TRUMBULL PUBLIC SCHOOLS

Trumbull, Connecticut

ACP AND CP MODELING AND REASONING IN MATHEMATICS
Grades 11-12
Mathematics Department
2021

Curriculum Writing Team

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ACP and CP Modeling and Reasoning in Mathematics
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CORE VALUES AND BELIEFS

The Trumbull High School community engages in an environment conducive to learning which believes that all students will **read** and **write effectively**, therefore communicating in an articulate and coherent manner. All students will participate in activities **that present problem-solving through critical thinking**. Students will use technology as a tool applying it to decision making. We believe that by fostering self-confidence, self-directed and student-centered activities, we will promote **independent thinkers and learners**. We believe **ethical conduct** to be paramount in sustaining the welcoming school climate that we presently enjoy.

Approved 8/26/2011

INTRODUCTION & PHILOSOPHY

The primary goal of Modeling and Reasoning in Mathematics is to have students develop mathematical proficiency in order to solve real-world problems. The students will build upon previous mathematical knowledge learned in Algebra II and apply these skills in future situations. Modeling and Reasoning in Mathematics provides an alternative senior mathematics course for students who do not intend to study calculus in college. This course reinforces necessary math skills and introduces new topics to prepare them adequately not only for college, but for real-world situations.

Topics covered in this course include: algebra, graphs and functions, systems of equations and inequalities, sets, sequences and series, and probability. Each of these topics will

be explored and students will be applying skills learned in these units to solve problems involving real-world data.

Modeling and Reasoning in Mathematics is designed to preview and prepare students for a standard non-calculus first-year Modeling and Reasoning in Mathematics course. Students will receive further instruction in previously studied Algebra II topics while also gaining an introduction to new topics such as sets, sequences and series, and probability. Students will be given instruction in reading and writing the notation, as well as vocabulary, associated with the various topics. Particular attention will be given to the understanding of the processes involved and the students' ability to communicate these procedures both orally and in written form.

COURSE GOALS

The following Course Goals derive from the 2010 Connecticut Core Standards for Mathematical Practice, which describe varieties of expertise that all teachers of mathematics will develop in their students. These practices rest on important "processes and proficiencies" that have long been valued in mathematics education.

At the completion of this course, students will:

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples.

They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

The following Course Goals derive from the 2016 International Society for Technology in Education Standards.

Standard 5: Computational Thinker
Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

5b. Students collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision making.

5c. Students break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem solving.

COURSE ENDURING UNDERSTANDINGS

Students will understand that...

- Algebra can be useful in solving everyday problems
- Systems of equations can be used to determine cost-effectiveness
- Putting elements into sets helps order and arrange the world
- Probability can be applied to help us make informed decisions in our lives

COURSE ESSENTIAL QUESTIONS

- How can we use Algebra concepts in everyday life?
- How can organizing and analyzing information be useful in our understanding of the world and the decisions we make?

COURSE KNOWLEDGE & SKILLS

Students will know…

- Basic algebraic vocabulary
- The order of operations
- How to solve and apply linear, quadratic, and exponential equations
- The process of graphing equations and functions
- The variety of methods used in solving systems of linear equations
- Methods to indicate sets, equal sets, and equivalent sets
- The meaning and differences between empirical, theoretical, compound, conditional, and binomial probability

Students will be able to…

- Solve linear and quadratic equations in one variable
- Evaluate a formula
- Solve application problems dealing with variation

- Solve systems of linear inequalities
- Solve application problems using linear programming
- Perform set operations such as complement, intersection, union, difference, and Cartesian product
- Apply Venn diagrams to represent data and solve problems
- Calculate odds against and event and odds in favor of an event
- Apply expected value to real life situations
- Calculate probabilities of events using tree diagrams
- Determine outcomes and probabilities of an event occurring using counting principle, permutations, and combinations

COURSE SYLLABUS

Course Name: Modeling and Reasoning in Mathematics

Course Level: College Preparatory/Advanced College Preparatory

Prerequisites:
Students enrolled in Introduction to Modeling and Reasoning in Mathematics should have successfully completed Advanced College Preparatory Algebra II with a B+ or better or College Preparatory Algebra II with a teacher recommendation.

