairs.
DAT-3.E	Interpret the *p*-value of a significance test for a population mean, including the mean difference between values in matched pairs.
DAT-3.F	Justify a claim about the population based on the results of a significance test for a population mean.
UNC-4.V	Identify an appropriate confidence interval procedure for a difference of two population means.
UNC-4.W	Verify the conditions to calculate confidence intervals for the difference of two population means.
UNC-4.X	Determine the margin of error for the difference of two population means.
UNC-4.Y	Calculate an appropriate confidence interval for a difference of two population means.
UNC-4.Z	Interpret a confidence interval for a difference of population means.
UNC-4.AA	Justify a claim based on a confidence interval for a difference of population means.
UNC-4.AB	Identify the effects of sample size on the width of a confidence interval for the difference of two means.
VAR-7.F	Identify an appropriate selection of a testing method for a difference of two population means.
VAR-7.G	Identify the null and alternative hypotheses for a difference of two population means.
VAR-7.H	Verify the conditions for the significance test for the difference of two population means.
VAR-7.I	Calculate an appropriate test statistic for a difference of two means.
DAT-3.G	Interpret the *p*-value of a significance test for a difference of population means.
DAT-3.H	Justify a claim about the population based on the results of a significance test for a difference of two population means in context.
VAR-1.J	Identify questions suggested by variation between observed and expected counts in categorical data.

VAR-8.A	Describe chi-square distributions.
VAR-8.B	Identify the null and alternative hypotheses in a test for a distribution of proportions in a set of categorical data.
VAR-8.C	Identify an appropriate testing method for a distribution of proportions in a set of categorical data.
VAR-8.D	Calculate expected counts for the chi-square test for goodness of fit.
VAR-8.E	Verify the conditions for making statistical inferences when testing goodness of fit for a chi-square distribution.
VAR-8.F	Calculate the appropriate statistic for the chi-square test for goodness of fit.
VAR-8.G	Determine the *p*-value for chi-square test for goodness of fit significance test.
DAT-3.I	Interpret the *p*-value for the chi-square test for goodness of fit.
DAT-3.J	Justify a claim about the population based on the results of a chi-square test for goodness of fit.
VAR-8.H	Calculate expected counts for two-way tables of categorical data.
VAR-8.I	Identify the null and alternative hypotheses for a chi-square test for homogeneity or independence.
VAR-8.J	Identify an appropriate testing method for comparing distributions in two-way tables of categorical data.
VAR-8.K	Verify the conditions for making statistical inferences when testing a chi-square distribution for independence or homogeneity.
VAR-8.L	Calculate the appropriate statistic for a chi-square test for homogeneity or independence.
VAR-8.M	Determine the *p*-value for a chi-square significance test for independence or homogeneity.
DAT-3.K	Interpret the *p*-value for the chi-square test for homogeneity or independence.
DAT-3.L	Justify a claim about the population based on the results of a chi-square test for homogeneity or independence.
VAR-1.K	Identify questions suggested by variation in scatter plots.
UNC-4.AC	Identify an appropriate confidence interval procedure for a slope of a regression model.

UNC-4.AD	Verify the conditions to calculate confidence intervals for the slope of a regression model.
UNC-4.AE	Determine the given margin of error for the slope of a regression model.
UNC-4.AF	Calculate an appropriate confidence interval for the slope of a regression model.
UNC-4.AG	Interpret a confidence interval for the slope of a regression model.
UNC-4.AH	Justify a claim based on a confidence interval for the slope of a regression model.
UNC-4.AI	Identify the effects of sample size on the width of a confidence interval for the slope of a regression model.
VAR-7.J	Identify the appropriate selection of a testing method for a slope of a regression model.
VAR-7.K	Identify appropriate null and alternative hypotheses for a slope of a regression model.
VAR-7.L	Verify the conditions for the significance test for the slope of a regression model.
VAR-7.M	Calculate an appropriate test statistic for the slope of a regression model.
DAT-3.M	Interpret the *p*-value of a significance test for the slope of a regression model.
DAT-3.N	Justify a claim about the population based on the results of a significance test for the slope of a regression model.

The following Unit Goals align with the 2010 Connecticut Core Standards.

