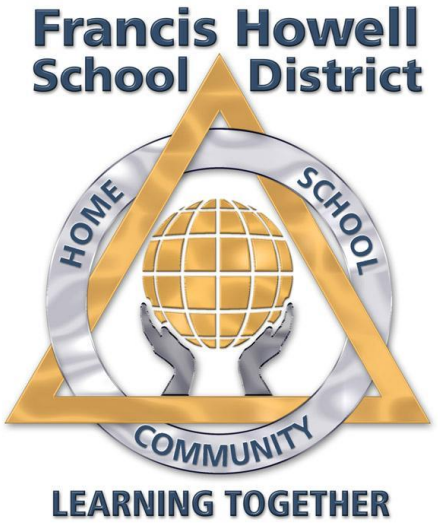
Pre Calculus

Curriculum



Board Approved:

Francis Howell School District

Mission Statement

The mission of the Francis Howell School District is to prepare students today for success tomorrow.

Vision Statement

Every student will graduate with college and career readiness skills.

Values

Francis Howell School District is committed to:

- Providing a consistent and comprehensive education that fosters high levels of academic achievement
- Operating safe and well-maintained facilities
- Providing a safe learning environment for all students
- Promoting parent, community, student, and business involvement in support of the school district
- Ensuring fiscal responsibility
- Developing responsible citizens
- Operating as a professional learning community
- Making appropriate use of technology

Francis Howell School District Graduate Goals

Upon completion of their academic study in the Francis Howell School District, students will be able to:

1. Gather, analyze and apply information and ideas.
2. Communicate effectively within and beyond the classroom.
3. Recognize and solve problems.
4. Make decisions and act as responsible members of society.

Mathematics Graduate Goals

Upon completion of their Mathematics study in the Francis Howell School District, students will be able to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Course Rationale

In order to be effective citizens in the 21st century, students need to understand mathematics. Students often encounter problem situations that require reasoning, computation, and communication. We regularly study the most efficient methods for reaching solutions, but also realize that examining different solution methods help develop more flexible problem solving skills. This course combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems

Course Description

Pre-AP Calculus is a year-long course with honors credit. This course combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics and other sciences, and/or engineering in college. If students do not plan on taking calculus, then they should not take this course. Instructional time will focus on four critical areas: (1) expand understanding of various function types and their graphs; (2) simplify and solve algebraic and trigonometric expressions, equations, and inequalities; (3) use and be able to identify characteristics of conics and their graphs; and (4) find limits of functions. *Prerequisite*: Successful completion of Honors Algebra 2 and teacher recommendation.

Pre AP Calculus Curriculum Team

Curriculum Committee

Danna Tedder	Francis Howell Central
Dena Rulo	Francis Howell Central
Tim Besse	Francis Howell North
Patty Bartell	Francis Howell North
Jennifer Pointer	Francis Howell High
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Mathematics Content Leader	Amy Ridling
Director of Student Learning	Dr. Chris Greiner
Chief Academic Officer	Nicole Whitesell
Superintendent	Dr. Mary Hendricks-Harris

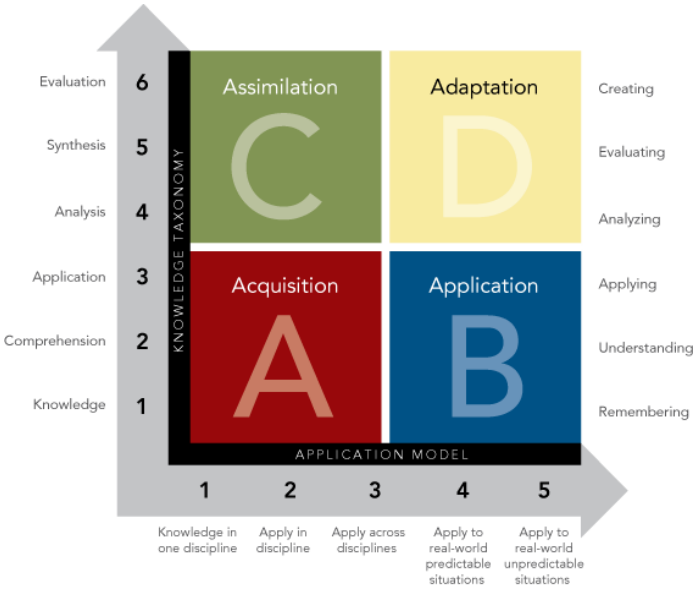
New Course Proposal: Pre AP Calculus New Course Proposal

Curriculum Notes

All FHSD performance tasks and sample learning activities are aligned not only to understandings and standards, but also the Rigor and Relevance Framework and 21st Century Skills. Information on these two things is provided below or by clicking on the hyperlinks.

Rigor and Relevance Framework

The Rigor/Relevance Framework is a tool developed by the International Center to examine curriculum, instruction, and assessment along the two dimensions of higher standards and student achievement.



The Rigor/Relevance Framework has four quadrants.
Quadrant A represents simple recall and basic understanding of knowledge for its own sake. Examples of Quadrant A knowledge are knowing that the world is round and that Shakespeare wrote Hamlet.

Quadrant C represents more complex thinking but still knowledge for its own sake. Quadrant C embraces higher levels of knowledge, such as knowing how the U.S. political system works and analyzing the benefits and challenges of the cultural diversity of this nation versus other nations.

Quadrants B and D represent action or high degrees of application. Quadrant B would include knowing how to use math skills to make purchases and count change. The ability to access information in wide-area network systems and the ability to gather knowledge from a variety of sources to solve a complex problem in the workplace are types of Quadrant D knowledge.

A	B	C	D
Students gather and store bits of knowledge and information. Students are primarily expected to remember or understand this knowledge.	Students use acquired knowledge to solve problems, design solutions, and complete work. The highest level of application is to apply knowledge to new and unpredictable situations.	Students extend and refine their acquired knowledge to be able to use that knowledge automatically and routinely to analyze and solve problems and create solutions.	Students have the competence to think in complex ways.

21st Century Skills

These skills have been pared down from 18 skills to what are now called the 4Cs. The components include critical thinking, communication, collaboration, and creativity. Critical thinking is focused, careful analysis of something to better understand and includes skills such as arguing, classifying, comparing, and problem solving. Communication is the process of transferring a thought from one mind to others and receiving thoughts back and includes skills such as choosing a medium (and/or technology tool), speaking, listening, reading, writing, evaluating messages. Collaboration is working together with others to achieve a common goal and includes skills such as delegating, goal setting, resolving conflicts, team building, decision-making, and managing time. Creativity is expansive, open-ended invention and discovery of possibilities and includes skills such as brainstorming, creating, designing, imagining, improvising, and problem-solving.

Standards

Standards aligned to this course can be found:

College Board Standards for College Success
College Board Standards

Common Course Fourth Course Standards
CCSS Fourth Course Standards

National Educational Technology Standards
ISTE Technology Standards

Units & Standards Overview

Semester 1 **Semester 2**

Unit 1: Functions	Unit 2: Right Triangle Trigonometry	Unit 3: Polynomial and Rational Functions	Unit 4: Graphing Trigonometric Functions and Inverses	Unit 5: Triangle Applications
Students will study the basic parent functions, their transformations, and characteristics. Students will be able to evaluate the domain and range of functions and their inverses. Students will be able to solve radical and absolute value equations and inequalities, and be able to evaluate the domain of each one.	Students will be able to find exact values of angles in both degrees and radians. They will use identity formulas to find values of non-special right triangles and simplify expressions. Students will find exact values of special angles using right triangles and the unit circle and then find the missing sides and angles of a right triangle.	Students will learn to solve and graph polynomial and rational equations, identifying key characteristics.	Students will use the characteristics of graphs such as amplitude, period, vertical translation, and phase shift to analyze and graph trigonometric functions and apply these characteristics to real-world situations. Students will then use the graphs of trigonometric functions to find the inverse of these functions and perform operations, such as, composition of functions.	Students will use the Law of Sines and Law of Cosines to solve oblique triangles and apply to real-world situations. They will find the area of triangles using Heron's formula or other trigonometric formulas.
PE Assessment: Unit 1 Functions Common Assessment	PE Assessment: Unit 2 Right Triangle Trigonometry	PE Assessment: Unit 3 Polynomial and Rational Functions	PE Assessment: Unit 4 Trigonometric Functions and Inverses	PE Assessment: Unit 5 Triangle Applications
PE Description: Students will graph a piece-wise function and identify transformations, domain and range in	PE Description: Students will use trigonometric definitions and exact values to simplify expressions and	PE Description: Students will find the key characteristics of a rational function including asymptotes, intercepts,	PE Description: Students will use the characteristics of graphs such as amplitude, period, vertical translation, and	PE Description: Students will use the Law of Sines, Law of Cosines, and area triangle formulas to find the area of a lake in a

