AP Calculus AB Mathematics Curriculum Francis Howell School District

Board Approved: February 17, 2011

Francis Howell School District Mission Statement

Francis Howell School District is a learning community where all students reach their full potential.

		Vision Statement		Page
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Francis Howell So	chool District is an educational leader that bui	lds excellence through a collaborativ	ve culture that values students, parents, employees, and	Е
	the co	ommunity as partners in learning.		*
				MER
		Values		GEF
				OR
Francis Howell Sch	nool District is committed to:			ΜΔΤ
•	Providing a consistent and comprehensive e	education that fosters high levels of a	cademic achievement for all	
•	Operating safe and well-maintained schools	3		1
•	Promoting parent, community, student, and	business involvement in support of t	he school district	
•	Ensuring fiscal responsibility			
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• Developing character and leadership

Francis Howell School District Graduate Goals

Upon completion of their academic study in the Francis Howell School District, students will be able to:

- 1. Gather, analyze and apply information and ideas.
- 2. Communicate effectively within and beyond the classroom.
- 3. Recognize and solve problems.
- 4. Make decisions and act as responsible members of society.

Mathematics Graduate Goals

Upon completion of their mathematics study in the Francis Howell School District, students will be able to:

- 1. Communicate mathematically
- 2. Reason mathematically
- 3. Make mathematical connections
- 4. Use mathematical representations to model and interpret practical situations

Mathematics Rationale for AP Calculus AB

In today's technological society, production and consumption of information, goods, and services continues to increase, necessitating advanced mathematical literacy skills, particularly for those in careers related to science, technology, engineering, and math (STEM). This increasing emphasis on STEM fields requires students to understand the mathematics of change: rates, accumulation, removal, growth, and decline, approximations, representations. Many physical situations are modeled and analyzed utilizing calculus. Calculus is the basis for more advanced study of many STEM fields. AP Calculus AB provides students with the necessary skills and meaningful applications to analyze phenomena encountered in the modern sciences.

Course Description for AP Calculus AB

This course is a college level course having many applications in engineering and the sciences. Topics include limits, derivatives and integration of a wide variety of functions, and applications of differentiation and integration. This is an advanced placement course that prepares the student to take the Calculus AB exam. AP Calculus AB may be taken for college credit.

Curriculum Team Lisa Jones FHHS Sharon Spoede FHC Steve Willott FHN

Secondary Content Leader Director of Student Learning Chief Academic Officer Superintendent Keiren Greenhouse Travis Bracht Mary Hendricks-Harris Dr. Pam Sloan

Content Area: Mathematics		Course: AP Calculus	AB	Strand: Functions, Graphs, Limits I	Pag PAC	
Learner Objectives: Student will analyze functions, graphs and limits					E *	
Concepts:	А.	Limits of functions (including one-sided limits)				GEI
		Students Should Know		S	Students Should Be Able to	MA
						_1

• An intuitive understanding of the limiting process	 Calculating limits using algebra Estimating limits from graphs or tables of data
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Instructional Support

Student Essential Vocabulary					
Limit	Rationalize	One-Sided Limit			

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Readiness & Equity Section 6				
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	MAT	
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	1	
	Problem Solving		4	
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		



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AB/BC AP CALCULUS STANDARD	Standard 1Analysis ofStandard 2Model/nume		
CONTENT	MA4 Patterns and relation		
PROCESS	1.6 Discover/evaluate re		
	1.10 Apply information,		
	3.6 Examine solutions f		
DOV	2		
DOK	2		
INSTRUCTIONAL	Non-Linguistic Representati		
STRATEGIES			
		Readiness & I	Equity Section
	SLA	= Sample Learning Activit	ies & SA = Sample Assessments
21 st Century Themes		Quantitative Literacy	Non Fiction Reading & Writing
Learning & Innovation	n Skills	Critical Thinking and	Enrichment Opportunity
		Problem Solving	
Information, Media, &	Technology Skills	ICT Literacy	Intervention Opportunity
Life & Career Skills		Initiative and Self	Gender, Ethnic, & Disability Equity
		Direction	

Sample Learning Activities

Sample Assessments

Learning Activity #2 :

 $\lim_{x\to 0}$

 $\frac{x-4}{x}$

Use your graphing calculator to complete the table for each function, then approximate the limit.

$$f(x) = \frac{x^2 - 5x + 6}{x^2 + 3x - 10}$$
1.

$$f(x) = \frac{x^2 - 5x + 6}{x^2 + 3x - 10} \approx \frac{1}{1.99 + 2.00 + 2.01 + 2.1 + 2.2}$$
1.

$$f(x) = \frac{f(x) = \frac{x^2 - 5x + 6}{x^2 + 3x - 10} \approx \frac{2}{1.99 + 2.00 + 2.01 + 2.1 + 2.2}$$
2.

$$f(x) = \frac{\sqrt{14 + x} - 3}{x^2 + 3x - 10} \approx \frac{2}{1.99 + 2.00 + 2.01 + 2$$

Assessment #2:

Evaluate each limit algebraically.

1.
$$\lim_{x \to 4} \frac{x^2 - 2x - 8}{x - 4} =$$

2.
$$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 + 2x - 15} =$$

3.
$$\lim_{x \to -5} \frac{\sqrt{14 + x} - 3}{x + 5} =$$

4.
$$\lim_{x \to 0} \frac{\frac{1}{8} + \frac{1}{x - 8}}{x} =$$

Find each one-sided limit.



PROCESS
Learn Activity #1 -- KEY:
1) table values: -0.176, -0.159, -0.144, ---, -0.141, -0.127, -0.111

$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + 3x - 10} \approx_{-0.142}$$
2) table values: 0.1676, 0.1671, 0.1667, ---, 0.1666, 0.1662, 0.1657

$$\lim_{x \to 3} \frac{\sqrt{14 + x} - 3}{x + 5} \approx_{0.1666}$$
3) table values: -0.060, -0.061, -0.062, ---, -0.063, -0.064, -0.066

$$\lim_{x \to 0} \frac{1}{4} + \frac{1}{x - 4} \approx_{-0.0625}$$
4) table values: -0.099, -0.049, -0.005, ---, 0.0049, 0.049, 0.0997

$$\lim_{x \to 0} \frac{1 - \cos x}{x} \approx_{-0.000}$$
5) table values: 0.993, 0.998, 0.999, ---, 0.999, 0.998, 0.993

$$\lim_{x \to 0} \frac{\sin x}{x} \approx_{-1.000}$$
6)
$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + 3x - 10} = \lim_{x \to 2} \frac{(x - 2)(x - 3)}{(x - 2)(x + 5)} = \frac{x - 3}{x + 5} = \frac{-1}{7} \approx -0.142857$$
7)
$$\lim_{x \to -5} \frac{\sqrt{14 + x} - 3}{x + 5} = \lim_{x \to -5} \frac{1}{\sqrt{14 + x} + 3} = \frac{1}{6} \approx 0.166$$

PROCESS	1.6 Discover/evaluate relationships	
	1.10	Apply information, ideas and skills
	3.6	Examine solutions from many perspectives
DOK	2	
LEVEL OF	Master	y Level – 70%
EXPECTATION		

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$$\lim_{x \to 0} \frac{1}{x} + \frac{1}{x} - \frac{1}{x} = \lim_{x \to 0} \frac{x - 4 + 4}{4x(x - 4)} = \lim_{x \to 0} \frac{1}{4(x - 4)} = \frac{-1}{16} = -0.0625$$

$$\frac{Activity's Alignment}{AB/BC AP}$$

$$\frac{AB/BC AP}{CALCULUS}$$

$$\frac{Standard 1 Analysis of functions}{Standard 2 Model/numerically/analytically}$$

$$\frac{CONTENT}{CONTENT} \frac{MA4}{MA4} Patterns and relationships}$$

$$\frac{PROCESS}{1.6} \frac{1.6}{Discover/valuate relationships}$$

$$\frac{1.0 Apply information, ideas and skills}{3.6} = \frac{1}{Examine solutions from many perspectives}$$

$$\frac{DOK}{INSTRAUCTIONAL}$$

$$\frac{Generating and Testing Hypotheses}{STRATFGIES}$$

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Student Resources	Teacher Resources	
General: Calculus of a Single Variable – 8 th edition	General: Calculus of a Single Variable – 8 th edition	
Enrichment:	Enrichment:	
Intervention:	Intervention:	
Enrichment:	Enrichment:	
Intervention:	Intervention:	

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	Course: AP Calculus AB	Strand: Functions, Graphs and Limits II
Learner Objectives: Student will analyze funct	ions, graphs and limits	

Concepts: В. Asymptotic and unbounded behavior

Students Should Know	Students Should Be Able to
• Understanding asymptotes in terms of graphical behavior	 Describing asymptotic behavior in terms of limits involving infinity Comparing relative magnitudes of functions and their rates of change – exponential growth, polynomial growth and logarithmic growth

Instructional	Support

		Instructiona	ll Support			Page
						PAG
		Student Essent	tial Vocabulary			E
Asymptote	Bounded Function	Unbounded Function	Infinite Limits	Limits at Infinity	End-Behavior	*
Exponential Growth	Polynomial Growth	Logarithmic Growth				MER
						GEF
						OR

Readiness & Equity Section			
SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments	
		Page PAG E MER GEF OR MAT 1



$\lim_{x \to \infty} \frac{4x^2 - x - 20}{x^2 - x - 36} = 0$	1) -4 2) 4	3) $-\infty$ 4) ∞ 5) 8/7 6) DNE 7) 0
$\lim_{x \to -36} \frac{4x^2 - 36}{2}$		
$\int_{x \to \infty}^{x \to \infty} x^2 - 2x - 15 = 4$ 3c) DNE 3d) -1/2 3e) 0 3f) 1 on as x approaches positive and negative infinity be horizontal asymptote(s)	 8) 0 9) x = 4 10) The limit of the <i>y</i>-value(s) of 	e function as x approaches positive and negative infinity is of the horizontal asymptote(s).
A ativitu's Alignment		Assessment's Alignment
andard 1 Analysis of functions	AB/BC AP CALCULUS STANDARD	Standard 1 Analysis of functions
 A4 Patterns and relationships 6 Discover/evaluate relationships 10 Apply information, ideas and skills 6 Examine solutions from many parametiyas 	CONTENT PROCESS	MA4Patterns and relationships1.6Discover/evaluate relationships1.10Apply information, ideas and skills3.6Examine solutions from many perspectives
ammarizing and Note taking	DOK LEVEL OF EXPECTATION	2 Mastery Level – 75%
$\frac{1}{2}$	3c) DNE 3d) -1/2 3e) 0 3f) 1 n as x approaches positive and negative infinity e horizontal asymptote(s). Activity's Alignment ndard 1 Analysis of functions A4 Patterns and relationships Discover/evaluate relationships 0 Apply information, ideas and skills Examine solutions from many perspectives mmarizing and Note taking	3c) DNE 3d) -1/2 3e) 0 3f) 1 n as x approaches positive and negative infinity the y-value(s) c activity's Alignment AB/BC AP ndard 1 Analysis of functions A4 Patterns and relationships Discover/evaluate relationships CONTENT PROCESS DOK Level OF EXPECTATION

	Readiness &	Equity Section	E
SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	МЕБ
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	ĢEF
	Problem Solving		ØR
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	МАТ
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	1
	Direction		I. I



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4.
$$\lim_{z \to \infty} \frac{e^{x}}{x^{2} + 3x + 4}$$
 5.
$$\lim_{z \to \infty} \frac{x^{2} + 3x + 4}{e^{x}}$$

6.
$$\lim_{z \to \infty} \frac{100(x^{2} + 3x + 4)}{e^{x}}$$

What can you hypothesize about the relative rates

of growth of
$$e^x$$
 and $x^2 + 3x + 4$?

