AP Calculus BC
Mathematics Curriculum
Francis Howell School District

Board Approved: June 2, 2011

Francis Howell School District
Mission Statement

Francis Howell School District is a learning community where all students reach their full potential.

Vision Statement

Francis Howell School District is an educational leader that builds excellence through a collaborative culture that values students, parents, employees, and the community as partners in learning.

Values

Francis Howell School District is committed to:

- Providing a consistent and comprehensive education that fosters high levels of academic achievement for all
- Operating safe and well-maintained schools
- Promoting parent, community, student, and business involvement in support of the school district
- Ensuring fiscal responsibility

- Developing character and leadership

Francis Howell School District Graduate Goals

Upon completion of their academic study in the Francis Howell School District, students will be able to:
1. Gather, analyze and apply information and ideas.
2. Communicate effectively within and beyond the classroom.
3. Recognize and solve problems.
4. Make decisions and act as responsible members of society.

Mathematics Graduate Goals

Upon completion of their mathematics study in the Francis Howell School District, students will be able to:
1. Communicate mathematically
2. Reason mathematically
3. Make mathematical connections
4. Use mathematical representations to model and interpret practical situations

Mathematics Rationale for AP Calculus BC

In today's technological society, production and consumption of information, goods, and services continues to increase, necessitating advanced mathematical literacy skills, particularly for those in careers related to science, technology, engineering, and math (STEM). This increasing emphasis on STEM fields requires students to understand the mathematics of change: rates, accumulation, removal, growth, and decline, approximations, representations. Many physical situations are modeled and analyzed utilizing calculus. Calculus is the basis for more advanced study of many STEM fields. AP Calculus BC provides students with the necessary skills and meaningful applications to analyze phenomena encountered in the modern sciences.

Course Description for AP Calculus BC

This course is a college level course having many applications in engineering and the sciences. Topics include limits, derivatives and integration of a wide variety of functions, and applications of differentiation and integration. This is an advanced placement course that prepares the student to take the Calculus AB exam. AP Calculus BC may be taken for college credit.

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Semester One

Section	Student-Friendly Learning Targets	Activity/topic	pg.	Assignment
1.2	I understand the limit process, can calculate limits with algebra, and can estimate limits from graphs or tables of data.	limits graphically & numerically	54	3,5,9,13,15,17,19,21, 23, 33
1.3		limits analytically	67	9,11,17,21,29,31,33, 37,41,45,57,61,67,73,77

3.5	I understand how to find limits at infinity and can analyze dominance of functions.	limits at infinity	205	1,3,5,21,27,55
3.5		dominance		
1.4	I understand the basic idea of continuity (that function values can be made as close as desired by taking sufficiently close values of the domain), understand continuity in terms of limits, and have a geometric understanding of graphs of continuous functions (IVT & EVT)	continuity	78	1,5,7,9,11,15,19,25,27,31,35,41,43,55,57,61,67,69,73,79
1.5	I understand asymptotes in terms of graphical behavior, can describe asymptotic behavior in terms of limits involving infinity, and can compare relative magnitudes of functions and their rates of change.	infinite limits	88	1,3,5,7,9,19,25,29,31,35,43,47,49
1.5		blueberry pancake recipe		
		review		
		test		
2.1	I can explain and determine the derivative graphically, numerically, and analytically, understand the interpretation of the derivative as an instantaneous rate of change, can calculate the derivative via a limit of a difference quotient, and understand the relationship between differentiability and continuity. I can also find the slope of a line at a point, can find the slope of a tangent line to a curve at a point, understand that the instantaneous rate of change is the limit of the average rate of change, and can approximate rate of change from graphs and tables of values.	local linearity	103	5,9,13,19,27,29,31,45,51,53,55,59,65,69,71, 81, 83, 95
2.1		limit defn. of derivative		
2.1		derivative & tangent line		

2.1 & 2.2		start diff. rules		
2.2	I can find the derivative of basic functions, know the rules for derivatives of sums, products, and quotients of functions, and can use the chain rule and perform implicit differentiation.	diff.rules	115	1,3,7,11,15,17,19,23,29,31,35,43,47,53,65,67,69,71,75,81, 97
2.2		finish notes and work day		
2.3		prod/quot rules	126	3,5,13,19,27,29,35,41,53,59,77,93,95,97
2.4		chain rule	137	1,7,9,15,19,43,47,51,53,55,61,65,67,73,91, 93
2.5		implicit differentiation	146	1,5,11,15,19,31,35,45
2.5		finish notes and hall walking for related rates		
2.6		related rates	154	1,5,11,13,25,31,33,41
formative				
		test		
3.1	I can analyze graphs including characteristics of monotonicity and concavity, can determine the absolute and local extrema, can use implicit differentiation to find the derivative of an inverse function, understand that the derivative is a rate of change that can be applied to varied contexts including velocity, speed, and acceleration, and understand the geometric representation of derivatives via slope fields and solution curves.	extrema	169	1,3,5,7,11,13,15,23,25,29,31,33,35,37, 41, 55, 57

3.2	I understand the corresponding characteristics of f and f ', understand the relationship between increasing and decreasing behavior of f and the sign of f ', can apply the MVT and understand its geometric consequences, and can translate between verbal descriptions and equations involving derivatives.	rolle's and MVT	176	1,9,13,15,21,27,33,39,43,47,49
3.3		incr/decr functions	186	5,9,15,27,31,37,39,45,55,57
3.3 cont'd				
3.4	I understand the corresponding characteristics of f, f ', and f '', understand the relationship between concavity of f and the sign of f '', and understand that POI are places where concavity changes.	concavity	195	5,13,19,23,31,49
3.6		curve sketching	215	1,3,5,7,29,33,47,
3.7		optimization	223	3,7,9,11,15,17,19,27
3.9		differentials	240	1,5,9,13,21,27,43,49,51
R		review		
R		review		
		review		
		test		
4.2	I can find the definite integral as a limit of a sum, can interpret a definite integral of a rate of change as the change of the quantity over the interval, and know basic properties of definite integrals such as additivity and linearity.	area	267	1,7,15,17,21,23,27,33,35,37,41,43,71
4.2		area		

4.3	I can use sums (right, left, midpoint, and trapezoidal) to approximate definite integrals of functions represented algebraically, graphically, and with tables of data	riemann sums/def integ	278	1,5,9,11,13,15,17,19,21,27,43
4.6		num'cal integ, simpson's	314	1,7,11,17,21,23
4.4	I can use the FTC to evaluate definite integrals and to represent a particular antiderivative and analyze functions so defined both analytically and graphically	fund thm of calc.	291	5,7,11,13,19,25,27,33,37,41,43,47,67,69,71,73,75,79,81,87
4.1		antideriv. indef integ	255	1,5,11,15,23,31,35,43,45,47,49,51,55,57,63,65,67,69
4.5	I can find antiderivatives following from derivatives of basic functions and can use substitution of variables.	integ by subst	304	1,7,11,15,21,31,39,47,49,55
		review		
		test		
5.1		ln and differentiation	329	7,9,11,17,19,21,29,33,41,47,57,71,81,83
5.2		ln and integration	338	1,5,11,19,21,27,29,31,33,43,51,61,63,67
5.3		inverse functions	347	1,5,9,11,13,21,51,61,73,75
5.4		exponential functions	356	1,3,5,7,9,27,33,37,45,49,51,55,59,85,87,93,97,99,101
5.5		bases other than e	366	1,3,5,7,9,31,37,45,57,63,69,73,79
5.6		inv. trig func. diff'tion	377	5,7,9,11,13,15,19,21,23,25,41,43,45,51,59
5.7		inv. trig func. int'ration	385	1,3,7,11,15,21,23,33,35,39,41,55
		review		
		test		

6.1	I can find the numerical solution of differential equations using Euler's method	slope fields and Euler's Method	409	1, 3,19,23,25,27,31,37,39,43,49,53,57,69,71,75,79
6.2		growth and decay	418	1,5,7,11,13,15,17,21,25,33,41,43,57,67
6.3	I can find specific antiderivatives using initial conditions, including motion along a line, and can solve separable differential equations and using them in modeling, particularly y ' =ky and exponential growth. Also, I can solve logistic differential equations and use them in modeling	separation of variables and logistic growth models	429	1,3,13,17,21,27,31,35,45,49,55, 67,69,73,75,79
		review		
		test		
ER		exam review		
E		exam		

Semester Two

Section	Student-Friendly Learning Targets	Activity/topic	pg.	Assignment
7.1	I can use appropriate integrals in a variety of applications and can adapt my knowledge and techniques to solve novel application problems, especially those involving accumulated change, area, volume of a solid with known cross sections, average value of a function, volume of revolution, and the distance traveled along a line.	area b/w 2 curves	452	1,3,5,13,17,19,27,37,57,59,73
7.2	I can find the volume of solids by slicing.	volume: discs	463	1,3,5,7,9,11,15,19,25,31,53,55,61,63
7.2		volume: washers		
7.2		volume: known cross-sections		

APFRQ	AP Free Response Question	AP Free Response Question		
7.3	I can find the volume of solids by using cylindrical shells.	volume: shells	472	3,5,9,13,17,21,45
7.4	I can find arc length and surface area.	arc length, surf. area	483	1,3,5,7,9,11,13,39,43
APFRQ	AP Free Response Question	AP Free Response Question		
8.2	I can perform integration by parts.	integration by parts	531	3,5,13,25,29,37,39,43,47,63,73
8.2-8.3	I can find antiverivatives involving combinations of trig functions.	trig integrals	540	3,7,9,11,15,21,25,37,47,53,55
8.3-8.4	I can find antiderivatives by using trigonometric substitution.	trig sublstitution	549	5,7,9,13,15,23,29,39,45
8.4-8.5	I can use the method of simple partial fractions	partial fractions	559	1,5,7,13,15,21,31
8.5				
8.7	I can use L'Hopital's Rule, including its use in determining limits and convergence of improper integrals and series	indeterminate forms	574	5,7,9,15,19,29,33,37,41,43,63
8.8	I can evaluate an improper integral as a limit of a definite integral	improper integrals	585	1,3,5,7,9,15,17
9.1	I can work with sequences.	sequences	602	7,15,29,31,33,37,47,51,53
9.2	I can use the limit of a sequence of partial sums to determine the convergence of a series.	series	612	1,3,9,11,17,19,31,39,51,53
9.3	I can find the sum of a series of constants, including geometric series, the harmonic series, p series, and alternating series, can find an error bound on an alternating series, can use the integral test, ratio test, and comparison test to determine convergence, and understand the relationship between improper integrals and the series made up of areas of rectangles.	integral test	620	1,7,11,15,19,21

9.4		comparing series	628	3, 7, 15, 17, 19, 23, 55,57
9.5		alternating series	636	1, 5, 9, 17, 21, 25, 29, 33,37, 41, 49,53
9.6		ratio and root tests	645	11, 15, 19, 27, 31, 33, 35, 37,39,41
9.7		taylor and maclaurin polynomials	656	1, 5, 9, 13, 17, 21, 25, 29, 33, 37
9.8		power series	666	1, 3,5, 9, 13, 15,17, 21, 25, 29
9.8		power series day 2		
9.9		functions-power series	674	1, 5, 9, 13, 17, 21, 25, 35, 37, 39
9.10.	I understand the convergence of a taylor polynomial as an approximation to another function, can find taylor series, know the Maclaurin series for the functions e^x, sinx, cosx, and 1/(1-x), can manipulate taylor series (including substitution, differentiation, antidifferentiation, and formation of new series from known ones), can define functions by power series, can determine the radius and interval of convergence of power series, and can find the Lagrange error bound for taylor polynomials	taylor and maclaurin series	685	1, 5, 9, 11, 17, 21, 25, 49, 51, 53
APFRQ	AP Free Response Question	AP Free Response Question		
T	I have reviewed conic sections.	conics and calculus	704	examine problems so that you are again familiar with conic sections
10.2	I can analyze plane curves given in parametric, polar, and vector forms	parametrics in the plane	716	1, 5, 9, 13, 17, 21, 25, 29,33
10.3	I can analyze plane curves given in parametric, polar, and vector forms, including velocity and acceleration. I canalso find derivatives of parametric, polar, and vector functions	parametrics and calculus	725	1, 5, 9, 13, 17, 21, 25, 33, 37,51, 61

10.4		polar graphs	736	1, 3, 5, 11, 13, 21-35 odds, 39, 41, 43, 49,
10.5	I can find the area of a region bounded by polar curves and the length of a curve (including in parametric form)	area&arc length, polar	745	3,7,9,13,23,27
VVF	I can work with functions that have vectors as their range elements.	vector valued functions		
AP Rev				
AP Exam				
5.8	I can do derivatives and antiderivatives of hyperbolic functions.	hyperbolic functions	396	1,13,15,23,37,39,41,47,49,51,53,55,63
7.5	I can use calculus to find work done.	work	493	1,3,9,13,15,17,19,21,23,25
7.6	I can use calculus to find the center of gravity.	center of gravity	504	1,3,7,9,13,15
7.7	I can use calculus to find force and pressure exerted by fluids.	force, fluid pressure	511	1,5,7,9,11,13,15,19
ER		exam review		
EXAM				

Content Area: Mathematics	Course: AP Calculus BC	Strand: 1
Learner Objectives: **Student will analyze functions, graphs and limits**		

Concepts: A. Limits of functions (including one-sided limits)

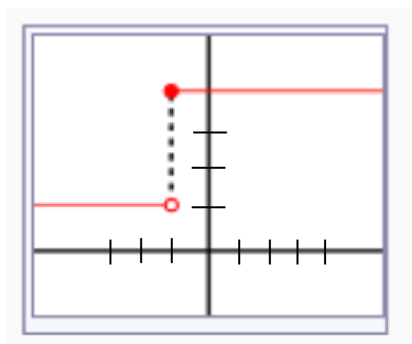
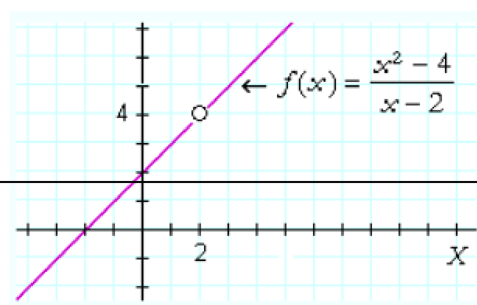
Students Should Know	Students Should Be Able to
• An intuitive understanding of the limiting process	• Calculating limits using algebra • Estimating limits from graphs or tables of data • **Apply L'Hopital's Rule in determining limits**

Instructional Support

Student Essential Vocabulary					
Limit	Rationalize	One-Sided Limit	**Indeterminate Forms**		

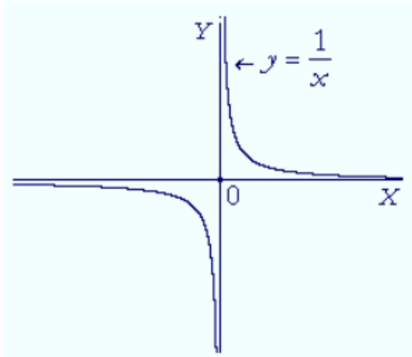
Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	

Learning & Innovation Skills	Critical Thinking and Problem Solving	Enrichment Opportunity	
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self Direction	Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments
Learning Activity #1 : 1. $\lim\limits_{x \to -1^-} f(x)$ = _______ 2. $\lim\limits_{x \to -1^+} f(x)$ = _______ 3. $\lim\limits_{x \to -1} f(x)$ = _______  4. $\lim\limits_{x \to 2^-} f(x)$ = _______  	**Assessment #1:** 1. $\lim\limits_{x \to -3} (x^2 + x - 2)$ = _______ 2. $\lim\limits_{x \to \frac{4\pi}{3}} (\sin x)$ = _______ 3. $\lim\limits_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + 3x - 10}$ = _______ 4. $\lim\limits_{x \to -3} \frac{x - 2}{2x + 6}$ = _______ 5. For the function $f(x) = \frac{x^2 - 9x + 20}{x^2 + x - 20}$, find the following (or state "does not exist"). a. $\lim\limits_{x \to -5^-} f(x)$ = _______ b. $\lim\limits_{x \to -5^+} f(x)$ = _______ c. $\lim\limits_{x \to -5} f(x)$ = _______ d. $\lim\limits_{x \to 5} f(x)$ = _______

5. $\lim_{x\to 2^+} f(x)$ = ______

6. $\lim_{x\to 2} f(x)$ = ______



7. $\lim_{x\to 0^-} f(x)$ = ______

8. $\lim_{x\to 0^+} f(x)$ = ______

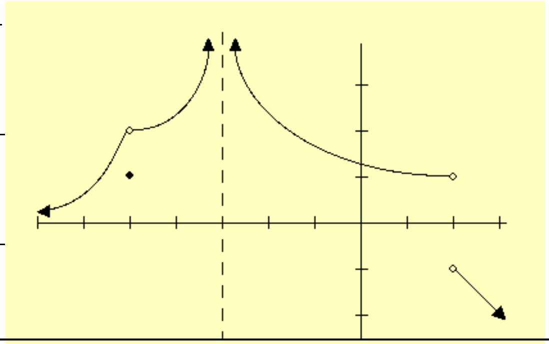
9. $\lim_{x\to 0} f(x)$ = ______

10. $\lim_{x\to -5} f(x)$

11. $\lim_{x\to 2^+} f(x)$ = ______

12. $\lim_{x\to +\infty} f(x)$ = ______

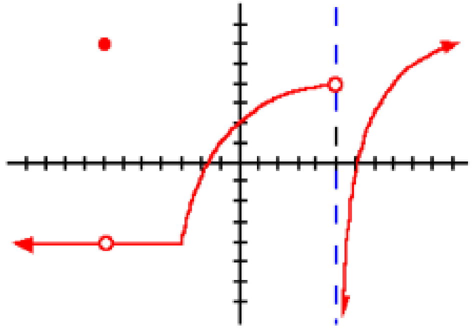
13. $\lim_{x\to -\infty} f(x)$ = ______



e. $\lim_{x\to 4} f(x)$ = ______

f. $\lim_{x\to -\infty} f(x)$ = ______

6. $\lim_{x\to -7} f(x)$ = ______



7. $f(-7)$ = ______

8. $\lim_{x\to 0} f(x)$ = ______

9. $\lim_{x\to -\infty} f(x)$ = ______

10. $\lim_{x\to 5^-} f(x)$ = ______

11. $\lim_{x\to 5^+} f(x)$ = ______

Assessment #1 -- KEY:

1) 4 2) $-\frac{\sqrt{3}}{2}$ 3) $-\frac{1}{7}$ 4) DNE

5a) ∞ 5b) $-\infty$ 5c) DNE 5d) 0 5e) $-\frac{1}{9}$ 5f) 1

6) -4 7) 6 8) 2 9) -4 10) 4 11) $-\infty$

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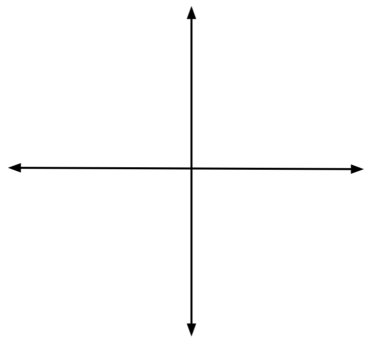
14. $\lim_{x \to -3} f(x) =$ ______

15. $\lim_{x \to 2} f(x) =$ _______

16. Graph the rational function $f(x) = \dfrac{x-1}{x^2-4}$ and find the following (or state "does not exist").

a. $\lim_{x \to 1} f(x) =$ _______

b. $\lim_{x \to 2^-} f(x) =$ _______



c. $\lim_{x \to 2^+} f(x) =$ _______

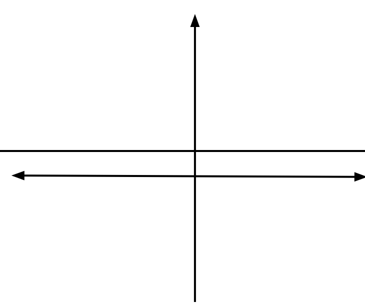
d. $\lim_{x \to -2} f(x) =$ _______

e. $\lim_{x \to 2} f(x) =$ _______

17. Graph the rational function $f(x) = \dfrac{x^2-2x-15}{x^2+x-6}$ and find the Following (or state "does not exist").

a. $\lim_{x \to 2^-} f(x) =$ _______

b. $\lim_{x \to 2^+} f(x) =$ _______



Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically
CONTENT	MA4 Patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

c. $\lim_{x\to 2} f(x)$ = _______

d. $\lim_{x\to -3} f(x)$ = _______

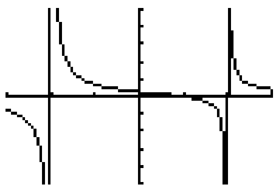
e. $\lim_{x\to 5} f(x)$ = _______

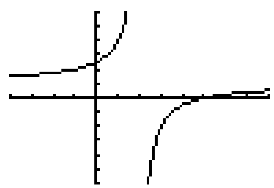
Learn Activity #1 -- KEY:

1) 1 2) 4 3) DNE 4) 4 5) 4 6) 4

7) $-\infty$ 8) ∞ 9) DNE 10) 2 11) -1 12) $-\infty$

13) 0 14) ∞ 15) DNE

16) 

17) 

(note: vertical asymptotes are dashed)

16a) 0 16b) $-\infty$

16c) ∞ 16d) DNE

16e) DNE

17a) ∞ 17b) $-\infty$

17c) DNE 17d) 8/5

17e) 0

Activity's Alignment

AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically
CONTENT	MA4 Patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
INSTRUCTIONAL STRATEGIES	Non-Linguistic Representation

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	
Learning & Innovation Skills	Critical Thinking and Problem Solving	Enrichment Opportunity	
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self Direction	Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #2 :

Use your graphing calculator to complete the table for each function, then approximate the limit.

1. $f(x) = \dfrac{x^2 - 5x + 6}{x^2 + 3x - 10}$

x	1.80	1.90	1.99	2.00	2.01	2.1	2.2
$f(x)$							

$\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 + 3x - 10} \approx$ ________

2. $f(x) = \dfrac{\sqrt{14 + x} - 3}{x + 5}$

x	-5.20	-5.10	-5.01	-5.00	-4.99	-4.90	-4.80
$f(x)$							

$\lim_{x \to -5} \dfrac{\sqrt{14 + x} - 3}{x + 5} \approx$ ________

3. $f(x) = \dfrac{\frac{1}{4} + \frac{1}{x - 4}}{x}$

x	-0.20	-0.10	-0.01	0.00	0.01	0.10	0.20
$f(x)$							

$\lim_{x \to 0} \dfrac{\frac{1}{4} + \frac{1}{x - 4}}{x} \approx$ ________

Assessment #2:

Evaluate each limit algebraically.

1. $\lim_{x \to 4} \dfrac{x^2 - 2x - 8}{x - 4} =$ ________

2. $\lim_{x \to 3} \dfrac{x^2 - 7x + 12}{x^2 + 2x - 15} =$ ________

3. $\lim_{x \to -5} \dfrac{\sqrt{14 + x} - 3}{x + 5} =$ ________

4. $\lim_{x \to 0} \dfrac{\frac{1}{8} + \frac{1}{x - 8}}{x} =$ ________

Find each one-sided limit.

5. $\lim_{x \to -3^+} \left(\dfrac{x - 1}{x + 3} \right) =$ ________

6. $\lim_{x \to -2^-} \left(\dfrac{x - 2}{x^2 - 4} \right) =$ ________

7. $\lim_{x \to \frac{\pi}{3}^+} \cot(3x) =$ ________

8. $\lim_{x \to 0^-} \dfrac{\sin x}{2x} =$ ________

4. $f(x) = \dfrac{1-\cos x}{x}$

x	-0.20	-0.10	-0.01	0.00	0.01	0.10	0.20
$f(x)$							

$\lim_{x\to 0} \dfrac{1-\cos x}{x} \approx$ ________

5. $f(x) = \dfrac{\sin x}{x}$

x	-0.20	-0.10	-0.01	0.00	0.01	0.10	0.20
$f(x)$							

$\lim_{x\to 0} \dfrac{\sin x}{x} \approx$ ________

Determine the exact value of each limit algebraically and compare it to your previous approximation.

6. $\lim_{x\to 2} \dfrac{x^2-5x+6}{x^2+3x-10} =$

7. $\lim_{x\to -5} \dfrac{\sqrt{14+x}-3}{x+5} =$

9. $\lim_{x\to 0^-}\left(1-\dfrac{1}{x^2}\right) =$ ________

10. $\lim_{x\to 1^-}\left(\dfrac{x+1}{x^2-1}\right) =$ ________

Assessment #2 -- KEY:

1) $\lim_{x\to 4}(x+2) = 6$

2) $\lim_{x\to 3} \dfrac{x-4}{x+5} = -\dfrac{1}{8}$

3) $\lim_{x\to -5} \dfrac{\sqrt{14+x}-3}{x+5} = \lim_{x\to -5} \dfrac{1}{\sqrt{14+x}+3} = \dfrac{1}{6}$

4) $\lim_{x\to 0} \dfrac{\frac{1}{8}+\frac{1}{x-8}}{x} = \lim_{x\to 0} \dfrac{x-8+8}{8x(x-8)} = \lim_{x\to 0} \dfrac{1}{8(x-8)} = -\dfrac{1}{64}$

5) $-\infty$ 6) $-\infty$ 7) ∞ 8) ½ 9) $-\infty$ 10) $-\infty$

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically
CONTENT	MA4 Patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
LEVEL OF EXPECTATION	Mastery level – 70%

8. $\lim_{x\to 0}\frac{\frac{1}{4}+\frac{1}{x-4}}{x} =$

Learn Activity #2 -- KEY:

1) table values: -0.176, -0.159, -0.144, ---, -0.141, -0.127, -0.111

$\lim_{x\to 2}\frac{x^2-5x+6}{x^2+3x-10} \approx$ -0.142

2) table values: 0.1676, 0.1671, 0.1667, ---, 0.1666, 0.1662, 0.1657

$\lim_{x\to -5}\frac{\sqrt{14+x}-3}{x+5} \approx$ 0.1666

3) table values: -0.060, -0.061, -0.062, ---, -0.063, -0.064, .-0.066

$\lim_{x\to 0}\frac{\frac{1}{4}+\frac{1}{x-4}}{x} \approx$ -0.0625

4) table values: -0.099, -0.049, -0.005, ---, 0.0049, 0.049, 0.0997

$\lim_{x\to 0}\frac{1-\cos x}{x} \approx$ 0.000

5) table values: 0.993, 0.998, 0.999, ---, 0.999, 0.998, 0.993

$\lim_{x\to 0}\frac{\sin x}{x} \approx$ 1.000

6) $\lim_{x\to 2}\frac{x^2-5x+6}{x^2+3x-10} = \lim_{x\to 2}\frac{(x-2)(x-3)}{(x-2)(x+5)} = \frac{x-3}{x+5} = \frac{-1}{7} \approx -0.142857$

7) $\lim_{x\to -5}\dfrac{\sqrt{14+x}-3}{x+5}=\lim_{x\to -5}\dfrac{1}{\sqrt{14+x}+3}=\dfrac{1}{6}\cong 0.166$

8) $\lim_{x\to 0}\dfrac{\frac{1}{4}+\frac{1}{x-4}}{x}=\lim_{x\to 0}\dfrac{x-4+4}{4x(x-4)}=\lim_{x\to 0}\dfrac{1}{4(x-4)}=\dfrac{-1}{16}=-0.0625$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically
CONTENT	MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
INSTRUCTIONAL STRATEGIES	Generating and Testing Hypotheses

Learning Activity #3

Theorem: L'Hôpital's Rule

Let *f* and *g* be functions that are differentiable on an open interval (*a*, *b*) containing *c*, except possibly at *c* itself. Assume that $g'(x) \neq 0$ for all *x* in (*a*, *b*), except possibly at *c* itself. If the limit of $f(x)/g(x)$ as *x* approaches *c* produces the indeterminate form 0/0, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies if the limit of *f*(*x*)/*g*(*x*) as *x* approaches *c* produces any one of the indeterminate forms ∞/∞**,** $(-\infty)/\infty$**,** $\infty/(-\infty)$ **,or** $(-\infty)/(-\infty)$**.**

There are actually seven indeterminate forms. These are the limits that produce 0/0 and ∞/∞ as well as those that produce $0\cdot\infty$, $\infty-\infty$, 1^{∞}, 0^{0} and ∞^{0}. In order to use L'Hôpital's Rule, one of the first two forms must exist. In the cases of the last four, they must each first be manipulated to have the form of 0/0 or ∞/∞, and only then can L'Hôpital's Rule be used.

Evaluate each limit:

a. $\lim_{x\to 0^+}\frac{1-\cos x}{x+x^2}$

b. $\lim_{x\to\infty}\frac{e^{2x}}{1+x^2}$

c. $\lim_{x\to\frac{\pi}{2}^-}(\tan x-\sec x)$

d. $\lim_{x\to\infty}x\ln\left(1+\frac{1}{x}\right)$

e. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$

f. $\lim_{x\to 0^+}(\sin x)^{x}$

g. $\lim_{x\to\frac{\pi}{2}^-}(\tan x)^{\cos x}$

Assessment #3

Use L'Hôpital's Rule to evaluate each of the following limits.

1. $\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}$

2. $\lim_{x\to\infty}\frac{x^2+5}{x+e^x}$

3. $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)$

4. $\lim_{x\to\infty}e^{-x}\ln x$

5. $\lim_{x\to 0^+}\sqrt{x^x}$

6. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$

Assessment #3 – KEY

1. 1 2. 0 3. ∞ 4. 0 5. 1 6. e^6

Assessment's Alignment

Learning Activity #3 – KEY

a. $\lim\limits_{x \to 0^+} \dfrac{1-\cos x}{x+x^2}$ $\left(\dfrac{0}{0} \text{ form}\right)$

$= \lim\limits_{x \to 0^+} \dfrac{\sin x}{1+2x}$ (application of L'Hôpital's Rule)

$= 0$

b. $\lim\limits_{x \to \infty} \dfrac{e^{2x}}{1+x^2}$ $\left(\dfrac{\infty}{\infty} \text{ form}\right)$

$= \lim\limits_{x \to \infty} \dfrac{2e^{2x}}{2x}$ $\left(\dfrac{\infty}{\infty} \text{ form}\right)$

$= \lim\limits_{x \to \infty} \dfrac{4e^{2x}}{2}$ (application of L'Hôpital's Rule)

$= \infty$

c. $\lim\limits_{x \to \frac{\pi}{2}^-} (\tan x - \sec x)$ $(\infty - \infty \text{ form})$

$= \lim\limits_{x \to \frac{\pi}{2}^-} \left(\dfrac{\sin x - 1}{\cos x}\right)$ $\left(\dfrac{0}{0} \text{ form}\right)$

$= \lim\limits_{x \to \frac{\pi}{2}^-} \left(\dfrac{\cos x}{-\sin x}\right)$ (application of L'Hôpital's Rule)

$= 0$

d. $\lim\limits_{x \to \infty} x \ln\left(1+\dfrac{1}{x}\right)$ $(\infty \cdot 0 \text{ form})$

AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
CONTENT	MA 1	number sense
	MA 4	patterns and relationships
PROCESS	1.6	discover/evaluate relationships
	1.10	apply information, ideas and skills
	3.3	apply one's own strategies
	3.4	evaluate problem-solving processes
	3.5	reason logically (inductive/deductive)
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 80%	

$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ $\left(\frac{0}{0} \text{ form}\right)$ $= \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{1}{x}}\right)\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$ (application of L'Hôpital's Rule) $= \lim_{x\to\infty}\left(\frac{1}{1+\frac{1}{x}}\right) = 1$ e. $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$ (1^∞ form) let $y = \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$, hence $\ln y = \lim_{x\to\infty} x\ln\left(1+\frac{2}{x}\right)$ $(\infty \cdot 0 \text{ form})$ $= \lim_{x\to\infty} \frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}$ $\left(\frac{0}{0} \text{ form}\right)$	

$$= \lim_{x\to\infty} \frac{\left(\dfrac{1}{1+\dfrac{2}{x}}\right)\left(-\dfrac{2}{x^2}\right)}{\left(-\dfrac{1}{x^2}\right)}$$

(application of L'Hôpital's Rule)

$$= \lim_{x\to\infty} \left(\frac{2}{1+\dfrac{2}{x}}\right) = 2$$

since $\ln y = 2$, it follows that $y = \lim_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x = e^2$

f. $\lim_{x\to 0^+} (\sin x)^x$ $(0^0 \quad \text{form})$

let $y = \lim_{x\to 0^+} (\sin x)^x$,

hence $\ln y = \lim_{x\to 0^+} x \ln(\sin x)$ $(0\cdot\infty \quad \text{form})$

$$= \lim_{x\to 0^+} \frac{\ln \sin x}{\dfrac{1}{x}} \qquad \left(\frac{\infty}{\infty} \quad \text{form}\right)$$

$$= \lim_{x\to 0^+} \frac{\dfrac{\cos x}{\sin x}}{-\dfrac{1}{x^2}}$$

(application of L'Hôpital's Rule)

$$= \lim_{x\to 0^+} \frac{-x^2\cos x}{\sin x} \qquad \left(\frac{0}{0} \quad \text{form}\right)$$

$$= \lim_{x\to 0^+} \frac{-x^2\sin x - 2x\cos x}{\cos x}$$

(application of L'Hôpital's Rule)

$= 0$

since $\ln y = 0$, it follows that $y = \lim_{x\to 0^+}(\sin x)^x = e^0 = 1$

g. $\lim_{x\to \frac{\pi}{2}^-}(\tan x)^{\cos x}$ $\quad (0^0 \quad \text{form})$

let $y = \lim_{x\to \frac{\pi}{2}^-}(\tan x)^{\cos x}$,

hence $\ln y = \lim_{x\to \frac{\pi}{2}^-} \cos x \ln(\tan x)$ $\quad (0\cdot\infty \quad \text{form})$

$$= \lim_{x\to \frac{\pi}{2}^-} \frac{\ln(\tan x)}{\sec x} \qquad \left(\frac{\infty}{\infty} \quad \text{form}\right)$$

$$= \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{\sec^2 x}{\tan x}}{\sec x \tan x}$$ (application of L'Hôpital's Rule)

$$= \lim_{x\to \frac{\pi}{2}^-} \frac{\sec x}{\tan^2 x} = \lim_{x\to \frac{\pi}{2}^-} \frac{\cos x}{\sin^2 x} = 0$$

since $\ln y = 0$, it follows that $y = \lim_{x\to \frac{\pi}{2}^-}(\tan x)^{\cos x} = e^0 = 1$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 3 Differential calculus
CONTENT	MA 1 number sense MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.3 apply one's own strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive)
DOK	2

INSTRUCTIONAL STRATEGIES	Guided Practice	

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 2**
Learner Objectives: Student will analyze functions, graphs and limits		

Concepts: B. Asymptotic and unbounded behavior

Students Should Know	Students Should Be Able to

• Understanding asymptotes in terms of graphical behavior	• Describing asymptotic behavior in terms of limits involving infinity • Comparing relative magnitudes of functions and their rates of change – exponential growth, polynomial growth and logarithmic growth • **Analyze planar curves including those given in parametric form, polar form and vector form**

Instructional Support

Student Essential Vocabulary					
Asymptote	Bounded Function	Unbounded Function	Infinite Limits	Limits at Infinity	End-Behavior
Exponential Growth	Polynomial Growth	Logarithmic Growth	**Polar form**	**Parametric form**	**Vector form**
Parameter	**Rectangular form**				

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #1 :	**Assessment #1:**

Learning Activity #1 :

1. State the vertical and horizontal asymptotes for $f(x)=\dfrac{x+4}{x^2-x-20}$.

   VA: _______ HA: _______ What is $\lim\limits_{x\to\infty}\dfrac{x+4}{x^2-x-20}$? _______

2. State the vertical and horizontal asymptotes for $f(x)=\dfrac{4x^2-36}{x^2-2x-15}$.

   VA: _______ HA: _______ What is $\lim\limits_{x\to\infty}\dfrac{4x^2-36}{x^2-2x-15}$? _______

3. For the function $f(x)=\dfrac{x^2+5x+6}{x^2+8x+15}$, find the following (or state "does not exist").

   a. $\lim\limits_{x\to -5^-} f(x)$ = _______ b. $\lim\limits_{x\to -5^+} f(x)$ = _______

   c. $\lim\limits_{x\to -5} f(x)$ = _______ d. $\lim\limits_{x\to -3} f(x)$ = _______

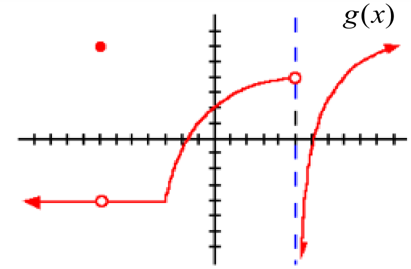
   e. $\lim\limits_{x\to -2} f(x)$ = _______ f. $\lim\limits_{x\to \infty} f(x)$ = _______

4. Explain the relationship between the horizontal asymptote(s) of a function and the limit of the function as x approaches positive or negative infinity.