interval notation.	apply arc length and area of a sector to a real-life application.	domain and range, then graph the function.	phase shift to analyze and graph trigonometric functions and apply these characteristics to real-world situations. Students will then use the graphs of trigonometric functions to find the inverse of these functions and perform operations, such as, composition of functions.	real-world application.
Unit 6 Logarithmic and Exponential Functions	Unit 7 Conics	Unit 8 Identities and Trigonometric Equations	Unit 9 Polar, Complex, and Vectors	Unit 10 Limits and Derivatives
Students will utilize properties of logarithms and exponents in order to solve, graph, and study the applications of logarithmic and exponential functions.	Students will analyze the characteristics and graphs of the various conic sections, including finding the intersection points of a system of conics.	Students will use identity formulas to simplify trigonometric expressions and use the formulas to establish an identity. They will find exact values of non-special angles using sum/difference formulas, double angle formulas, half-angle formulas, Pythagorean identity formulas, and Fundamental identities. Students will solve trigonometric equations using algebra skills and trigonometry, including exact values and identity formulas.	Students will extend their concept of coordinates to include graphing on the polar coordinate plane. Students graph polar coordinates and convert between polar and rectangular coordinates. They will graph polar equations and polar form of linear equations and circles. Students will write complex numbers in polar form and find products, quotients, powers and roots of complex numbers in polar form. Students will add and subtract vectors graphically and algebraically. They will find a position vector, unit vector, the magnitude of a vector. Students will apply properties of vectors to	Students will explore the concepts of limits and derivatives, which are foundations for Calculus. Students will learn about the meanings of, and the properties for determining limits and derivatives, and connect them to graphing rational and polynomial functions. Students will find limits and derivatives algebraically, and with the aid of technology.

			real-world situations.	
PE Assessment: Unit 6 Logarithmic and Exponential Functions	PE Assessment: Unit 7 Conics	PE Assessment: Unit 8 Identities and Trigonometric Equations	PE Assessment: Unit 9 Polar, Complex, and Vectors	PE Assessment: Unit 10 Limits and Derivatives
PE Description: Students will solve a logarithmic inequality and give the solution in interval notation. Students will apply logarithms to make predictions for an exponential decay scenario.	PE Description: Students will identify each conic section in a system of equations in general form and then graph the system. Students will solve a system containing a parabola and ellipse algebraically.	PE Description: Students will be able to find exact values using identity formulas, establish an identity statement and solve a trigonometric equation.	PE Description: Students will graph polar equations by checking for symmetry and use the graphing calculator to plot points. Students will use DeMoivre's Theorem to simplify an expression.	PE Description: Students will analyze a piece-wise function and identify various limits, including one-sided limits and limits to infinity. Students will also find a limit of a function algebraically, using the rationalizing technique. Students will use the first and second derivative to find the critical points and point of inflection of a polynomial function.

Course Map

	Unit Description	PE Summary	PE Standards
Semester 1 Unit 1: Functions 3 weeks (14 days)	Students will study the basic parent functions, their transformations, and characteristics. Students will be able to evaluate the domain and range of functions and their inverses. Students will be able to solve radical and absolute value equations and inequalities, and be able to evaluate the domain of each one.	Students will graph a piecewise function and identify transformations, domain and range in interval notation.	**IMV.2.4.1 IMV.2.4.3 IMVI.1.2.3 IMVI.1.1.1**
Semester 1 Unit 2: Right Triangle Trigonometry 4 weeks (19 days)	Students will be able to find exact values of angles in both degrees and radians. They will use identity formulas to find values of non-special right triangles and simplify expressions. Students will find exact values of special angles using right triangles and the unit circle and then find the missing sides and angles of a right triangle.	Students will use trigonometric definitions and exact values to simplify expressions and apply arc length and area of a sector to a real-life application.	**IMVI.1.2.3 IMVI.2.1.4 IMV.4.1.4**
Semester 1 Unit 3: Polynomial and Rational Functions 3-4 weeks (17 days)	Students will learn to solve and graph polynomial and rational equations, identifying key characteristics.	Students will find the key characteristics of a rational function including asymptotes, intercepts, domain and range, then graph the function.	**IMVI.1.1.2 FIF.7.d**
Semester 1 Unit 4: Graphing Trigonometric Functions and Inverses	Students will use the characteristics of graphs such as amplitude, period, vertical translation, and phase shift to analyze and graph trigonometric functions and apply these characteristics to real-world situations. Students will then use the graphs of	Students will use the characteristics of graphs such as amplitude, period, vertical translation, and phase shift to analyze and graph trigonometric functions and apply these characteristics to real-world situations. Students will then use the graphs of	**IMVI.2.1.5 IMVI.2.2.1**

3 weeks (15 days)	trigonometric functions to find the inverse of these functions and perform operations, such as, composition of functions.	trigonometric functions to find the inverse of these functions and perform operations, such as, composition of functions.	
Semester 1 Unit 5: Triangle Applications **1 - 2 weeks (7 days)**	Students will use the Law of Sines and Law of Cosines to solve oblique triangles and apply to real-world situations. They will find the area of triangles using Heron's formula or other trigonometric formulas.	Students will use the Law of Sines, Law of Cosines, and area triangle formulas to find the area of a lake in a real-world application.	**IMVI.2.1.8**
Semester 2 Unit 6: Logarithmic and Exponential Functions **3 weeks (14 days)**	Students will utilize properties of logarithms and exponents in order to solve, graph, and study the applications of logarithmic and exponential functions.	Students will solve a logarithmic inequality and give the solution in interval notation. Students will apply logarithms to make predictions for an exponential decay scenario.	**IMV.2.1.3** **IMV.2.1.4**
Semester 2 Unit 7: Conics **2 weeks (9 days)**	Students will analyze the characteristics and graphs of the various conic sections, including finding the intersection points of a system of conics.	Students will identify each conic section in a system of equations in general form and then graph the system. Students will solve a system containing a parabola and ellipse algebraically.	**IMVI.3.1.2** **IMVI.3.1.3**
Semester 2 Unit 8: Identities and Trigonometric Equations **4 weeks (18-20 days)**	Students will use identity formulas to simplify trigonometric expressions and use the formulas to establish an identity. They will find exact values of non-special angles using sum/difference formulas, double angle formulas, half-angle formulas, Pythagorean identity formulas, and Fundamental identities. Students will solve trigonometric equations using algebra skills and trigonometry, including exact values and identity formulas.	Students will be able to find exact values using identity formulas, establish an identity statement and solve a trigonometric equation.	**IMVI.2.1.7** **IMVI.2.2.3**