Learning Activity #2 -- KEY

x	Δx	$Y_1 = e^x$	ΔY_1	$Y_2 = x^2 + 3x + 4$
1		2.7		8
2	1	7.4	4.7	14
4	2	54.6	47.2	32
7	3	1096.6	1042.0	74
13	6	442413.4	441316.8	212

a) > b) > c) >

Note: for questions 1-3, the functions are not shown on the calculator image but larger values of *x* can be "traced" with appropriate window setting.

1) ∞ 2) 0 3) 0

The exponential function grows much faster than the logarithmic function.

The exponential function grows much faster than the polynomial.

PROCESS	1.6
	1.10
	3.6
DOK	2
LEVEL OF	Maste
EXPECTATION	

	Activity's Alignment
AB/BC AP	Standard 1 Analysis of
CALCULUS	
STANDARD	
CONTENT	MA4 Patterns and relation
PROCESS	1.6 Discover/evaluate re
	1.10 Apply information, i
	3.6 Examine solutions
DOK	3
INSTRUCTIONAL	Generating and Testing Hyp
STRATEGIES	

	Student Resources		Teacher Resources	
General:	Calculus of a Single Variable – 8th edition	General:	Calculus of a Single Variable – 8th edition	
	Enrichment:		Enrichment:	
	Intervention:		Intervention:	Page PAG E *
NOTE: These section	ns will be partially completed during the curriculum writin	g process and finalize	d during the year one review process.	GEF OR MAT

Content Area: Mathematics	Course: AP Calculus AB	Strand: Functions, Graphs, Limits III
Learner Objectives: Student will analyze funct	ions, graphs and limits	

Concepts: C. Continuity as a property of functions

		_Page
Students Should Know	Students Should Be Able to	PAG
• An intuitive understanding of continuity	 Understanding continuity in terms of limits Geometric understanding graphs of continuous functions – Intermediate Value Theorem and Extreme Value Theorem 	E * MER
		GEF -OR
		MAT

Instructional Support

Student Essential Vocabulary

Continuity Discontinuity Removable Discontinuity	Non-Removable Discontinuity
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	Readiness & Equity Section					
SLA	= Sample Learning Activit	ties & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing				
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity				
	Problem Solving					
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity				
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity				
	Direction					

Sample Learning Activities	Sample Assessments	Page
		PAG
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		GEF
		ИАТ
		1



$\lim_{x \to -5} f(x) = f(-2.5)$ 1A) 0 B) -1 C) 3 D) DNE 2A) undefined b) ∞ c) because $f(-3)$ is not defined 3A) 1 b) DNE c) because $\lim_{x \to -2} f(x)$ DNE Aa) 1 b) DNE c) because $\lim_{x \to -5} f(x) = f(-5)$ (A) 0 B) -1 C) DNE D) undefined (A) 2 B) -1 C) DNE D) UNE (A) 4A) 2 B) -1 C) DNE (A) 4A) 2 B			
2a) undefined b) ∞ c) because $f(-3)$ is not defined 3a) 1 b) DNE c) because $\lim_{x \to -3} f(x)$ DNE 4a) 1 b) 2 c) because $\lim_{x \to -5} f(x) \neq f(-5)$ 4a) 1 b) 2 c) because $\lim_{x \to -5} f(x) \neq f(-5)$ Activity's Alignment Ab/BC AP CALCULUS STANDARD CONTENT MA 2 Geometric and spatial sense MA 4 Patterns and relationships 1.10 Apply information, ideas and skills 3.6 Examine solutions from many perspectives ADK 2 INSTRUCTIONAL Identifying Similarities and Differences STRATEGIES ACK 2 INSTRUCTIONAL Identifying Similarities and Differences ADK 2 ADK 2 ADK 2 ACK ADK ADK	1a) 2 b) 2 c) because $\lim_{x \to -2.5} f(x) = f(-2.5)$	1A) 0 B) -	1 C) 3 D) DNE
2a) Indefinedb) ∞ c) because $\lim_{x \to 2} f(x)$ Indefined3a) 1b) DNEc) because $\lim_{x \to 2} f(x)$ DNE4a) 1b) 2c) because $\lim_{x \to -5} f(x) \neq f(-5)$ c) DNE4a) 1b) 2c) because $\lim_{x \to -5} f(x) \neq f(-5)$ c) DNE5) $f(x)$ is discontinuous at $x = -2$ because $\lim_{x \to -2} f(x)$ does not exist, $f(x) \neq f(3)$ $\lim_{x \to -2} f(x) \neq f(3)$ $f(x) \neq f(3)$ $\lim_{x \to -2} f(x) \neq f(3)$ $f(x) \neq f(3)$ $\int f(x) = f(3)$ $f(x) \neq f(3)$ $\int f(x) = f(3)$ $f(x) \neq f(3)$ $\int f(x) = f(3)$ $f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(x) = f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3 because \lim_{x \to -2} f(x) = \int f(3)f(x) \Rightarrow discontinuous at x = 3f(x) \Rightarrow disco$	(2) undefined (b) (2) (c) here $f(2)$ is not defined	2A) 0 B) -	1 C) 3 D) DNE
3a) 1b) DNEc) because $\lim_{x\to 2^+} f(x)$ DNE4A) 2B) -1C) DNED) undefined4a) 1b) 2c) because $\lim_{x\to -5^+} f(x) \neq f(-5)$ 5) $f(x)$ is discontinuous at $x = -2$ because $\lim_{x\to -2^+} f(x)$ does not exist, $f(x) \neq f(3)$ $\overline{Activity's Alignment}$ $\overline{Adigan drl 1}$ Analysis of functions $\overline{Adigan drl 1}$ Analysis of functions $\overline{CALCULUS}$ $\overline{CALCULUS}$ $\overline{Adigan drl 1}$ Analysis of functions $\overline{Adigan drl 1}$ Analysis of functions $\overline{CONTENT}$ MA 2Geometric and spatial sense $\overline{MA 4}$ Patterns and relationships $\overline{MA 4}$ Patterns and relationshipsPROCESS1.6Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.6 $\overline{Examine solutions from many perspectives}$ \overline{DOK} 2 1.0 Apply information, ideas and presentives 1.10 1.0 $\overline{Apply information, ideas and skills}3.6\overline{DOK}21.0\overline{Apply information, ideas and skills}3.6\overline{Examine solutions from many perspectives}\overline{DOK}21.0\overline{Apply information, ideas and skills}3.6\overline{Examine solutions from many perspectives}\overline{DOK}21.0\overline{Apply information, ideas and \overline{Apply information, ideas and skills}3.6\overline{DOK}21.0\overline{Apply information, ideas and skills}3.6\overline{DOK}21.0\overline{Apply information, ideas and skills}1.0Apply information information information information information information information information info$	(-3) undefined b) \sim c) because $f(-3)$ is not defined	3A) 2 B) -	1 C) DNE D) undefined
4a) 1 b) 2 c) because $\lim_{x\to -5} f(x) \neq f(-5)$ Activity's Alignment AB/BC AP CALCULUS STANDARD CONTENT MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 2 INSTRUCTIONAL STRATEGIES STATEGIES (A) (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x)$ does not exist, $\lim_{x\to -3} f(x) \neq f(3)$. (A) (x) is discontinuous at $x = 3$ because $\lim_{x\to -2} f(x)$ does not exist, $\lim_{x\to -3} f(x) \neq f(3)$. (A) (x) is discontinuous at $x = 3$ because $\lim_{x\to -2} f(x) \neq f(3)$. (A) (x) is discontinuous at $x = 3$ because $\lim_{x\to -2} f(x) = 1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = 1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = 1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = 1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = 1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = 1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -1$. (x) is discontinuous at $x = -2$ because $\lim_{x\to -2} f(x) = -$	3a) 1 b) DNE c) because $\lim_{x \to 2} f(x)$ DNE	4A) 2 B) -	1 C) DNE D) undefined
Activity's AlignmentAB/BC AP CALCULUS STANDARDStandard 1Analysis of functionsCONTENTMA 2Geometric and spatial sense MA 4Patterns and relationshipsPROCESS1.6Discover/evaluate relationships1.10Apply information, ideas and skills 3.6Examine solutions from many perspectives 4.1Support decisionsDOK2INSTRUCTIONAL STRATEGIESIdentifying Similarities and DifferencesDOKDOK2	4a) 1 b) 2 c) because $\lim_{x \to -5} f(x) \neq f(-5)$	5) $f(x)$ is discontin f(x) is discontin	nuous at $x = -2$ because $\lim_{x \to -2} f(x)$ does not exist, nuous at $x = 3$ because $\lim_{x \to 3} f(x) \neq f(3)$.
AB/BC AP CALCULUS STANDARD Standard 1 Analysis of functions CONTENT MA 2 Geometric and spatial sense MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 2 INSTRUCTIONAL STRATEGIES Identifying Similarities and Differences	Activity's Alignment		A
CALCULUS STANDARDMA 2 MA 2 PROCESSGeometric and spatial sense MA 4 Patterns and relationshipsCALCULUS STANDARDStandard 1 CALCULUS STANDARDPROCESS1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.6 4.1 Support decisionsMA 2 PROCESSGeometric and spatial sense MA 4 Patterns and relationshipsDOK2INSTRUCTIONAL STRATEGIESIdentifying Similarities and Differences	AB/BC AP Standard 1 Analysis of functions		Assessment's Alignment Standard 1 Analysis of functions
OTRINUMEMA 2Geometric and spatial senseSTANDARDCONTENTMA 4Patterns and relationshipsMA 4Patterns and relationshipsPROCESS1.6Discover/evaluate relationshipsMA 4Patterns and relationships1.10Apply information, ideas and skills3.6Examine solutions from many perspectives1.6Discover/evaluate relationships3.6Examine solutions from many perspectives1.10Apply information, ideas and skills3.6Examine solutions from many perspectives4.1Support decisions3.6Examine solutions from many perspectives4.1Support decisionsDOK24.1Support decisionsDOK2INSTRUCTIONAL STRATEGIESIdentifying Similarities and DifferencesDOK2LEVEL OF EXPECTATIONMastery Level – 75%EXPECTATION	STANDARD	CALCULUS	Standard 1 Analysis of functions
MA 4Patterns and relationshipsPROCESS1.6Discover/evaluate relationships1.10Apply information, ideas and skills3.6Examine solutions from many perspectives4.1Support decisionsDOK2INSTRUCTIONAL STRATEGIESIdentifying Similarities and Differences	CONTENT MA 2 Geometric and spatial sense	STANDARD	
PROCESS1.6Discover/evaluate relationships1.10Apply information, ideas and skills3.6Examine solutions from many perspectives4.1Support decisionsDOK2INSTRUCTIONAL STRATEGIESIdentifying Similarities and DifferencesDOK2LEVEL OF EXPECTATIONLEVEL OF EXPECTATION	MA 4 Patterns and relationships	CONTENT	MA 2 Geometric and spatial sense
1.10Apply information, ideas and skills3.6Examine solutions from many perspectives4.1Support decisionsDOK2INSTRUCTIONAL STRATEGIESIdentifying Similarities and DifferencesSTRATEGIES00K1.10Apply information, ideas and skills3.6Examine solutions from many perspectives4.1Support decisions00K21.10Mastery Level – 75%2EXPECTATION	PROCESS 1.6 Discover/evaluate relationships		MA 4 Patterns and relationships
3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 2 INSTRUCTIONAL STRATEGIES Identifying Similarities and Differences DOK 2 LEVEL OF EXPECTATION Mastery Level – 75%	1.10 Apply information, ideas and skills	PROCESS	1.6 Discover/evaluate relationships
4.1 Support decisions DOK 2 INSTRUCTIONAL STRATEGIES Identifying Similarities and Differences DOK 2 LEVEL OF EXPECTATION Mastery Level – 75%	3.6 Examine solutions from many perspectives		1.10 Apply information, ideas and skills
DOK 2 INSTRUCTIONAL STRATEGIES Identifying Similarities and Differences DOK 2 LEVEL OF Mastery Level – 75% EXPECTATION	4.1 Support decisions	-11	3.6 Examine solutions from many perspectives
INSTRUCTIONAL Identifying Similarities and Differences STRATEGIES LEVEL OF EXPECTATION	DUK 2 DISTRUCTIONAL LINE Construction of Difference	DOK	2
EXPECTATION EXPECTATION	STRATECIES	LEVEL OF	A Mastery Level – 75%
	STRATEOIES	EXPECTATION	

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	Readiness &	Equity Section	Inc.
SL	A = Sample Learning Activit	ties & SA = Sample Assessments	1
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	

AP Calculus AB

Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities

Learning Activity #2 :

Intermediate Value Theorem

If the function f(x) is continuous on [a, b], and y is a number between f(a) and f(b), then there exists at least one number c in the open interval (a, b) such that f(c) = y.