Assessment #1:

Use the graph of $g(x)$ at right to complete the following:

1. $\lim\limits_{x\to -\infty} g(x)$ = _______
2. $\lim\limits_{x\to 5^-} g(x)$ = _______
3. $\lim\limits_{x\to 5^+} g(x)$ = _______
4. $\lim\limits_{x\to \infty} g(x)$ = _______



For the function $f(x)=\dfrac{x^2-2x-15}{x^2-x-12}$, find the following (or state "does not exist").

5. $\lim\limits_{x\to -3^+} f(x)$ = _______ 6. $\lim\limits_{x\to 4} f(x)$ = _______

7. $\lim\limits_{x\to 5} f(x)$ = _______ 8. $\lim\limits_{x\to -\infty} f(x)$ = _______

9. What is the vertical asymptote of $f(x)$? ___________

10. Explain the relationship between the horizontal asymptote(s) of a

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function and the limit of the function as x approaches positive or negative infinity.

Learning Activity #1 -- KEY

1) VA: $x = 5$, HA: $y = 0$, $\lim_{x\to\infty} \frac{x+4}{x^2 - x - 20} = 0$

2) VA: $x = 5$, HA: $y = 4$, $\lim_{x\to\infty} \frac{4x^2 - 36}{x^2 - 2x - 15} = 4$

3a) ∞ 3b) $-\infty$ 3c) DNE 3d) -1/2 3e) 0 3f) 1

4) The limit of the function as x approaches positive and negative infinity is the y-value(s) of the horizontal asymptote(s).

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
CONTENT	MA4	patterns and relationships
PROCESS	1.6 1.10 3.6	discover/evaluate relationships apply information, ideas and skills examine solutions from many perspectives
DOK	2	
INSTRUCTIONAL STRATEGIES	Summarizing and Note taking	

Assessment #1 -- KEY

1) -4 2) 4 3) $-\infty$ 4) ∞ 5) 8/7 6) DNE 7) 0

8) 0 9) $x = 4$

10) The limit of the function as x approaches positive and negative infinity is the y-value(s) of the horizontal asymptote(s).

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
CONTENT	MA4	patterns and relationships
PROCESS	1.6 1.10 3.6	discover/evaluate relationships apply information, ideas and skills examine solutions from many perspectives
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 70%	

Sample Learning Activities	Sample Assessments

Learning Activity #2 :

Relative Magnitudes and Rates of Change

Complete the table:

x	Δx	$Y_1 = e^x$	ΔY_1	$Y_2 = x^2 + 3x + 4$	ΔY_2	$Y_3 = \ln x$	ΔY_3
1	-----		-----		-----		-----
2							
4							
8							
16							

Use the values in the table above to compare each set of ratios. Insert “<” or “>”.

a. $\frac{\Delta Y_1}{\Delta x} \square \frac{\Delta Y_2}{\Delta x}$ b. $\frac{\Delta Y_1}{\Delta x} \square \frac{\Delta Y_3}{\Delta x}$ c. $\frac{\Delta Y}{\Delta x} \square \frac{\Delta Y_3}{\Delta x}$

Use your calculator to graph the following and make a conjecture about the value of each limit:

1. $\lim_{z\to\infty} \frac{e^x}{\ln x}$ 2. $\lim_{z\to\infty} \frac{\ln x}{e^x}$ 3. $\lim_{z\to\infty} \frac{\ln(x^{10})}{e^x}$

What can you hypothesize about the relative rates of growth of e^x and $\ln x$?

Assessment #2:

1) $\lim_{z\to\infty} \frac{\ln x^{100}}{0.01e^x} =$ 2) $\lim_{z\to\infty} \frac{1000x^2 + 300x}{0.001e^x} =$

3) $\lim_{z\to\infty} \frac{0.005e^x}{\ln x} =$ 4) $\lim_{z\to\infty} \frac{0.003e^x}{100x^3 + 10x^2} =$

Assessment #2 -- KEY

1) 0 2) 0 3) ∞ 4) ∞

Assessment’s Alignment

Use your calculator to graph the following and make a conjecture about the value of each limit:

4. $\lim_{z\to\infty} \frac{e^x}{x^2+3x+4}$ 5. $\lim_{z\to\infty} \frac{x^2+3x+4}{e^x}$ 6. $\lim_{z\to\infty} \frac{100(x^2+3x+4)}{e^x}$

What can you hypothesize about the relative rates of growth of e^x and x^2+3x+4 ?

Learning Activity #2 -- KEY

x	Δx	$Y_1 = e^x$	ΔY_1	$Y_2 = x^2+3x+4$	ΔY_2	$Y_3 = \ln x$	ΔY_3
1	-----	2.7	-----	8	-----	0	-----
2	1	7.4	4.7	14	6	0.69	0.69
4	2	54.6	47.2	32	18	1.39	0.70
7	3	1096.6	1042.0	74	42	1.95	0.56
13	6	442413.4	441316.8	212	138	2.56	0.61

a) > b) > c) >

Note: for questions 1 – 3, the functions are not shown on the calculator image but larger values of *x* can be "traced" with appropriate window setting.

1) ∞ 2) 0 3) 0

The exponential function grows much faster than the logarithmic function.

4) ∞ 5) 0 6) 0

The exponential function grows much faster than the polynomial.

AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
LEVEL OF EXPECTATION	Mastery level – 70%

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	3
INSTRUCTIONAL STRATEGIES	Generating and Testing Hypotheses

Sample Learning Activities	Sample Assessments

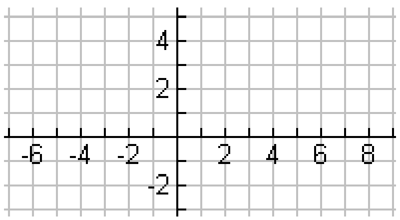
Learning Activity #3:

Parametric Equations

Use the accompanying table of values to sketch the curve described by the parametric equations:

$x(t) = t^2 - 4$ and $y(t) = \frac{t}{2}$, $-2 \le t \le 3$

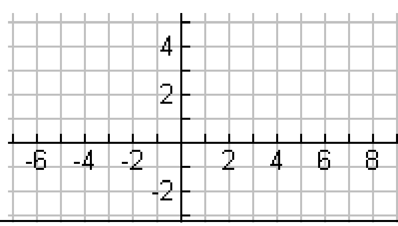
t	-2	-1	0	1	2	3
x						
t						



Use the accompanying table of values to sketch the curve described by the parametric equations:

$x(t) = 4t^2 - 4$ and $y(t) = t$, $-1 \le t \le \frac{3}{2}$

t	-2	-1	0	1	2	3
x						
t						



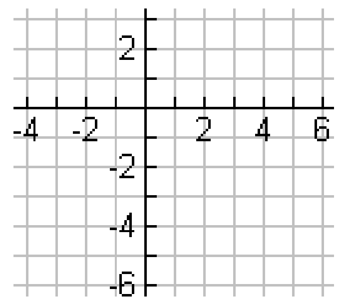
Assessment #3:

Consider the parametric equations $x = \sqrt{t}$ and $y = 1 - t$,

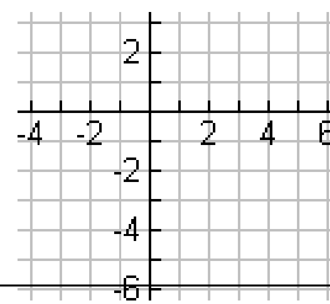
a. Complete the table.

t	0	1	2	3	4
x					
t					

b. Plot the points (x, y) generated in the table, and sketch a graph of the parametric equations. Indicate the orientation of the graph.



c. Find the rectangular equation by eliminating the parameter, and sketch its graph. Compare the graph in part "b" with the graph of the rectangular equation.



What do you notice about the graphs of the two sets of parametric equations shown above?

Now, eliminate the parameter in each pair of equations graphed above.

What do you notice about the resulting equations? Why does this happen?

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA1 number sense MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA1 number sense MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 3**
Learner Objectives: Student will analyze functions, graphs and limits		

Concepts: C. Continuity as a property of functions

Students Should Know	Students Should Be Able to
• An intuitive understanding of continuity	• Understanding continuity in terms of limits • Geometric understanding graphs of continuous functions – Intermediate Value Theorem and Extreme Value Theorem

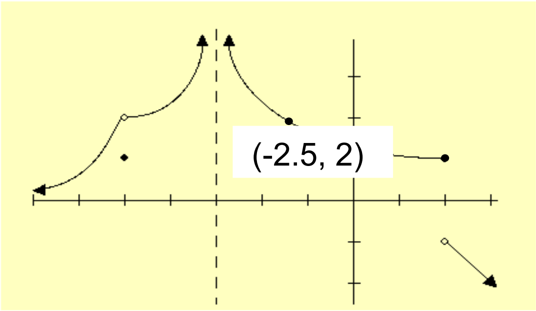
Instructional Support

Student Essential Vocabulary			
Continuity	Discontinuity	Removable Discontinuity	Non-Removable Discontinuity

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #1 :



Use the graph of *f* shown above to answer the following:

1. a) $f(-2.5) =$ _______ b) $\lim_{x \to -2.5} f(x) =$ _______

   c) *f* is continuous at $x = -2.5$ because ____________________

   ____

2. a) $f(-3) =$ _______ b) $\lim_{x \to -3} f(x) =$ _______

   c) *f* is ***not continuous*** at $x = -3$ because ____________________

   ____

3. a) $f(2) =$ _______ b) $\lim_{x \to 2} f(x) =$ _______

   c)) *f* is ***discontinuous*** at $x = 2$ because ____________________

   ____

4. a) $f(-5) =$ _______ b) $\lim_{x \to -5} f(x) =$ _______

Assessment #1:

Use the graph of the function $f(x)$ shown at the right to answer questions 1-6.

1. $\lim_{x \to 3} f(x) =$

   A. 0 B. -1 C. 3 D. does not exist

2. $f(3) =$

   A. 0 B. -1 C. 3 D. undefined

3. $\lim_{x \to -2^+} f(x) =$

   A. 2 B. -1 C. does not exist D. undefined

4. $\lim_{x \to -2} f(x) =$

   A. 2 B. -1 C. does not exist D. undefined

5. Give all values of *x* at which $f(x)$ is discontinuous and explain why.

____

____

c) f is ***not continuous*** at x = -5 because ______________________

____

Learning Activity #1 -- KEY

1a) 2 b) 2 c) because $\lim_{x \to -2.5} f(x) = f(-2.5)$

2a) undefined b) ∞ c) because f(-3) is not defined

3a) 1 b) DNE c) because $\lim_{x \to 2} f(x)$ DNE

4a) 1 b) 2 c) because $\lim_{x \to -5} f(x) \neq f(-5)$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
INSTRUCTIONAL STRATEGIES	Identifying Similarities and Differences

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Assessment #1 -- KEY

1A) 0 B) -1 C) 3 D) DNE

2A) 0 B) -1 C) 3 D) DNE

3A) 2 B) -1 C) DNE D) undefined

4A) 2 B) -1 C) DNE D) undefined

5) $f(x)$ is discontinuous at x = -2 because $\lim_{x \to -2} f(x)$ does not exist,

$f(x)$ is discontinuous at x = 3 because $\lim_{x \to 3} f(x) \neq f(3)$.

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
LEVEL OF EXPECTATION	Mastery level –75%

Readiness & Equity Section

SLA = Sample Learning Activities & SA = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities

Learning Activity #2 :

Intermediate Value Theorem

If the function $f(x)$ is continuous on [a, b], and y is a number between $f(a)$ and $f(b)$, then there exists at least one number c in the open interval (a, b) such that $f(c) = y$.

Restate this theorem using $G'(t)$ instead of $f(x)$.

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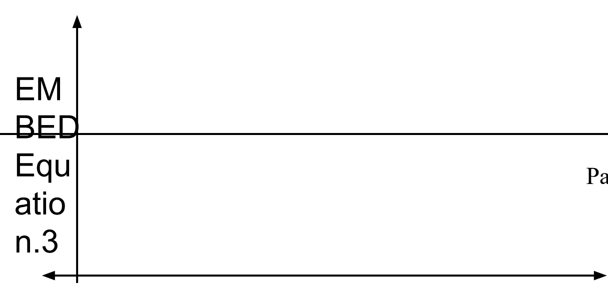
An example from the 2008 AP Free Response Question 2 (part c):

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

Sketch a graph for the table of values above.



Sample Assessments

Assessment #2:

Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.

$$f(x) = x^2 - 6x + 8, \quad [0, 3], \quad f(c) = 0$$

Assessment #2 -- KEY

$f(x)$ is continuous on [0, 3], and $f(3) < f(c) < f(0)$.

$f(c) = c^2 - 6c + 8 = (c - 4)(c - 2) = 0$ when $c = 2$ or $c = 4$. The solution for c is $c = 2$ only since 4 is not in the interval (0, 3).

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA2 geometric and spatial MA4 patterns and relationships

PROCESS	1.6	discover/evaluate relationships
	1.10	apply information, ideas and skills
	3.6	examine solutions from many perspectives
	4.1	support details
DOK	2	
LEVEL OF EXPECTATION	Mastery level –75%	

For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.

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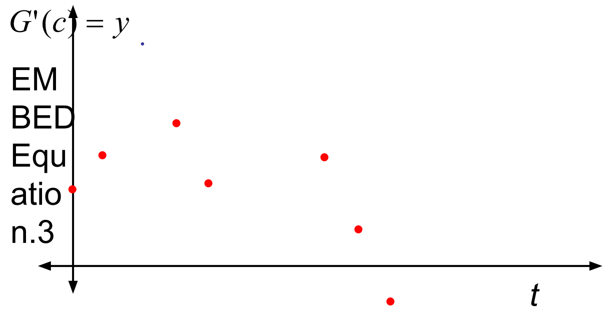
Learning Activity #2 -- KEY

If the function $G'(t)$ is continuous on [a, b], and *y* is a number between $G'(a)$ and $G'(b)$, then there exists at least one number *c* in the open interval (a, b) such that $G'(c) = y$.



(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
CONTENT	MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.6 examine solutions from many perspectives 4.1 support details
DOK	3
INSTRUCTIONAL STRATEGIES	Cues, Questions and Advanced Organizers

Student Resources	Teacher Resources
General: **Enrichment:**	**General:** **Enrichment:**

Intervention:	**Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 4**
Learner Objectives: Students will calculate, interpret and analyze derivatives		

Concepts: A. Concept of the derivative

Students Should Know	Students Should Be Able to
• Relationship between differentiability and continuity	• Find a derivative presented graphically, numerically and analytically • Find a derivative interpreted as an instantaneous rate of change • Find a derivative defined as the limit of the difference quotient • **Find derivatives of parametric, polar and vector functions**

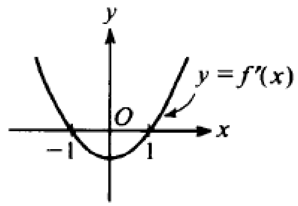
Instructional Support

Student Essential Vocabulary					
Differentiability	Average Rate of Change	Instantaneous Rate of Change	Difference Quotient	Higher-Order Derivative	Implicit Differentiation
Parametric Form	**Polar Form**	**Vector Form**	**Velocity Vector**	**Acceleration Vector**	

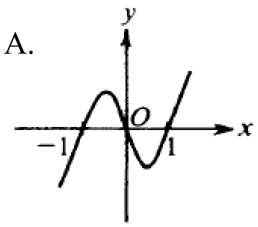
Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

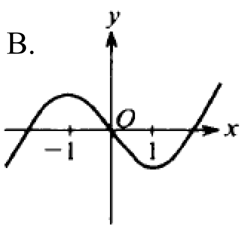
Sample Learning Activities	Sample Assessments

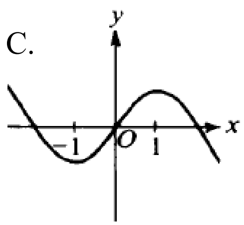
Learning Activity #1 :

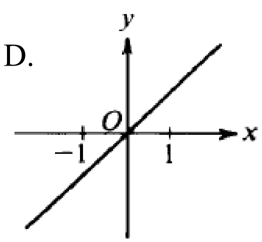


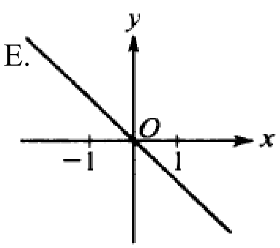
1. The graph of the <u>derivative</u> of *f* is shown in the figure above. Which of the following could be the graph of *f* ?

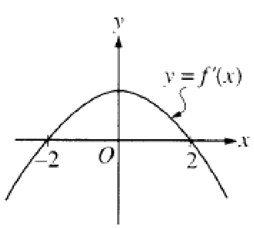
A. 

B. 

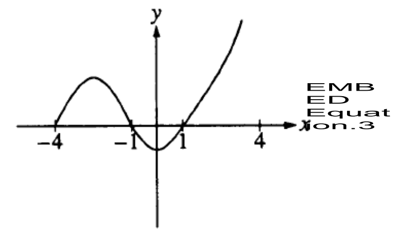
C. 

D. 

E. 