Semester 2 **Unit 9: Polar, Complex, and Vectors** **3 weeks** **(15 days)**	Students will extend their concept of coordinates to include graphing on the polar coordinate plane. Students graph polar coordinates and convert between polar and rectangular coordinates. They will graph polar equations and polar form of linear equations and circles. Students will write complex numbers in polar form and find products, quotients, powers and roots of complex numbers in polar form. Students will add and subtract vectors graphically and algebraically. They will find a position vector, unit vector, the magnitude of a vector. Students will apply properties of vectors to real-world situations.	Students will graph polar equations by checking for symmetry and use the graphing calculator to plot points. Students will use DeMoivre's Theorem to simplify an expression.	**IMVI.3.2.2** **IMVI.2.2.2**
Semester 2 **Unit 10: Limits and Derivatives** **3 weeks** **(14 days)**	Students will explore the concepts of limits and derivatives, which are foundations for Calculus. Students will learn about the meanings of, and the properties for determining limits and derivatives, and connect them to graphing rational and polynomial functions. Students will find limits and derivatives algebraically, and with the aid of technology.	Students will analyze a piece-wise function and identify various limits, including one-sided limits and limits to infinity. Students will also find a limit of a function algebraically, using the rationalizing technique. Students will use the first and second derivative to find the critical points and point of inflection of a polynomial function.	**EK 1.A12** **EK 1.1C2** **EK 2.2A1** **EK 2.1C2**

Unit 1: Functions

Content Area: Mathematics	Course: Pre AP Calculus	UNIT: Functions
Unit Description: In this unit, students will study the basic parent functions, their transformations, and characteristics. Students will be able to evaluate the domain and range of functions, and their inverses. They will be able to solve radical and absolute value equations and inequalities, and be able to evaluate the domain of each one.		**Unit Timeline:** 14 Days

DESIRED Results
Transfer Goal - ***Students will be able to independently use their learning to……*** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Expressions for functions can be written in multiple forms, where each form provides a different insight into the relationship between two variables.
2. Expressions, verbal descriptions, graphs, and tables of values for a function are all connected and provide insight into the relationship between two variables.
3. Functions can be built from simpler functions, from transformations, or inverses; the analysis of how a function is built can provide insight into the relationship between two variables and can be used to solve real world problems.
4. The properties of functions and function operations are used to model and analyze real-world applications and quantitative relationships.

5. Radical equations and absolute value equations can be solved by isolating the radical or absolute value. This process may introduce extraneous solutions.
6. The same techniques used to transform the graphs other functions can be applied to the graphs of square root functions and absolute value functions

Essential Questions: ***Students will keep considering…***

- What information does the form of the function give us about the relationship between the two variables?
- How are the graph, the table of values, the expression, and the verbal description of a function related?
- Why study functions and how was this function built?
- How are the properties of functions and functional operations useful?

Students will know/understand ...	Students Will Be Able to ...	Standard
Functions are defined within a specific domain and range **Function** - a relation that assigns to each element x in the set A exactly one element y in the set B. **Interval notation -** A notation for representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included. **Domain -** The set of values of the independent variable(s) for which a function or relation is defined. Typically, this is the set of x-values that give rise to real y-values. **Range** - the output values of a function	**Determine the Domain and Range of functions, using interval notation**	**IMVI.1.1.1**

Analyzing graphs of functions provides a visual relationship between two variables. **Continuity** - The graph of f is a connected curve with no jumps, gaps, or holes. **Discontinuity** - a point at which a function is discontinuous or undefined. **Symmetry** - Symmetry is a type of invariance: the property that something does not change under a set of transformations. A mapping of the object onto itself which preserves the structure. **Even function** - A **function** with a graph that is symmetric with respect to the y-axis. A **function** is **even** if and only if f(–x) = f(x). **Odd function** - A **function** is odd if and only if $f(-x) = -f(x)$, (so all of the signs are switched), then the function is odd. **Piecewise functions -** a function that is defined by two or more equations over a specified domain.	Recognize and describe continuity and symmetry, and connect these concepts to functions and their graphs, including piecewise functions	IMVI.1.1.4
Simple functions can be transformed to create more complicated functions. **Parent functions (linear, quadratic, absolute value, radical, cubic, rational)** - the simplest **function** of a family of **functions** that preserves the **definition** (or shape) of the entire family. **Transformation** - rigid **transformations** that do not change the shape or size of the preimage and **non-rigid transformations** that change the size but not the shape of	**Apply basic function transformations to a parent function, and interpret the results of these transformations verbally, graphically, and numerically**	**IMVI.1.2.3**

the preimage. **Vertical translation** - shift the graph up or down: f(x) $\pm$k **Horizontal translation** - shift the graph left or right: f(x $\pm$ h) **Reflection** - a transformation that creates a mirror image. ie) -f(x) reflects over x-axis, f(-x) reflects over y-axis **Dilation (stretch/compression)** - changes the size of a figure. af(x) -> vertical stretch when a > 1, vertical compression when 0 < a < 1; f(bx) -> horizontal compression when a > 1, horizontal stretch when 0 < b < 1		
Two functions can be combined to create new functions. **Arithmetic combination** - just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to create new functions. **Restrictions** - a condition that imposes a constraint on the possible values of a variable or on the domain of arguments of a function	Find and interpret the sum, difference, product, and quotient of two functions, and indicate relevant domain and range for the resulting function	IMVI.1.2.1

When you compose two functions and the result is the identity function, then those functions are inverses. **Composition of functions** - In mathematics, **function composition** is the pointwise application of one **function** to the result of another to produce a third **function**. ... The resulting composite **function** is denoted g ∘ f : X → Z, **defined** by (g ∘ f)(x) = g(f(x)) for all x in X. **Inverse function** - a **function** that "reverses" another **function**: if the **function** f applied to an input x gives a result of y, then applying its **inverse function** g to y gives the result x, and vice versa. i.e., f(x) = y if and only if g(y) = x. **One-to-one** - A **function** for which every element of the range of the **function** corresponds to exactly **one** element of the domain. **Horizontal Line Test** - a method that can be used to determine if a function is a one-to-one function. This means that, for every y-value in the function, there is only one unique x-value.	Form the composition of two functions, indicate the relevant domain and range for the resulting function, and determine whether they are inverses	IMVI.1.2.2
Piecewise Functions may contain a variety of functions over a specified domain and range and may be represented in multiple ways. **Interval of increasing or decreasing** - **Intervals** of **increase** and decrease are the domain of a function where its value is getting larger or smaller, respectively. For a function f(x) over an **interval** where , f(x) is **increasing** if and f(x) is decreasing if . For f(x) over a given **interval**, if f(x) is **increasing** and if f(x) is decreasing. **Constant Interval** - if for any x_1 and x_2 in the **interval**, f (x_1) = f (x_2).	**Analyze a problem situation to determine or interpret reasonable domain and range values for piecewise functions representing the situation, and represent them verbally, graphically, tabularly, and symbolically**	**IMV.2.4.1** **IMV.2.4.3**

Relative maximum - The highest point in a particular section of a graph. **Relative minimum** - The lowest point in a particular section of a graph.		
Radical and absolute value equations and inequalities may be solved algebraically and graphically. **Radical function** - contains a **radical** expression with the independent variable (usually x) in the radicand. **Absolute value function** - a **function** that contains an algebraic expression within **absolute value** symbols. **Extraneous solutions** - is a **solution** that emerges from the process of solving the problem but is not a valid **solution** to the original problem.	Use properties of radicals and absolute value functions to solve equations and inequalities, and identify extraneous solutions, when they occur	IMV.2.3.5 IMV.2.4.2

Unit 1: Assessment

EVIDENCE of LEARNING			
Understanding 1, 2, 3	Standards **IMV.2.4.1** **IMVI.1.2.3** **IMVI.1.1.1**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will graph a piecewise function and identify transformations, domain and range in interval notation. Unit 1 Assessment - Functions **Teacher will assess:** Students should know how to graph a piecewise function which include quadratic, radical and absolute value functions and interpret key characteristics. **Performance:** **Mastery:** Score of 75% or above **Scoring Guide:** *See assessment*	**R/R Quadrant** **21 Century** C Critical Thinking

Unit 1: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1, 2, 3	**IMVI.1.1.1** IMVI.1.2.1 IMVI.1.2.	**1. Lesson Title:** Functions - Simultaneous RoundTable **Learning Objective:** Students will be able to find the domain and range, the composition, and the inverse of functions.	Cooperative Learning	B Collaboration