CCS.ELA-Literacy.RST.11-12.10	By the end of the course, read and comprehend statistical/technical texts in the grades 11-CCR text complexity band independently and proficiently.
CCS.ELA-Literacy.WHST.11-12.1	Write arguments based on predetermined research questions and corresponding analyses.
CCS.ELA-Literacy.WHST.11-12.1.a	Introduce precise, knowledgeable research questions, establish the significance of the research findings, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence.
CCS.ELA-Literacy.WHST.11-12.1.b	Develop possible interpretations for findings, acknowledging strengths, limitations, possible biases, and generalizability of the findings.
CCS.ELA-Literacy.WHST.11-12.1.d	Establish and maintain a formal style (particularly based on APA style guidelines) as well as an objective and dispassionate reporting of findings.

CCS.ELA-Literacy.WHST.11-12.1.e	Provide a concluding discussion section that highlights "what the researchers see in the findings."
CCS.ELA-Literacy.WHST.11-12.2	Write a detailed methodology section for an original research project.
CCS.ELA-Literacy.WHST.11-12.2.a	Introduce the research topic and organize complex ideas, concepts, and information in an easy-to-read and sequential manner so that each section (introduction, review of literature, methodology, review, and discussion) builds on that which precedes it to create a unified whole.
CCS.ELA-Literacy.WHST.11-12.2.b	Develop the research paper thoroughly by selecting only the most significant and relevant findings from the given research study.
CCS.ELA-Literacy.WHST.11-12.2.c	Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify relationships among complex ideas and concepts, with each section anticipating the next.
CCS.ELA-Literacy.WHST.11-12.2.d	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.
CCS.ELA-Literacy.WHST.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to the research.
CCS.ELA-Literacy.WHST.11-12.5	Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach throughout the research process.
CCS.ELA-Literacy.WHST.11-12.6	Use technology, including Microsoft Excel and Minitab, to generate raw findings to be interpreted; then use technology such as Google Docs to collaborate in writing with peers and the teacher.
CCS.ELA-Literacy.WHST.11-12.7	Conduct a large, cohesive research study to address a set of research questions, developing the research questions, devising a data collection procedure, carefully selecting procedures for statistical analysis, and effectively interpreting results.
CCS.ELA-Literacy.WHST.11-12.8	Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and

	audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation to write a review of literature.
CCS.ELA-Literacy.WHST.11-12.9	Draw evidence from informational texts to support analysis, reflection, and research.

Unit Essential Questions

- What is statistical inference?
- What are statistical tests?
- What does it mean to test a claim?
- What does it mean to gather statistical evidence?
- What is the difference between significant and statistically significant?
- What is the logic associated with conducting statistical tests?
- How does one select the appropriate statistical test to answer a given research question?
- How does one devise a data collection procedure that produces data conducive to a given statistical test?
- How does one connect research questions, methodology and data collection, data management, statistical tests, and the interpretation of results into one cohesive structure?
- How does probability underlie statistical tests?
- What role do sampling distributions play in statistical inference?
- What role do data collection/sampling procedures play in statistical inference?
- What are the types of errors that can occur when testing statistical hypotheses?
- When is a finding acceptable?
- When is a finding noteworthy?
- In which ways can statistical tests be untrustworthy?
- How is bias introduced into the process of conducting statistical procedures?
- What thresholds of error are allowed in different procedures?
- How is estimation related to statistical inference?
- How are large sample tests different from smaller sample tests?
- How does the organization of data play a role in the types of statistical tests used?
- How do tests for means differ from those for proportions?
- How can one conduct statistical tests for the relationships between two variables?
- How can one transform inherently non-linear data to linear data to perform statistical procedures?
- How does research in fields outside of statistics rely on statistical inference?
- How does statistical inference combine all of the knowledge gained in previous units?
- Why are statistical inference tests so important to research?
- What are ethical procedures for statistical inference?
- What are the conditions that need to be met in order for a statistical test to be appropriate?

- If certain conditions are not met, how does this influence the results?
- Are there ways to be robust against the violation of certain conditions?
- How can one increase the generalizability of a finding?
- How can one combine all of the knowledge gained in the course into a cohesive understanding of statistics?