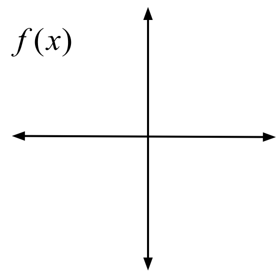


Assessment #1:

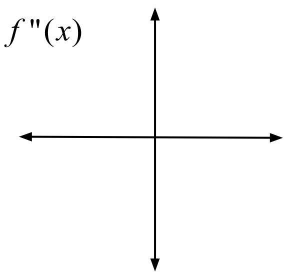


Given the graph of $f'(x)$ shown above, on the axes provided, sketch graphs that could be $f(x)$ and $f''(x)$.

a. $f(x)$

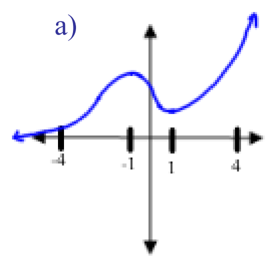


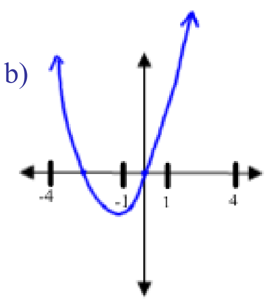
b. $f''(x)$



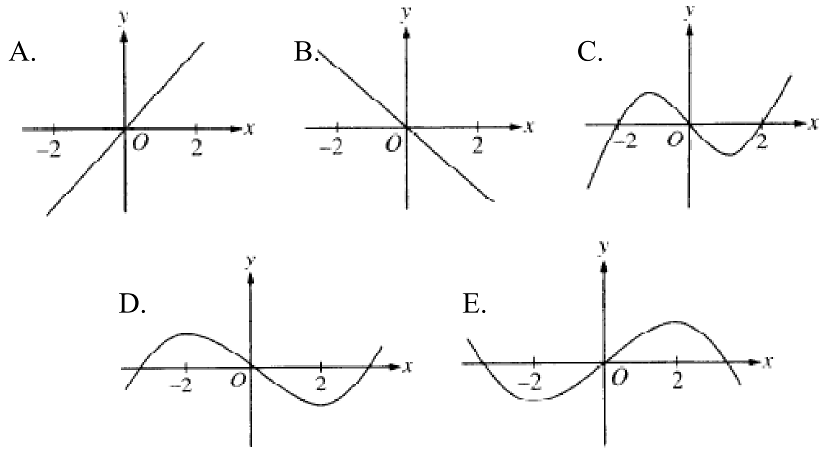
Assessment #1 – KEY

Graphs may vary

a) 

b) 

2. The graph of the derivative of *f* is shown in the figure above. Which of the following could be the graph of *f* ?



Learning Activity #1 – KEY

AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA1 number sense MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation

1) B 2) E

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA1 number sense MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
LEVEL OF EXPECTATION	Mastery level – 85%

Sample Learning Activities	Sample Assessments

Learning Activity #2 :

The Definition of a Derivative can also be written like this

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

On the AP exam, you may need to recognize this alternate form of the derivative. Use the two forms to find the value of each:

1. $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$
2. $\lim_{h \to 0} \frac{5(x+h)^4 - 5x^4}{h}$
3. $\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2}+h\right) - \cos\frac{\pi}{2}}{h}$
4. $\lim_{h \to 0} \frac{\tan(\pi+h) - \tan\pi}{h}$
5. $\lim_{h \to 0} \frac{\sin(\pi+h) - \sin\pi}{h}$
6. $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$ where $f(x) = 5(2x+3)^2$

Learning Activity #2 – KEY

1) $3x^2$ 2) $20x^3$ 3) $-\sin\left(\frac{\pi}{2}\right) = -1$

4) $\sec^2(\pi) = 1$ 5) $\cos\pi = -1$ 6) $f'(x) = 20(2x+3)$

Assessment #2:

1. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?
   I. f is continuous at $x = 3$.
   II. f is differentiable at $x = 3$.
   III. $f(3) = 7$.

   A. None B. II only C. III only
   D. I and III only E. I, II, and III

2. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is
   A. undefined.
   B. continuous but not differentiable.
   C. differentiable but not continuous.
   D. neither continuous nor differentiable.
   E. both continuous and differentiable.

3. If f is a differentiable function, then $f'(a)$ is given by which of The following?
   I. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
   II. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
   III. $\lim_{x \to a} \frac{f(x+h) - f(x)}{h}$

$f'(2) = 140$

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA1 MA2 MA4	number sense geometric and spatial patterns and relationships
PROCESS	1.6 1.10 3.5 3.6 4.1	discover/evaluate relationships apply information, ideas and skills reason logically (inductive/deductive) examine solutions from many perspectives support details
DOK	2	
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation	

A. I only B. II only C. I and II only
D. I and III only E. I, II, and III

4. $\lim_{h\to 0} \dfrac{\tan 3(x+h) - \tan 3x}{h} =$

A. 0 B. $3\sec^2(3x)$ C. $\sec^2(3x)$
D. $3\cot(3x)$ E. nonexistent

5. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

A. 0 B. $\frac{1}{2}$ C. 1 D. The limit does not exist.
E. Cannot be determined from the information given.

Assessment #2 – KEY

1) A 2) E 3) E 4) B 5) B

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA1 MA2 MA4	number sense geometric and spatial patterns and relationships

PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
LEVEL OF EXPECTATION	Mastery level – 75%

Learning Activity #3:

Differentiation and Parametric Form

Parametric Form of the Derivative –

If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of C at (x, y) is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \qquad \frac{dx}{dt} \neq 0.$$

Parametric Form of Higher-order Derivatives –

Furthermore,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt} \quad \text{and} \quad \frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{\frac{d}{dt}\left[\frac{d^2y}{dx^2}\right]}{dx/dt}$$

1. Find dy/dx for the curve given by $x = \sin t$ and $y = \cos t$.

Assessment #3:

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, and find the slope and concavity (if possible) at the given point.

   $x = t^2 + 3t + 2$ and $y = 2t$ at the point where $t = 0$

2. Consider the parametric equations $x = 2\cot(t)$ and $y = 2\sin^2 t$. Find each of the following:

   a. $\frac{dx}{dt} =$

   b. $\frac{dy}{dt} =$

   c. $\frac{dy}{dx} =$

   d. $\frac{d^2y}{dx^2} =$

2. For the curve given by $x = \sqrt{t}$ and $y = \frac{1}{4}\left(t^2 - 4\right)$, $t \geq 0$; find the slope and concavity at the point (2, 3).

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA1 number sense MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
INSTRUCTIONAL STRATEGIES	Guided Practice

d. Write an equation of the line tangent to the graph of the parametric equations at each of the following points on the curve:

(i) $t = \frac{2\pi}{3}$ (ii) $t = \frac{\pi}{2}$ (iii) $t = \frac{\pi}{4}$

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA1 number sense MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support details
DOK	2
LEVEL OF EXPECTATION	Mastery level –85%

Student Resources	Teacher Resources
General:	**General:**
Enrichment:	**Enrichment:**
Intervention:	**Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 5**
Learner Objectives: Students will calculate, interpret and analyze derivatives		

Concepts: B. Derivative at a point

Students Should Know	**Students Should Be Able to**
• Slope of a curve at a point – points that are vertical tangent, points at which there are no tangent	• Tangent line to a curve at a point • **Numerical solution of differential equations using Euler's method** • Instantaneous rate of change as the limit of average rate of change • Approximate rate of change from graphs and tables of values

Instructional Support

Student Essential Vocabulary					
Tangent Line	Instantaneous Rate of Change	Secant Line	Average Rate of Change	Point-Slope Form of a Linear Equation	**Euler's Method**
Numerical Approximation					

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments
Learning Activity #1 : 1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$ A. -6 B. -3 C. 3 D. 6 E. 8 2. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is A. $y = 2x + 1$ B. $y = x + 1$ C. $y = x$ D. $y = x - 1$ E. $y = 0$ **Learning Activity #1 – KEY** 1) C 2) B	**Assessment #1:** 1. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$ A. 3 B. 1 C. -1 D. -3 E. -5 2. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point (1, 5) is A. $13x - y = 8$ B. $13x + y = 18$ C. $x - 13y = 64$ D. $x + 13y = 66$ E. $-2x + 3y = 13$ **Assessment #1 – KEY**

1) D 2) B

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 3	Differential calculus
CONTENT	MA1	number sense
	MA2	geometric and spatial
	MA4	patterns and relationships
PROCESS	1.6	discover/evaluate relationships
	1.10	apply information, ideas and skills
	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
DOK	2	
INSTRUCTIONAL STRATEGIES	Homework and Practice	

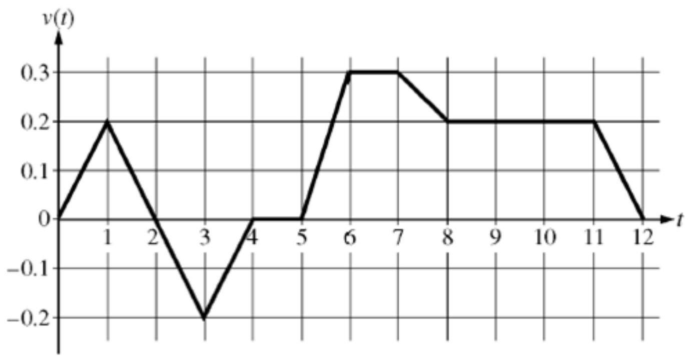
Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 3	Differential calculus
CONTENT	MA1	number sense
	MA2	geometric and spatial
	MA4	patterns and relationships
PROCESS	1.6	discover/evaluate relationships
	1.10	apply information, ideas and skills
	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 80%	

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #2

1.



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes.

During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

a. Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

b. Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

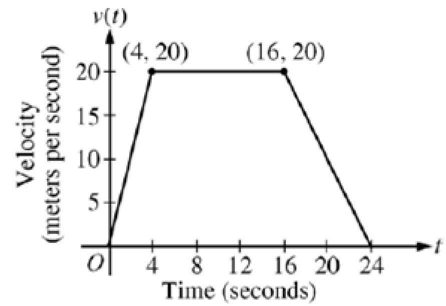
2.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by the twice differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

Assessment #2:

1.



For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

2.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius $(^{\circ}C)$, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

Assessment #2 – KEY

Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 PM ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.

Learning Activity #2 – KEY

1a) $a(7.5) = v'(7.5) = \frac{-.1}{1} = -0.1$ miles per minute

1b) since $v(t) = 0 \ @ \ t = 2$ and $v(t)$ changes from positive to negative,

Caren turns around at $t = 2$ minutes

2) $L'(t) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8$ people per hour

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA1 MA2 MA4	number sense geometric and spatial patterns and relationships
PROCESS	1.6 1.10 3.5 3.6	discover/evaluate relationships apply information, ideas and skills reason logically (inductive/deductive) examine solutions from many perspectives
DOK	2	
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation	

1) $v'(4)$ does not exist since on (0, 4) $v'(t) = 5$ and on (4, 16) $v'(t) = 0$ and $5 \neq 0$.

$v'(20) = \frac{-20}{8} = -2.5$ meters per second per second

2) $T'(7) \approx \frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{2} = -3.5$ degrees Celsius per centimeter

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA1 MA2 MA4	number sense geometric and spatial patterns and relationships
PROCESS	1.6 1.10 3.5 3.6	discover/evaluate relationships apply information, ideas and skills reason logically (inductive/deductive) examine solutions from many perspectives
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 70%	

Learning Activity #3:

Euler's Method –

A numerical approach to approximating the particular solution (x_i, y_i) of the differential equation $y' = F(x, y)$ that passes through the point (x_0, y_0). From the given information, you know that the graph of the solution passes through the point (x_0, y_0) and has a slope of $F(x_0, y_0)$ at this point. This gives you a "starting point" for approximating the solution.

From this point, you can proceed in the direction indicated by the slope. Using a step *h*, move along the tangent line until you arrive at the point (x_i, y_i), using the fact that where $x_n = x_{n-1} + h$, $y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$.

1. Use Euler's Method to approximate y (1) for the differential equation $y' = x - y$ passing through the point (0.5, 1). Use a step of $h = 0.1$.

2. Given $y' = 2x + \frac{y}{2}$, find an approximation for $y(2)$, given $y(1) = 3$. Use 5 iterations of Euler's Method with equal step sizes.

Learning Activity #3 – KEY

1. $y' = x - y$ and $y(0.5)=1$.

Assessment #3:

1. AP Free Response 2005 – Q4c

   Consider the differential equation $\frac{dy}{dx} = 2x - y$.

   Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

2. AP Free Response 2007B – Q5 c & d

   Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

   c. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.

Let $x_0 = 0.5$ and $y_0 = 1$, using $h = 0.1$ we get

$x_1 = 0.6 \Rightarrow y_1 = 1 + 0.1[.5 - 1] = 0.95$

$\Rightarrow x_2 = 0.7 \Rightarrow y_2 = 0.95 + 0.1[0.6 - 0.95] = 0.915$

$\Rightarrow x_3 = 0.8 \Rightarrow y_3 = 0.915 + 0.1[0.7 - 0.915] = 0.8935$

$\Rightarrow x_4 = 0.9 \Rightarrow y_4 = 0.8935 + 0.1[0.8 - 0.8935] = 0.8842$

$\Rightarrow x_5 = 1.0 \Rightarrow y_5 = 0.8842 + 0.1[0.9 - 0.8842] = 0.88578$

Therefore $y(1) \approx 0.886$

2. $y' = 2x + \frac{y}{2}$ and $y(1) = 3$.

Let $x_0 = 1$ and $y_0 = 3$, using $h = \frac{2-1}{5} = 0.2$ we get

$x_1 = 1.2 \Rightarrow y_1 = 3 + 0.2[2(1) + 3/2] = 3.7$

$\Rightarrow x_2 = 1.4 \Rightarrow y_2 = 3.7 + 0.2[2(1.2) + 3.7/2] = 4.55$

$\Rightarrow x_3 = 1.6 \Rightarrow y_3 = 4.55 + 0.2[2(1.4) + 4.55/2] = 5.565$

$\Rightarrow x_4 = 1.8 \Rightarrow y_4 = 5.565 + 0.2[2(1.6) + 5.565/2] = 6.7615$

$\Rightarrow x_5 = 2.0 \Rightarrow y_5 = 6.7615 + 0.2[2(1.8) + 6.7615/2] = 8.15765$

Therefore $y(2) \approx 8.158$

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 2 Standard 6	Analysis of functions Model numerically/analytically Differential equations/slope fields
CONTENT	MA 1 MA 4 MA 5	number sense patterns and relationships mathematical systems

d. Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k.

Assessment #3 – KEY

1. $f(-0.2) \approx f(0) + f'(0)(-0.2) = 1 + (-1)(-0.2) = 1.2$

$f(-0.4) \approx f(-0.2) + f'(-0.2)(-0.2) \approx 1.2 + (-1.6)(-0.2) = 1.52$

2. c. $f(\tfrac{1}{2}) \approx f(0) + f'(0) \cdot \tfrac{1}{2} = -2 + (-3) \cdot \tfrac{1}{2} = -\tfrac{7}{2}$

$f'(\tfrac{1}{2}) \approx 3(\tfrac{1}{2}) + 2(-\tfrac{7}{2}) + 1 = -\tfrac{9}{2}$

$f(1) \approx f(\tfrac{1}{2}) + f'(\tfrac{1}{2}) \cdot \tfrac{1}{2} = -\tfrac{7}{2} + (-\tfrac{9}{2}) \cdot (\tfrac{1}{2}) = -\tfrac{23}{4}$

d. $g'(0) \approx 3(0) + 2(k) + 1 = 2k + 1$

$g(1) \approx g(0) + g'(0) \cdot 1 = k + (2k + 1) = 3k + 1 = 0$

$k = -\tfrac{1}{3}$

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 2 Standard 6	Analysis of functions Model numerically/analytically Differential equations/slope fields
CONTENT	MA 1	number sense

PROCESS	1.6 discover/evaluate relationships 1.8 organize data and ideas 3.2 apply others' strategies 3.5 reason logically (inductive/deductive)
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation

	MA 4 patterns and relationships MA 5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 1.8 organize data and ideas 3.2 apply others' strategies 3.5 reason logically (inductive/deductive)
DOK	2
LEVEL OF EXPECTATION	Mastery level – 85%

General:

Enrichment:

Intervention:

General:

Enrichment:

Intervention:

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	Course: AP Calculus BC	Strand: 6
Learner Objectives: **Students will calculate, interpret and analyze derivatives**		

Concepts: C. Derivative as a function

Students Should Know	Students Should Be Able to
● Corresponding characteristics of graphs of *f* and *f''* ● Equations involving derivatives	● Relationship between the increasing and decreasing behavior of *f* and the sign of *f'* ● The Mean Value Theorem and its geometric interpretation ● Utilize relationships between *f* and *f'* to determine relative extrema

Instructional Support

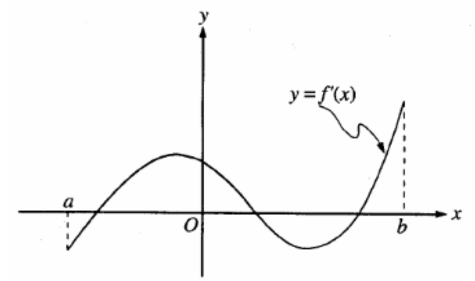
Student Essential Vocabulary					
Increasing	Decreasing	Absolute Extrema	Relative Extrema	Tangent Line	Secant Line

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA =** Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #1 :

1.



The graph of f', the derivative of f, is shown above. Which of the following describes all relative extrema of f on the open interval (a, b)?

A. One relative maximum and two relative minima
B. Two relative maxima and one relative minimum
C. Three relative maxima and one relative minimum
D. One relative maximum and three relative minimum
E. Three relative maxima and two relative minima

2. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

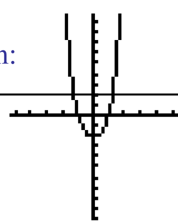
A. $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ C. $(0, \infty)$

D. $(-\infty, 0)$ E. $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

Now, graph f to verify your conclusion.

Learning Activity #1 – KEY

1) A 2) C Graph:



Assessment #1:

The function f given by $f(x) = x^3 + 12x - 24$ is

A. Increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$.
B. Decreasing for $x < 0$, increasing for $x > 0$.
C. Increasing for all x.
D. Decreasing for all x.
E. Decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$.