	2	**Activity:** In pods, students will work on question number 1; once they are finished the students will pass their paper to the left. This students will then check their classmates work and initial if it is correct. Once the entire pod feels that question 1 is correct, they will now work on question number 2. Continue this process until all 4 questions are done. Functions - Kagan RoundTable		Communication
5	IMV.2.3.5 IMV.2.4.2	**2. Lesson Title:** Need to Knows!!! How to Solve Radical and Absolute Value Equations and Inequalities **Learning Objective:** Students will be able to identify the important aspects to solving radical and absolute value equations and inequalities. **Activity:** The students will write down the "Need to Knows". What are the similarities and differences to solving radical and absolute value equations and inequalities? Need to Knows - Solving Radical and Absolute Value	Summarizing and Note Taking Advance Organizer	A Communication
1, 2, 3, 6	**IMVI.1.1.1** **IMVI.1.2.3** IMVI.1.2.1 **IMV.2.4.1** **IMV.2.4.3**	**3. Lesson Title:** Outline of Functions **Learning Objective:** The students will be able to create an outline of the chapter and what important concepts we covered. **Activity:** Each pod of students will receive a stack of post-its. The students will be asked to write different concepts, ideas, vocabulary, graphs, etc. on the post-its and place in the center of the pod. Once they are finished, the students will organize the post-its into categories. Final the students will create their own outline of the unit based on their post-its. The teacher can then show the students their version of an outline for the unit, and compare and see what the differences are. Outline of Functions	Summarizing and Note Taking Cues, Questions, and Advance Organizer Identifying Similarities and Differences Cooperative Learning - Jot Thoughts	C Communication Collaboration

1, 2, 6	IMVI.1.1.4 **IMV.2.4.1** **IMV.2.4.3**	**4. Lesson Title:** Investigating Piecewise Functions **Learning Objective:** Students will be able to graph piecewise functions and state the appropriate domain and range in interval notation. **Activity:** Students will complete the investigation on piecewise functions independently or with a partner. Following the directions, students will use tables to determine a set of points for each function, then physically graph each piece individually, cut out the pieces within the restricted domain and merge them all onto a single graph. At the conclusion of the activity, students will continue to practice graphing piecewise functions to demonstrate their understanding. Piecewise Functions Investigation	Providing Practice Nonlinguistic Representation	B, C Critical Thinking

Unit 1: Resources

UNIT RESOURCES
Teacher Resources: ● *Kuta software* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos*
Student Resources: ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Khan Academy - Functions* ● *Cliff's Notes - Functions* ● *Study Island* ● *Spark Notes - Functions*
Vocabulary: **Function** - a relation that assigns to each element x in the set A exactly one element y in the set B. **Interval notation -** A notation for representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses

and/or brackets are used to show whether the endpoints are excluded or included.
Domain - The set of values of the independent variable(s) for which a function or relation is defined. Typically, this is the set of x-values that give rise to real y-values.
Range - the output values of a function
Continuity - The graph of f is a connected curve with no jumps, gaps, or holes.
Discontinuity - a point at which a function is discontinuous or undefined.
Symmetry - Symmetry is a type of invariance: the property that something does not change under a set of transformations. A mapping of the object onto itself which preserves the structure.
Even function - A **function** with a graph that is symmetric with respect to the y-axis. A **function** is **even** if and only if f(–x) = f(x).
Odd function - A **function** is odd if and only if $f(-x) = -f(x)$, (so all of the signs are switched), then the function is odd.
Piecewise functions - a function that is defined by two or more equations over a specified domain.
Parent functions (linear, quadratic, absolute value, radical, cubic, rational) - the simplest **function** of a family of **functions** that preserves the **definition** (or shape) of the entire family.
Transformation - rigid **transformations** that do not change the shape or size of the preimage and **non-rigid transformations** that change the size but not the shape of the preimage.
Vertical translation - shift the graph up or down: f(x) $\pm$k
Horizontal translation - shift the graph left or right: f(x $\pm$ h)
Reflection - a transformation that creates a mirror image. ie) -f(x) reflects over x-axis, f(-x) reflects over y-axis
Dilation (stretch/compression) - changes the size of a figure. af(x) -> vertical stretch when a > 1, vertical compression when 0 < a < 1; f(bx) -> horizontal compression when a > 1, horizontal stretch when 0 < b < 1
Arithmetic combination - just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to create new functions.
Restrictions - a condition that imposes a constraint on the possible values of a variable or on the domain of arguments of a function
Composition of functions - In mathematics, **function composition** is the pointwise application of one **function** to the result of another to produce a third **function**. ... The resulting composite **function** is denoted $g \circ f : X \to Z$, **defined** by $(g \circ f)(x) = g(f(x))$ for all x in X.
Inverse function - a **function** that "reverses" another **function**: if the **function** f applied to an input x gives a result of y, then applying its **inverse function** g to y gives the result x, and vice versa. i.e., f(x) = y if and only if g(y) = x.
One-to-one - A **function** for which every element of the range of the **function** corresponds to exactly **one** element of the domain.
Horizontal Line Test - a method that can be used to determine if a function is a one-to-one function. This means that, for every y-value in the function, there is only one unique x-value.
Interval of increasing or decreasing - **Intervals** of **increase** and decrease are the domain of a function where its value is getting larger or smaller, respectively. For a function f(x) over an **interval** where , f(x) is **increasing** if and f(x) is decreasing if . For f(x) over a given **interval**, if f(x) is **increasing** and if f(x) is decreasing.
Constant Interval - if for any x_1 and x_2 in the **interval**, $f(x_1) = f(x_2)$.
Relative maximum - The highest point in a particular section of a graph.
Relative minimum - The lowest point in a particular section of a graph.

Radical function - contains a **radical** expression with the independent variable (usually x) in the radicand.
Absolute value function - a **function** that contains an algebraic expression within **absolute value** symbols.
Extraneous solutions - is a **solution** that emerges from the process of solving the problem but is not a valid **solution** to the original problem.

Unit 2: Right Triangle Trigonometry

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Right Triangle Trigonometry**
Unit Description: Students will be able to find exact values of angles in both degrees and radians. They will use identity formulas to find values of non-special right triangles and simplify expressions. Students will find exact values of special angles using right triangles and the unit circle and then find the missing sides and angles of a right triangle.		**Unit Timeline:** 19 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. The measure of an angle may be represented in degrees and in radians.
2. Arc length of a circle and area of a sector of a circle are a function of the radius and the central angle.
3. Linear speed and angular speed may be applied to real-world situations.
4. The Fundamental Identities may be used to find the values of trigonometric functions and applied to real-world situations.
5. The Complementary Angle Theorem may be applied to simplify trigonometric expressions.
6. Exact values of trigonometric functions may be found using right triangles.
7. Missing sides and angles of a right triangle may be found using trigonometry and applied to real-world situations.

Essential Questions: ***Students will keep considering…***

- How do you convert an angle in radian measure to degrees?
- How do you calculate the angular speed of a race car on a circular track?
- How can you find the height of Lincoln's face of the caricature on Mount Rushmore given two sightings?
- How do you find the exact value of a trigonometric function?
- How can one find the missing sides and angles of a triangle?

