TRUMBULL PUBLIC SCHOOLS

Trumbull, Connecticut

Calculus

Advanced College-Preparatory
Mathematics Department
Trumbull High School

2016

(Last revision date: 2014)

Curriculum Writing Team

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Calculus
Advanced College-Preparatory
Table of Contents

The Trumbull Board of Education will continue to take Affirmative Action to ensure that no persons are discriminated against in its employment.

CORE VALUES AND BELIEFS

The Trumbull School Community engages in an environment conducive to learning which believes that all students will **read** and **write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION

Advanced College-Preparatory Calculus is designed for seniors who have completed ACP PreCalculus and wish to be introduced to a college calculus course. ACP Calculus reinforces PreCalculus topics and covers topics in a first-year Calculus course. Students taking this class will not be prepared to take an Advanced Placement test in Calculus in May; however, they will be prepared for college-level Calculus. ACP Calculus will prepare students for 21st-century success by incorporating technology and problem-solving while students work independently and with their peers. This course is aligned with the 2010 Connecticut Core Standards for Mathematics. In addition, the course exceeds the scope of the CCS; all units therefore align with TPS-created Performance Standards as well.

ACP Calculus begins with a review of concepts from Algebra I, Geometry, Algebra II, and PreCalculus necessary to the study of Calculus. Calculus topics begin with the study of limits. The derivative is introduced through the study of the tangent line problem, and the limit definition of the derivative is developed. Students become proficient at both explicit and implicit derivatives of polynomial, rational, trigonometric, exponential, and logarithmic functions. Derivative applications are studied through motion, curve fitting, extrema, and related rates. The course continues with anti-differentiation and its applications. Graphing calculators are used throughout the course as a tool for exploration and discovery as well as a problem-solving tool to execute complicated computations and to visualize theoretical concepts.

PHILOSOPHY

Success in mathematics depends upon active involvement in a variety of interrelated experiences. When students participate in stimulating learning opportunities, they can reach their full potential.

The Trumbull Mathematics Program embraces these goals for all students.
The successful mathematician will:

- Acquire the factual knowledge necessary to solve problems
- Gain procedural proficiency in problem solving
- Demonstrate a perceptual understanding of problems posed
- Make meaningful mathematical connections to his or her world
- Solve problems utilizing a variety of strategies
- Utilize technology to improve the quality of the problem-solving process

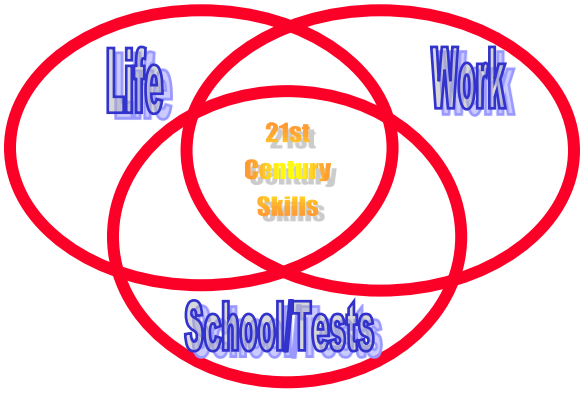
- Communicate effectively using mathematical terminology, both independently and collaboratively
- Use sound mathematical reasoning by utilizing the power of conjecture and proof in his or her thinking
- Become a reflective thinker through continuous self-evaluation
- Become an independent, self-motivated, lifelong learner

The Trumbull Mathematics Program promotes the empowerment of students and encourages students to embrace the skills needed to become successful in the 21st century. Students expand their mathematical abilities by investigating real-world phenomena. Through such experiences, students can access the beauty and power of mathematics and truly appreciate the impact mathematics has on the world in which they live.

Developed by Trumbull K-12 Math Committee, June 2004; revised and approved April 2011

Mathematics instruction must:

- Blend the concrete with the abstract, the practical with the theoretical, and the routine with the non-routine.
- Teach students to search for, find, and represent patterns.
- Instill in students an appreciation for the intrinsic beauty of mathematics.
- Encourge students to reason, analyze, make connections, and self-assess.
- Immerse students in the learning process through questioning, technology, manipulatives, cooperative, and individual activities.



Information, Media And Technology Skills

1. Use real-world digital and other research tools to access, evaluate and effectively apply information appropriate for authentic tasks.

Learning and Innovation Skills

2. Work independently and collaboratively to solve problems and accomplish goals
3. Communicate information clearly and effectively using a variety of tools/media in varied contexts for a variety of purposes.
4. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits, and working/learning conditions.
5. Effectively apply the analysis, synthesis, and evaluative processes that enable productive problem solving.