Assessment #1 – KEY

Choice C

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA1 number sense MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA1 number sense MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support decisions
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #2 :

See "Relating a Function and its Derivative" activity cards. Appendix : A

Learning Activity #2 – KEY

See Appendix

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA2 MA4	geometric and spatial patterns and relationships
PROCESS	1.6 1.10 3.5 3.6 4.1	discover/evaluate relationships apply information, ideas and skills reason logically (inductive/deductive) examine solutions from many perspectives support decisions
DOK	2	
INSTRUCTIONAL STRATEGIES	Nonlinguistic representation	

Assessment #2:

The ***derivative*** of *f* is $x^4(x-2)(x+3)$. At how many points will the graph of *f* have a relative maximum?

A. None B. One C. Two D. Three E. Four

Assessment #2 – KEY

B

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA2 MA4	geometric and spatial patterns and relationships
PROCESS	1.6 1.10 3.5 3.6 4.1	discover/evaluate relationships apply information, ideas and skills reason logically (inductive/deductive) examine solutions from many perspectives support decisions
DOK	2	
LEVEL OF EXPECTATION	Mastery level –75%	

Student Resources	Teacher Resources
General: **Enrichment:**	**General:** **Enrichment:**

Intervention:	**Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Learner Objectives: Students will calculate, interpret and analyze derivatives

Concepts: D. Second derivatives

Students Should Know	Students Should Be Able to
• Corresponding characteristics of the graphs of *f*, *f'*, and *f"* • Points of inflection as places where concavity changes	• Relationship between the concavity of *f* and the sign of *f"*

Instructional Support

Student Essential Vocabulary					
Point of Inflection	Concave Upward	Concave Downward			

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments
Learning Activity #1 : 1. What is the x-coordinate of the point of inflection on the graph of $y=\frac{1}{3}x^3+5x^2+24$? A. 5 B. 0 C. $-\frac{10}{3}$ D. -5 -10 2. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$ A. -1 only B. 2 only C. -1 and 0 only D. -1 and 2 only E. -1, 0 and 2 only 3. The graph of the function $y=x^3+6x^2+7x-2\cos x$ changes concavity at $x=$ A. -1.58 B. -1.63 C. -1.67 D. -1.89 E. -2.33 **Learning Activity #1 – KEY** 1) D 2) C 3) D	**Assessment #1:** 1. Identify all intervals on which the graph of the function $f(x)=\frac{4}{3}x^3-x^2-3x$ is either concave up or concave down. 2. Find all ***points*** of inflection for the graph of the function $f(x)=\frac{1}{2}x^4-4x^3+x-1$. **Assessment #1 – KEY** 1) $f'(x)=4x^2-2x-3 \Rightarrow f''(x)=8x-2 \Rightarrow f''(x)=0$ @ $x=\frac{1}{4}$ The graph of $f(x)$ is concave upward on $\left(\frac{1}{4},\infty\right)$ since $f''(x)>0$ on the interval and the graph of $f(x)$ is concave downward on $\left(-\infty,\frac{1}{4}\right)$ since $f''(x)<0$ on the interval. 2) $f'(x)=2x^3-12x^2+1 \Rightarrow f''(x)=6^2x-24x \Rightarrow f''(x)=0$ @ $x=$ 0 and $x=4$. Points of inflection exist at (0, -1) and (4, -125) since the concavity of the graph changes at that location. We know this since $f''(x)>0$ on the intervals $(-\infty,-1)$ and (0, 2) and $f''(x)<0$ on the intervals (-1, 0) and $(2,\infty)$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support decisions
DOK	2
INSTRUCTIONAL STRATEGIES	Homework and Practice

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery level – 70%

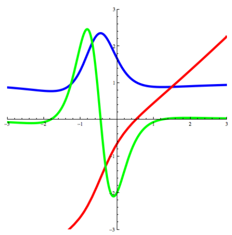
Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities

Learning Activity #2 :

1. See "It's a Match-up" activity cards. Appendix : B

2. For the graph shown at the right, determine which graph represents f , f', and f'' .



Learning Activity #2 – KEY

1) See Appendix

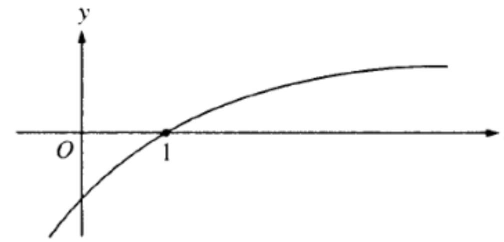
2) *f* is the red graph, *f* ' is the blue graph and *f* " is the green graph

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 3	Analysis of functions Differential calculus
CONTENT	MA2 MA4	geometric and spatial patterns and relationships
PROCESS	1.6 1.10 3.5 3.6 4.1	discover/evaluate relationships apply information, ideas and skills reason logically (inductive/deductive) examine solutions from many perspectives support decisions
DOK	3	

Sample Assessments

Assessment #2:

1. The graph of a twice-differentiable function *f* is shown in the figure at the right. Which of the following is true?



A. $f(1) < f'(1) < f''(1)$

B. $f(1) < f''(1) < f'(1)$

C. $f'(1) < f(1) < f''(1)$

D. $f''(1) < f(1) < f'(1)$

E. $f''(1) < f'(1) < f(1)$

2. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

A. $x < 0$

B. $x > 0$

C. $x < -2$ or $x > -\frac{2}{3}$

D. $\frac{2}{3} < x < 2$

E. $x < \frac{2}{3}$ or $x > 2$

Assessment #2 – KEY

1) D 2) D

INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 3 Differential calculus
CONTENT	MA2 geometric and spatial MA4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 4.1 support decisions
DOK	3
LEVEL OF EXPECTATION	Mastery level – 75%

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Student Resources	**Teacher Resources**
General:	**General:**
Enrichment:	**Enrichment:**
Intervention:	**Intervention:**

Content Area: Mathematics	Course: AP Calculus BC	Strand: 8
Learner Objectives: Students will calculate, interpret and analyze derivatives		

Concepts: E. Applications of derivatives

Students Should Know	Students Should Be Able to
● All theorems, properties and relationships needed to apply the concepts of functions and their first and second derivatives.	● Analysis of curves, including the notions of monotonicity and concavity ● Optimization, both absolute (global) and relative (local) extrema ● Modeling rates of change, including related rates problems ● Use of implicit differentiation to find the derivative of an inverse functions ● Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration ● **Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration** ● Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Instructional Support

Student Essential Vocabulary					
Position Function	Velocity Function	Acceleration Function	Relative Extrema	Absolute Extrema	Monotonic
Optimize	Slope Field	Differential Equation	General Solution	Particular Solution	Related Rates
Implicitly Defined Functions	Explicitly Defined Functions	**Parametric Form**	**Polar Form**	**Vector Form**	**Velocity Vector**
Acceleration Vector					

Readness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

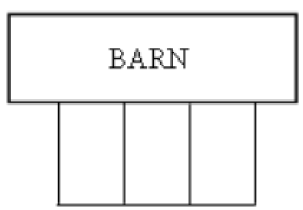
Sample Learning Activities

Learning Activity #1 :

1. If $y = 2x - 8$, what is the minimum value of the product *xy*?

   A. -16 B. -8 C. -4 D. 0 E. 2

2. A farmer has 80 feet of fence and wants to make three identical pens. No fence will be needed on one side since the pens will attach to the barn as shown in the diagram. What dimensions (for the total enclosure) will make the area of the pens as large as possible?



3. A manufacturer wants to design an open rectangular box with a volume of 256 square inches. What dimensions will produce a box that will require the least amount of material to produce it?

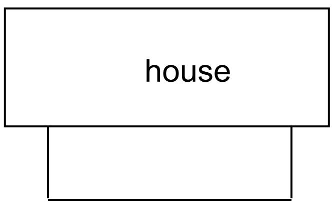
Learning Activity #1 – KEY

Sample Assessments

Assessment #1:

1. Find two rational numbers whose product is 192 and whose sum is a minimum.

2. A family plans to fence in their backyard in order for their dog to be able to run free. They will attach the fence to the back of their house as shown in the diagram. They want the dog to have 800 square feet of area in which to run. How much fence should they purchase in order to use the least fence?



3. A manufacturer wants to design an open rectangular box having a square base and a surface area of 108 square inches. What dimensions will produce a box of maximum volume?

1) B

2) The dimensions of the total enclosure that has a maximum area are 40 feet by 10 feet (the 40 foot side runs parallel to the side of the barn).

3) The dimensions of the box of maximum volume is 8 inches by 8 inches by 4 inches.

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 4	Position, speed, acceleration
	Standard 5	Related rates
	Standard 6	Differential equations/slope fields
CONTENT	MA 1	number sense
	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.7	evaluate information
	1.10	apply information, ideas and skills
	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
	3.7	evaluate strategies
DOK	2	
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation	

Assessment #1 – KEY

1) The numbers are both $8\sqrt{3}$.

2) The dimensions of the enclosure with the minimum amount of fence are 40 feet by 20 feet (the 40 foot side runs parallel to the house).

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 4	Position, speed, acceleration
	Standard 5	Related rates
	Standard 6	Differential equations/slope fields
CONTENT	MA 1	number sense
	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.7	evaluate information
	1.10	apply information, ideas and skills
	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
	3.7	evaluate strategies
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 75%	

Readiness & Equity Section
SLA = Sample Learning Activities & **SA** = Sample Assessments

21^{st} Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities

Learning Activity #2

1. The radius of a circle is decreasing at a constant rate of 0.1 centimeters per second. In terms of the circumference *C*, what is the rate of change of the area of the circle, in square centimeters per second?

   A. $-(0.2)\pi C$ B. $-(0.1)C$ C. $-\frac{(0.1)C}{2\pi}$ D. $(0.1)^2 C$ E. $(0.1)^2 \pi C$

2. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer when it is 100 meters past the intersection?

   A. 49.15 B. 57.60 C. 57.88 D. 59.20 E. 67.40

Learning Activity #2 – KEY

1) B 2) A

Activity's Alignment

Sample Assessments

Assessment #2:

1. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

   A. $\frac{1}{\pi}$ B. $\frac{1}{2}$ C. $\frac{2}{\pi}$ D. 1 E. 2

2. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

   A. $-\frac{7}{8}$ ft per min B. $-\frac{7}{24}$ ft per min C. $\frac{7}{24}$ ft per min D. $\frac{7}{8}$ ft per min E. $\frac{21}{25}$ ft per min

3. Population grows according to the equation $\frac{dy}{dt} = ky$, where *k* is a constant and *t* is measured in years. If the population doubles every 10 years, then the value of *k* is

   A. 0.069 B. 0.200 C. 0.301 D. 3.322 E. 5.000

AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 4	Position, speed, acceleration
	Standard 5	Related rates
	Standard 6	Differential equations/slope fields
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	1.10	apply information, ideas and skills
	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
	3.7	evaluate strategies
DOK	3	
INSTRUCTIONAL STRATEGIES	Homework and practice	

Assessment #2 – KEY

1) D 2) D 3) A

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 4	Position, speed, acceleration
	Standard 5	Related rates
	Standard 6	Differential equations/slope fields
CONTENT	MA 1	number sense
	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.7	evaluate information
	1.10	apply information, ideas and skills
	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
	3.7	evaluate strategies
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 75%	

Sample Learning Activities	Sample Assessments

Learning Activity #3

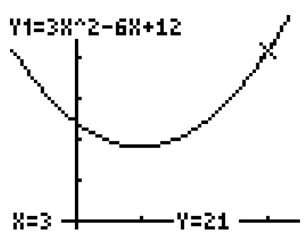
The maximum acceleration on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

A. 9 B. 12 C. 14 D. 21 E. 40

Now, graph the acceleration function and determine which characteristics of the graph support your answer.

Learning Activity #3 – KEY

$a(t) = 3t^2 - 6t + 12$ critical number for $a(t)$ (when $a'(t)$ = 0) is $t = 1$.
$a(0) = 12$, $a(1) = 9$, $a(3) = 21$ so maximum acceleration on the interval [0, 3] is 21.



Assessment #3:

A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

A. -5.19 B. 0.74 C. 1.32 D. 2.55 E. 8.13

Note: Be sure to show equations for velocity and acceleration in the work you do.

Assessment #3 – KEY

D -- see justification below

$v(t) = s'(t) = 4\sin t - t$ $a(t) = v'(t) = s''(t) = 4\cos t - 4$

$a(t) = 0 \Rightarrow \cos t = 0.25 \Rightarrow t \approx 1.318$

$v(1.318) = 4\sin(1.318) - 1.318 \approx 2.55$

Assessment's Alignment

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 4 Position, speed, acceleration Standard 5 Related rates Standard 6 Differential equations/slope fields
CONTENT	MA 1 number sense MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 3.7 evaluate strategies
DOK	3
INSTRUCTIONAL STRATEGIES	Homework and practice

AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 4 Position, speed, acceleration Standard 5 Related rates Standard 6 Differential equations/slope fields
CONTENT	MA 1 number sense MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 3.7 evaluate strategies
DOK	2
LEVEL OF EXPECTATION	Mastery level –75%

Sample Learning Activities	Sample Assessments

Learning Activity #4 :

Free Response Question 3 – 2010 Exam

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

a. Find the speed of the particle at time $t = 3$ seconds.

b. Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

c. Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

d. There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.
   (i) The two values of t when that occurs.
   (ii) The slopes of the lines tangent to the particle's path at that point.
   (iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$.

Learning Activity #4 – KEY

a. speed = $\sqrt{(x'(3))^2 + (y'(3)^2)} = 2.828$ meters per second

b. $x'(t) = 2t - 4$

Distance = $\int_0^4 \sqrt{(2t-4)^2 + (te^{t-3} - 1)^2}\, dt$ = 11.587 or 11.588 meters

Assessment #4:

1. In the xy-plane, the graph of the parametric equations $x = 5t + 2$, and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

   A. $\frac{3}{5}$ B. $\frac{5}{3}$ C. 3 D. 5 E. 13

2. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^3 - t$ and its y-coordinate is $(2t-1)^3$. The acceleration vector of the particle at $t = 1$ is

   A. (0, 1) B. (2, 3) C. (2, 6) D. (6, 12) E. (6, 24)

3. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

   A. $\int_0^1 \sqrt{t^2 + 1}\, dt$ B. $\int_0^1 \sqrt{t^2 + t}\, dt$ C. $\int_0^1 \sqrt{t^4 + t^2}\, dt$

   D. $\frac{1}{2}\int_0^1 \sqrt{4 + t^4}\, dt$ E. $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\, dt$

Assessment #4 – KEY

1. A
2. E
3. C

c. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \neq 0$. This occurs at

$t = 2.20794$.

Since $x'(2.20794) > 0$, the particle is moving toward the right at time

$t = 2.207$ or 2.208.

d. $x(t) = 5$ at $t = 1$ and $t = 3$

At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = 0.432.$

At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3} = 1$.

$Y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\,dt = 4$.

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 4	Position, speed, acceleration
	Standard 9	Integral calculus
CONTENT	MA 1	number sense
	MA 5	mathematical systems

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 4	Position, speed, acceleration
	Standard 9	Integral calculus
CONTENT	MA 1	number sense
	MA 5	mathematical systems
PROCESS	1.6	discover/evaluate relationships
	3.2	apply other's strategies
	3.4	evaluate problem-solving processes
DOK	2	
LEVEL OF EXPECTATION	Mastery level –80%	

PROCESS	1.6 discover/evaluate relationships 3.2 apply other's strategies 3.4 evaluate problem-solving processes	
DOK	2	
INSTRUCTIONAL STRATEGIES	Nonlinguistic Representation	

Student Resources	Teacher Resources
General:	**General:**
Enrichment:	**Enrichment:**
Intervention:	**Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 9**
Learner Objectives: The student will calculate, interpret, and apply integrals		

Concepts: A. Calculate definite integrals

Students Should Know	Students Should Be Able to
● Definite integral as limit of Riemann sums ● Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f' \int_a^b f' (x)dx = f(b) - f(a)$	● Basic properties of definite integrals – examples include additivity and linearity

Instructional Support

Student Essential Vocabulary					
Definite Integral	Riemann sum	Antiderivative	Differentiate	Integrate	Limiting Behavior
Upper Limit of Integration	Lower Limit of Integration				

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #1 :

If $\int_0^5 f(x)dx = 10$, $\int_5^7 f(x)dx = 3$, and $\int_3^5 f(x)dx = 2$, find the value of each.

1. $\int_0^7 f(x)dx$

2. $\int_5^0 f(x)dx$

3. $\int_5^5 f(x)dx$

4. $\int_0^5 f(x)dx$

5. $\int_0^3 f(x)dx$

Learning Activity #1 – KEY

1) 13 2) -10 3) 0 4) 30 5) 8

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 3 Standard 6 Standard 9 Standard 10	Differential calculus Differential equations/slope fields Integral calculus Area and volume
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships

Assessment #1:

If *f* is an ***<u>even</u>*** function, then $\int_{-2}^{2} f(x)dx$ is always equal to

A. $-\int_{-2}^{2} f(x)dx$ B. $\int_0^2 f(x)dx$ C. $2\int_0^2 f(x)dx$

D. $\int_0^4 f(x)dx$ E. $\frac{1}{2}\int_0^4 f(x)dx$

Assessment #1 – KEY

C

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 3 Standard 6 Standard 9 Standard 10	Differential calculus Differential equations/slope fields Integral calculus Area and volume
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships
PROCESS	1.7 1.10 3.4 3.7	evaluate information apply information, ideas and skills evaluate problem-solving processes evaluate strategies
DOK	2	
LEVEL OF EXPECTATION	Mastery level –80%	

PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.4 evaluate problem-solving processes 3.7 evaluate strategies
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic representation

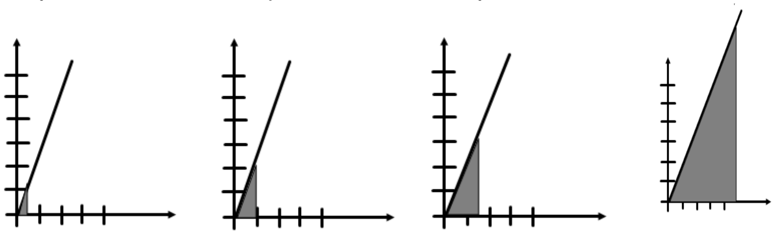
Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	**Sample Assessments**

Learning Activity #2 :

Evaluate the following by using geometric area formulas:

$\int_0^{0.5} 2x\,dx$ $\int_0^{1} 2x\,dx$ $\int_0^{1.5} 2x\,dx$ $\int_0^{t} 2x\,dx$



$A(0.5) =$ $A(1) =$ $A(1.5) =$ $A(\text{t}) =$

Learning Activity #2 – KEY

$A(0.5) = 0.25$, $A(1) = 1$, $A(1.5) = 2.25$, $A(t) = x^2$

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 3 Standard 6 Standard 9 Standard 10	Differential calculus Differential equations/slope fields Integral calculus Area and volume
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships
PROCESS	1.7 1.10 3.4 3.7	evaluate information apply information, ideas and skills evaluate problem-solving processes evaluate strategies
DOK	2	

Assessment #2:

If $\int_1^3 f(x)dx = 5$, find each of the following values:

1. $\int_1^3 [f(x) - 7]\,dx =$

A. -14 B. -9 C. -5 D. -2 E. 3

2. $\int_{-1}^{1} f(x+2)\,dx =$

A. -2 B. 1 C. 5 D. 7 E. 10

Assessment #2 – KEY

1) B 2) C

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 3 Standard 6 Standard 9 Standard 10	Differential calculus Differential equations/slope fields Integral calculus Area and volume
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships
PROCESS	1.7 1.10 3.4 3.7	evaluate information apply information, ideas and skills evaluate problem-solving processes evaluate strategies
DOK	3	

INSTRUCTIONAL STRATEGIES	Nonlinguistic representation	LEVEL OF EXPECTATION	Mastery level –75%

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

Content Area: Mathematics	Course: AP Calculus BC	Strand: 10
Learner Objectives: **The student will calculate, interpret, and apply integrals**		

Concepts: B. Fundamental Theorem of Calculus

Students Should Know	Students Should Be Able to
• The relationship between a function and its antiderivative $\int_a^b f(x)dx = F(b) - F(a)$ • $\frac{d}{dx}\int_a^{f(x)} g(t)dt = g(f(x)) \cdot f'(x)$	• Use the Fundamental Theorem of Calculus to evaluate definite integrals. • Use the Fundamental Theorem of Calculus to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Instructional Support

Student Essential Vocabulary					
Antiderivative	Definite Integral	Chain Rule	Particular Antiderivative	Upper Limit of Integration	Lower Limit of Integration
Derivative of a Function Defined by an Integral					

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	

Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities

Learning Activity #1 :

Evaluate the following definite integrals by finding the antiderivative of the integrand.