Students will know/understand ...	Students Will Be Able to ...	Standard
Trigonometric function: a function whose rule is given by a trigonometric ratio. **sine** - ratio of the length of the opposite leg to the length of the hypotenuse. **cosine**-ratio of the length of the adjacent leg to the length of the hypotenuse. **tangent**-ratio of the length of the opposite leg to the length of the adjacent leg. **cotangent**-ratio of the length of the adjacent leg to the length of the opposite leg. **secant**-ratio of the length of the hypotenuse to the length of the adjacent leg. **cosecant**-ratio of the length of the hypotenuse to the length of the opposite leg	**Apply the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.**	**IMVI.4.1.4**
angle of elevation - angle created between the horizon and the line of sight to the object above. **angle of depression -** angle created between the	Apply properties of trigonometric ratios in solving mathematical and real-world problems	IMV.4.1.5

horizon and the line of sight to the object below.		
Standard position of an angle: an angle is in standard position when its vertex is at the origin and one ray is on the positive x-axis. **Initial side of an angle:** The initial side of an angle is the ray on the positive x-axis **Angle of rotation:** the angle formed by rotating the terminal side and keeping the initial side in place. **Coterminal angles:** angles in standard position with the same terminal side. **Reference angle:** the positive acute angle formed by the terminal side of an angle and the x-axis.	Develop and apply the definition of the sine and cosine functions of the degree measure of a general angle in standard position in relation to the values of the y- and x-coordinates, respectively, of points on the terminal side of the angle.	IMV.2.1.1
Radian measure: The measure of the central angle of a circle when the subtended arc is the same measure as the radius.	Develops radian measure of angles, measures angles in both degrees and radians, and converts between these measures	IMV.2.1.2
Exact Value - the value determined using special right triangles and reference angles **Arc length** - The distance along the **arc** (part of the circumference of a **circle**, or of any curve). **Sector of a circle** - is the part of a **circle** enclosed by two radii of a **circle** and their intercepted arc	Define the trigonometric functions as functions of the radian measure of a general angle, and describe them as functions of real numbers. Apply arc length and area of a sector to real-world circular motion problems.	IMVI.2.1.3
Unit circle - a circle with a radius of one unit. **Special Angle** - an angle whose reference angle is 30̊,	Develop and apply the values of the trigonometric functions at 0, π/6, π/4, π/3, π/2 radians and their multiples.	IMVI.2.1.4

60°, 45°, or is a quadrantal angle. **Quadrantal angle -** an angle whose terminal side lies on the x- or y- axis	Find the exact value of the special angles on the unit circle.	

Unit 2: Assessment

EVIDENCE of LEARNING

Understanding 2 4 5 6	**IMVI.2.1.3** **IMVI.2.1.4** **IMV.4.1.4**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will use trigonometric definitions and exact values to simplify expressions and apply arc length and area of a sector to a real life application. Unit 2 Triangle Trigonometry Assessment **Teacher will assess:** Students should know exact values, trigonometric definitions, and how to apply formulas to find arc length and area. **Performance:** **Mastery:** Score of 75% or above **Scoring** Unit 2 Triangle Trigonometry Assessment	**R/R Quadrant 21 Century** C Critical Thinking Communication

Unit 2: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1	IMV.2.1.2	**1. Lesson Title:** Radians and Degrees Inside-Outside Circle **Learning Objective:** Students will be able to convert between degrees and radians. **Activity:** Each student will be given a problem card. Students will form two circles with each person facing a partner. Students will quiz each other using their card. When the teacher prompts, students will trade cards and upon instruction from teacher, students on the inside circle will rotate a given direction. This will repeat for several rounds. Radians & Degrees Inside-Outside Circle	Cooperative Learning Cues and Questions	C Collaboration Communication Critical Thinking
6	**IMVI.2.1.4**	**2. Lesson Title:** Exact Value Bingo **Learning Objective:** Students will find the exact value of special angles. **Activity:** After completing the guided notes explaining exact values**,** students will be given a Bingo game card and will fill the card with exact values (i.e., 1, ½ , -2, etc) . Teacher will read off problems and students will mark the answer on their bingo sheet. Students who obtain a “bingo” within a specified time wins. Exact Values Guided notes Exact Values filled in notes Bingo card Questions PP Instructions and problems	Providing Practice	B Critical Thinking

7	**IMV.4.1.4** ISTE 5.a	**3. Lesson Title:** Solving Right Triangles **Learning Objective:** Students will be able to find the missing sides and/or angles of a right triangle. **Activity:** Kahoot! is a game-based classroom response system. Games are displayed on a shared screen – for example a smart TV, a laptop or an interactive whiteboard. Players join in using their own device – whether that is a smartphone, iPad, laptop, or desktop doesn't matter, as long as they have a browser and good internet connection. Players do NOT need a kahoot account to play. Solving Right Triangles kahoot	Providing Practice Feedback	C Critical Thinking Technology
6, 7	**IMV.4.1.4** IMV.4.1.5	**4. Lesson Title:** The Lunar Lander - Descending From the Moon **Learning Objective:** Students will use trigonometric function rules to solve real-world problems, graph and analyze functions to determine a relationship between two variables. **Activity:** The Lunar Lander - Descending From the Moon Activity Students will apply trigonometric functions when using data from the liftoff of the Lunar Lander from the Moon's surface. Students are asked to evaluate functions and use trigonometric ratios to find distances and angle measures. They are also asked to determine a function describing the angle of elevation in terms of height and a function describing the angle of elevation in terms of time. Students are then asked to graph the function of angle in terms of time, describe how the angle of elevation changes as time elapses and explain why. Students will work in small groups or pairs to solve the problem. Students are encouraged to perform calculations individually and verify their answers with other members of their group. Students may be asked to demonstrate their answers to the last question as well as complete an Extension of the activity.	Problem-based Providing Practice	C Communication Critical Thinking

Unit 2: Resources

UNIT RESOURCES
Teacher Resources: ● *Kuta software* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos*
Student Resources: ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Khan Academy - Right Triangle Trig* ● *Cliff's Notes - Trigonometry* ● *Study Island*
angle of elevation:angle created between the horizon and the line of sight to the object above. **angle of depression:** angle created between the horizon and the line of sight to the object below. **Standard position of an angle:** an angle is in standard position when its vertex is at the origin and one ray is on the positive x-axis. **Initial side of an angle:** the initial side of an angle is the ray on the positive x-axis **Angle of rotation:** the angle formed by rotating the terminal side and keeping the initial side in place. **Coterminal angles:** angles in standard position with the same terminal side. **Reference angle:** the positive acute angle formed by the terminal side of an angle and the x-axis. **Trigonometric function:**a function whose rule is given by a trigonometric ratio. **sine of an angle**- ratio of the length of the opposite leg to the length of the hypotenuse. **cosine of an angle**-ratio of the length of the adjacent leg to the length of the hypotenuse. **tangent of an angle**-ratio of the length of the opposite leg to the length of the adjacent leg. **cotangent of an angle**-ratio of the length of the adjacent leg to the length of the opposite leg. **secant of an angle**-ratio of the length of the hypotenuse to the length of the adjacent leg. **cosecant of an angle**-ratio of the length of the hypotenuse to the length of the opposite leg **Radian measure:** the measure of the central angle of a circle when the subtended arc is the same measure as the radius.

Exact Value of an angle: the value determined using special right triangles and reference angles
Arc length: the distance along the **arc** (part of the circumference of a **circle**, or of any curve).
Sector of a circle: the part of a circle enclosed by two radii of a circle and their intercepted arc
Unit circle: a circle with a radius of one unit.
Special Angle: an angle whose reference angle is 30°, 60°, 45°, or is a quadrantal angle.
Quadrantal angle:an angle whose terminal side lies on the x- or y- axis

Unit 3: Polynomial and Rational Functions

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Polynomial and Rational Functions**
Unit Description: In this unit, students will learn to solve and graph polynomial and rational equations.		**Unit Timeline:** 17 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Polynomial functions can be sketched using key characteristics like end behavior and intercepts.
2. Polynomial functions can be solved and graphed using a variety of algebraic methods.
3. Complex zeros can affect the shape of the graph of polynomial functions.
4. Rational functions can be verbally, graphically, numerically, and algebraically represented and can be rewritten so that intercepts and asymptotes may be more easily found.
5. Rational functions may be graphed using key characteristics such as asymptotes and intercepts.

Essential Questions: ***Students will keep considering…***

- How are the graph, the table of values, the expression, and the verbal description of a polynomial function related?
- How does the form of a polynomial determine how the intercepts are found and how the function is graphed?

- In what ways can rewriting a function be useful?
- In what ways can rational functions be rewritten, represented (verbally, graphically, numerically, and algebraically), and analyzed?