Restate this theorem using G'(t) instead of f(x).

An example from the 2008 AP Free Response Question 2 (part c):

t (hours)	0	1	3	4	7	8	9
L(t) (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time *t* is modeled

by a twice-differentiable function L for $0 \le t \le 9$. Values of L(t) at various times t are shown in the table above.



Assessment #2:

Verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.

Sample Assessments

$$f(x) = x^2 - 6x + 8$$
, [0, 3], $f(c) = 0$

Assessment #2 -- KEY

f(x) is continuous on [0, 3], and $f(3) \le f(c) \le f(0)$.

	Assessment's Alignment	
AB/BC AP	Standard 1 Analysis of functions	
CALCULUS		
STANDARD		
CONTENT	MA 2 Geometric and spatial sense	
	MA 4 Patterns and relationships	Page
PROCESS	1.6 Discover/evaluate relationships	PAG
	1.10 Apply information, ideas and skills	E
	3.6 Examine solutions from many perspectives	*
	4.1 Support decisions	MEF
DOK	2	GEF
LEVEL OF	Mastery Level – 75%	D R
EXPECTATION		МАТ
$f(c) = c^2 - 6c + 8$	= (c-4)(c-2) = 0 when $c = 2$ or $c = 4$. The	1
solution for c is c	= 2 only since 4 is not in the interval $(0, 3)$.	-



For $0 \le t \le 9$, what is the fewest number of times at which L'(t) must equal 0? Give a reason for your answer.

Learning Activity #2 -- KEY

If the function G'(t) is continuous on [a, b], and y is a number between G'(a) and G'(b), then there exists at least one number c in the open interval (a, b) such that G'(c) = y. EM BED Equ atio n.3 • t

(c) L is differentiable on [0, 9] so the Mean Value Theorem implies L'(t) > 0 for some t in (1, 3) and some t in (4, 7). Similarly, L'(t) < 0 for some t in (3, 4) and some t in (7, 8). Then, since L' is continuous on [0, 9], the Intermediate Value Theorem implies that L'(t) = 0 for at least three values of t in [0, 9].

OR

The continuity of *L* on [1, 4] implies that *L* attains a maximum value there. Since L(3) > L(1) and L(3) > L(4), this maximum occurs on (1, 4). Similarly, *L* attains a minimum on (3, 7) and a maximum on (4, 8). *L* is differentiable, so L'(t) = 0 at each relative extreme point on (0, 9). Therefore L'(t) = 0 for at least three values of *t* in [0, 9].

[*Note:* There is a function L that satisfies the given conditions with L'(t) = 0 for exactly three values of t.]

	Activity's Alignment
AB/BC AP	Standard 1 Analysis of functions
CALCULUS	
STANDARD	
CONTENT	MA 2 Geometric and spatial sense
	MA 4 Patterns and relationships
PROCESS	1.6 Discover/evaluate relationships
	1.10 Apply information, ideas and skills
	3.6 Examine solutions from many perspectives
	4.1 Support decisions
DOK	3
INSTRUCTIONAL	Cues, Questions and Advanced Organizers
STRATEGIES	

	Student Resources		Teacher Resources	
General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition	
				Page
				PAG
Enrichment:		Enrichment:		E
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Intervention:		Intervention:		GEF
				OR
				—MAT

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process

Content Area: Mathematics	Course: AP Calculus AB	Strand: Derivatives I
Learner Objectives: Students will calculate, int	terpret and analyze derivatives	

Concepts: A. Concept of the derivative

Students Should Know	Students Should Be Able to
• Relationship between differentiability and continuity	 Find a derivative presented graphically, numerically and analytically Find a derivative interpreted as an instantaneous rate of change Find a derivative defined as the limit of the difference quotient

Instructional Support

	Student Essential Vocabulary					
Differentiability	Difference Quotient	Higher-Orde	Higher-Order Derivative		tantaneous Rate of Change	
Average Ra	te of Change	Implicit Differentiation				Page
						PAG
						E
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						MER
						GEF
						OR
						MAT
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AP Calculus AB

Readiness & Equity Section				
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing		
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity		
	Problem Solving			
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity		
	Direction			





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	Standard 1 Analysis of functions
CALCULUS	Standard 3 Differential calculus
STANDARD	Standard 5 Differential calculus
CONTENT	MA 1 Number sense
001012101	MA 2 Geometric and spatial sense
	MA 4 Patterns and relationships
PROCESS	1.6 Discover/evaluate relationships
	1.10 Apply information, ideas and skills
	3.5 Reason logically (inductive/deductive)
	3.6 Examine solutions from many perspectives
	4.1 Support decisions
DOK	2
INSTRUCTIONAL	Nonlinguistic Representation
STRATEGIES	

Readiness & Equity Section				
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes Quantitative Literacy Non Fiction Reading & Writing				
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity		
	Problem Solving			
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity		
	Direction			

Sample Learning Activities	Sample Assessments
	Page
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	Activity's Alignment	$\tan 3(r+h) - \tan 3r$
AB/BC AP CALCULUS STANDARD	Standard 1Analysis of functionsStandard 3Differential calculus	$\lim_{h \to 0} \frac{\lim_{h \to 0} \frac{(x+h)^2 \tan 3x}{h}}{h} =$
CONTENT	MA 1Number senseMA 2Geometric and spatial senseMA 4Patterns and relationships	A. 0 D. $3\cot(3x)$ B. $3\sec^2(3x)$ C. $\sec^2(3x)$ E. nonexistent
PROCESS	 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions 	5. What is $\lim_{h \to 0} \frac{8\left(\frac{1}{2} + h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?
DOK	2	
STRATEGIES		A. 0 B. $\frac{1}{2}$ C. 1 D. The limit does not exist. E. Cannot be determined from the information given. Assessment #2 – KEY 1) A 2) E 3) E
		4) B 5) B
		Assessment's Alignment
		AB/BC APStandard 1Analysis of functionsCALCULUSStandard 3Differential calculusSTANDARD
		CONTENT MA 1 Number sense MA 2 Geometric and spatial sense MA 4 Patterns and relationships

PROCESS	 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions
DOK	2
LEVEL OF EXPECTATION	Mastery Level – 70%

Student Resources		Teacher Resources	
General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition
Enrichment:		Enrichment:	
Intervention		Intervention	

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

					Page PAG E
Content Ar	ea: Matl	hematics	Course: AP Calculus AB	Strand: Derivatives II	
Learner Objectives: Students will calculate, interpret and analyze derivatives			GEF		
Concepts:	B.	Derivative at a point			мат 1
Students Should Know	Students Should Be Able to				
--	--				
• Slope of a curve at a point – points that are vertical tangent, points at which there are no tangent	 Tangent line to a curve at a point Instantaneous rate of change as the limit of average rate of change Approximate rate of change from graphs and tables of values 				

Student Essential Vocabulary					
Tangent LineInstantaneous Rate of ChangeAverage Rate of ChangeSecant Line					
Point-Slope Form of a Linear Equation					

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	Readiness &	Equity Section	, in the second s	
SLA	SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing		
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity		
	Problem Solving			
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		
fhsd.acad.kg		AP Calculus AB February 2011		-

Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments			
Learning Activity #1 :	Assessment #1:			
1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$ A6 B3 C. 3 D. 6 E. 8	1. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$ A. 3 B. 1 C1 D3 E5			
 2. An equation of the line tangent to the graph of y = x + cos x at the point (0, 1) is A. y = 2x + 1 B. y = x + 1 C. y = x D. y = x - 1 E. y = 0 	2. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point (1, 5) is A. $13x - y = 8$ B. $13x + y = 18$ C. $x - 13y = 64$ D. $x + 13y = 66$ E. $-2x + 3y = 13$			
Learning Activity #1 – KEY				
1) C 2) B	Assessment #1 – KEY			
	1) D 2) B			
Activity's AlignmentAB/BC APStandard 1Analysis of functionsCALCULUSStandard 3Differential calculusSTANDARDAnalysis of functionsAnalysis of functions				
CONTENT MA 1 Number sense	Assessment's Alignment			
MA 2 Geometric and spatial sense MA 4 Patterns and relationships	AB/BC AP Standard 1 Analysis of functions			
	CALCULUS Standard 3 Differential calculus			
	STANDARD			
	CONTENT MAI Number sense			
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PROCESS	1.6 Discover relationships1.10 Apply ideas and skills		MA 2 Geometric and spatial sense MA 4 Patterns and relationships
	3.5 Reason logically	PROCESS	1.6 Discover relationships
DOK	2		1.10Apply ideas and skills3.5Reason logically
INSTRUCTIONAL	Homework and Practice		3.6 Examine multiple perspectives
STRATEGIES		DOK	2
		LEVEL OF	Mastery Level – 80%
		EXPECTATION	

Readiness & Equity Section					
SLA	SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing			
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity			
	Problem Solving				
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity			
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity			
	Direction				

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Sample Learning Activities	Sample Assessments



Caren rides her bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time t = 12 minutes.

During the time interval $0 \le t \le 12$ minutes, her velocity v(t), in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

- a. Find the acceleration of Caren's bicycle at time t = 7.5 minutes. Indicate units of measure.
- b. Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

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t (hours)	0	1	3	4	7	8	9
L(t) (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time *t* is modeled by the twice differentiable function *L* for $0 \le t \le 9$. Values of L(t) at various times *t* are shown in the table above.

Assessment #2:

1.

2.



For each of v'(4) and v'(20), find the value or explain why it does not exist. Indicate units of measure.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature T(x), in degrees Celsius ${}^{(\square C)}$, of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Estimate T'(7). Show the work that leads to your answer. Indicate units of measure.