General Description of Course Content:
Introduction to Modeling and Reasoning in Mathematics is a course designed for students who have completed Algebra II and are not planning to take calculus in college. It is designed as preparation for the first year of non-calculus Modeling and Reasoning in Mathematics courses. Students will learn about the following topics: algebra, graphs, functions, systems of equations and inequalities, sets, and probability. Real world applications and connections between topics are stressed. This class is intended for students who have not taken PreCalculus.

Assessment:
Students are evaluated by their performance on classroom problem sets, journal tasks, tests,quizzes, projects, and departmental midyear and final exams.

Text and Supplementary Materials:
1. A Survey of Mathematics with Applications, Pearson Education, Inc., by Angel, Abbott, and Runde, 2009.
2. TI-84 plus calculators
3. Microsoft Word, Excel

UNIT 1
Algebra, Graphs, and Functions

Unit Goal
At the completion of this unit students will be able to:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.
A-CED **Creating Equations**
Create equations that describe numbers or relationships
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A-REI Reasoning with Equations and Inequalities
Understand solving equations as a process of reasoning and explain the reasoning
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Solve equations and inequalities in one variable
4. Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Unit Essential Questions
How do we use the language of algebra to solve real-world problems?

Focus Questions:
1. How do we visually represent real-world problems?
2. How can we use quadratic equations to answer questions?
3. What are the differences between equations and inequalities?

Scope and Sequence

1. Order of Operations
2. Solving Linear Equations and Ratios
3. Using Formulas to solve for different variables
4. Writing Linear Equations to solve real-world applications
5. Using Direct, Inverse and Joint Variation to solve word problems
6. Solving and Graphing Linear Inequalities and Compound inequalities
7. Using graphs of Linear Equations to solve real-world examples
8. Solving Quadratic Equations using Factoring and the Quadratic Formula
9. Functions and their Graphs

Assured Assessments

Homework, Classwork Problem Sets, Quizzes, and a Unit Test.

Resources

Core- Textbook: A Survey of Mathematics with Applications, Pearson Addison Wesley, by Allen R. Angel, Christine D. Abbot, Dennis C. Runde
Supplement- Khan Academy

Time Allotment

25 days

Unit 2

Systems of Linear Equations and Inequalities

Unit Goal

At the completion of this unit students will be able to:
The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.

A-REI Reasoning with Equations and Inequalities

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Unit Essential Questions
What is the importance of the intersection of two lines?

Focus Questions:
1. How is a situation represented with a system of equations?
2. What does the solution of a system of two linear equations represent?
3. How do systems of linear functions model real-world data?
4. How is the solution to a system of linear inequalities represented?
5. What is the connection between the feasible region and the solution in linear programming? (ACP only)

Scope and Sequence
1. Solve systems of linear equations by graphing.
2. Solve systems of linear equations by substitution and elimination.
3. Solve systems of linear inequalities by graphing.
4. Use linear programming to determine a feasible region by graphing
5. Use linear programming to write a system and then solve a real-world problem by finding the minimum cost or maximum profit.

Assured Assessments
Homework, Classwork Problem Sets, Quizzes, and a Unit Test.

Resources
Core- Textbook: A Survey of Mathematics with Applications, Pearson Addison Wesley, by Allen R. Angel, Christine D. Abbot, Dennis C. Runde
Supplement- Khan Academy
Multiple websites that offer Linear programming problems with solutions.
TI-84 Graphing calculator
Teacher resource package

Time Allotment
20 days

Unit 3
Sets

Unit Goal
At the completion of this unit students will be able to:
1. Determine the difference between sets, equal sets, equivalent sets and subsets.
2. Construct Venn diagrams given different sets of data.
3. Use set operations such as complement, intersection, union and the difference of two sets.
4. Solve real world applications using Venn diagrams.

Unit Essential Questions
How are sets used to sort and classify?