Students will know/understand ...	Students Will Be Able to ...	Standard
Root - a solution to a quadratic or polynomial equation **Zero** - a solution, or root, to a quadratic or polynomial equation. A zero produces the x-intercepts to a function	Formulate equations and inequalities based on quadratic functions, solve them using factoring, completing the square, and technology, and interpret the solutions in terms of the context of the problem	IMV.1.2.5
The quadratic formula may be used to assist in finding solutions of a polynomial function.	Develop the quadratic formula, and apply it to the solution of quadratic equations and interpret the nature of the roots	IMV.1.2.6
Ending behavior - the ending behavior of the graph of f(x) as x approaches positive infinity or negative infinity. The degree and the leading coefficient of a polynomial function determine the end behavior of the graph. **Infinity notation -** the notation used to indicate the ending behaviors for a function	Analyze and describe graphs of polynomial functions by examining their intercepts, zeros, domain and range, relative maximum/minimum, and ending behavior	IMV.1.1.2
Quadratic equations with real coefficients can have complex solutions.	Solve quadratic equations with real coefficients over the set of complex numbers	IMV.1.3.3
Descarte's Rule of Signs - a technique for determining an the number of possible positive or negative real roots of a polynomial. **Fundamental Theorem of Algebra** - states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.	Determine the number and nature of solutions to polynomial equations with real coefficients over the complex numbers	IMVI.1.1.3
Rational Root Theorem - in a polynomial function, f(x), every possible rational root of f(x) must be in the form $\pm p/q$, where p is a factor of the constant term, and q is a	Identify situations involving functions for which there is no elementary algorithm to find zeros (ie the Rational Root Theorem)	IMVI.1.1.5

factor of the leading coefficient.		
Polynomial functions can have multiple related representations.	**Identify and apply relationships among significant points of a function (zeros, max/min points), the graph of the function, the nature and number of the function's zeros, and the symbolic representation of the function**	**IMVI.1.1.2**
Rational expressions can be added, subtracted, multiplied and divided. *Rational equations and inequalities can be solved algebraically.* *Partial fraction decomposition allows us to look at a rational expression in a different form* **Partial fraction decomposition** - the process of writing a rational expression as a sum of two or more simpler rational expressions	Perform operations, evaluate, and solve rational equations and inequalities with linear and quadratic denominators, including the decomposition of partial fractions	IMV.2.3.3
Rational functions can be graphed and analyzed. **Horizontal asymptote** - the line y = b, where f(x) approaches a value b, as the x-value of f(x) approaches positive or negative infinity. **Vertical asymptote** - the line x = a at which a function does not exist **Oblique(slant) asymptote** - in a rational function, an asymptote in the form of y = mx + b that occurs when the degree of the numerator is exactly one more than the degree of the denominator **Point of Discontinuity (hole in the graph)** -moments within a function that are undefined and appear as a	**Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior**	**FIF.7.D**

break or hole in a graph. A point of discontinuity is created when a function is presented as a fraction and the function is able to be reduced by a factor of x - n, and the discontinuity occurs as x = n. **Removable Discontinuity** - also known as a hole in the graph **Nonremovable Discontinuity** - a type of discontinuity that exists when there is jump discontinuity		

Unit 3: Assessment

EVIDENCE of LEARNING

Understanding	Standards	**Unit Performance Assessment**:	**R/R Quadrant 21 Century**
4, 5	**IMVI.1.1.2** **FIF.7.D**	**Description of Assessment Performance Task(s):** Students will find the key characteristics of a rational function including asymptotes, intercepts, domain and range, then graph the function. Unit 3 Assessment - Rational Functions **Teacher will assess:** Students should know the key characteristics of a rational function (vertical, horizontal and oblique asymptotes, points of discontinuity, x- and y-intercepts, domain and range) and how to graph the function. **Performance:** **Mastery:** Score of 75% or above **Scoring Guide:** See assessment	C Critical Thinking

Unit 3: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
4, 5	**F.IF.7.D**	**1. Lesson Title:** Rational Functions **Learning Objective:** Students will be able to graph rational functions, identifying zeros, points of discontinuity, domain and asymptotes**.** **Activity:** In pods students will work on question number 1; once they are finished the students will pass their paper to the left. This students will then check their classmates work and initial if it is correct. Once the entire pod feels that question 1 is correct, they will now work on question number 2. Continue this process until all 4 questions are done. Simultaneous Roundtable - Rational Functions	Cooperative Learning Feedback Providing Practice	B Collaboration Communication
1	IMV.1.1.2	**2. Lesson Title:** End Behavior of Polynomial Functions **Learning Objective:** Students will be able to determine end behavior of polynomial functions by exploring graphs, comparing similarities and differences, and formulating a definition and visual representation of how to determine end behavior. **Activity:** Students are provided the exploration worksheet to guide them in discovering end behavior of polynomial functions. The worksheet provides guided instructions with cues and questions leading them to the discovery of how changes to the degree and leading coefficient affect end behavior. The end of the exploration prompts students to formulate a rule, visual or summary of determining end behavior for any polynomial function. Students should share their discovery per the teacher's chosen method.	Similarities and Differences Cues and Questions	C Critical Thinking Communication

		End Behavior Exploration		
2, 3	IMV.1.2.5 IMV.1.2.6 IMV.1.1.3 IMVI.1.1.5 IMV.1.3.3 **IMVI.1.1.2**	**3. Lesson Title:** Polynomial Functions **Learning Objective:** The students will be able to solve higher degree polynomial functions by factoring. Students will be able to write polynomial functions given the zeros. **Activity:** Paper Slide Video - In pods the students will receive one problem to solve. The students will solve the problem. The students will then create a paper slide using paper or whiteboards. The students will write their problem and the process they used to solve the problem (they can use multiple sheets of paper or whiteboards). One student will teach and be recorded (use phone) but only record the whiteboard and the students hands while they are explaining. Once each group is completed, you can share the videos with the class. The videos should be short 1-2 minutes. Paper Slide Video - Polynomial Functions	Non-linguistics Summarizing Providing Practice	C Collaboration Communication Creativity
4	IMV.2.3.3	**4. Lesson Title:** Partial Fraction Decomposition **Learning Objective:** Students will be able to decompose partial fractions. **Activity:** Divide the class into pairs. Designate one student from each pair to be the scribe and the other as the sage, or allow students to choose their first roles. Provide a problem or task to each sage, which he will explain to his scribe. Instruct the sage to describe the task or problem to the scribe so that the sage can reach the correct solution. Allow students to discuss the problem and to ask questions for clarification. Sages cannot help scribes write the solutions, and scribes can only write what sages say. Have the two students in each pair switch positions. Have the scribe become the sage and vice verse for the second task. Repeat the procedure so each student has a chance to fulfill both roles at least once. Repeat the rotation as time allows. Have students turn in papers in pairs, so you can see the both the desired outcome and the actual results. Sage and Scribe - Partial Fraction Decomposition	Cooperative Learning Providing Practice	B Communication Collaboration

UNIT RESOURCES

Teacher Resources:

- *Kuta software*
- *Precalculus with Limits, A Graphing Approach, Larson, 2005*
- *Trigonometry, Sullivan, 2009*
- *Desmos - Rational Functions*
- *Desmos - Polynomial Functions*

Student Resources:

- *Precalculus with Limits, A Graphing Approach, Larson, 2005*
- *Trigonometry, Sullivan, 2009*
- *Cliffs Notes - Polynomials*
- *Khan Academy - Polynomials*
- *Khan Academy - Rational Functions*
- *SparkNotes - Polynomial and Rational Functions*

Vocabulary:

Root - a solution to a quadratic or polynomial equation
Zero - a solution, or root, to a quadratic or polynomial equation. A zero produces the x-intercepts to a function
Ending behavior - the ending behavior of the graph of f(x) as x approaches positive infinity or negative infinity. The degree and the leading coefficient of a polynomial function determine the end behavior of the graph.
Infinity notation - the notation used to indicate the ending behaviors for a function
Descarte's Rule of Signs - a technique for determining an the number of possible positive or negative real roots of a polynomial.
Fundamental Theorem of Algebra - states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.
Rational Root Theorem - in a polynomial function, f(x), every possible rational root of f(x) must be in the form $\pm p/q$, where p is a factor of the constant term, and q is a factor of the leading coefficient.
Partial fraction decomposition - the process of writing a rational expression as a sum of two or more simpler rational expressions
Horizontal asymptote - the line y = b, where f(x) approaches a value b, as the x-value of f(x) approaches positive or negative infinity.
Vertical asymptote - the line x = a at which a function does not exist
Oblique(slant) asymptote - in a rational function, an asymptote in the form of y = mx + b that occurs when the degree of the numerator is

exactly one more than the degree of the denominator
Point of Discontinuity (hole in the graph) -moments within a function that are undefined and appear as a break or hole in a graph. A point of discontinuity is created when a function is presented as a fraction and the function is able to be reduced by a factor of x - n, and the discontinuity occurs as x = n.
Removable Discontinuity - also known as a hole in the graph
Nonremovable Discontinuity - a type of discontinuity that exists when there is jump discontinuity