Assessment #2 – KEY

1)
$$v'(4)$$
 does not exist since on (0, 4) $v'(t) = 5$ and on (4, 16) $v'(t) = 0$

and

Use the data in the tapeople waiting in lin computations that lease Learning Activity #2 a(7.5) = v'(7.5) 1a) 1b) since $v(t) = 0$ @ negative, Caren turns arous	able to estimate the rate at which the number of e was changing at 5:30 PM ($t = 5.5$). Show the ad to your answer. Indicate units of measure. - KEY $= \frac{1}{1} = -0.1$ miles per minute t = 2 and $v(t)$ changes from positive to and at $t = 2$ minutes	$5 \neq 0$ $v'(20) = \frac{-20}{8}$ $T'(7) \approx \frac{T(8)}{8}$	= -2.5 meters per second per second $\frac{-T(6)}{-6} = \frac{55-62}{2} = -3.5$ degrees Celsius per centimeter
2) $L'(t) \approx \frac{L(7) - L(7)}{7 - 4}$	$\frac{(4)}{3} = \frac{150 - 126}{3} = 8$ people per hour		
	Activity's Alignment		Assessment's Alignment
AB/BC AP	Standard 1 Analysis of functions	AB/BC AP	Standard I Analysis of functions
CALCULUS	Standard 3 Differential calculus	CALCULUS	Standard 5 Differential calculus
STANDARD		SIANDAKD	MA 1 Number sense
CONTENT	MA 1 Number sense	CONTENT	MA 1 Number sense MA 2 Geometric and anoticl sense
	MA 2 Geometric and spatial sense		MA 4 Patterns and relationships
	MA 4 Patterns and relationships	PROCESS	1.6 Discover relationships
PROCESS	1.6 Discover relationships	IROCLOS	1.10 Apply ideas and skills
	1.10 Apply ideas and skills		3.5 Reason logically
	3.5 Reason logically		3.6 Examine multiple perspectives
	3.6 Examine multiple perspectives	DOK	2
DOK	2	LEVEL OF	Mastery Level – 70%
INSTRUCTIONAL	Nonlinguistic Representation	EXPECTATION	
STRATEGIES			·J

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	Student Resources		Teacher Resources
General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8th edition
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Enrichment:		Enrichment:	
Intervention:		Intervention:	

Content Area: Mathematics	Course: AP Calculus AB	Strand: Derivatives III
Learner Objectives: Students will calculate, in	terpret and analyze derivatives	

Concepts: C. Derivative as a function

Students Should Know	Students Should Be Able to
 Corresponding characteristics of graphs of <i>f</i> and <i>f</i>" Equations involving derivatives 	 Relationship between the increasing and decreasing behavior of <i>f</i> and the sign of <i>f</i>' The Mean Value Theorem and its geometric interpretation Utilize relationships between <i>f</i> and <i>f</i>' to determine relative extrema

		Student Essen	ii Support		
Increasing	Decreasing	Absolute Extrema	Relative Extreme	Tangent Line	Secont Line
mercasing	Decreasing	Ausolule Extrema	Kelauve Extrema		Secalit Lille

Readiness & Equity Section			
SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments



Assessment #1:

The function f given by
$$f(x) = x^3 + 12x - 24$$
 is

- A. Increasing for x < -2, decreasing for -2 < x < 2, increasing for x > 2.
- B. Decreasing for x < 0, increasing for x > 0.
- C. Increasing for all *x*.
- D. Decreasing for all *x*.
- E. Decreasing for x < -2, increasing for -2 < x < 2, decreasing for x > 2.

Assessment #1 – KEY

С

	Α	ssessment's Alignment	1
AB/BC AP	Standar	rd 1 Analysis of functions	
CALCULUS	Standar	rd 3 Differential calculus	
STANDARD			
CONTENT	MA 2	Geometric and spatial sense	
	MA 4	Patterns and relationships	
PROCESS	1.6	Discover/evaluate relationships	
	1.10	Apply information, ideas and skills	Page
	3.5	Reason logically (inductive/deductive)	PAG
	3.6	Examine solutions from many perspectives	IE
	4.1	Support decisions	*
DOK	2		MER
LEVEL OF	Master	y Level – 80%	GEF
EXPECTATION] OR
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AP Calculus AB

	Activity's Alignment
AB/BC AP	Standard 1 Analysis of functions
CALCULUS	Standard 3 Differential calculus
STANDARD	
CONTENT	MA 2 Geometric and spatial sense
	MA 4 Patterns and relationships
PROCESS	1.6 Discover/evaluate relationships
	1.10 Apply information, ideas and skills
	3.5 Reason logically (inductive/deductive)
	3.6 Examine solutions from many perspectives
	4.1 Support decisions
DOK	2
INSTRUCTIONAL	Nonlinguistic Representation
STRATEGIES	
STRALOILS	

Readiness & Equity Section			
SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities		Sample Assessments	
Learning Activity #2	:	Assessment #2:	
See "Relating a Functi	on and its Derivative" activity cards. Appendix : A	The <u>derivative</u> of of f have a relative	<i>f</i> is $x^4(x-2)(x+3)$. At how many points will the graph remaximum?
Learning Activity #2	– KEY	A. None	B. One C. Two D. Three E. Four
See Appendix		Assessment #2 – F	KEY
		В	
	Activity's Alignment		Assossment's Alignment
AB/BC AP CALCULUS	Standard 1 Analysis of functions Standard 3 Differential calculus	AB/BC AP	Standard 1 Analysis of functions
STANDARD		CALCULUS	Standard 3 Differential calculus
CONTENT	MA 2 Geometric and spatial sense MA 4 Patterns and relationships	CONTENT	MA 2 Geometric and spatial sense MA 4 Patterns and relationships
PROCESS	 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions 	PROCESS	 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives
DOK	2	DOK	4.1 Support decisions
INSTRUCTIONAL	Homework and Practice	DOK	2
STRATEGIES			

LEVEL OF EXPECTATION	Mastery Level – 75%

	Student Resources		Teacher Resources
General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition
Enrichment:		Enrichment:	
Intervention		Intervention	
intervention.		intervention.	

Content Area: Mathematics	Course: AP Calculus AB	Strand: Derivative IV
Learner Objectives: Students will calculate, interpret and analyze derivatives		

Concepts: D. Second derivatives

Students Should Know	Students Should Be Able to
 Corresponding characteristics of the graphs of <i>f</i>, <i>f</i>', and <i>f</i>'' Points of inflection as places where concavity changes 	• Relationship between the concavity of <i>f</i> and the sign of <i>f</i> "

		Student Essen	tial Vocabulary	Page
Point of Inflection	Concave Upward	Concave Downward		PAG
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Readiness & Equity Section				
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing		
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity		
	Problem Solving			
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity		
	Direction			

Sample Learning Activities	Sample Assessments	
Learning Activity #1 :	Assessment #1:	
1. What is the <i>x</i> -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^{3} + 5x^{2} + 24$?	1. Identify all intervals on which the graph of the function $f(x) = \frac{4}{3}x^3 - x^2 - 3x$ is either concave up or concave down.	
A. 5 B. 0 C. $-\frac{10}{3}$ D5 -10	2. Find all <u><i>points</i></u> of inflection for the graph of the function $f(x) = \frac{1}{2}x^4 - 4x^3 + x - 1$	Pag PA E
2. If $f''(x) = x(x+1)(x-2)^2$, then the graph of <i>f</i> has inflection points when $x =$	Assessment #1 – KEY	I* ME GE
A1 only B. 2 only C1 and 0 only D1 and 2 only E1, 0 and 2 only	1) $f'(x) = 4x^2 - 2x - 3 \Rightarrow f''(x) = 8x - 2 \Rightarrow f''(x) = 0$ (a) $x = -\frac{1}{4}$	рк МА 1

 3. The graph of the f concavity at x = A1.58 B Learning Activity #1 1) D 2) C 	Function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes 1.63 C1.67 D1.89 E2.33 - KEY 3) D	The graph of $f''(x) > 0$ on the interval $\left(-\infty, \frac{1}{4}\right)_{\text{sinc}}$ 2) $f'(x) = 2x^3 - 0$ and $x = 4$. Point concavity of the $f''(x) > 0$ on	$f(x)$ is concave upward on $\left(\frac{1}{4},\infty\right)$ since and the graph of $f(x)$ is concave downward on e f''(x) < 0 on the interval. $-12x^2 + 1 \Rightarrow f''(x) = 6^2 x - 24x \Rightarrow f''(x) = 0$ @ $x =$ nts of inflection exist at (0, -1) and (4, -125) since the he graph changes at that location. We know this since the intervals $(-\infty, -1)$ and $(0, 2)$ and $f''(x) < 0$ on
	Activity's Alignment		$1 (2 \infty)$
AB/BC AP CALCULUS STANDARD	Standard 1Analysis of functionsStandard 3Differential calculus	the intervals (-	$(1, 0)$ and $(2, \infty)$
CONTENT	MA 2Geometric and spatial senseMA 4Patterns and relationships		
PROCESS	1.6 Discover/evaluate relationships		Assessment's Alignment
	3.5 Reason logically (inductive/deductive)	AB/BC AP	Assessment's Angliment
	3.6 Examine solutions from many perspectives4.1 Support decisions	CALCULUS STANDARD	Standard 3 Differential calculus Page
DOK	2	CONTENT	MA 2 Geometric and spatial sense PAC
INSTRUCTIONAL	Homework and Practice		MA 4 Patterns and relationships
STRATEGIES		PROCESS	1.6Discover/evaluate relationshipsImage: Metric constraints1.10Apply information, ideas and skillsMEI3.5Reason logically (inductive/deductive)GEI3.6Examine solutions from many perspectivesOR4.1Support decisionsMAT
		DOK	2 1
		LEVEL OF EXPECTATION	Mastery Level – 75%

Readiness & Equity Section				
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing		
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity		
	Problem Solving			
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity		
	Direction			

Learning Activity #2	2	Assessment #2:
 See "It's a Match For the graph sho right, determine v represents f, f 	-up" activity cards. Appendix : B we at the which graph f'' , and f''' .	1. The graph of a twice- differentiable function f is shown in the figure at the right. Which of the following is true? A. $f(1) < f'(1) < f''(1)$ B. $f(1) < f''(1) < f''(1)$
 Learning Activity #2 1) See Appendix 2) <i>f</i> is the red graph, <i>j</i> 	f' is the blue graph and f'' is the green graph	C. $f'(1) < f(1) < f''(1)$ E. $f''(1) < f'(1) < f(1)$ D. $f''(1) < f'(1)$ 2. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for $x > -\frac{2}{2}$
AB/BC AP CALCULUS STANDARD CONTENT PROCESS DOK INSTRUCTIONAL STRATEGIES	Activity's AlignmentStandard 1Analysis of functionsStandard 3Differential calculusMA 2Geometric and spatial senseMA 4Patterns and relationships1.6Discover/evaluate relationships1.10Apply information, ideas and skills3.5Reason logically (inductive/deductive)3.6Examine solutions from many perspectives4.1Support decisions3Nonlinguistic Representation	A. $x < 0$ B. $x > 0$ C. $x < -2$ or 3 $\frac{2}{3} < x < 2$ E. $x < \frac{2}{3}$ or $x > 2$ Assessment #2 – KEY 1) D 2) D

AB/BC AP Standard 1 Analysis of functions CALCULUS Standard 3 Differential calculus STANDARD MA 2 Geometric and spatial sense MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION X		Assessment's Angnment
CALCULUS STANDARD Standard 3 Differential calculus CONTENT MA 2 Geometric and spatial sense MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION	AB/BC AP	Standard 1 Analysis of functions
STANDARD CONTENT MA 2 Geometric and spatial sense MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION State of the second sec	CALCULUS	Standard 3 Differential calculus
CONTENT MA 2 Geometric and spatial sense MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION Support decisions	STANDARD	
MA 4 Patterns and relationships PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION Support decisions	CONTENT	MA 2 Geometric and spatial sense
PROCESS 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION		MA 4 Patterns and relationships
1.10 Apply information, ideas and skills 3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION	PROCESS	1.6 Discover/evaluate relationships
3.5 Reason logically (inductive/deductive) 3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION		1.10 Apply information, ideas and skills
3.6 Examine solutions from many perspectives 4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION Support decisions		3.5 Reason logically (inductive/deductive)
4.1 Support decisions DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION		3.6 Examine solutions from many perspectives
DOK 3 LEVEL OF Mastery Level – 75% EXPECTATION 9	DOK	4.1 Support decisions
EXPECTATION Mastery Level – 75%	DOK	3 Martare Land 750/
EXPECIATION	LEVEL OF	Mastery Level – 75%