Life and Career Skills

6. Value and demonstrate personal responsibility, character, cultural understanding, and ethical behavior.

COURSE DESCRIPTION

ACP Calculus begins with an in-depth look at rational and polynomial functions, rational inequalities, trigonometry, and various other types of functions. The course continues with the study of limits. The derivative is introduced through the study of the tangent line problem. Students become proficient at both explicit and implicit derivatives of polynomial, rational, and trigonometric functions. Derivative applications are studied through motion, extrema, and related rates. The course continues with anti-differentiation. A graphing calculator is required for this course.

COURSE GOALS

The Standards for Mathematical Practice describe varieties of expertise that all teachers of mathematics will develop in their students.

These practices rest on important "processes and proficiencies" that have long been valued in mathematics education.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to

compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning.

They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.

They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

COURSE ENDURING UNDERSTANDINGS

Students will understand that . . .

- calculus is the study of change.
- calculus can be used to extend our mathematical boundaries.
- formal definitions and graphical interpretations can help us understand limits and continuity.
- formal definitions, applications, and properties of derivatives and integrals are essential to the study of calculus.

COURSE ESSENTIAL QUESTIONS

- What is calculus, and what is its importance?
- What is a limit, and how can it be interpreted?
- What is a derivative, and how can it be applied?
- What is an integral, and how can it be applied?

COURSE KNOWLEDGE & SKILLS

Students will understand . . .

- the definition of the limit of a function.
- how continuity and differentiability can be determined.
- average rate of change.
- instantaneous rate of change.
- the derivative.
- the anti-derivative.
- the Fundamental Theorem of Calculus.

Students will be able to . . .

- determine the limit of a function graphically and algebraically.
- sketch functions using Calculus.
- use implicit differentiation to solve related rates problems.
- use extrema to solve optimization problems.
- use integration to calculate the area underneath a curve and the area between curves.

COURSE SYLLABUS

Course Name

Calculus

Level

Advanced College-Preparatory

Prerequisites

Completion of ACP PreCalculus with a C or better and teacher recommendation, or successful completion of Honors PreCalculus.

Materials Required

TI-84 graphing calculator

General Description of the Course

ACP Calculus begins with an in-depth look at rational and polynomial functions, rational inequalities, trigonometry, and various other types of functions. The course continues with the study of limits. The derivative is introduced through the study of the tangent line problem. Students become proficient at both explicit and implicit derivatives of polynomial, rational, and trigonometric functions. Derivative applications are studied through motion, extrema, and related rates. The course continues with anti-differentiation. A graphing calculator is required for this course.

Assured Assessments

Students will be evaluated by their performance on tests, quizzes, homework, problem sets, journals, APBAs, and departmental midterm and final examinations.

Core Text

Lial, Margaret L., Raymond N. Greenwell, and Nathan P. Ritchey. *Calculus with Applications.* New York: Pearson, 2011. Print.

Unit 1: Algebra and Functions

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-RN The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-APR Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x) + r(x) / b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-REI Reasoning with Equalities and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A-CED Creating Equations

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints for equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal

description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-BF Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

G-SRT Similarity, Right Triangles, and Trigonometry

Define trigonometric ratios and solve problems involving right triangles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

The following Performance Standards are TPS-created.

Expressions, Exponents, and Radicals

- Use interval notation to describe sets of numbers.
- Find rational zeros of a polynomial.
- Add two rational expressions with literal numerators.
- Use fractional and negative exponents to simplify rational expressions.
- Simplify complex radical expressions.
- Rationalize numerator and denominator of a fraction involving radicals with the use of conjugate.

Derivatives of Exponential and Logarithmic Functions

- Graph exponential function $f(x) = a^x$ and $f(x) = e^x$.
- Graph logarithmic function $f(x) = \ln x$ and use it to solve exponential and logarithmic equations.

Essential Questions

- How can algebraic properties be used to solve, evaluate, simplify, and rewrite algebraic expressions and equations?
- What does the slope of a line indicate about the line?
- What information does the equation of a line give?
- How are equations and graphs related?
- How are functions evaluated?
- How are expressions involving exponents and logarithms related?
- What is the relationship between exponential and logarithmic functions?
- How can positive and negative angles of all sizes be represented on a unit circle?

- How are the *x* and *y* coordinates of a point related to the angles and their trigonometric functions?
- What is radian measure and how does one convert between degrees and radians?
- How can algebraic operations be used to simplify trigonometric expressions?
- How can trigonometric identities be used to simplify expressions and solve equations?