Unit 4: Graphing Trigonometric Functions and Inverses

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Graphing Trigonometric Functions and Inverses**
Unit Description: Students will use the characteristics of graphs such as amplitude, period, vertical translation, and phase shift to analyze and graph trigonometric functions and apply these characteristics to real-world situations. Students will then use the graphs of trigonometric functions to find the inverse of these functions and perform operations, such as, composition of functions.		**Unit Timeline:** 15 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. The graphs of the trigonometric functions have certain characteristics.
2. Changing the characteristics of trigonometric functions transforms the graphs.
3. The range determines where a trigonometric function is one-to-one and determines the existence of inverse trigonometric functions and composite inverse trigonometric functions.
4. The graphs of trigonometric functions are used to model data tables.

Essential Questions: ***Students will keep considering…***

- How can the Tacoma Narrows Bridge collapse be modeled by a sinusoidal function?
- How do changes in the characteristics of trigonometric function effect the graph?

- How do the range of the trigonometric functions determine the existence of inverse trigonometric functions and composite inverse trigonometric functions?
- How can real-world data tables be modeled using the graphs of the trigonometric functions?

Students will know/understand ...	Students Will Be Able to ...	Standard
periodicity - the distance required for the function to complete one full cycle **amplitude** - the height from the x-axis to its maximum or minimum or half the distance between the minimum and maximum values of the range. **interval** - one-fourth of the period of the graph **zeros** - x-intercepts of the graph	**Construct the graphs of the trigonometric functions and describe their behavior, including period, amplitude, and zeros.**	**IMVI.2.1.5**
transformation of a graph - a vertical translation, horizontal shift, reflection, and stretch or compression of the graph of a trigonometric function **vertical translation** - shifting the parent graph up or down in the direction of the y-axis. **phase shift**- the amount a graph shifts horizontally from the original graph.	**Construct the graphs of the trigonometric functions using transformations.** **Determine the characteristics of trigonometric functions, including period, amplitude, vertical translation, and phase shift, from the graph and the equation.**	**IMVI.2.2.1**
Inverse trigonometric function - the inverse functions of the trigonometric functions (with suitably restricted domains). Specifically, they are the inverses of the sine, cosine, tangent, cotangent, secant, and cosecant functions, and are used to obtain an angle from any of the angle's trigonometric ratios. **Composite function** - a composition of functions	Define and graph inverses of trigonometric functions with appropriately restricted domains. Find the exact value of inverse trigonometric functions. Find the exact value of composite trigonometric functions.	IMVI.2.1.6

Unit 4: Assessment

EVIDENCE of LEARNING

Understanding	Standards		R/R Quadrant 21 Century
1, 2, 3, 4	**IMVI.2.1.5** **IMVI.2.2.1**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will use their knowledge of trigonometric functions and transformations to graph and label characteristics of a trigonometric function and evaluate composite trigonometric functions. Unit 4 Graphing Trig Functions and Finding Inverses **Teacher will assess:** Teacher will assess students' knowledge of the characteristics of a given graph and their ability to graph the given function correctly and their ability to evaluate composite trigonometric functions. **Performance:** **Mastery:** Students will score 75% or above. **Scoring Guide:**. Unit 4 Graphing Trig Functions and Finding Inverses	C Critical Thinking Communication

Unit 4: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1	**IMVI.2.2.1**	**1. Lesson Title:** Graphing Trig Functions **Learning Objective:** Students will be able to analyze and graph trig functions. **Activity:** Using Roundtable - graphs, students will complete a roundtable cooperative learning structure. Working in groups of four, each student will answer the question in box 1. When all are finished, they will pass their paper to the left where the next student will check the work and then work the problem in box 2. Students will proceed until all problems are finished. Finally, students will graph the function on the back page.	Cooperative Learning	C Collaboration Critical Thinking
2	**IMVI.2.1.5**	**2. Lesson Title:** Matching Trig Graphs **Learning Objective:** Students will be able to analyze graphs and match the graph with the appropriate equation.	Nonlinguistic representation, Identifying Similarities and Differences	C Collaboration Critical Thinking

		Activity: After a **guided notes** lesson on phase shift, half of the class will have cards with graphs while the other half will have cards with equations. Students will walk around the room and match appropriate graphs and equations. When completed students will trade cards with other students and repeat. Matching Graphs		
3	IMVI.2.1.6	**3. Lesson Title:** Inverse Trig Functions **Learning Objective:** Students will match the trigonometric inverse function with its correct angle value. **Activity:** After a guided notes lesson, each student will be given a problem card. Students will form two circles with each person facing a partner. Students will quiz each other using their card. When the teacher prompts, students will trade cards and upon instruction from teacher, students on the inside circle will rotate a given direction. This will repeat for several rounds. Inverse Functions Inside-Outside Circle	Cooperative Learning	C Collaboration Critical Thinking
2	**IMVI.2.1.5**	**4. Lesson Title:** Graphing Trig Functions with Transformations	Providing Practice	B

	IMVI.2.2.1 ISTE 5.c	**Learning Objective:** Students will build a visual understanding of amplitude, period, and phase shift in this introduction to trigonometric graphing. They will use this understanding to find models for given graphs of the sine function. **Activity:** **Desmos graphing activity** **Check for Understanding:** Student's work will be submitted and checked by Desmos and/or the teacher.		Critical Thinking Technology
1, 2, 3, 4	**IMVI.2.1.5** **IMVI.2.2.1** ISTE 5.b	**5. Lesson Title:** Earth, Can you hear me now? **Learning Objective:** Students will use trigonometric functions to model periodic behavior or real-life data, make predictions on the properties of a function, graph a sine curve, and determine domain and range using graphs and tables. **Activity:** Earth, Can you hear me now? Activity Student will work in groups of 2 - 4 to complete the NASA PBL activity. Problem: In preparation for human exploration missions to Mars, NASA has been sending spacecrafts to orbit the planet and rovers to explore the surface. Vital information has been gathered about the flight to Mars and about the planet	Problem-based Cooperative Learning Cues & Questions	C/D Communication Collaboration Critical Thinking Technology

		itself. Since Mars and the Earth orbit the sun at different rates, the distance between Mars and the Earth is constantly changing. In one proposed Mars exploration plan, astronauts will remain on the surface of Mars for approximately 18 months. The ability to communicate between Mars and Earth will be extremely critical. Students will use provided worksheets and guided questions to model and solve the problem.		

Unit 4: Resources

UNIT RESOURCES
Teacher Resources: ● *Kuta software* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos - Trigonometry*
Student Resources: ● *Cliffs Notes - Trigonometry* ● *Khan Academy - Trigonometry* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Study Island* ● *Spark Notes - Trigonometry*
Vocabulary: **periodicity:** the distance required for the function to complete one full cycle

amplitude: Amplitude is the height from the x-axis to its maximum or minimum or half the distance between the minimum and maximum values of the range.
interval: one-fourth of the period of the graph
zeros: x-intercepts of the graph
transformation of a graph: a vertical translation, horizontal shift, reflection, and stretch or compression of the graph of a trigonometric function
vertical translation: shifting the parent graph up or down in the direction of the y-axis.
phase shift: the amount a graph shifts horizontally from the original graph.
Inverse trigonometric function: the inverse functions of the trigonometric functions (with suitably restricted domains). Specifically, they are the inverses of the sine, cosine, tangent, cotangent, secant, and cosecant functions, and are used to obtain an angle from any of the angle's trigonometric ratios.
Composite function: a composition of functions

Unit 5: Triangle Applications

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Triangle Applications**
Unit Description: Students will use the Law of Sines and Law of Cosines to solve oblique triangles and apply to real-world situations. They will find the area of triangles using Heron's formula or other trigonometric formulas.		**Unit Timeline:** 7 days

DESIRED Results
<u>Transfer Goal</u> - ***Students will be able to independently use their learning to……*** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. The Law of Sines and the Law of Cosines may be used to solve triangles and applied to real-world problems.
2. Heron's Formula and trigonometric formulas may be used to find the area of a triangle.