	Student Resources		Teacher Resources
General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition
Enrichment:		Enrichment:	
T ()		T ()	
Intervention:		Intervention:	

Content Area: Mathematics	Course: AP Calculus AB	Strand: Derivatives V
Learner Objectives: Students will calculate	interpret and analyze derivatives	

Concepts: E. Applications of derivatives

Students Should Know	Students Should Be Able to
 All theorems, properties and relationships needed to apply the concepts of functions and their first and second derivatives. 	 Analysis of curves, including the notions of monotonicity and concavity Optimization, both absolute (global) and relative (local) extrema Modeling rates of change, including related rates problems Use of implicit differentiation to find the derivative of an inverse functions Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Student Essential Vocabulary				Pag		
Position Function	Velocity Function	Acceleration Function	Relative Extrema	Absolute Extrema	Monotonic	PAC
Optimize	Slope Field	Differential Equation	General Solution	Particular Solution	Related Rates	Ē
Implicitly Defined Functions Explicitly Defined Functions				*		
			ME			
						GEI

Readiness & Equity Section				
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing		
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity		
	Problem Solving			
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity		
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity		
	Direction			

Sample Example Assessments	Sample Learning Activities	Sample Assessments
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Learning Activity #1 :

1. If y = 2x - 8, what is the minimum value of the product *xy*?

A. -16 B. -8 C. -4 D. 0 E. 2

2. A farmer has 80 feet of fence and wants to make three identical pens. No fence will be needed on one side since the pens will attach to the barn as shown in the diagram. What dimensions (for the total enclosure) will make the area of the pens as large as possible?



3. A manufacturer wants to design an open rectangular box with a volume of 256 square inches. What dimensions will produce a box that will require the least amount of material to produce it?

Learning Activity #1 - KEY

- 1) B
- 2) The dimensions of the total enclosure that has a maximum area are 40 feet by 10 feet (the 40 foot side runs parallel to the side of the barn).
- 3) The dimensions of the box of maximum volume is 8 inches by 8 inches by 4 inches.

Assessment #1:

- 1. Find two rational numbers whose product is 192 and whose sum is a minimum.
- 2. A family plans to fence in their backyard in order for their dog to be able to run free. They will attach the fence to the back of their house as shown in the diagram. They want the dog to have 800 square feet of area in which to run. How much fence should they purchase in order to use the least fence?



- 3. A manufacturer wants to design an open rectangular box having a square base and a surface area of 108 square inches. What dimensions will produce a box of maximum volume?
- Assessment #1 KEY
- 1) The numbers are both $8\sqrt{3}$.
- The dimensions of the enclosure with the minimum amount of fence are 40 feet by 20 feet (the 40 foot side runs parallel to the house).

			Assessment's Alignment
		AB/BC AP	Standard 1 Analysis of functions
	Activity's Alignment	CALCULUS	Standard 2 Model numerically/analytically
AB/BC AP	Standard 1 Analysis of functions	STANDARD	Standard 3 Differential calculus
CALCULUS	Standard 2 Model numerically/analytically		Standard 4 Position, speed, acceleration
STANDARD	Standard 3 Differential calculus		Standard 5 Related rates
	Standard 4 Position, speed, acceleration		Standard 6 Differential equations/slope fields
	Standard 5 Related rates	CONTENT	MA 1 Number sense
	Standard 6 Differential equations/slope fields		MA 2 Geometric and spatial sense
CONTENT	MA 1 Number sense		MA 4 Patterns and relationships
	MA 2 Geometric and spatial sense	PROCESS	1.7 Evaluate information
	MA 4 Patterns and relationships		1.10 Apply information, ideas, and skills
PROCESS	1.7 Evaluate information		3.5 Apply one's own strategies
	1.10 Apply information, ideas, and skills		3.6 Examine solutions from many perspectives
	3.5 Apply one's own strategies		3.7 Evaluate strategies
	3.6 Examine solutions from many perspectives	DOK	2
	3.7 Evaluate strategies	LEVEL OF	Mastery Level – 75%
DOK	2	EXPECTATION	
INSTRUCTIONAL	Nonlinguistic Representation		•
STRATEGIES	-		

	Readiness &	Equity Section	
SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	Page
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	PAG
	Problem Solving		É
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	/*
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	MER
	Direction		ĠEF
			OR
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Sample Learning Activities	Sample Assessments	MAT
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Learning Activity #2	Assessment #2:
1. The radius of a circle is decreasing at a constant rate of 0.1 centimeters per second. In terms of the circumference <i>C</i> , what is the rate of change of the area of the circle, in square centimeters per second? A. $-(0.2)\pi C$ B. $-(0.1)C$ C. $-\frac{(0.1)C}{2\pi}$	 The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is
D. $(0.1)^2 C$ E. $(0.1)^2 \pi C$	A. $\frac{1}{\pi}$ B. $\frac{1}{2}$ C. $\frac{2}{\pi}$ D. 1 E. 2
2. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer when it is 100 meters past the intersection?	2. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?
A. 49.15 B. 57.60 C. 57.88 D. 59.20 E. 67.40	A. $-\frac{7}{8}$ ft per min B. $-\frac{7}{24}$ ft per min C. $\frac{7}{24}$ ft per min 7 21
Learning Activity #2 – KEY	D. $\overline{8}$ ft per min E. $\overline{25}$ ft per min
1) B 2) A	3. Population grows according to the equation $\frac{dy}{dt} = ky$, where <i>k</i> is a constant and <i>t</i> is measured in years. If the population doubles every 10 years, then the value of <i>k</i> is
	A. 0.069 B. 0.200 C. 0.301 D. 3.322 E. 5.000
	Assessment #2 – KEY
	1) D 2) D 3) A

			Assessment's Alignment
	Activity's Alignment	AB/BC AP	Standard 1 Analysis of functions
AB/BC AP	Standard 1 Analysis of functions	CALCULUS	Standard 2 Model numerically/analytically
CALCULUS	Standard 2 Model numerically/analytically	STANDARD	Standard 3 Differential calculus
STANDARD	Standard 3 Differential calculus		Standard 4 Position, speed, acceleration
	Standard 4 Position, speed, acceleration		Standard 5 Related rates
	Standard 5 Related rates		Standard 6 Differential equations/slope fields
	Standard 6 Differential equations/slope fields	CONTENT	MA 1 Number sense
CONTENT	MA 1 Number sense		MA 2 Geometric and spatial sense
	MA 2 Geometric and spatial sense		MA 4 Patterns and relationships
	MA 4 Patterns and relationships	PROCESS	1.7 Evaluate information
PROCESS	1.7 Evaluate information		1.10 Apply information, ideas, and skills
	1.10 Apply information, ideas, and skills		3.5 Apply one's own strategies
	3.5 Apply one's own strategies		3.6 Examine solutions from many perspectives
	3.6 Examine solutions from many perspectives		3.7 Evaluate strategies
	3.7 Evaluate strategies	DOK	2
DOK	3	LEVEL OF	Mastery Level – 75%
INSTRUCTIONAL	Homework and practice	EXPECTATION	
STRATEGIES			
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	Readiness &	Equity Section	
SI	LA = Sample Learning Activit	ties & SA = Sample Assessments	
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	1
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments
Learning Activity #3	Assessment #3:
The maximum acceleration on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is A. 9 B. 12 C. 14 D. 21 E. 40 Now, graph the acceleration function and determine which characteristics of the graph support your answer. Learning Activity #3 – KEY $a(t) = 3t^2 - 6t + 12$ critical number for $a(t)$ (when $a'(t) = 0$) is $t = 1$. a(0) = 12, a(1) = 9, a(3) = 21 so maximum acceleration on the interval [0, 3] is 21.	A particle moves along a line so that at time t, where $0 \le t \le \pi$, its $s(t) = -4\cos t - \frac{t^2}{2} + 10$ position is given by particle when its acceleration is zero? A5.19 B. 0.74 C. 1.32 D. 2.55 E. 8.13 Note: Be sure to show equations for velocity and acceleration in the work you do. Assessment #3 – KEY D see justification below $v(t) = s'(t) = 4\sin t - t$ $a(t) = v'(t) = s''(t) = 4\cos t - 4$ $a(t) = 0 \Rightarrow \cos t = 0.25 \Rightarrow t \approx 1.318$ $v(1.318) = 4\sin(1.318) - 1.318 \approx 2.55$
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			Assessment's Alignment
		AB/BC AP	Standard 1 Analysis of functions
	Activity's Alignment	CALCULUS	Standard 2 Model numerically/analytically
AB/BC AP	Standard 1 Analysis of functions	STANDARD	Standard 3 Differential calculus
CALCULUS	Standard 2 Model numerically/analytically		Standard 4 Position, speed, acceleration
STANDARD	Standard 3 Differential calculus		Standard 5 Related rates
	Standard 4 Position, speed, acceleration		Standard 6 Differential equations/slope fields
	Standard 5 Related rates	CONTENT	MA 1 Number sense
	Standard 6 Differential equations/slope fields		MA 2 Geometric and spatial sense
CONTENT	MA 1 Number sense		MA 4 Patterns and relationships
	MA 2 Geometric and spatial sense	PROCESS	1.7 Evaluate information
	MA 4 Patterns and relationships		1.10 Apply information, ideas, and skills
PROCESS	1.7 Evaluate information		3.5 Apply one's own strategies
	1.10 Apply information, ideas, and skills		3.6 Examine solutions from many perspectives
	3.5 Apply one's own strategies		3.7 Evaluate strategies
	3.6 Examine solutions from many perspectives	DOK	2
	3.7 Evaluate strategies	LEVEL OF	Mastery Level – 75%
DOK	3	EXPECTATION	
INSTRUCTIONAL	Homework and practice		1
STRATEGIES			
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Student Resources		Teacher Resources
Calculus of a Single Variable – 8th edition	General:	Calculus of a Single Variable – 8 th edition
	Enrichment:	
	Interventions	
	Intervention:	
	Student Resources Calculus of a Single Variable – 8 th edition	Student Resources General: Calculus of a Single Variable – 8 th edition Furichment: Intervention: Intervention:

Content Area: Mathematics	Course: AP Calculus AB	Strand: Derivatives VI
Learner Objectives: The student will calculate,	interpret, and apply integrals	

Concepts: A. Calculate definite integrals

Students Should Know		Students Should Be Able to				
Definite integral as limit of Riemann sums		Basic properties of de	finite integrals – exampl	es include additivity and		
• Definite integral of the rate of change of a quantity over an interval		linearity				
interpreted as the c	hange of the quantity over	the interval:				Page
$\int_{a}^{b} f' \int_{a}^{b} f' (w) dv$	$\mathbf{r} = \mathbf{f}(\mathbf{h}) - \mathbf{f}(\mathbf{a})$					PAG
$a^{3} = a^{3} = (x)ax - f(b) - f(a)$					E	
						_/*
						MER
		Instruction	al Support			GEF
					OR	
Student Essen		tial Vocabulary			MAT	
Definite Integral	Riemann sum	Antiderivative	Differentiate	Integrate	Limiting Behavior	_1
Upper Limit of Integration Lower Limit		of Integration				