Scope and Sequence

- Tests of significance
    - Logic of significance testing, null and alternative hypotheses; *p*-values; one-and two-sided tests; concepts of Type I and Type II errors; concept of power
    - Large sample test for a proportion
    - Large sample test for a difference between two proportions
    - Test for a mean
    - Test for a difference between two means (unpaired and paired)
    - Chi-square test for goodness of fit, homogeneity of proportions, and independence (one- and two-way tables)
    - Test for the slop of a least-squares regression line
    - Transformations to achieve linearity: logarithmic and power transformations

Assured Assessments

Formative Assessments:

- Students will participate in classroom discourse, self-reflections of progress, checked homework, checkpoint quizzes, quizlets (small homework quizzes), group quizzes, and the teacher's observation of progress.
- Students will also be given reading and writing assignments aimed at developing their ability to engage with scientific literature. Each of these assessments will count as either a homework or a quiz grade.

Summative Assessments:

- Students will take quizzes throughout the unit, with multiple-choice and free-response quizzes given on each of the four textbook chapters.
- Students will also take a summative test at the end of each textbook chapter to assess content knowledge.
- Students will also work individually or in a group (of up to five students) on two original research projects.
    - One research project, before the College Board Examination, will focus on creating a set of research questions, choosing statistical inference procedures that address each question, devising a data collection procedure that is conducive to the chosen inference procedures, conducting the procedures, and then reporting and interpreting the results in the style of an academic research paper. They will be required to write and submit a project proposal consisting of a review of literature, proposed research questions, and a methodological plan; this assignment will count as a quiz grade.
    - The second research project, after the College Board Examination, will be the most extensive, comprehensive research project of the year; the project and its write-up are expected to be at the level of academic work suitable for publication. For this project,

students will write a proposal consisting of a review of relevant literature on a topic, a set of research questions, methodology, and appropriate statistical procedures. The first draft of the proposal will count as a quiz grade. After feedback on the proposal, students will submit a revised version for a second quiz grade. The final written project will be worth three test grades.

Resources

Core

- Starnes, Daren E., and Josh Tabor. *The Practice of Statistics*. 6th ed. New York: Bedford, Freeman, & Worth, 2018. Print. Chps. 9-12.
- Sternstein, Martin. *Barron's AP Statistics*. 10th ed. New York: Barron's, 2019. Print.
- Minitab software
- TI-84 emulator software

Supplemental

- American Statistical Association. *This is Statistics*. https://thisisstatistics.org/. Web.
- Huff, Darrell. *How to Lie with Statistics*. New York: Norton, 1954. Print.
- Keshav, S. "How to Read a Paper." *ACM SIGCOMM Computer Communication Review* 37.3 (July 2007): 83-84. http://ccr.sigcomm.org/online/files/p83-keshavA.pdf. Web.
- Klingenberg, Bernhard. "Statistics: The Art & Science of Learning from Data." http://www.artofstat.com/. 2017. Web.
- LOCUS Project. "LOCUS: Levels of Conceptual Understanding in Statistics." https://locus.statisticseducation.org/. 2019. Web.
- Molesky, Jason. "StatsMonkey." http://www.apstatsmonkey.com/StatsMonkey/Statsmonkey.html. 2012. Web.
- National Science Foundation. "STATS4STEM." https://www.stats4stem.org/. Web.
- O'Neil, Cathy. *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*. New York: Penguin, 2016. Print.
- Purugganan, Mary, and Jan Hewitt. "How to Read a Scientific Article." *Cain Project for Engineering and Professional Communication*. 2004. http://www.owlnet.rice.edu/~cainproj/courses/HowToReadSciArticle.pdf. Web.
- "Rossman/Chance Applet Collection." http://www.rossmanchance.com/applets/. Web.
- "Stapplets." https://www.stapplet.com/. Web.
- "StatCrunch." https://www.statcrunch.com/. Web.
- "StatKey." http://www.lock5stat.com/StatKey/. Web.
- TheStatsMedic. "Stats Medic." https://www.statsmedic.com/. 2019. Web.
- Subramanyam, R.V. "Art of Reading a Journal Article: Methodically and Effectively." *Journal of Oral and Maxillofacial Pathology* 17.1 (January – April 2013): 65-70. https://www.researchgate.net/publication/236612069_Art_of_reading_a_journal_article_Methodically_and_effectively. Web.

Time Allotment

- Approximately 13 weeks

CULMINATING ACTIVITY

After students take the AP Statistics Examination, the remainder of the year is focused on the completion of a final, comprehensive research project. Over the course of the year, the students completed three extensive scientific research studies and wrote technical reports of their findings in the style of academic literature. Through completing these studies, the students developed their: analytic, interpretive, and communicative skillsets; capacity to do authentic research; deep knowledge of the content; and the connection between their work and the real work of statisticians and data analysts in the field. The final research study is the most extensive, widest in scope, and requires students to combine what they have learned over the course of the entire year. This culminating project requires students to design, implement, and analyze a comprehensive research study.