Content (Scope and Sequence)

1. Algebraic Reference: Linear, Polynomial, and Rational Functions
   a. Simplifying and factoring polynomial and rational expressions
   b. Solving polynomials and rational equations and inequalities
   c. Evaluating and simplifying exponential expressions
   d. Simplifying radical expressions
   e. Rationalizing the denominator in radical expressions
   f. Rationalizing the numerator in radical expressions
   g. Writing and graphing linear equations
   h. Finding the domain and range of a function
   i. Evaluating functions
2. Exponential and Logarithmic Functions
   a. Graphing and solving basic exponential functions
   b. Graphing and solving logarithmic functions
   c. Converting between exponential and logarithmic expressions
   d. Using the properties of logarithms to simplify expressions
   e. Solving exponential functions using logarithms
3. Trigonometric Functions
   a. Unit circle
      i. Converting degree measures to radians
      ii. Finding exact values for trigonometric functions for special angles
   b. Simplifying trigonometric expressions
   c. Solving trigonometric equations

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.

4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Summer Packet
Summer Packet Quick Quizzes
Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 5 weeks

Unit 2: Limits

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-APR Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $^{a(x)}$ / $b(x)$ in the form $q(x)$ + $^{r(x)}$ / $b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-BF Building Functions

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

The following Performance Standards are TPS-created.

Functions, Graphs, and Limits

- Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity.
- Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
- Use graphical calculators to verify and estimate limits.
- Determine values of limits of a given function (e.g., a particular number, does not exist, etc.).
- Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.

Derivative Applications

- Know how asymptotes are related to an infinite limit.

Essential Questions

- What is a limit?
- How does one calculate and interpret limits?
- How does continuity depend on limits?
- How are a rational function's domain, discontinuities, zeros, and holes found?
- How are limits to infinity used to locate horizontal asymptotes?

Content (Scope and Sequence)

1. Finding limits graphically
2. Estimating limits
3. Evaluating limits algebraically
4. Properties of limits
5. One-sided limits
6. Continuity
7. Limits to infinity
8. Rational functions
   a. Transforming $1/x$
   b. Graphing by finding discontinuities, zeros, and horizontal asymptotes using infinite limits

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 3 weeks

Unit 3: The Derivative

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

A-SSE Seeing Structure in Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-APR Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $^{a(x)}$ / $b(x)$ in the form $q(x)$ + $^{r(x)}$ / $b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

The following Performance Standards are TPS-created.

<u>The Derivative</u>

- Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability.
- Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function at a particular point.
- Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Use derivatives to solve problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.
- Understand the relationships between differentiability and continuity.

Essential Questions

- How does one determine the slope of a function at a given point?
- How does the concept of a limit lead to a derivative?
- How does one determine the derivative of a function?
- What is the relationship between differentiability and continuity?
- How does the derivative represent an instantaneous rate of change?

Content (Scope and Sequence)

1. Average rate of change
2. Instantaneous rate of change (alternate form optional)
3. Definition of the derivative
4. Equation of tangent line
5. Equation of secant line
6. Differentiability and continuity

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 2 weeks

Unit 4: Calculating the Derivative

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-RN The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-APR Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x) + r(x) / b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-REI Reasoning with Equalities and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

The following Performance Standards are TPS-created.

The Derivative

- Know the chain rule and applications to the calculation of the derivative of a variety of composite functions.
- Compute derivatives of higher orders.
- Use differentiation to solve related rates problems in a variety of pure and applied contexts involving variables that change in respect to time.

Derivatives of Exponential and Logarithmic Functions

- Calculate derivatives of exponential functions.
- Calculate derivatives of logarithmic functions.

Essential Questions

- In what types of problems do various differentiation rules apply?
- How can a function be transformed prior to differentiation to apply a simpler differentiation rule?
- How can derivatives be applied to solve motion problems?
- What do the first and second derivatives tell us about a function?

Content (Scope and Sequence)

1. Finding derivatives using sum, constant, product, quotient, and power rules
2. Finding second derivatives and applying them to velocity problems
3. Finding derivatives using the chain rule
4. Finding derivatives of all trigonometric functions, logarithmic functions, and exponential functions

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.

4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 6 weeks

Unit 5: Graphs and the Derivative

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-RN The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-APR Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x) + r(x) / b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-CED Creating Equations

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI Reasoning with Equalities and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

F-IF Interpreting Functions

Understand the concept of a function and use function notation.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-BF Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

The following Performance Standard is TPS-created.