1. $\int_1^4 \left(3\sqrt{x} + x\right) dx$

2. $\int_0^2 \left|2x - 1\right| dx$

3. Find the area bounded by the function and the x-axis:

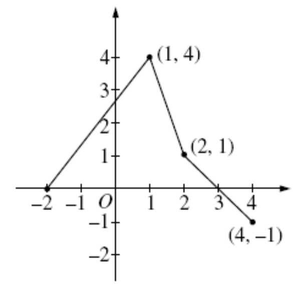
a. $f(x) = 4 - x^2$ b. $g(x) = \sin x$ on the interval $[0, 2\pi]$

Learning Activity #1 – KEY

1) 21.5   2) 2.5   3a) $\frac{32}{3}$ b) 4

Sample Assessments

Assessment #1:



1. The graph of the function f, consisting of three line segments is shown at the right.

Let $g(x) = \int_1^x f(t)\,dt$

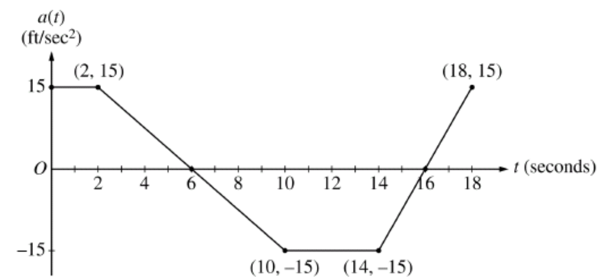
a. Compute $g(4)$ and $g(-2)$.

b. Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

c. Find the absolute minimum value of g on the closed interval [-2, 4]. Justify your answer.

d. The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 3 Differential calculus Standard 6 Differential equations/slope fields Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.4 evaluate problem-solving processes 3.7 evaluate strategies
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic representation



2. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec ^{2}, is the piecewise linear function defined by the graph above.

a. Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

b. At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

c. On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

d. At what time in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

Assessment #1 – KEY

1a) $g(4) = 2.5$, $g(-2) = -6$

1b) $g'(1) = f(1) = 4$

1c) $x = 3$ is the only critical number, where $g'(x) = f(x) = 0$, so the only candidates for an absolute minimum are $g(-2)$, $g(3)$, and $g(4)$. Because $g(-2) = -6$, $g(3) = 3$, and $g(4) = 2.5$, $g(-2) = -6$ is the absolute minimum on this interval.

1d) For a point of inflection, $g''(x) = f`(x)$ much change sign, so the graph of $g'(x) = f(x)$ must change from increasing to decreasing or from decreasing to increasing. This only occurs at $x = 1$, so only at $x = 1$ is there a point of inflection for g.

2a) Yes, because $v'(t) = a(t)$ is positive at $t = 2$.

2b) At $t = 12$, because $\int_0^{12} a(t)\,dt = 0$. Thus, $v(12) = v(0) + \int_0^{12} a(t)\,dt$ = 55

2c) The absolute maximum velocity on [0, 18] must occur at an endpoint or at a critical number. Only at the critical number $t = 6$ does $v'(t) = a(t)$ change from positive to negative, therefore this is the only critical number where there is a relative maximum. Then checking the values of $v(0) = 55$, $v(6) = v(0) + \int_0^{6} a(t)\,dt = 55 + 60 = 115$, and $v(18) = v(0) + \int_0^{18} a(t)\,dt = 55 + 60 - 180 + 15 = -50$, We see that $v(6)$ is the absolute maximum velocity on [0, 18].

	2d) Never. The local minimum occurs at $t = 16$ because only there does $v'(t) = a(t)$ change from negative to positive. Because $v(16) = v(0) + \int_0^{16} a(t)\,dt = 10$, $v(t)$ is always positive on [0, 18].

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 3 Differential calculus Standard 6 Differential equations/slope fields Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.4 evaluate problem-solving processes 3.7 evaluate strategies
DOK	3
LEVEL OF EXPECTATION	Mastery level –70%

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #2 :

The Fundamental Theorem of Calculus – If f is continuous on an open interval I containing x, then $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. (Note: the chain rule still applies)

Use the Fundamental Theorem to evaluate each of the following:

1. $\frac{d}{dx}\int_3^x \sqrt{t}\,dt$
2. $\frac{d}{dx}\int_3^{x^2} \sqrt{t}\,dt$
3. $\frac{d}{dx}\int_{\sin x}^{4} \sqrt{t}\,dt$
4. $\frac{d}{dx}\int_{x^3}^{\cos x} \sqrt{t}\,dt$

Learning Activity #2 – KEY

1) $\sqrt{x}$

2) $\sqrt{x^2}\cdot 2x = 2x^2$

3) $-\sqrt{\sin x}(\cos x)$

4) $\sqrt{\cos x}(-\sin x) - \sqrt{x^3}\cdot 3x^2$

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 3 Standard 6 Standard 9 Standard 10	Differential calculus Differential equations/slope fields Integral calculus Area and volume
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships

Assessment #2:

Find the derivative of $\int_0^{x^{10}} \cos\sqrt{t}\,dt$

Assessment #2 – KEY

$\cos\sqrt{x^{10}}\cdot 10x^9$

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 3 Standard 6 Standard 9 Standard 10	Differential calculus Differential equations/slope fields Integral calculus Area and volume
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships
PROCESS	1.7 1.10 3.4 3.7	evaluate information apply information, ideas and skills evaluate problem-solving processes evaluate strategies
DOK	2	
LEVEL OF EXPECTATION	Mastery level –80%	

PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.4 evaluate problem-solving processes 3.7 evaluate strategies	
DOK	3	
INSTRUCTIONAL STRATEGIES	Nonlinguistic representation	

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Student Resources	Teacher Resources
General:	**General:**
Enrichment:	**Enrichment:**
Intervention:	**Intervention:**

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 11**
Learner Objectives: The student will calculate, interpret, and apply integrals		

Concepts: C. Techniques of antidifferentiation

Students Should Know	Students Should Be Able to

• Antiderivatives following directly from derivatives of basic functions.	• Evaluate antiderivatives by substitution of variables (including change of limits for definite integrals). • **Evaluate integrals using integration by parts and simple partial fractions (non-repeating linear factors only)** • **Evaluate improper integrals (as limits of definite integrals)**

Instructional Support

Student Essential Vocabulary					
Anitderivative	Substitution Technique	Upper Limit of Integration	Lower Limit of Integration	**Integration by Parts**	**Tabular Method**
Partial Fraction Decomposition		**Improper Integral**	**Infinite Discontinuity**		

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	**Sample Assessments**

Learning Activity #1 :

If the substitution $x = 2\sin y$ is made, then how would

$$\int_0^2 x^3 \sqrt{4 - x^2}\, dx$$

be re-written?

Learning Activity #1 – KEY

$$\int_0^{\frac{\pi}{2}} 32\sin^3 y\cos^2 y\, dy$$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 3 Differential calculus Standard 6 Differential equations/slope fields Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.4 evaluate problem-solving processes 3.7 evaluate strategies
DOK	2
INSTRUCTIONAL STRATEGIES	Homework and practice

Assessment #1:

Evaluate the integral: $\int \frac{e^{3x}}{1+e^{3x}}\, dx$

Assessment #1 – KEY

$$\frac{1}{3}\ln\left(1+e^{3x}\right)+C$$

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 3 Differential calculus Standard 6 Differential equations/slope fields Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.7 evaluate information 1.10 apply information, ideas and skills 3.4 evaluate problem-solving processes 3.7 evaluate strategies
DOK	2
LEVEL OF EXPECTATION	Mastery level – 85%

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21^{st} Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	

Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments
Learning Activity #2 :	**Assessment #2:**

Learning Activity #2 :

Use the substitution method to find the value of each integral.

1. $\int x\sqrt{x^2+1}\,dx$

2. $\int x\sqrt{x+2}\,dx$

3. $\int \frac{-x}{(x^2-4)^3}\,dx$

Learning Activity #2 – KEY

1) $\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}+C$

2) $\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$

3) $\frac{1}{2\left(x^2-4\right)^2}+C$

Activity's Alignment

Assessment #2:

If the substitution $u=1+\sqrt{x}$ is made, then $\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$ is equivalent to which one of the following?

A. $2\int_1^2 \frac{u-1}{u}\,du$ B. $2\int_1^2 \frac{(u-1)^2}{u}\,du$ C. $2\int_0^1 \left(1-\frac{1}{u}\right)du$

D. $\int_1^2 \left(2u-4+\frac{2}{u}\right)du$ E. $2\int_0^2 \frac{(u-1)^2}{u}\,du$

Assessment #2 – KEY

B

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 3	Differential calculus
	Standard 6	Differential equations/slope fields
	Standard 9	Integral calculus
	Standard 10	Area and volume
CONTENT	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.7	evaluate information
	1.10	apply information, ideas and skills
	3.4	evaluate problem-solving processes

AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 3	Differential calculus
	Standard 6	Differential equations/slope fields
	Standard 9	Integral calculus
	Standard 10	Area and volume
CONTENT	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.7	evaluate information
	1.10	apply information, ideas and skills
	3.4	evaluate problem-solving processes
	3.7	evaluate strategies
DOK	2	
NSTRUCTIONAL STRATEGIES	Homework and practice	

Learning Activity #3 :

Use Integration by Parts to evaluate each:

1. $\int x \cos x \, dx$ 2. $\int x \sec^2 x \, dx$

Learning Activity #3 – KEY

1. $\int x \cos x dx$

let $u = x \ and \ dv = \cos x dx$ **then** $du = dx \ and \ v = \sin x$

Integration by Parts:

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

	3.7	evaluate strategies
DOK	3	
LEVEL OF EXPECTATION	Mastery level –70%	

Assessment #3:

Use Integration by Parts to evaluate $\int x^2 \sin x dx$

Assessment # 3 – KEY

Applying Integration by Parts twice (or using the tabular method) yields the solution:

$$\int x^2 \sin x dx = -x^2 \cos x + \int 2x \cos x dx$$

$$= -x^2 \cos x + 2x \sin x - \int 2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

2. $\int x\sec^2 xdx$

let $u = x \ and \ dv = \sec^2 xdx$ **then** $du = dx \ and \ v = \tan x$

Integration by Parts:

$$\int x\sec^2 xdx = x\tan x - \int \tan xdx = x\tan x + \ln|\cos x| + C$$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 11 Antidifferentiation by parts
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.6 examine solutions from many perspectives
DOK	2
NSTRUCTIONAL STRATEGIES	Guided practice

Learning Activity #4:

Use Partial Fraction Decomposition to rewrite each integrand and evaluate each integral:

1. $\int \frac{1}{x^2 + 2x - 3}dx$

2. $\int \frac{1}{x^2 - 3x - 10}dx$

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 11 Antidifferentiation by parts
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.6 examine solutions from many perspectives
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

Assessment #4:

Use Partial Fractions to evaluate: $\int \frac{1}{x^2 - 6x + 8}dx$

Assessment #4 – KEY

Learning Activity #4 – KEY

1. $$\int \frac{1}{x^2+2x-3}dx = -\frac{1}{4}\int \frac{1}{x+3}dx + \frac{1}{4}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{4}\ln|x+3| + \frac{1}{4}\ln|x-1| + C = \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$$

2. $$\int \frac{1}{x^2-3x-10}dx = \frac{1}{7}\int \frac{1}{x-5}dx - \frac{1}{7}\int \frac{1}{x+2}dx$$

$$= \frac{1}{7}\ln|x-5| - \frac{1}{7}\ln|x+2| + C = \frac{1}{7}\ln\left|\frac{x-5}{x+2}\right| + C$$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 9 Integral calculus
CONTENT	MA 4 patterns and relationships MA 5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.6 examine solutions from many perspectives
DOK	2
NSTRUCTIONAL STRATEGIES	Guided practice

Learning Activity #5:

Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

$$\int \frac{1}{x^2-6x+8}dx = \int \frac{1}{(x-4)(x-2)}dx$$

Using partial fractions:

$$\frac{1}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2},\ A = -1/2,\ B = 1/2$$

$$-\frac{1}{2}\int \frac{1}{x-4}dx + \frac{1}{2}\int \frac{1}{x-2}dx = -\frac{1}{2}\ln|x-4| + \frac{1}{2}\ln|x-2| + C = \frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$$

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 9 Integral calculus
CONTENT	MA 4 patterns and relationships MA 5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.6 examine solutions from many perspectives
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

Assessment #5:

1. $\int_0^{\infty} x^2 e^{-x^3}\,dx$ 2. $\int_3^4 \frac{1}{\sqrt{x-3}}\,dx$ 3. $\int_3^4 \frac{1}{(x-3)^{3/2}}\,dx$

Learning Activity #5 – KEY

1. converges to 1/3 2. converges to 2 3. diverges

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 9 Integral calculus
CONTENT	MA 4 patterns and relationships MA 5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 1.8 organize data and ideas 3.2 apply others' strategies 3.7 evaluate strategies
DOK	3
NSTRUCTIONAL STRATEGIES	Guided practice

Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

$$\int_1^{\infty} \frac{x}{(1+x^2)^2}\,dx$$

Assessment #5 – KEY

$\int_1^{\infty} \frac{x}{(1+x^2)^2}\,dx$ converges to ¼

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 9 Integral calculus
CONTENT	MA 4 patterns and relationships MA 5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 1.8 organize data and ideas 3.2 apply others' strategies 3.7 evaluate strategies
DOK	3
LEVEL OF EXPECTATION	Mastery level – 80%

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	Course: AP Calculus BC	Strand: 12
Learner Objectives: **The student will calculate, interpret, and apply integrals**		

Concepts: D. Applications of Definite and Indefinite Integrals

Students Should Know	Students Should Be Able to
• The relationship between a function and its antiderivative $\int_a^b f(x)dx = F(b) - F(a)$ • Average value of a function *f* is $\frac{\int_a^b f(x)dx}{b-a}$ (i.e. "integral over interval") • Total distance traveled: $D = \int_a^b \lvert v(t) \rvert dt$	• Find specific antiderivatives using initial conditions, including applications to motion along a line and total distance traveled. • Solve separable differential equations and use them to model (including exponential growth, i.e. $y' = ky$). • **Solve logistic differential equations and use them in modeling** • Calculate the area of a region. • **Calculate area of a region bounded by polar curves** • **Calculate the length of a curve given in parametric form** • Calculate the volume of a solid with known cross-sections. • Calculate the volume of a solid of revolution. • Calculate accumulated change from a rate of change or average value of a function (including applications of physical, biological and economic situations).

Instructional Support

Student Essential Vocabulary					
Integrals as Areas	Distance vs. Displacement	Position	Velocity	Acceleration	Differential Equation
Initial Value Problem	General Solution	Particular Solution	Known Cross-Section	Solid of Revolution	**Logistic Differential Equation**
Logistic Growth	**Carrying Capacity**	**Polar Equation**	**Pole**	**Parameter**	**Parametric Equation**
Eliminate the Parameter		**Plane Curve**			

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #1 :

1. Find the average value of $f(x)=3x^2-2x$ on [1, 4]. Then find the value of x guaranteed by the Mean Value Theorem for Integrals.

2. What is the average value of $y=x^2\sqrt{x^3+1}$ on the interval [0, 2]?

   A. $\frac{26}{9}$ B. $\frac{52}{9}$ C. $\frac{26}{3}$ D. $\frac{52}{3}$ E. 24

Learning Activity #1 – KEY

1) average value: 16; $x = 2\frac{2}{3}$
2) A

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 6	Differential equations/slope fields
	Standard 9	Integral calculus
	Standard 10	Area and volume

Assessment #1:

Find the particular solution of $\frac{dy}{dx}=\frac{x}{y}$ through (-2, -1) and identify the domain.