AP Calculus AB

Readiness & Equity Section					
SLA = Sample Learning Activities & SA = Sample Assessments					
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing			
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity			
	Problem Solving				
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity			
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity			
	Direction				

Sample Assessments	Page
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	Sample Assessments

Learning Activity #1 :Assessment #1:If
$$\int_{a}^{b} f(x) dx = 3$$
, and $\int_{a}^{b} f(x) dx = 2$, find the value of each.1. $\int_{a}^{b} f(x) dx$...1. $\int_{a}^{b} f(x) dx$...2. $\int_{a}^{b} f(x) dx$...2. $\int_{a}^{b} f(x) dx$...3. $\int_{a}^{b} f(x) dx$...4. $\int_{0}^{b} f(x) dx$...5. $\int_{0}^{b} f(x) dx$...6. $\int_{0}^{b} f(x) dx$...7. $\int_{a}^{b} f(x) dx$...8. $\int_{a}^{b} f(x) dx$...9. $\int_{a}^{b} f(x) dx$...9. $\int_{a}^{b} f(x) dx$...1. $\int_{a}^{b} f(x) dx$...1. $\int_{a}^{b} f(x) dx$...2. $\int_{a}^{b} f(x) dx$...3. $\int_{a}^{b} f(x) dx$...4. $\int_{0}^{b} f(x) dx$...5. $\int_{0}^{b} f(x) dx$...1. 13 2) -103)04) 132) -103)04) 132) -103)05. $a f(x) dx$...11 $a ctivity * a Lignment$ 12 $a ctivity * a Lignment$ 132) -103)014 $b ctivity * a Lignment$ 15 $a ctivity * a Lignment$ 16 $a ctivity * a Lignment$ 17 $b cvaluate robolem -solving processes18 $b ctivity * a Lignment$ 19 $b ctivity * a Lignment$ 10 $b ctivity * a Lignment$ 10 $b ctivity * a Lignment$ 11 $b ctivity * a Lignment$ 12$

	MA 4	Patterns and relationships
PROCESS	1.7	Evaluate information
	1.10	Apply information, ideas and skills
	3.4	Evaluate problem-solving processes
	3.7	Evaluate strategies
DOK	2	
INSTRUCTIONAL	Nonlin	guistic representation
STRATEGIES		

Readiness & Equity Section					
SLA = Sample Learning Activities & SA = Sample Assessments					
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing			
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity			
	Problem Solving				
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity			
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity			
	Direction				

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Sample Learning Activities	Sample Assessments	MER
		GEF
		OR
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Learning Activity #2 : Assessment #2: Evaluate the following by using geometric area formulas: If $\int_{1}^{3} f(x) dx = 5$, find each of the following values: $\int_{0}^{0.5} 2x \, dx$ $\int_0^1 2x \, dx \qquad \int_0^{1.5} 2x \, dx$ $\int_{0}^{t} 2x \, dx$ 1. $\int_{1}^{3} [f(x) - 7] dx =$ ł A. -14 B. -9 C. -5 D. -2 E. 3 $\int_{-1}^{1} f(x+2) \, dx =$ A(1) =A(1.5) =A(0.5) =A(t) =A. -2 B. 1 C. 5 D. 7 E. 10 Learning Activity #2 – KEY Assessment #2 – KEY $A(0.5) = 0.25, A(1) = 1, A(1.5) = 2.25, A(t) = x^{2}$ 2) C 1) B **Activity's Alignment** AB/BC AP Standard 3 Differential calculus **Assessment's Alignment** CALCULUS Standard 6 Differential equations/slope fields Standard 3 Differential calculus AB/BC AP **STANDARD** Standard 9 Integral calculus Differential equations/slope fields CALCULUS Standard 6 Standard 10 Area and volume **STANDARD** Standard 9 Integral calculus MA 2 Geometric and spatial sense CONTENT Area and volume Standard 10 Patterns and relationships MA 4 CONTENT MA 2 Geometric and spatial sense PROCESS 1.7 Evaluate information MA 4 Patterns and relationships Apply information, ideas and skills 1.10 PROCESS 1.7 Evaluate information 3.4 Evaluate problem-solving processes 1.10 Apply information, ideas and skills 3.7 Evaluate strategies 3.4 Evaluate problem-solving processes 2 DOK 3.7 Evaluate strategies INSTRUCTIONAL Nonlinguistic representation DOK 3 **STRATEGIES** LEVEL OF Mastery Level – 75%

EXPECTATION

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Student Resources	Teacher Resources
General: Calculus of a Single Variable – 8 th edition	General: Calculus of a Single Variable – 8 th edition
Enrichment: Intervention:	Enrichment: Intervention:

Content Area: Mathematics	Course: AP Calculus AB	Strand: Integral I
Learner Objectives: The student will calculate	, interpret, and apply integrals	

Concepts: B. Fundamental Theorem of Calculus

Students Should Know	Students Should Be Able to
• The relationship between a function and its antiderivative $\int_{a}^{b} f(x)dx = F(b) - F(a)$ • $\frac{d}{dx} \int_{a}^{f(x)} g(t)dt = g(f(x)) \cdot f'(x)$	 Use the Fundamental Theorem of Calculus to evaluate definite integrals. Use the Fundamental Theorem of Calculus to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Student Essential Vocabulary						
Antiderivative	Definite Integral	Chain Rule	Particular Antiderivati	ve Lowe	r Limit of Integration	Page
Derivative of a Function	n Defined by an Integral	Upper Limit of Integration				PAG
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						*
						MER
Readiness & Equity Section					GEF	
SLA = Sample Learning Activities & SA = Sample Assessments					ΦR	
21 st Century Themes		Quantitative Literacy	Non Fiction Reading & Writ	ting		MAT
Learning & Innovation Sk	cills	Critical Thinking and	Enrichment Opportunity			1
		Problem Solving				₽
Information, Media, & Te	chnology Skills	ICT Literacy	Intervention Opportunity			

Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments	
Learning Activity #1 :	Assessment #1:	
Evaluate the following definite integrals by finding the antiderivative of the integrand. $\int_{1}^{4} (3\sqrt{x} + x) dx$ 1.	1. The graph of the function <i>f</i> , consisting of three line segments is shown at the right. Let $g(x) = \int_{1}^{x} f(t) dt$ $(1, 4)$ $(2, 1)$ $(2, 1)$ $(2, 1)$ $(4, -1)$	
	a. Compute $g(4)$ and $g(-2)$.	
2. $\int_{0}^{2} 2x - 1 dx$	b. Find the instantaneous rate of change of g , with respect to x , at $x = 1$.	
3. Find the area bounded by the function and the <i>x</i> -axis:	 c. Find the absolute minimum value of g on the closed interval [-2, 4]. Justify your answer. 	
a. $f(x) = 4 - x^2$ b. $g(x) = \sin x$ on the interval $[0, 2\pi]$ Learning Activity #1 – KEY	d. The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.	
1) 21.5 2) 2.5 3a) $\frac{32}{3}$ b) 4	1 (^{a(t)})	
fhsd.acad.kg A	AP Calculus AB $(105eC^2)$ February 2011 (18, 15) (18, 15)	
		2. A car is traveling on a straight road v
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	Activity's Alignment	For $0 \le t \le 18$ seconds, the car's ac
AB/BC AP	Standard 1 Analysis of functions	piecewise linear function defined by
CALCULUS	Standard 3 Differential calculus	a Is the velocity of the car increasing a
STANDARD	Standard 6 Differential equations/slope fields	a. Is the velocity of the car mercasing a
	Standard 9 Integral calculus	
	Standard 10 Area and volume	$1 h A \neq a = b \neq \pm i = a = i = \pm b = i = \pm a = a = 1 0 \le t \le t$
CONTENT	MA 2 Geometric and spatial sense	b. At what time in the interval $0 = t = 1$
	MA 4 Patterns and relationships	of the car 55 ft/sec? why?
PROCESS	1.7 Evaluate information	$0 \leq t \leq 12$
	1.10 Apply information, ideas and skills	c. On the time interval $0 \le t \le 18$, what
	3.4 Evaluate solutions from many perspectives	velocity, in ft/sec, and at what time d
	3.7 Evaluate strategies	
DOK	2	d. At what time in the interval $0 \le t \le$
INSTRUCTIONAL	Nonlinguistic representation	to zero? Justify your answer.
STRATEGIES		
		Assessment #1 – KEY
		1a) $g(4) = 2.5, g(-2) = -6$ 11
		1c) $x = 3$ is the only critical number, whe
		candidates for an absolute minimum $g(-2) = -6$, $g(3) = 3$, and $g(4) = 2.5$, $g(3) = 3$, $g(4) = 2$,

- with velocity 55 ft/sec at time t = 0. ecceleration a(t), in ft/sec², is the the graph above.
- t = 2 seconds? Why or why not?
- 18, other than t = 0, is the velocity
- at is the car's absolute maximum oes it occur? Justify your answer.
- ¹⁸, if any, is the car's velocity equal Page PAG

a)
$$g(4) = 2.5, g(-2) = -6$$
 1b) $g'(1) = f(1) = 4$

ere g'(x) = f(x) = 0, so the only are g(-2), g(3), and g(4). Because (-2) = -6 is the absolute minimum

on this interval.

- 1d) For a point of inflection, g''(x) = f'(x) much change sign, so the graph of g'(x) = f(x) must change from increasing to decreasing or from decreasing to increasing. This only occurs at x = 1, so only at x = 1 is there a point of inflection for g.
- 2a) Yes, because v'(t) = a(t) is positive at t = 2.

2b) At
$$t = 12$$
, because $\int_{0}^{12} a(t) dt = 0$. Thus, $v(12) = v(0) + \int_{0}^{12} a(t) dt = 55$

2c) The absolute maximum velocity on [0, 18] must occur at an endpoint or at a critical number. Only at the critical number t = 6 does v'(t) = a(t) change from positive to negative, therefore this is the only critical number where there is a relative maximum. Then

checking the values of
$$v(0) = 55$$
, $v(6) = v(0) + \int_0^6 a(t) dt = 55 + 60$
= 115, and $v(18) = v(0) + \int_0^{18} a(t) dt = 55 + 60 - 180 + 15 = -50$,

We see that v(6) is the absolute maximum velocity on [0, 18].