Focus Questions:
1. What is the difference between union and intersection?
2. What is the empty set and universal set?
3. What is the complement of a set?
4. What is a subset and proper subset?
5. How is the union and intersection represented symbolically?
6. How does a Venn diagram represent data?
7. How do Venn diagrams solve set logic problems?

Scope and Sequence
1. Set notation and vocabulary.
2. Elements of a set
3. Determine subsets and proper subsets from a Universal set
4. Null set/empty set
5. Complement of a set
6. Find the intersection, union and difference between sets
7. Use Venn diagrams to solve set logic problems
8. Apply sets to sort and classify data to solve real-world problems

Assured Assessments
Homework, Classwork Problem Sets, Quizzes, and a Unit Test.

Resources
Core- Textbook: A Survey of Mathematics with Applications, Pearson Addison Wesley, by Allen R. Angel, Christine D. Abbot, Dennis C. Runde
TI-84 Graphing calculator
Teacher resource package

Time Allotment
20 day

UNIT 4
Probability

Unit Goal
At the completion of this unit students will be able to:

The following Unit Goals align with the 2010 Connecticut Core Standards for Mathematics.
Conditional Probability and the Rules of Probability

S-CP Conditional Probability and the rules of Probabilty

Understand independence and conditional probability and use them to interpret data
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2.Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
Use the rules of probability to compute probabilities of compound events in a uniform probability model.
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. Use permutations and combinations to compute probabilities of compound events and solve problems.

S-MD Using Probability to Make Decisions

2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fastfood restaurant.

6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

Unit Essential Questions

How does probability help to explain the world around us?

Focus Questions:

1. What is the difference between empirical and theoretical probability?
2. How is probability determined?
3. What is an equally likely outcome?
4. What is the difference between odds and probability?
5. What is the relationship between odds against and odds in favor?
6. How is probability used to determine expected value?
7. How is an "or" probability problem calculated?
8. How is an "and" probability problem calculated?
9. What makes two events independent?

Scope and Sequence

1. Calculate and distinguish between empirical and theoretical probability.
2. Define equally likely outcomes and determine the probability of simple events.
3. Determine the odds of an event in favor or against.
4. Calculate the expected value of an event.
5. Use Tree diagrams to calculate probabilities of events occurring.
6. Apply formulas for "or" and "and" probabilities.
7. Determine probability given a certain event has already occurred.
8. Use Combinations and Permutations to determine the number of possible outcomes.
9. Determining the probability of an event occurring using Combinations.

Assured Assessments

Homework, Classwork Problem Sets, Quizzes, and a Unit Test.

Resources

Core- Textbook: A Survey of Mathematics with Applications, Pearson Addison Wesley, by Allen R. Angel, Christine D. Abbot, Dennis C. Runde
TI-84 Graphing calculator
Teacher resource package

Time Allotment

25 days

COURSE CREDIT

One-half credit in Mathematics
One class period daily for a half a year

PREREQUISITES

Successful completion of ACP Algebra II of Honors Algebra II, or a B+ or higher in CP Algebra II with teacher recommendation

TEXT

A Survey of Mathematics with Applications, Pearson Addison Wesley, by Allen R. Angel, Christine D. Abbot, Dennis C. Runde