Assessment #1 – KEY

Particular solution: $y=-\sqrt{x^2-3}$; domain: $\left(-\infty,-\sqrt{3}\right)$

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 6	Differential equations/slope fields
	Standard 9	Integral calculus
	Standard 10	Area and volume
CONTENT	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.6	discover/evaluate relationships
	1.10	apply information, ideas and skills
	3.1	identify and define problems
	3.4	evaluate problem-solving processes
	3.5	reason logically (inductive/deductive)
	3.7	evaluate strategies
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 80%	

CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships	
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.1 identify and define problems 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 3.7 evaluate strategies	
DOK	2	
INSTRUCTIONAL STRATEGIES	Homework and practice	

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	**Sample Assessments**

Learning Activity #2 :

By U.S. law, yogurt must contain 100 million bacteria per gram. At noon, Some sterilized milk is inoculated with a yogurt culture so that the milk is inoculated with a yogurt culture so that the milk contains 400 bacteria per gram. Suppose the bacteria growth rate is proportional to the number of bacteria present and that at 1 pm, there are 1600 bacteria per gram. At 7 pm, how many bacteria are there per gram? At what time does the culture legally become yogurt?

Learning Activity #2 – KEY

At 7 pm there are 6,553,600 bacteria per gram. It is legally yogurt 8.9695 hours after noon.

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 6 Differential equations/slope fields Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.1 identify and define problems 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 3.7 evaluate strategies
DOK	2
INSTRUCTIONAL STRATEGIES	Cues, questions, and advanced organizers.

Assessment #2:

Solve the following analytically:

Suppose the population *y* of a hive of wasps is growing at a rate proportional to the population. On May 1, there were 10 wasps and on May 31, there were 50. If growth continues like this, how long after May 1 will the population reach 100 wasps?

Assessment #2 – KEY

42.92 days after May 1

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 6 Differential equations/slope fields Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 1.10 apply information, ideas and skills 3.1 identify and define problems 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 3.7 evaluate strategies
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

Assessment #3:

Learning Activity #3:

A pond has a carrying capacity of 500 fish. Assume the population growth is proportional to the product of the number of fish in the pond and the number of fish the pond could still sustain. Assume there is a logistic growth constant $k = 0.4$ and that time is measured in months.

a. Find the fish population model $P(t)$, if the initial population is 50 fish.

b. How long does it take for the fish population to reach 250?

Learning Activity #3 – KEY

a. A logistic differential equation $\frac{dP}{dt} = kP\left(1-\frac{P}{L}\right)$ **leads to a solution** $P(t0 = \frac{L}{1+be^{-kt}}$ **where the variables represent the following: *L* is the carrying capacity, *P* is the number in the population at time *t* and *k* is the growth constant. Also,** $b = \frac{P(0)-L}{-P(0)} = \frac{L-P(0)}{P(0)}$.

In this specific case, *L* = 500, *k* = 0.4, and *P*(0) = 50. This leads to $b = \frac{50-500}{-50} = \frac{500-50}{50} = 9$ **and the differential equation yields** $\frac{dP}{dt} = 0.4P\left(1-\frac{P}{500}\right)$ **which has the solution** $P(t) = \frac{500}{1+9e^{-0.4t}}$

b. Setting *P*(*t*) = 250 and solving for time *t*, we have: $250 = \frac{500}{1+9e^{-0.4t}}$

which leads to $t = \frac{\ln\left(\frac{1}{9}\right)}{-0.4} \approx 5.493$ **months.**

The growth rate of a population of wolves in a newly established preserve is modeled by $\frac{dP}{dt} = 0.08P(100-P)$, where t is measured in years.

a. What is the carrying capacity for the wolves in this preserve?

b. What is the wolf population when the population is growing the fastest?

c. What is the rate of change of the population when the population is growing the fastest?

d. If $P(0) = 3$, what value does P approach as t grows infinitely large?

Assessment #3 – KEY

a. 100 wolves

b. The wolf population is growing the fastest when population is half of carrying capacity therefore, 50 wolves.

c. When *P* = 50, $\frac{dP}{dt} = 0.008(5)(100-50) = 20$ **wolves per year. So the growth rate is about 20 wolves that year (the derivative is an instantaneous growth rate).**

d. 100, since that is the carrying capacity.

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 2 Standard 3 Standard 6 Standard 8	Analysis of functions Model numerically/analytically Differential calculus Differential equations/slope fields
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships
PROCESS	3.5	reason logically (inductive/deductive)
DOK	2	
INSTRUCTIONAL STRATEGIES	Summarizing and Note Taking, Homework and Practice, & Nonlinguistic Representation	

Learning Activity #4:

Graph the curve described by $x(t) = t^2 - 5t$ and $y(t) = 2t - 1$, for t in [0, 6].

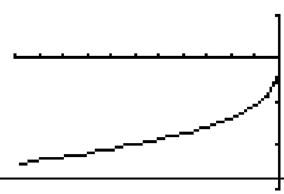
a. note the direction/orientation of the graph,

b. rewrite the equations in rectangular form, and

c. find the length of the curve on this interval.

Learning Activity #4 – KEY

a. from upper right to lower left



Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 2 Standard 3 Standard 6 Standard 8	Analysis of functions Model numerically/analytically Differential calculus Differential equations/slope fields
CONTENT	MA 2 MA 4	geometric and spatial sense patterns and relationships
PROCESS	3.2 3.5	apply others' strategies reason logically (inductive/deductive)
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 80%	

Assessment #4:

Set up an integral for the arc length of the plane curve defined by $x(t) = t^3$ and $y(t) = 2t^2$ for t in [0, 1].

Assessment #4 – KEY

$$\int_0^1 \sqrt{\left(3t^2\right)^2 + \left(4t\right)^2}\, dt$$

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1 Standard 2 Standard 3	Analysis of functions Model numerically/analytically Differential calculus

b. For simplicity, let $x(t)$ and $y(t)$ be denoted as x and y, respectively. Solving y for t,

we get $t = \frac{y+1}{2}$ **. Substituting this for t in the equation for x, we have**

$x = \left(\frac{y+1}{2}\right)^2 - 5\left(\frac{y+1}{2}\right)$ **which can be simplified to**

$x = \frac{1}{4}\left(y^2 - 8y - 9\right)$

c. The length of a parametrically defined curve is generally given as

$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ **. Taking the derivative of each parametric equation and substituting gives us** $\int_0^6 \sqrt{(2t-5)^2 + (2)^2}\, dt \approx 23.085$ **.**

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 4 Position, speed, acceleration Standard 9 Integral calculus
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	3.2 apply others' strategies
DOK	2
INSTRUCTIONAL STRATEGIES	Summarizing and Note Taking, Homework and Practice, & Nonlinguistic Representation

	Standard 4 Position, speed, acceleration Standard 9 Integral calculus
CONTENT	MA 2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	3.2 apply others' strategies
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

Learning Activity #5:

Set up an integral to find the area inside the smaller loop of the limaçon $r = 2\cos\theta + 1$.

Learning Activity #5 – KEY

The area of a region bounded by a polar curve is generally given as $\frac{1}{2}\int_a^b (r(\theta))^2\,d\theta$. The inner loop is traced out for values of θ from $\frac{2\pi}{3}$ to $\frac{4\pi}{3}$, so $\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (2\cos\theta + 1)^2\,d\theta$ will give the area of this inner loop.

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 9	Integral calculus
CONTENT	MA 2	geometric and spatial sense
	MA 4	patterns and relationships

Assessment #5:

Set up an integral for the area inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$.

Assessment #5 – KEY

These curves intersect at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ and so, the area of this region is given by $\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1^2 - (1 - \cos\theta)^2\right)d\theta$.

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 9	Integral calculus
CONTENT	MA 2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	3.2	apply others' strategies
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 80%	

PROCESS	3.2 apply others' strategies
DOK	2
INSTRUCTIONAL STRATEGIES	Summarizing and Note Taking, Homework and Practice, & Nonlinguistic Representation

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	Course: AP Calculus BC	Strand: 13
Learner Objectives: **The student will calculate, interpret, and apply integrals**		

Concepts: E. Numerical Approximations to Definite Integrals

Students Should Know	Students Should Be Able to
● Understand that the definite integral can be approximated by a finite sum of areas of geometric regions.	● Use Riemann sums (left, right, midpoint, trapezoidal) to approximate the definite integral of functions represented algebraically, graphically, and/or by a table of values.

Instructional Support

Student Essential Vocabulary					
Left-Riemann Sum	Right-Riemann Sum	Mid-Point Riemann Sum	Trapezoidal Rule		

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #1 :

1. A tank is being filled with water using an old pump that slows down at it runs. The table below gives the rate at which the pump pumps at 10 minute intervals. If the tank is initially empty, estimate how much water is in the tank after 80 minutes.

Elapsed time (minutes)	0	10	20	30	40	50	60	70	80	90
Rate (gallons/minute)	42	40	38	35	35	32	28	20	19	10

2. Use the data table below to approximate $\int_{10}^{90} f(x)\,dx$ with the indicated methods.

x	10	30	52	70	90
$f(x)$	30	22	14	20	48

a. Left Riemann sum with 4 subintervals

b. Right Riemann sum with 4 subintervals

c. Midpoint Riemann sum with 2 subintervals of equal length.

Assessment #1:

Use 4 subintervals to find the Left Riemann Sum to approximate the area bounded by the x-axis, $y = x + 2$, and $x = 4$.

Assessment #1 – KEY

13.5

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus
	Standard 9	Integral calculus
	Standard 10	Area and volume
CONTENT	MA2	geometric and spatial sense
	MA 4	patterns and relationships
PROCESS	1.8	organize data and ideas
	1.10	apply information, ideas and skills
	2.1	plan and make presentations

d. Trapezoidal Rule with 4 subintervals.

3. Use the data table below to approximate $\int_{20}^{120} f(x)\,dx$ with the indicated methods.

x	20	40	45	60	70	80	95	100	120
$f(x)$	23	18	17	15	14	12	9	6	3

a. Left Riemann sum with 5 subintervals

b. Right Riemann sum with 5 subintervals

c. Midpoint Riemann sum with 2 subintervals of equal length.

d. Trapezoidal Rule with 8 subintervals.

4. Complete the data table below and approximate $\int_{2}^{8} f(x)\,dx$ where $f(x) = 16 - x^2$ with the indicated methods.

x	2	3	3.5	4	6	6.5	8
$f(x)$							

a. Left Riemann sum with 3 subintervals of equal length.

	3.5	reason logically (inductive/deductive)
	3.6	examine solutions from many perspectives
	3.7	evaluate strategies
	4.1	support decisions
DOK	2	
LEVEL OF EXPECTATION	Mastery level – 80%	

b. Right Riemann sum with 3 subintervals of equal length.

c. Midpoint Riemann sum with 2 subintervals of equal length.

d. Trapezoidal Rule with 3 subintervals of equal length.

5. Complete the data table below and approximate $\int_{-1}^{15}(x^2+3x)\,dx$ with the indicated methods.

x	-1	3	7	11	15
$f(x)$					

a. Left Riemann sum with 4 subintervals

b. Right Riemann sum with 4 subintervals

c. Midpoint Riemann sum with 2 subintervals of equal length.

d. Trapezoidal Rule with 4 subintervals.

Use the Fundamental Theorem of Calculus to evaluate each of the following:

6. $\int_{-1}^{15}\left(x^2+3x\right)dx$

7. $\int_{2}^{8}\left(16-x^2\right)dx$

Learning Activity #1 – KEY

1) Results my vary but the left Riemann sum is 2890, the right Riemann sum is 2570, and the trapezoidal approximation is 2730.

2a) 1736 b) 2636 c) 1680 d) 2186

3a) 1480 b) 1080 c) 1300 d) 1297.5

4a) -16 b) -136 c) -67.5 d) -76

5a) 960 b) 1024 c) 1376 d) 992

6) 1461.333 7) -72

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA2 geometric and spatial sense MA 4 patterns and relationships
PROCESS	1.8 organize data and ideas 1.10 apply information, ideas and skills 2.1 plan and make presentations 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 3.7 evaluate strategies 4.1 support decisions
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic representation

Readness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments
Learning Activity #2 : 1. Find each of the following approximating sums to approximate the area under $f(x) = -x^2 + 5$ between $x = 0$ and $x = 2$, using 5 subintervals: a. Right Riemann Sum b. Left Riemann Sum c. Now, average the approximations from parts *a* and *b* and determine whether this average is equivalent to the Midpoint Riemann Sum or Trapezoidal Rule. Do not actually find value of these approximations. Explain your answer. d. Is your approximation from part *a* an overapproximation or an underapproximation for the actual area defined? Explain your answer. e. Is your approximation from part *b* an overapproximation or an	**Assessment #2:** The function *f* is continuous on the interval [2, 8] and has values that are given in the table below. x: 2, 5, 7, 8 $f(x)$: 10, 30, 40, 20 Using the 3 subintervals available in the table, what is the approximate value of the integral $\int_2^8 f(x)\,dx$? Use the Trapezoidal Rule. **Assessment #2 – KEY** 160

x	2	5	7	8
$f(x)$	10	30	40	20

Assessment's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 1	Analysis of functions
	Standard 2	Model numerically/analytically
	Standard 3	Differential calculus

underapproximation for the actual area defined? Explain your answer.

Learning Activity #2 – KEY

1a) 6.48 b) 8.08 c) 7.28

d) under, because the function is decreasing on [0, 2]

e) over, because the function is decreasing on [0, 2]

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA2 geometric and spatial sense MA4 patterns and relationships
PROCESS	1.8 organize data and ideas 1.10 apply information, ideas and skills 2.1 plan and make presentations 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 3.7 evaluate strategies 4.1 support decisions
DOK	2
INSTRUCTIONAL STRATEGIES	Nonlinguistic representation

	Standard 9 Integral calculus Standard 10 Area and volume
CONTENT	MA2 geometric and spatial sense MA4 patterns and relationships
PROCESS	1.8 organize data and ideas 1.10 apply information, ideas and skills 2.1 plan and make presentations 3.5 reason logically (inductive/deductive) 3.6 examine solutions from many perspectives 3.7 evaluate strategies 4.1 support decisions
DOK	2

LEVEL OF EXPECTATION	Mastery level – 80%

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

Content Area: Mathematics	**Course: AP Calculus BC**	**Strand: 14**
Learner Objectives: The student will apply approximations and infinite series.		

Concepts: A. Series of Constants

Students Should Know	Students Should Be Able to
• **A series is defined as a sequence of partial sums** • **Convergence is defined in terms of the limit of the sequence of partial sums**	• **Use technology to explore convergence and divergence** • **Geometric series with applications, including decimal expansion** • **Harmonic series and alternating series with error bound** • **Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of *p*-series** • **Apply L'Hopital's Rule to determine the convergence of improper integrals and series**

	• **Tests of convergence including the n^{th} term test, ratio test, root test, direct comparison and limit comparison**

Instructional Support

Student Essential Vocabulary					
Sequence	**Series**	**Partial Sum**	**Convergent Series**	**Divergent Series**	**Geometric Series**
***p*-series**	**Harmonic Series**	**Improper Integral**	**n^{th} term test**	**Ratio Test**	**Root Test**
Direct Comparison Test		**Telescoping Series**			

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	**Sample Assessments**

Learning Activity #1 :

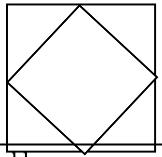
Theorem: Convergence of a Geometric Series

An infinite geometric series with common ratio *r* is of the form $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \ldots$ **and will converge if** $0 < |r| < 1$ **and diverge if** $|r| \geq 1$ **. If the series converges, the sum of the infinite series is** $\frac{a}{1-r}$ **. Determine the convergence of each series in (a) – (c) and find the sum of any that converge.**

a. $\sum_{n=0}^{\infty} \frac{3}{2^n}$ b. $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$ c. $\sum_{n=1}^{\infty} \frac{5}{4^n}$

d. For their wedding, a couple chooses a cake that has fruit filling. Each tier of the cake consists of a layer of cake, covered with the fruit filling, and then topped with another layer of cake. The entire tier of cake is then coated in frosting. When the wedding cake is assembled, several tiers are placed on top of one another, generally getting smaller and smaller as they move up the cake. The various tiers of the cake are held together with frosting as well.

When a cake decorator makes such a cake, the fruit does not Usually extend all the way out to the edges,, but we are going to assume that is does in order to make this problem a little simpler.



The figure at left shows 2 layers of a stacked wedding cake, when viewed from above. Each tier is a square. The outermost (bottom) square tier has a top area

Assessment #1:

For which of these possible values of *k* will both $\sum_{n=0}^{\infty} 3\left(\frac{k}{4}\right)^n$ and $\sum_{n=0}^{\infty} \left(\frac{2}{k}\right)^n$ converge?

A. 2 B. 3 C. 4 D. 5 E. 6

Assessment #1 – KEY

Answer: B. 3. This is the only choice given for the value of *k* that will result in both ratios of the given geometric series to fall between 0 and 1.

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery level – 75%

(just the top face, not the bottom, nor the sides) of 64 square inches. The size of the next (top) layer is determined by joining the midpoints of the sides of the tier below it.

d. Find the sum of the top areas of both tiers (again, just the tops). This will help the cake decorator determine the amount of filling that will need to be used. You see, the total area of the tops of the tiers will be the same as the area to be covered in fruit, between the layers.

e) Find the sum of the top areas of all such layers if there were 5 tiers.

f) Determine the convergence or divergence of the sum of the top areas of all such layers if there were an infinite number of tiers. Write a geometric series to represent this infinite sum. If the total are does converge, be sure to state the sum.

Learning Activity #1 – KEY

The Convergence of a Geometric Series Theorem will be used in this solution.

a. $\sum_{n=0}^{\infty}\frac{3}{2^n}=\sum_{n=0}^{\infty}3\left(\frac{1}{2}\right)^n$ **, so *a* = 3, and** $r=\frac{1}{2}<1$ **, so the series converges**

to $\frac{3}{1-\frac{1}{2}}=6$ **.**

b. $\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n$ **diverges since** $r=\frac{3}{2}>1$ **.**

c. $\sum_{n=1}^{\infty}\frac{5}{4^n}=\sum_{n=1}^{\infty}5\left(\frac{1}{4}\right)^n$ **, so $a = 5$, and** $r=\frac{1}{4}<1$ **, so the infinite series that begins at $n = 0$ will converge to** $\frac{5}{1-\frac{1}{4}}=\frac{20}{3}$ **. However, the given series begins at $n = 1$, so the term at $n = 0$** $\left(\frac{5}{4}\right)$ **must be subtracted from this sum. Thus, the given series converges to** $\frac{20}{3}-\frac{5}{4}=\frac{65}{12}$ **.**

d. 64 + 32 = 96 square inches

e. 64 + 32 + 16 + 8 + 4 = 124 square inches

f. The sum is represented by the series $\sum_{n=0}^{\infty}64\left(\frac{1}{2}\right)^n$ **and the sum is** $\frac{64}{1-\frac{1}{2}}=128$ **square inches.**

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 2 Model numerically/analytically Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems

PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions
DOK	2
INSTRUCTIONAL STRATEGIES	Identifying Similarities and Differences, Homework and Practice, Nonlinguistic Representation

Learning Activity #2:

Theorem: The nth Term Test

If the sequence $\{a_n\}$ does not converge to 0, then the series $\sum_{n=1}^{\infty} a_n$ will not converge.