Derivative Applications

- Use differentiation to sketch, by hand, graphs of functions. Identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.

Essential Questions

- How can calculus be used to find characteristics of a curve?
- How can calculus be used to sketch a curve?
- What information does calculus give one concerning graphs of functions?

Content (Scope and Sequence)

1. Finding intervals where a function is increasing or decreasing
2. Finding critical points and points of inflection of a function

3. Finding relative extrema
4. Finding intervals where a function is concave up or concave down
5. Sketching curves by hand
6. Mystery curves

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 5-6 weeks

Unit 6: Applications of the Derivative

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-Q Quantities

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-CED Creating Equations

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

A-REI Reasoning with Equalities and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

F-IF Interpreting Functions

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-BF Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F-LE Linear, Quadratic, and Exponential Models

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

G-SRT Similarity, Right Triangles, and Trigonometry

Prove theorems involving similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-GMD Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G-MG Modeling with Geometry

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

The following Performance Standards are TPS-created.

The Derivative

- Find the derivatives of defined functions through implicit differentiation in a wide variety of problems.

Derivative Applications

- Use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.

Essential Questions

- How does one use the first and second derivatives?
- How does implicit differentiation allow one to find derivatives of functions that are not defined or written explicitly as a function of a single variable?
- How can calculus be used to solve optimization and related rates problems?
- How can one solve real-world problems using calculus?

Content (Scope and Sequence)

1. Finding absolute extrema
2. Solving maximum and minimum problems
3. Finding derivatives implicitly
4. Solving related rates problems

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Tootsie Roll Pop Lab
Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 5-6 weeks

Unit 7: Integration and Applications

Performance Standards

The following Performance Standards align with the 2010 Connecticut Core Standards for Mathematics.

N-RN The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-Q Quantities

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.

A-SSE Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1/15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

F-IF Interpreting Functions

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F-TF Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

The following Performance Standards are TPS-created.

Integrals

- Find the anti-derivative of a function.
- Demonstrate knowledge of the Fundamental Theorem of Calculus and use it to interpret integrals as anti-derivatives.
- Use the general power, the exponential rule, and the log rule to calculate anti-derivatives (indefinite integrals).
- Know the definition of the definite integral.
- Evaluate definite integrals and apply the Fundamental Theorem of Calculus to find the area bounded by two graphs.
- Apply the definition of the integral to model problems in geometry, physics, economics, and so forth.

Essential Questions

- What is integral calculus?
- How does one find an anti-derivative?
- How does one determine the area under a curve?
- What does the integral represent?
- How is integration related to differentiation through the Fundamental Theorem of Calculus?
- How does one determine the area between two curves?

Content (Scope and Sequence)

1. Using basic integration rules to find anti-derivatives
2. Using integration by substitution
3. Approximating the area under a curve using rectangles
4. Finding the exact area under a curve using integrals
5. Understanding and using the Fundamental Theorem of Calculus
6. Calculating the area between curves
7. Volume (optional; time-permitting)

Instructional/Teaching Strategies

Brainstorming, comprehension questions based on instruction, cuing expected behavior, direct instruction, discussion groups, encouraging students to clarify and expand ideas, peer modeling, question-and-answer sessions, refocusing students, research using technology, restating and rephrasing key concepts, wait time

Technology Competency Standards

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Assured Experiences (Projects)

Problem Sets
Homework
Mid-Unit Quizzes
End-of-Unit Test

Time Allocation

Approximately 6 weeks

TEACHER GUIDE

Unit One: Functions

Part 1

Sections:

R.1 Polynomials
R.2 Factoring
R.3 Rational Expressions
R.4 Equations
R.5 Inequalities
R.6 Exponents
R.7 Radicals
1.1 Slopes and Equations of Lines
2.1 Properties of Functions

Part 2

2.4 Exponential Functions
2.5 Logarithmic Functions

Part 3

13.1 Trigonometric Functions (with supplemental topics as indicated in Scope and Sequence)

Unit Two: Limits

3.1 Limits
3.2 Continuity (with supplemental topics as indicated in Scope and Sequence)
2.3 Rational Functions

Unit Three: The Derivative

3.3 Rates of Change
3.4 Definition of the Derivative

Unit Four: Calculating the Derivative

4.1 Techniques for Finding Derivatives
4.2 Derivatives of Products and Quotients
13.2 Derivatives of Trigonometric Functions
4.3 The Chain Rule
4.4 Derivatives of Exponential Functions
4.5 Derivatives of Logarithmic Functions