SUPPLEMENTARY MATERIALS/RESOURCES/TECHNOLOGY

Department and teacher prepared materials

TI-84 Plus graphing calculators

CURRENT REFERENCES

2010 Connecticut Core Standards for Mathematics

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

ASSURED STUDENT PERFORMANCE RUBRICS

Rubric 2: Write Effectively

Category/ Weight	Exemplary 4 Student work:	Goal 3 Student work:	Working Toward Goal 2 Student work:	Needs Support 1-0 Student work:
Purpose X_____	• Establishes and maintains a clear purpose • Demonstrates an insightful understanding of audience and task	• Establishes and maintains a purpose • Demonstrates an accurate awareness of audience and task	• Establishes a purpose • Demonstrates an awareness of audience and task	• Does not establish a clear purpose • Demonstrates limited/no awareness of audience and task
Organization X_____	• Reflects sophisticated organization throughout • Demonstrates logical progression of ideas • Maintains a clear focus • Utilizes effective transitions	• Reflects organization throughout • Demonstrates logical progression of ideas • Maintains a focus • Utilizes transitions	• Reflects some organization throughout • Demonstrates logical progression of ideas at times • Maintains a vague focus • May utilize some ineffective transitions	• Reflects little/no organization • Lacks logical progression of ideas • Maintains little/no focus • Utilizes ineffective or no transitions
Content X_____	• Is accurate, explicit, and vivid • Exhibits ideas that are highly developed and enhanced by specific details and examples	• Is accurate and relevant • Exhibits ideas that are developed and supported by details and examples	• May contain some inaccuracies • Exhibits ideas that are partially supported by details and examples	• Is inaccurate and unclear • Exhibits limited/no ideas supported by specific details and examples
Use of Language X_____	• Demonstrates excellent use of language • Demonstrates a highly effective use of standard writing that enhances communication • Contains few or no errors. Errors do not detract from meaning	• Demonstrates competent use of language • Demonstrates effective use of standard writing conventions • Contains few errors. Most errors do not detract from meaning	• Demonstrates use of language • Demonstrates use of standard writing conventions • Contains errors that detract from meaning	• Demonstrates limited competency in use of language • Demonstrates limited use of standard writing conventions • Contains errors that make it difficult to determine meaning

Rubric 3: Problem Solving through Critical Thinking

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Understanding X_____	Student demonstrates clear understanding of the problem and the complexities of the task	Student demonstrates sufficient understanding of the problem and most of the complexities of the task	Student demonstrates some understanding of the problem but requires assistance to complete the task	Student demonstrates limited or no understanding of the fundamental problem after assistance with the task
Research X_____	Student gathers compelling information from multiple sources including digital, print, and interpersonal	Student gathers sufficient information from multiple sources including digital, print, and interpersonal	Student gathers some information from few sources including digital, print, and interpersonal	Student gathers limited or no information
Reasoning and Strategies X_____	Student demonstrates strong critical thinking skills to develop a comprehensive plan integrating multiple strategies	Student demonstrates sufficient critical thinking skills to develop a cohesive plan integrating strategies	Student demonstrates some critical thinking skills to develop a plan integrating some strategies	Student demonstrates limited or no critical thinking skills and no plan
Final Product and/or Presentation X_____	Solution shows deep understanding of the problem and its components. Solution shows extensive use of 21st Century Technology Skills.	Solution shows sufficient understanding of the problem and its components. Solution shows sufficient use of 21st Century Technology Skills.	Solution shows some understanding of the problem and its components. Solution shows some use of 21st Century Technology Skills.	Solution shows limited or no understanding of the problem and its components. Solution shows limited or no use of 21st Century Technology Skills.

Rubric 5: Independent Learners And Thinkers

Category/Weight	Exemplary 4	Goal 3	Working Toward Goal 2	Needs Support 1-0
Proposal X_____	Student demonstrates a strong sense of initiative by generating compelling questions, creating uniquely original projects/work.	Student demonstrates initiative by generating appropriate questions, creating original projects/work.	Student demonstrates some initiative by generating questions, creating appropriate projects/work.	Student demonstrates limited or no initiative by generating few questions and creating projects/work.
Independent Research & Development X_____	Student is analytical, insightful, and works independently to reach a solution.	Student is analytical, and works productively to reach a solution.	Student reaches a solution with direction.	Student is unable to reach a solution without consistent assistance.
Presentation of Finished Product X_____	Presentation shows compelling evidence of an independent learner and thinker. Solution shows deep understanding of the problem and its components. Solution shows extensive and appropriate application of 21st Century Skills.	Presentation shows clear evidence of an independent learner and thinker. Solution shows adequate understanding of the problem and its components. Solution shows adequate application of 21st Century Skills.	Presentation shows some evidence of an independent learner and thinker. Solution shows some understanding of the problem and its components. Solution shows some application of 21st Century Skills.	Presentation shows limited or no evidence of an independent learner and thinker. Solution shows limited or no understanding of the problem. Solution shows limited or no application of 21st Century Skills.