2d) Never. The local minimum occurs at t = 16 because only there does v'(t) = a(t) change from negative to positive. Because

$$v(16) = v(0) + \int_0^{10} a(t) dt = 10, v(t) \text{ is always positive on } [0, 18].$$

Assessment's Alignment				
AB/BC AP	Standard 1	Analysis of functions		
CALCULUS	Standard 3	Differential calculus		
STANDARD	Standard 6	Differential equations/slope fields		
Standard 9 Integral calculus				

Page PAG E MER GEF OR MAT **1**

	Standard 10 Area and volume
CONTENT	MA 2 Geometric and spatial sense
	MA 4 Patterns and relationships
PROCESS	1.7 Evaluate information
	1.10 Apply information, ideas and skills
	3.4 Evaluate solutions from many perspectives
	3.7 Evaluate strategies
DOK	3
LEVEL OF	Mastery Level – 70%
EXPECTATION	

	Readiness &	Equity Section	
SI	A = Sample Learning Activi	ties & SA = Sample Assessments	
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	Pag
	Direction		PAG
			E
Sample Learning Activities		Sample Assessments	*
			ME

1

Learning Activity #	*2:	Assessment #2:	
The Fundament interval <i>I</i> contain still applies) Use the Fundament 1. $\frac{d}{dx} \int_{3}^{x} \sqrt{t} dt$ 3. $\frac{d}{dx} \int_{\sin x}^{4} \sqrt{t} dt$ Learning Activity # 1) \sqrt{x} 3) $-\sqrt{\sin x} (\cos x)$	al Theorem of Calculus – If <i>f</i> is continuous on an open ing <i>x</i> , then $\frac{d}{dx} \int_{a}^{x} f(t) dt = f(x)$. (Note: the chain rule antal Theorem to evaluate each of the following: 2. $\frac{d}{dx} \int_{3}^{x^{2}} \sqrt{t} dt$ 4. $\frac{d}{dx} \int_{x^{3}}^{\cos x} \sqrt{t} dt$ 22 – KEY 2) $\sqrt{x^{2}} \cdot 2x = 2x^{2}$ 4) $\sqrt{\cos x} (-\sin x) - \sqrt{x^{3}} \cdot 3x^{2}$	Find the derivation of the derivative of the de	tive of $\int_0^{x^{10}} \cos \sqrt{t} dt$ KEY
	Activity's Alignment		Assessment's Alignment
AB/BC AP CALCULUS STANDARD	Standard 1Analysis of functionsStandard 3Differential calculusStandard 6Differential equations/slope fieldsStandard 9Integral calculusStandard 10Area and volume	AB/BC AP CALCULUS STANDARD	Standard 1Analysis of functionsStandard 3Differential calculusStandard 6Differential equations/slope fieldsStandard 9Integral calculusStandard 10Area and volume
CONTENT	MA 2 Geometric and spatial sense	CONTENT	MA 2 Geometric and spatial sense
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	MA 4 Patterns and relationships	MA 4 Patterns	and relationships
PROCESS	1.7 Evaluate information	PROCESS 1.7 Evaluate	information
	1.10 Apply information, ideas and skills	1.10 Apply in	formation, ideas and skills
	3.4 Evaluate solutions from many perspectives	3.4 Evaluate	solutions from many perspectives
	3.7 Evaluate strategies	3.7 Evaluate	strategies
DOK	3	DOK 2	
INSTRUCTIONAL	Nonlinguistic representation	LEVEL OF Mastery Level –	30%
STRATEGIES		EXPECTATION	

Student Resources		Teacher Resources	
alculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition	
	Enrichment:		
	Intomontion		
	intervention:		
	Student Resources alculus of a Single Variable – 8 th edition	Student Resources General: alculus of a Single Variable – 8 th edition General: Enrichment: Intervention:	

NOTE: These sections will be partially co	ompleted during the curriculum writing process and	finalized during the year one review process.	Pag PAC E *
Content Area: Mathematics	Course: AP Calculus AB	Strand: Integrals II	GEF
Learner Objectives: The student will	calculate, interpret, and apply integrals		
			1

Concepts: C. Techniques of antidifferentiation

Students Should Know	Students Should Be Able to
• Antiderivatives following directly from derivatives of basic functions.	• Evaluate antiderivatives by substitution of variables (including change of limits for definite integrals).

Instructional Support

Student Essential Vocabulary			
Anitderivative	Substitution Technique	Upper Limit of Integration	Lower Limit of Integration

Page PAG E * MER GEF OR MAT

Readiness & Equity Section			
SLA = Sample Learning Activities & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	1
Learning & Innovation Skills	Critical Thinking and Problem Solving	Enrichment Opportunity	

Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self Direction	Gender, Ethnic, & Disability Equity	

Sample Learning Activities		Sample Assessments		
Learning Activity #1	:	Assessment #1:		
If the substitution $x = 2 \sin y$ is made, then how would $\int_{0}^{2} x^{3} \sqrt{4 - x^{2}} dx$		Evaluate the integral: $\int \frac{e^{3x}}{1+e^{3x}} dx$		
be re-written?		Assessment #1 – KEY		
Learning Activity #1	– KEY	$\frac{1}{3}\ln(1+e^{3x})+C$		
$\int_0^{\frac{\pi}{2}} 32\sin^3 y \cos^2 y dx$	ly			
		Assessment's Alignment		
AB/BC AP CALCULUS	Activity's Alignment Standard 1 Analysis of functions Standard 3 Differential calculus Standard 4 Differential calculus	AB/BC AP CALCULUS STANDARD	Standard 1Analysis of functionsStandard 3Differential calculusStandard 6Differential equations/slope fieldsStandard 9Integral calculusStandard 10Area and volume	
SIANDARD	Standard 6Differential equations/stope fieldsStandard 9Integral calculusStandard 10Area and volume	CONTENT	MA 2 Geometric and spatial sense MA 4 Patterns and relationships	
CONTENT	MA 2Geometric and spatial senseMA 4Patterns and relationships	PROCESS	1.7Evaluate information1.10Apply information, ideas and skills	
PROCESS	 Evaluate information Apply information, ideas and skills Evaluate solutions from many perspectives 	DOK	3.4Evaluate solutions from many perspectives3.7Evaluate strategies2C	
DOK	3.7 Evaluate strategies	LEVEL OF EXPECTATION	Mastery Level – 85%	
INSTRUCTIONAL STRATEGIES	Homework and practice		 	

Readiness & Equity Section					
SLA	= Sample Learning Activit	ties & SA = Sample Assessments			
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing			
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity			
	Problem Solving				
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity			
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity			
	Direction				

Sample Learning Activities	Sample Assessments
Learning Activity #2 :	Assessment #2:
Use the substitution method to find the value of each integral.	$\int_{1}^{1} \frac{\sqrt{x}}{\sqrt{x}} dx$
$\int x\sqrt{x^2+1}dx$	If the substitution $u = 1 + \sqrt{x}$ is made, then $\int_0^{1} 1 + \sqrt{x} dx$ is equivalent to which one of the following?
2. $\int x \sqrt{x+2} dx$	A. $2\int_{1}^{2} \frac{u-1}{u} du$ B. $2\int_{1}^{2} \frac{(u-1)^{2}}{u} du$ C. $2\int_{0}^{1} \left(1-\frac{1}{u}\right) du$
$\int \frac{-x}{\left(x^2-4\right)^3} dx$	D. $\int_{1}^{2} \left(2u - 4 + \frac{2}{u} \right) du$ E. $2 \int_{0}^{2} \frac{(u - 1)^{2}}{u} du$
	Assessment #2 – KEY
Learning Activity #2 – KEY	B
$ \begin{vmatrix} \frac{1}{3}(x^2+1)^{\frac{3}{2}} + C \\ 1 & 2 \end{vmatrix} = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C $	

3) $\frac{1}{2(x^2-4)^2}+C$			Assessment's Alignment
	Activity's Alignmont	AB/BC AP	Standard I Analysis of functions
AB/BC AP CALCULUS STANDARD	Standard 1Analysis of functionsStandard 3Differential calculusStandard 6Differential equations/slope fields	STANDARD	Standard 3Differential calculusStandard 6Differential equations/slope fieldsStandard 9Integral calculusStandard 10Area and volume
	Standard 9Integral calculusStandard 10Area and volume	CONTENT	MA 2 Geometric and spatial sense MA 4 Patterns and relationships
CONTENT	MA 2Geometric and spatial senseMA 4Patterns and relationships	PROCESS	1.7 Evaluate information1.10 Apply information, ideas and skills
PROCESS	1.7 Evaluate information1.10 Apply information, ideas and skills	Dov	3.4Evaluate solutions from many perspectives3.7Evaluate strategies
	3.4 Evaluate solutions from many perspectives 3.7 Evaluate strategies	DOK LEVEL OF	3
DOK	2	EXPECTATION	/0/0
NSTRUCTIONAL STRATEGIES	Homework and practice		

			Page
	Student Resources	Teacher Resources	PAG
General:	Calculus of a Single Variable – 8 th edition	General: Calculus of a Single Variable – 8 th edit	ion E
			*
			MER
Enrichment:		Enrichment:	GEF
			OR
Interventions		Intermention	MAT
Intervention:		Intervention:	1

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

Content Area: Mathematics	Course: AP Calculus AB	Strand: Integrals 3
Learner Objectives: The student will calculate, interpret, and apply integrals		

Concepts: D. Applications of Definite and Indefinite Integrals

Students Should Know	Students Should Be Able to		
 The relationship between a function and its antiderivative	 Find specific antiderivatives using initial conditions, including applications to motion along a line and total distance traveled. Solve separable differential equations and use them to model (including exponential growth, i.e. y' = ky). Calculate the area of a region. Calculate the volume of a solid with known cross-sections. Calculate the volume of a solid of revolution. Calculate accumulated change from a rate of change or average value of a function (including applications of physical, biological and economic situations). 		
Instructional Support			

						GEF
		Student Essen	tial Vocabulary			ØR
Integrals as Areas	Distance vs. Displacement	Position	Velocity	Acceleration	Differential Equation	
Initial Value Problem	General Solution	Particular Solution	Known Cross-Section	Solid of Revolution		

Readiness & Equity Section					
SLA	= Sample Learning Activit	ties & SA = Sample Assessments			
21 st Century Themes Quantitative Literacy Non Fiction Reading & Writing					
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity			
	Problem Solving				
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity			
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity			
	Direction				

Sample Learning Activities	Sample Assessments	
Learning Activity #1 :	Assessment #1:	
1. Find the average value of $f(x) = 3x^2 - 2x$ on [1, 4]. Then find the value of x guaranteed by the Mean Value Theorem for Integrals.	Find the particular solution of $\frac{dy}{dx} = \frac{x}{y}$ through (-2, -1) and identify the domain.	Page PAG
2. What is the average value of $y = x^2 \sqrt{x^3 + 1}$ on the interval [0, 2]?	Assessment #1 – KEY	E * MFR
$\frac{26}{52}$ $\frac{52}{26}$ $\frac{52}{52}$	Particular solution: $y = -\sqrt{x^2 - 3}$; domain: $(-\infty, -\sqrt{3})$	GEF
A. 9 B. 9 C. 3 D. 3 E. 24		MAT
Learning Activity #1 – KEY		1

1) average value: 16; 2) A	$x = \frac{2\frac{2}{3}}{3}$		Assessment's Alignment
Activity's Alignment		AB/BC AP	Standard 2 Model numerically/analytically
AB/BC AP CALCULUS STANDARD	Standard 2Model numerically/analyticallyStandard 3Differential calculusStandard 6Differential equations/slope fieldsStandard 9Integral calculusStandard 10Area and volume	CALCULUS STANDARD	Standard 3Differential calculusStandard 6Differential equations/slope fieldsStandard 9Integral calculusStandard 10Area and volumeMA 2Geometric and spatial senseMA 4Patterne and relationships
CONTENT	MA 2 Geometric and spatial sense MA 4 Patterns and relationships	PROCESS	MA 4 Patterns and relationships 1.6 Discover/evaluate relationships 1.10 Analysis formationships
PROCESS	 1.6 Discover/evaluate relationships 1.10 Apply information, ideas and skills 3.1 Identify and define problems 3.4 Evaluate problem-solving processes 2.5 Descent being limited in the time of the limited set of the li		 3.1 Identify and define problems 3.4 Evaluate problem-solving processes 3.5 Reason logically (inductive/deductive) 3.7 Evaluate strategies
	3.5 Reason logically (inductive/deductive) 3.7 Evaluate strategies	DOK LEVEL OF	2 Mastery Level – 80%
DOK INSTRUCTIONAL STRATEGIES	2 Homework and practice	EXPECTATION	