In their pursuit of answering their own authentic research questions – which they are required to develop after carefully analyzing existing literature in the field of their choice – they will be required to adhere to the ethics expected of researchers in that field. In addition, the final technical report is expected to be at the level of research articles submitted for publication. The final technical report will adhere strictly to the citation style of the American Psychological Association (APA) and will be required to have each of the following sections, which appear in most research articles: Abstract, Introduction, Review of Relevant Literature, Methodology, Results, Discussion, Conclusion, Limitations, Directions for Future Research, and References. In addition to submitting a final technical report, the students will be required to present their findings in the style of a research conference presentation.

Time Allotment

- Approximately 5 weeks

COURSE CREDIT

One credit in Mathematics
One class period daily for a full year

PREREQUISITES

Grade of B or better in ACP Algebra II or successful completion of H Algebra II.

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

Department- and teacher-prepared materials

TI-84 Plus graphing calculators

College Board website including past Advanced Placement Statistics tests

CURRENT REFERENCES

College Board: Advanced Placement Statistics:
https://apcentral.collegeboard.org/courses/ap-statistics

ASSURED STUDENT PERFORMANCE RUBRICS

- Trumbull High School School-Wide Writing Rubric (attached)
- Trumbull High School School-Wide Problem-Solving Rubric (attached)
- Trumbull High School School-Wide Independent Learning and Thinking Rubric (attached)

Trumbull High School School-Wide Writing Rubric

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_______	• Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task	• Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task	• Establishes a purpose • Demonstrates an awareness of audience and task	• Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_______	• Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions	• Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions	• Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions	• Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_______	• Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples	• Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples	• May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples	• Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_______	• Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning	• Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors Most errors do not detract from meaning	• Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning	• Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Trumbull High School School-Wide Problem-Solving Rubric

Category/ Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_______	• Student demonstrates clear understanding of the problem and the complexities of the task	• Student demonstrates sufficient understanding of the problem and most of the complexities of the task	• Student demonstrates some understanding of the problem but requires assistance to complete the task	• Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_______	• Student gathers compelling information from multiple sources including digital, print, and interpersonal	• Student gathers sufficient information from multiple sources including digital, print, and interpersonal	• Student gathers some information from few sources including digital, print, and interpersonal	• Student gathers limited or no information
Reasoning and Strategies X_______	• Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	• Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	• Student demonstrates some critical thinking skills to develop a plan integrating some strategies	• Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_______	• Solution shows deep understanding of the problem and its components • Solution shows extensive use of 21st-century technology skills	• Solution shows sufficient understanding of the problem and its components • Solution shows sufficient use of 21st-century technology skills	• Solution shows some understanding of the problem and its components • Solution shows some use of 21st-century technology skills	• Solution shows limited or no understanding of the problem and its components • Solution shows limited or no use of 21st-century technology skills

Trumbull High School School-Wide
Independent Learning and Thinking Rubric

Category/ Weight	**Exemplary 4**	**Goal 3**	**Working Toward Goal 2**	**Needs Support 1-0**
Proposal X_______	• Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work	• Student demonstrates initiative by generating appropriate questions, creating original projects/work	• Student demonstrates some initiative by generating questions, creating appropriate projects/work	• Student demonstrates limited or no initiative by generating few questions and creating projects/work
Independent Research & Development X_______	• Student is analytical, insightful, and works independently to reach a solution	• Student is analytical, and works productively to reach a solution	• Student reaches a solution with direction	• Student is unable to reach a solution without consistent assistance
Presentation of Final Product X_______	• Presentation shows compelling evidence of an independent learner and thinker • Solution shows deep understanding of the problem and its components • Solution shows extensive and appropriate application of 21st-century skills	• Presentation shows clear evidence of an independent learner and thinker • Solution shows adequate understanding of the problem and its components • Solution shows adequate application of 21st-century skills	• Presentation shows some evidence of an independent learner and thinker • Solution shows some understanding of the problem and its components • Solution shows some application of 21st-century skills	• Presentation shows limited or no evidence of an independent learner and thinker • Solution shows limited or no understanding of the problem and its components • Solution shows limited or no application of 21st-century skills