Identify which of the following series do not converge according to the nth Term Test or state that the nth Term Test is inconclusive.

a. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ b. $\sum_{n=1}^{\infty} \frac{1}{n}$ c. $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n}$

d. $\sum_{n=1}^{\infty} 2^n$ e. $\sum_{n=1}^{\infty} \frac{n!}{2(n!)+1}$ f. $\sum_{n=1}^{\infty} \frac{n^2}{n-2}$

Assessment #2

Identify which of the following series do not converge according to the nth Term Test or state that the nth Term Test is inconclusive.

a. $\sum_{n=1}^{\infty} \frac{3n}{n+2}$ b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ c. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

d. $\sum_{n=1}^{\infty} 2n$ e. $\sum_{n=1}^{\infty} \frac{(n+1)!}{n!+1}$ f. $\sum_{n=1}^{\infty} (3n)^n$

Assessment #2 – KEY

a. series diverges since $\lim_{n\to\infty}\left(\frac{n}{n+2}\right) = 3 \neq 0$

b. this test is inconclusive since $\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$

c. this test is inconclusive since $\lim_{n\to\infty} \frac{1}{n^2} = 0$

Learning Activity #2 – KEY

a. series diverges since $\lim_{n\to\infty}\left(\frac{n}{n+2}\right)=1\neq 0$

b. this test is inconclusive since $\lim_{n\to\infty}\left(\frac{1}{n}\right)=0$

c. this test is inconclusive since $\lim_{n\to\infty}\left(\frac{\cos(\pi\, n)}{n}\right)=0$

d. series diverges since $\lim_{n\to\infty}2^n \to \infty \neq 0$

e. series diverges since $\lim_{n\to\infty}\left(\frac{n!}{2(n!)+1}\right)=\frac{1}{2}\neq 0$

f. series diverges since $\lim_{n\to\infty}\frac{n^2}{n-2}\to \infty \neq 0$

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive)

d. series diverges since $\lim_{n\to\infty}2n \to \infty \neq 0$

e. series diverges since $\lim_{n\to\infty}\frac{(n+1)!}{n!+1}\to \infty \neq 0$

f. series diverges since $\lim_{n\to\infty}(3n)^n \to \infty \neq 0$

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery level – 80%

	4.1 support decisions	
DOK	2	
INSTRUCTIONAL STRATEGIES	Identifying Similarities and Differences, Homework and Practice, Nonlinguistic Representation	

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Learning Activity #3:

Theorem: Direct Comparison Test

Let $0 < a_n < b_n$ for all *n* beyond some value. If the series $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ does as well. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ does as well.

Theorem: Limit Comparison Test

If a_n and b_n are both positive and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ for some finite and positive real number *L*, then the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or diverge.

Apply the Direct Comparison Test or the Limit Comparison Test to determine the convergence of each of the following series:

a. $\sum_{n=1}^{\infty} \frac{1}{5+3^n}$ **b.** $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$ **c.** $\sum_{n=1}^{\infty} \frac{3n^2+2n-5}{2n^5+6}$

Learning Activity #3 – KEY

Assessment #3:

Apply the Direct Comparison Test or The Limit Comparison Test to determine the convergence or divergence of each series.

a. $\sum_{n=1}^{\infty} \frac{5^n}{3n-2}$ b. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ c. $\sum_{n=1}^{\infty} \frac{n}{3n^2+2}$

Assessment #3 – KEY

a. $\frac{5^n}{3n-2} > \frac{5^n}{3n}$ and $\sum_{n=1}^{\infty} \frac{5^n}{3n}$ **diverges by the nth term test, hence** $\sum_{n=1}^{\infty} \frac{5^n}{3n-2}$ **diverges by Direct Comparison.**

b. $\frac{1}{3n^2+2} < \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ **converges by *p*-series, *p* = 2 > 1, hence** $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$ **converges by Direct Comparison.**

c. $\lim_{n\to\infty} \frac{\frac{n}{3n^2+2}}{\frac{1}{n}} = \lim_{n\to\infty} \left(\frac{n}{3n^2+2}\right)\left(\frac{n}{1}\right) = \frac{1}{3}$ **, which is both finite and positive, also** $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ **is the divergent Harmonic Series, hence**

a. $\frac{1}{5+3^n} < \frac{1}{3^n}$ **and** $\sum_{n=1}^{\infty} \frac{1}{3^n}$ **converges since it is a geometric series with**

$r = \frac{1}{3} < 1$ **, hence** $\sum_{n=1}^{\infty} \frac{1}{5+3^n}$ **converges by Direct Comparison.**

b. $\frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n}}$ **and** $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ **diverges since it is a *p*-series with** $p = \frac{1}{2}$**,**

$0 < \frac{1}{2} \le 1$ **, hence** $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$ **diverges by Direct Comparison.**

c. $\lim_{n\to\infty} \frac{\frac{3n^2+2n-5}{2n^5+6}}{\frac{n^2}{n^5}} = \lim_{n\to\infty}\left(\frac{3n^2+2n-5}{2n^5+6}\right)\left(\frac{n^5}{n^2}\right) = \frac{3}{2}$ **, and** $\frac{3}{2}$ **is both finite and positive, also** $\sum_{n=1}^{\infty} \frac{n^2}{n^5} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ **is a convergent *p*-series with** $p = 3 > 1$**, hence** $\sum_{n=1}^{\infty} \frac{3n^2+2n-5}{2n^5+6}$ **converges by Limit Comparison.**

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems

$\sum_{n=1}^{\infty} \frac{n}{3n^2+2}$ **diverges by Limit Comparison.**

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery level – 75%

PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions
DOK	2
INSTRUCTIONAL STRATEGIES	Identifying Similarities and Differences, Homework and Practice, Nonlinguistic Representation

Learning Activity #4:

Theorem: The Integral Test

If *f* is positive, continuous, and decreasing for $x \geq 1$ **and** $a_n = f(n)$**, then** $\sum_{n=1}^{\infty} a_n$ **and** $\int_1^{\infty} f(x)\,dx$ **either both converge or both diverge.**

Definition:

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ **is called a *p*-series and will converge if** $p > 1$ **and will diverge if** $0 < p \leq 1$**. If *p* = 1, we call this series the Harmonic Series.**

Use the Integral Test to determine the convergence or divergence of the following series:

Assessment #4:

Use the Integral Test to determine the convergence or divergence of each of the following series:

a. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

c. Choose the best answer.

If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

A. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

B. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

C. $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

D. $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

E. $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

Assessment #4 – KEY

a. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ b. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

Learning Activity #4 – KEY

a. $\int_1^{\infty} \frac{dx}{x^2+1} = \lim_{b\to\infty} \int_1^b \frac{dx}{x^2+1} = \lim_{b\to\infty} \arctan x \Big|_1^b$

$= \lim_{b\to\infty} [\arctan b - \arctan 1] = \frac{\pi}{2}$ **,**

therefore, this improper integral converges.

Also, $f(x) = \frac{1}{x^2+1}$ **is positive, continuous, and decreasing for** $x \geq 1$ **and** $a_n = f(n)$ **, hence** $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ **converges by the Integral Test.**

b. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ **which converges by *p*-series,** $p = \frac{3}{2} > 1$ **.**

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems

a. $\int_1^{\infty} \frac{x^2\,dx}{x^3+1} = \lim_{b\to\infty} \int_1^b \frac{x^2\,dx}{x^3+1} = \lim_{b\to\infty} \ln \sqrt[3]{x^3+1}\Big|_1^b$

$= \lim_{b\to\infty} \left[\ln \sqrt[3]{b^3+1} - \ln \sqrt[3]{2}\right] \to \infty$ **,**

therefore, this improper integral diverges.

Also, $f(x) = \frac{x^2}{x^3+1}$ **is positive, continuous, and decreasing for** $x \geq 1$ **and** $a_n = f(n)$ **, hence** $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ **diverges by the Integral Test.**

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{5}}}$ **which diverges by *p*-series,** $p = \frac{1}{5} \leq 1$ **.**

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series
CONTENT	MA1 number sense MA2 geometric and spatial sense MA4 patterns and relationships MA5 mathematical systems
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery level – 75%

PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.4 evaluate problem-solving processes 3.5 reason logically (inductive/deductive) 4.1 support decisions	
DOK	2	
INSTRUCTIONAL STRATEGIES	Identifying Similarities and Differences, Homework and Practice, Nonlinguistic Representation	

Student Resources	Teacher Resources
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	Course: AP Calculus BC	Strand: 15
Learner Objectives: The student will apply approximations and infinite series.		

Concepts: B. Taylor Series

Students Should Know	Students Should Be Able to
• A Taylor Series is a particular sequence of partial sums developed from derivatives.	• Investigate Taylor polynomial approximations with graphical demonstrations of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve • Generate a Taylor series for a function, centered at $x = a$ • Generate a Maclaurin series for a function to include e^x, sin x, cos x, and $1/(1 - x)$ • Manipulate Taylor series using techniques of substitution, differentiation, antidifferentiation and the formation of new series from known series • Generate a power series for a given function • Determine the radius and interval of convergence of power series • Apply the Lagrange error bound for Taylor polynomials

Instructional Support

Student Essential Vocabulary					
Polynomial Approximations	Taylor Polynomial Approximations	Maclaurin Polynomial Approximations	Remainder of a Taylor Polynomial	Lagrange Form of the Remainder	Lagrange Error Bound
Convergent Series	Divergent Series	Power Series	Geometric Power Series	Taylor Series	Maclaurin Series
Radius of Convergence	Interval of Convergence	Endpoint Convergence	Differentiation and/or Integration of Power Series		

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA =** Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	Sample Assessments

Introductory Activity: Compare the graphs of $y = \sin x$, $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$, and $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ **Learning Activity #1 – KEY** Students observe that $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ approximates $y = \sin x$ on a smaller domain than $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ and hypothesize that adding more terms to the polynomial will result in better approximations to the sine function around $x = 0$. **Activity's Alignment**	**No Assessment for Introductory Activity.**

AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series Standard 13 Taylor polynomials
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply others' strategies 3.6 examine solutions from many perspectives
DOK	1
INSTRUCTIONAL STRATEGIES	Identifying Similarities and Differences, Generating and Testing Hypotheses

Readiness & Equity Section			
SLA = Sample Learning Activities & **SA** = Sample Assessments			
21st Century Themes		Non Fiction Reading & Writing	
Learning & Innovation Skills		Enrichment Opportunity	
Information, Media, & Technology Skills		Intervention Opportunity	
Life & Career Skills		Gender, Ethnic, & Disability Equity	

Sample Learning Activities	**Sample Assessments**

Learning Activity #1:

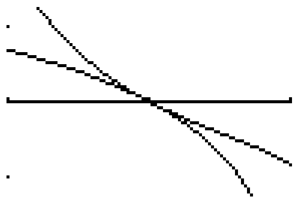
1. Generate the sixth-degree Taylor polynomial about $x = 0$ (Maclaurin) for $\cos x$.

2. Let f be the function given by $f(x) = \ln(4\text{-}x)$. Generate the third degree Taylor polynomial for f about $x = 3$. Compare the graphs of $f(x)$ and $P_3(x)$ about $x = 3$.

Learning Activity #1 – KEY

1. $\cos x \approx P_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

2. $P_3(x) = -(x-3) + \frac{(x-3)^2}{2} - \frac{(x-3)^3}{3}$

Graph comparison: 

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 12 Standard 13	Sequence/series Taylor polynomials
CONTENT	MA 4	patterns and relationships
PROCESS	1.6 3.2 3.6	discover/evaluate relationships apply one's own strategies examine solutions from many perspectives
DOK	2	

Assessment #1:

1. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

A. $1 - \frac{1}{2} + \frac{1}{24}$ B. $1 - \frac{1}{2} + \frac{1}{4}$ C. $1 - \frac{1}{3} + \frac{1}{5}$

D. $1 - \frac{1}{4} + \frac{1}{8}$ E. $1 - \frac{1}{6} + \frac{1}{120}$

2. The coefficient of x^6 in the Taylor polynomial for $f(x) = \sin(x^2)$ is

A. $-\frac{1}{6}$ B. 0 C. $\frac{1}{120}$

D. $\frac{1}{6}$ E. 1

3. Let f be the function given by $f(x) = \ln(3\text{-}x)$. The third-degree Taylor polynomial for f about $x = 2$ is

A. $-(x-2) + \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

B. $-(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

C. $(x-2) + (x-2)^2 + (x-2)^3$

D. $(x-2) + \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3}$

INSTRUCTIONAL STRATEGIES	Guided Practice

Learning Activity #2:

1. Use the Maclaurin series for cos x with substitution to find the Maclaurin series for $f(x) = \cos 2x$.

2. Use the identity $\cos^2 x = \dfrac{1+\cos 2x}{2}$ and the Maclaurin series for cos $2x$ found above to determine the Maclaurin series for $\cos^2 x$.

3. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, determine the series representation for $f\,'(1)$.

4. Determine the interval of convergence for $\sum_{n=1}^{\infty} \dfrac{(x-2)^n}{n \cdot 3^n}$.

Learning Activity #2 – KEY

1. $\cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}$ for all real x, then by substitution,

$$\cos 2x = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}$$

2. $\cos^2 x = \dfrac{1+\cos 2x}{2} = \dfrac{1}{2} + \dfrac{\cos 2x}{2} = \dfrac{1}{2} + \dfrac{1}{2}\sum_{n=0}^{\infty} \dfrac{(-1)^n (2x)^{2n}}{(2n)!}$

E. $(x-2) - \dfrac{(x-2)^2}{2} + \dfrac{(x-2)^3}{3}$

Assessment #1 – KEY

1. E 2. A 3. A

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series Standard 13 Taylor polynomials
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply one's own strategies 3.6 examine solutions from many perspectives
DOK	Standard 12 Standard 13
LEVEL OF EXPECTATION	Mastery level – 66% (2 out of the 3)

Assessment #2a:

1. The Taylor series for sin$x$ about $x = 0$ is $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots$. If f is a function such that $f\,'(x) = \sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

A. $\dfrac{1}{7!}$ B. $\dfrac{1}{7}$ C. 0 D. $\dfrac{1}{7!}$ E. $-\dfrac{1}{7!}$

3. $\sum_{n=1}^{\infty} na_n$

4. Use the ratio test to determine the possible interval of convergence, then test the endpoints: $x = -1$ produces a convergent series and $x = 5$ yields a divergent series, therefore $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges for $-1 \le x < 5$.

Activity's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series Standard 13 Taylor polynomials
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply one's own strategies 3.6 examine solutions from many perspectives
DOK	3
INSTRUCTIONAL STRATEGIES	Homework and Practice

2. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

A. $-3 < x \le 3$ B. $-3 \le x \le 3$ C. $-2 < x < 4$

D. $-2 \le x < 4$ E. $0 \le x \le 2$

3. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

A. $-3 < x < -1$ B. $-3 \le x < -1$ C. $-3 \le x \le -1$

D. $-1 \le x < 1$ E. $-1 \le x \le 1$

Assessment #2a – KEY

1. D 2. C 3. A

Assessment's Alignment	
AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series Standard 13 Taylor polynomials
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply one's own strategies 3.6 examine solutions from many perspectives
DOK	3
LEVEL OF EXPECTATION	Mastery level – 66% (2 out of the 3)

Assessment #2b:

Learning Activity #3:

1. Use the remainder in Taylor's Theorem to obtain a Lagrange error bound for the error of the approximation: $\arcsin(0.4) \approx 0.4 + \frac{(0.4)^3}{2 \cdot 3}$
2. Determine the degree of the Maclaurin polynomial required for the error in the approximation of $f(x) = e^x$ at $x = 0.6$ to be less than 0.001.

Learning Activity #3 – KEY

1. $R_3 \le 7.82 \times 10^{-3}$

2. Degree 5

Activity's Alignment		
AB/BC AP CALCULUS STANDARD	Standard 12 Standard 13	Sequence/series Taylor polynomials
CONTENT	MA 4	patterns and relationships

2010 FRQ6

$$f(x) = \begin{cases} \frac{\cos x - 1}{x^2} & \text{for } x \neq 0 \\ -\frac{1}{2} & \text{for } x = 0 \end{cases}$$

The function f, defined above, has derivatives of all orders. Let g be the function defined by $g(x) = 1 + \int_0^x f(t)\,dt$.

a. Write the first three nonzero terms and the general term of the Taylor series for $\cos x$ about $x = 0$. Use this series to write the first three nonzero terms and the general term of the Taylor series for f about $x = 0$.

b. Use the Taylor series for f about $x = 0$ found in part (a) to determine whether f has a relative maximum, relative minimum, or neither at $x = 0$. Give a reason for your answer.

c. Write the fifth-degree Taylor polynomial for g about $x = 0$.

d. The Taylor series for g about $x = 0$, evaluated at $x = 1$, is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for g about $x = 0$ to estimate the value of $g(1)$. Explain why this estimate differs from the actual value of $g(1)$ by less than $\frac{1}{6!}$.

Assessment #2b – KEY

Assessment's Alignment

PROCESS	1.6 discover/evaluate relationships 3.2 apply one's own strategies 3.6 examine solutions from many perspectives
DOK	2
INSTRUCTIONAL STRATEGIES	Guided Practice

AB/BC AP CALCULUS STANDARD	Standard 12 Sequence/series Standard 13 Taylor polynomials
CONTENT	MA 4 patterns and relationships
PROCESS	1.6 discover/evaluate relationships 3.2 apply one's own strategies 3.6 examine solutions from many perspectives
DOK	3
LEVEL OF EXPECTATION	Mastery level – 70%

Student Resources	**Teacher Resources**
General: **Enrichment:** **Intervention:**	**General:** **Enrichment:** **Intervention:**

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.