				MAT
Readiness & Equity Section				1
SLA = Sample Learning Activities & SA = Sample Assessments				
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing		

AP Calculus AB

Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments	
Learning Activity #2 :	Assessment #2:	
By U.S. law, yogurt must contain 100 million bacteria per gram. At noon, Some sterilized milk is inoculated with a yogurt culture so that the milk is inoculated with a yogurt culture so that the milk contains 400 bacteria per gram. Suppose the bacteria growth rate is proportional to the number of bacteria present and that at 1 pm, there are 1600 bacteria per gram. At 7 pm, how many bacteria are there per gram? At what time does the culture legally become yogurt?	Solve the following analytically: Suppose the population <i>y</i> of a hive of wasps is growing at a rate proportional to the population. On May 1, there were 10 wasps and on May 31, there were 50. If growth continues like this, how long after May 1 will the population reach 100 wasps?	Page PAG E
Learning Activity #2 – KEY	Assessment #2 – KEY	* MER
At 7 pm there are 6,553,600 bacteria per gram. It is legally yogurt 8.9695 hours after noon.	42.92 days after May 1	gef or mat 1
And and the		

	Activity's Alignment	Assessment's Alignment			
AB/BC AP	Standard 2 Model numerically/analytically	AB/BC AP	Standard 2 Model numerically/analytically		
CALCULUS	Standard 3 Differential calculus	CALCULUS	Standard 3 Differential calculus		
STANDARD	Standard 6 Differential equations/slope fields	STANDARD	Standard 6 Differential equations/slope fields		
	Standard 9 Integral calculus		Standard 9 Integral calculus		
	Standard 10 Area and volume		Standard 10 Area and volume		
CONTENT	MA 2 Geometric and spatial sense	CONTENT	MA 2 Geometric and spatial sense		
	MA 4 Patterns and relationships		MA 4 Patterns and relationships		
PROCESS	1.6 Discover/evaluate relationships	PROCESS	1.6 Discover/evaluate relationships		
	1.10 Apply information, ideas and skills		1.10 Apply information, ideas and skills		
	3.1 Identify and define problems		3.1 Identify and define problems		
	3.4 Evaluate problem-solving processes		3.4 Evaluate problem-solving processes		
	3.5 Reason logically (inductive/deductive)		3.5 Reason logically (inductive/deductive)		
	3.7 Evaluate strategies		3.7 Evaluate strategies		
DOK	2	DOK	2		
INSTRUCTIONAL	Cues, questions, and advanced organizers.	LEVEL OF	Mastery Level – 80%		
STRATEGIES		EXPECTATION			
	·		·		

	Student Resources		Teacher Resources			
General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition			
				D		
Enrichment:		Enrichment:		Page		
				E		
				*		
Intervention:		Intervention:		MER		
				GEF		
				OR		
		1.0		MAT		

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.

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Content Ar	ea: Mat	hematics	Course: AP Calculus	AB	Strand: Integrals IV		
Learner Objectives: The student will calculate, interpret, and apply integrals							
Concepts:	Concepts: E. Numerical Approximations to Definite Integrals						
		Students Should Know		S	students Should Be Able to	OR MAT	
						1	

•	Understand that the definite integral can be approximated by a finite sum of areas of geometric regions.	•	Use Riemann sums (left, right, midpoint, trapezoidal) to approximate the definite integral of functions represented algebraically, graphically, and/or by a table of values.

Instructional Support

Student Essential Vocabulary							
Left-Riemann Sum	Right-Riemann Sum	Mid-Point Riemann Sum	Trapezoidal Rule				

Readiness & Equity Section							
SLA = Sample Learning Activities & SA = Sample Assessments							
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing	OR				
Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	МАТ				
	Problem Solving		1				
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	A				

AP Calculus AB

Page PAG

E *

Life & Career Skills Initiative and Self Direction						itiati Di	ve an rectio	Gender, Ethnic, &	Disability Equity					
Sample Learning Activities												Sample Assessments		
Lear 1.	Learning Activity #1 : 1. A tank is being filled with water using an old pump that slows down as it runs. The table below gives the rate at which the pump pumps at 10 minute intervals. If the tank is initially empty, estimate how much water is in the tank after 80 minutes. $ \frac{\text{Elapsed time}}{\text{Rate}} 0 10 20 30 40 50 60 70 80 90 10 20 38 35 35 32 28 20 19 10 10 20 38 35 35 32 28 20 19 10 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90 10 20 10 10 10 10 10 1$						that soump ate ho 70 20	lows pum ow m 80 19	dow ps at uch 90 10	Sample Assessments Assessment #1: Use 4 subintervals to find the Left Riemann Sum to approximate the area bounded by the x-axis, $y = x + 2$, and $x = 4$. Assessment #1 – KEY 13.5 Mathematical Standard 1 Analysis of functions CALCULUS Standard 1 Analysis of functions CALCULUS Standard 2 Model numerically/analytically Standard 3 Differential calculus Standard 3 Differential calculus Standard 3 Differential calculus Standard 9 Integral calculus				
indic	ated methods.		w to a	pprox	iiiiai	C			wit	in the			CONTENT	Standard 10Area and volumeMA 2Geometric and spatial senseMA 4Patterns and relationships
	$\begin{array}{c} x \\ f(x) \end{array}$ a. Left Rier	10 30 mann sum	30 22 with 4	2 subin	5 1 terv	2 4 als		70 20		90 48			PROCESS	1.8Organize data and ideas1.10Apply information, ideas and skills2.1Plan and make presentations3.5Reason logically (inductive/deductive)3.6Examine solutions from many perspectives3.7Evaluate strategies4.1Support decisions
	b. Right Riemann sum with 4 subintervalsc. Midpoint Riemann sum with 2 subintervals of equal length.d. Trapezoidal Rule with 4 subintervals.									ngth.		DOK LEVEL OF EXPECTATION	2 Mastery Level – 80% MAT 1	



- c. Midpoint Riemann sum with 2 subintervals of equal length.
- d. Trapezoidal Rule with 3 subintervals of equal length.
- 5. Complete the data table below and approximate $\int_{-1}^{1} (x + y) dx$ the indicated methods.

 $\int_{-1}^{15} (x^2 + 3x) \, dx \quad \text{with}$

x	-1	3	7	11	15
f(x)					

- a. Left Riemann sum with 4 subintervals
- b. Right Riemann sum with 4 subintervals
- c. Midpoint Riemann sum with 2 subintervals of equal length.
- d. Trapezoidal Rule with 4 subintervals.

Use the Fundamental Theorem of Calculus to evaluate each of the following:

6.
$$\int_{-1}^{15} (x^2 + 3x) dx$$
 7. $\int_{2}^{8} (16 - x^2) dx$

Learning Activity #1 – KEY

Page PAG E MER GEF OR MAT 1

1) Results my var is 2570, and th	ry but the left R ne trapezoidal ap	iemann su proximat	um is 2890, the right Riemann sum ion is 2730.
2a) 1736 b	o) 2636 o	c) 1680	d) 2186
3a) 1480 b	o) 1080 o	:) 1300	d) 1297.5
4a) -16 b)	o) -136 c) -67.5	d) -76
5a) 960 b)) 1024 .	:) 1376	d) 992
6) 1461.333	7	7) -72	
	Activ	ity's Alig	nment
AB/BC AP	Standard	1 An	alysis of functions
CALCULUS	Standard	2 Ma	odel numerically/analytically
SIANDARD	Standard	5 D11 9 Int	egral calculus
	Standard	9 Int 10 Ar	ea and volume
CONTENT	MA 2 G	eometric	and spatial sense
	MA4 P	atterns an	d relationships
PROCESS	1.8 C	rganize d	ata and ideas
	1.10 A	pply info	rmation, ideas and skills
	2.1 P	lan and m	ake presentations
	3.5 K	xamine so	olutions from many perspectives
	3.7 E	valuate st	rategies
	4.1 S	upport de	cisions
DOK	2		
INSTRUCTION	IAL Nonlingu	istic repre	sentation
STRATEGIES			

Readiness & Equity Section						
SLA = Sample Learning Activities & SA = Sample Assessments						
21 st Century Themes	Quantitative Literacy	Non Fiction Reading & Writing				

Learning & Innovation Skills	Critical Thinking and	Enrichment Opportunity	
	Problem Solving		
Information, Media, & Technology Skills	ICT Literacy	Intervention Opportunity	
Life & Career Skills	Initiative and Self	Gender, Ethnic, & Disability Equity	
	Direction		

Sample Learning Activities	Sample Assessments			
Learning Activity #2 :	Assessment #2:			
1. Find each of the following approximating sums to approximate the area under $f(x) = -x^2 + 5$ between $x = 0$ and $x = 2$, using 5 subintervals:	The function f is continuous on the interval [2, 8] and has values that are given in the table below.			
a. Right Riemann Sum	x 2 5 7 8 f(x) 10 30 40 20			
b. Left Riemann Sum	Using the 3 subintervals available in the table, what is the approximate			
c. Now, average the approximations from parts <i>a</i> and <i>b</i> and determine whether this average is equivalent to the Midpoint Riemann Sum or Trapezoidal Rule. Do not actually find value of these approximations. Explain your answer.	value of the integral $\int_{2}^{8} f(x) dx$? Use the Trapezoidal Rule. Assessment #2 – KEY 160			
 d. Is your approximation from part <i>a</i> an over approximation or an under approximation for the actual area defined? Explain your answer. 	Assessment's AlignmentAB/BC APStandard 1Analysis of functionsCALCULUSStandard 2Model numerically/analyticallySTANDARDStandard 3Differential calculusStandard 9Integral calculus			
e. Is your approximation from part <i>b</i> an over approximation or an under approximation for the actual area defined? Explain your answer.	Standard 10 Area and volume CONTENT MA 2 Geometric and spatial sense MA 4 Patterns and relationships			
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Learning Activity #2	- KEY	PROCESS	1.8 Organize data and ideas
			1.10 Apply information, ideas and skills
la) 6.48 b) 8.0	8 c) 7.28		2.1 Plan and make presentations
d)	for ation in the second of [0, 2]		3.5 Reason logically (inductive/deductive)
a) under, because the	function is decreasing on [0, 2]		3.6 Examine solutions from many perspectives
e) over because the fi	inction is decreasing on [0, 2]		3.7 Evaluate strategies
c) over, because the re	inclion is decreasing on [0, 2]	DOK	
		LEVEL OF	Z Mastery Level – 80%
		EXPECTATION	Mastery Lever - 6070
	Activity's Alignment		
AB/BC AP	Standard 1 Analysis of functions		
CALCULUS	Standard 2 Model numerically/analytically		
STANDARD	Standard 3 Differential calculus		
	Standard 9 Integral calculus		
	Standard 10 Area and volume		
CONTENT	MA 2 Geometric and spatial sense		
	MA 4 Patterns and relationships		
PROCESS	1.8 Organize data and ideas		
	1.10 Apply information, ideas and skills		
	2.1 Plan and make presentations		
	3.5 Reason logically (inductive/deductive)		
	3.6 Examine solutions from many perspectives		
	3.7 Evaluate strategies		Page
	4.1 Support decisions		PAG
DOK	2		E
INSTRUCTIONAL	Homework and practice		*
STRATEGIES			MER
			GEF
			MAT

Student Resources	Teacher Resources

General:	Calculus of a Single Variable – 8 th edition	General:	Calculus of a Single Variable – 8 th edition
Enrichment:		Enrichment:	
Intervention:		Intervention:	

NOTE: These sections will be partially completed during the curriculum writing process and finalized during the year one review process.