***Midterm exam**

Unit Five: Graphs and the Derivative

5.1 Increasing and Decreasing Functions
5.2 Relative Extrema
5.3 Higher Derivatives, Concavity, and the Second Derivative Test
5.4 Curve Sketching (with supplemental topics as indicated in Scope and Sequence)

Unit Six: Applications of the Derivative

6.1 Absolute Extrema
6.2 Applications of Extrema
6.4 Implicit Differentiation
6.5 Related Rates

Unit Seven: Integration and Applications

7.1 Anti-derivatives
7.2 Substitution
13.3 Integrals of Trigonometric Functions
7.3 Area and the Definite Integral
7.4 The Fundamental Theorem of Calculus
7.5 The Area between Two Curves
8.2 Volume and Average Value (optional; time-permitting)

COURSE CREDIT

One credit in Mathematics

One class period daily for a full year

PREREQUISITES

Completion of ACP PreCalculus with a C or better and teacher recommendation, or successful completion of Honors PreCalculus.

TEXT

Lial, Margaret L., Raymond N. Greenwell, and Nathan P. Ritchey. *Calculus with Applications.* New York: Pearson, 2011. Print.

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

Department- and teacher-prepared materials, including worksheets on rational functions, transformations of functions, trigonometry differentiation and integration, and mystery curves

TI-84 Plus graphing calculators

Larson, Ron, and Bruce H. Edwards. *Calculus of a Single Variable*. 5th ed. Boston: Brooks/Cole, 2014. Print.

CURRENT REFERENCES

2010 Connecticut Core Standards for Mathematics

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

AP Calculus Learning Essential Questions

http://www.troyareasd.org/Page/707

Fairfield Public Schools Intro to Calculus 50 Curriculum

http://cdn.fairfieldschools.org/curriculum/math-2014/Intro%20to%20Calculus%2050.pdf

ASSURED STUDENT PERFORMANCE RUBRICS

- Trumbull High School School-Wide Writing Rubric
- Trumbull High School School-Wide Problem-Solving Rubric
- Trumbull High School School-Wide Independent Learning and Thinking Rubric

Rubric 2: Write Effectively

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	• Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task	• Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task	• Establishes a purpose • Demonstrates an awareness of audience and task	• Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_____	• Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions	• Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions	• Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions	• Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_____	• Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples	• Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples	• May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples	• Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_____	• Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning	• Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors. Most errors do not detract from meaning	• Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning	• Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Rubric 3: Problem Solving through Critical Thinking

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	Student demonstrates clear understanding of the problem and the complexities of the task	Student demonstrates sufficient understanding of the problem and most of the complexities of the task	Student demonstrates some understanding of the problem but requires assistance to complete the task	Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_____	Student gathers compelling information from multiple sources including digital, print, and interpersonal	Student gathers sufficient information from multiple sources including digital, print, and interpersonal	Student gathers some information from few sources including digital, print, and interpersonal	Student gathers limited or no information
Reasoning and Strategies X_____	Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	Student demonstrates some critical thinking skills to develop a plan integrating some strategies	Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_____	Solution shows deep understanding of the problem and its components. Solution shows extensive use of 21st Century Technology Skills.	Solution shows sufficient understanding of the problem and its components. Solution shows sufficient use of 21st Century Technology Skills.	Solution shows some understanding of the problem and its components. Solution shows some use of 21st Century Technology Skills.	Solution shows limited or no understanding of the problem and its components. Solution shows limited or no use of 21st Century Technology Skills.

Rubric 5: Independent Learners And Thinkers

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work.	Student demonstrates initiative by generating appropriate questions, creating original projects/work.	Student demonstrates some initiative by generating questions, creating appropriate projects/work.	Student demonstrates limited or no initiative by generating few questions and creating projects/work.
Independent Research & Development X_____	Student is analytical, insightful, and works independently to reach a solution.	Student is analytical, and works productively to reach a solution.	Student reaches a solution with direction.	Student is unable to reach a solution without consistent assistance.
Presentation of Finished Product X_____	Presentation shows compelling evidence of an independent learner and thinker. Solution shows deep understanding of the problem and its components. Solution shows extensive and appropriate application of 21st Century Skills.	Presentation shows clear evidence of an independent learner and thinker. Solution shows adequate understanding of the problem and its components. Solution shows adequate application of 21st Century Skills.	Presentation shows some evidence of an independent learner and thinker. Solution shows some understanding of the problem and its components. Solution shows some application of 21st Century Skills.	Presentation shows limited or no evidence of an independent learner and thinker. Solution shows limited or no understanding of the problem. Solution shows limited or no application of 21st Century Skills.