Essential Questions: *Students will keep considering…*

- How do you apply the Law of Sines and the Law of Cosines to solve real-world problems?
- How do you find the area of an oblique triangle?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
Oblique Triangle: a non-right triangle **Law of Sines:** The *law of sines* states that the ratio between the length of the side opposite an angle and the sine of that angle is the same for all interior angles in the same triangle**.** **Law of Cosines:** For any △ABC, where *a* is the length of the side opposite angle A, *b* is the length of the side opposite angle B, and *c* is the length of the side opposite angle C, $a^2 = b^2 + c^2 - \cos(\text{angle A})$ **Area of a triangle: A = ½ a·b sinC** **Heron's Formula: Area =** $\sqrt{s(s-a)(s-b)(s-c)}$	**IMVI.2.1.8**	**Apply the trigonometric ratios, the Law of Sines and the Law of Cosines to find the measures of unknown sides and angles in triangles.** **Find the area of oblique triangles**.	**IMVI.2.1.8**

where s is the semiperimeter			

Unit 5: Assessment

EVIDENCE of LEARNING			
Understanding 1 2	Standards **IMVI.2.1.8**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will use the Law of Sines, Law of Cosines, and triangle area formulas to find the area of a lake in a real world application. Unit 5 Triangle Applications Assessment **Teacher will assess:** Students' use of the Law of Sines and Law of Cosines to find the missing sides of a triangular object in order to apply the trigonometric area formulas. **Performance:** **Mastery:** Students will score a 75% or above. **Scoring Guide:**	**R/R Quadrant 21st Century** C Critical Thinking Creativity

		Unit 5 Triangle Applications Assessment	

Unit 5: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1, 2	**IMVI.2.1.8**	**1. Lesson Title**: Law of Sines and Cosines **Learning Objective:** Students will be able to apply the Law of Sines and Law of Cosines and find area of a triangle using trigonometry. **Activity:** After guided notes, (Law of Sines Lesson 1, Law of Sines Lesson 2), students will use the Sum It Up to practice applying the law of sines and cosines. Students complete individual problems, find the sum of their individual solutions, work with other students to check answers, and check with teacher to see if their sum is correct. If not, students will rotate through the problems and discuss in order to get the correct solutions.	Cooperative Learning	C Collaboration Critical Thinking
1, 2	**IMVI.2.1.8**	**2. Lesson Title:** Area of a Triangle, Law of Sines of Cosines Pairs-Share **Learning Objective:** Students will find the area of a triangle and solve oblique triangles using the Law of Sines and Law of Cosines. **Activity:** After a guided notes lesson**,** students will use a Pairs-Share activity to find the area of a triangle and use the Law of Sines and Law of Cosines to solve an oblique triangle. Student 1 will work problem number 1 and Student 2 will coach and encourage. After Student 1 completes the first problem, then Student 2 will work problem 2 and Student 1 will coach and encourage. The activity will proceed in this fashion until completed.	Cooperative Learning	C Collaboration Critical Thinking

UNIT RESOURCES

Teacher Resources:

- *Kuta software*
- *Precalculus with Limits, A Graphing Approach, Larson, 2005*
- *Trigonometry, Sullivan, 2009*

Student Resources:

- *Khan Academy - Law of Sines/Cosines*
- *Precalculus with Limits, A Graphing Approach, Larson, 2005*
- *Trigonometry, Sullivan, 2009*
- *Cliffs Notes - Law of Sines/Cosines*
- *Spark Notes - Law of Sines/Cosines*

Vocabulary:
Oblique Triangle: a non-right triangle. A triangle which does not contain a 90 degree angle.
Law of Sines: The *law of sines* states that the ratio between the length of the side opposite an angle and the sine of that angle is the same for all interior angles in the same triangle**.**
Law of Cosines: For any △ABC, where *a* is the length of the side opposite angle A, *b* is the length of the side opposite angle B, and *c* is the length of the side opposite angle C, $a^2 = b^2 + c^2 - \cos(\text{angle A})$
Area of a triangle: A = ½ a·b sinC
Heron's Formula:
Area =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where s is the semiperimeter
Semiperimeter: One half the perimeter of a triangle.

Unit 6: Logarithmic and Exponential Functions

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Logarithmic and Exponential Functions**
Unit Description: In this unit, students will utilize properties of logarithms and exponents in order to solve, graph, and study the applications of logarithmic and exponential functions.		**Unit Timeline:** 14 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Logarithms provide a way to work with inverses of exponential functions.
2. Exponents are used to solve logarithmic equations and inequalities.
3. Logarithms are used to solve exponential equations and inequalities.
4. Exponential and logarithmic functions can be graphed and analyzed.
5. Exponential functions can be used to solve real-world problems representing things such as growth, decay or compound interest.

Essential Questions: ***Students will keep considering…***

- How are exponents and logarithms related?
- How are logarithmic and exponential functions related?
- How can logarithmic and/or exponential functions model real-life situations?

Students will know/understand ...	Students Will Be Able to ...	Standard
Expressions with rational exponents represent radical expressions, vice versa. **Real Exponents** - exponents that are real numbers	Extend the properties of rational exponents to real exponents, relating expression with rational exponents to the corresponding radical expressions.	IMV.2.1.1
Exponential Function is $f(x) = a^x$ where a > 0, a ≠ 1, and *x* is any real number.	Approximate solutions to an exponential equation, and relate the solutions to the points of intersection of the graph of the exponential equation and the graph of a horizontal line.	IMV.2.1.2
Domain is the set of all values of the independent variable for which the function is defined.	**Analyze a problem situation modeled by an exponential function, formulates an equation or inequality, and solves the problems.**	**IMV.2.1.3**
Compound Interest - After *t* years, the balance *A* in an account with principal *P* and annual interest rate *r* (in decimal form) is given by the following formulas. 1. For *n* compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ 2. For continuous compounding: $A = Pe^{rt}$ **Exponential Growth** - occurs when the growth rate of the value of a mathematical function is proportional to the function's current value, resulting in its growth with time being an exponential function, i.e., a function in which the time value is the exponent. **Exponential Decay** - When a population or group of something is declining, and the amount that decreases is proportional to the size of the population. In exponential decay, the total value decreases but the proportion that leaves remains constant over time.	**Use exponential functions to solve problems involving compound interest and exponential growth and decay in mathematics and real-world contexts.**	**IMV.2.1.4**
Ending behavior - the ending behavior of the graph of	Graph and analyze the behavior of exponential	IMV.2.1.5

f(x) as x approaches positive infinity or negative infinity. **Infinity notation -** the notation used to indicate the ending behaviors for a function	functions.	
Logarithmic Functions - For x > 0, a > 0, and $a \neq 1$, $y = log_a x$ if and only if $x = a^y$.	Define a logarithm as a solution to an exponential equation, and recognize the inverse relationship between functions defined by logarithms and exponential expressions, showing this relationship graphically.	IMV.2.2.1
Natural Logarithm - For x > 0, $y = ln\, x$ if and only if $x = e^y$.	Solve problems by applying properties of logarithms $\left[log\, xy = log\, x + log\, y;\ log\left(\frac{x}{y}\right) = log\, x - log\, y,\right]$ and $\left[log\,(x^a) = a log\,(x)\right]$to construct equivalent forms of a logarithmic expression.	IMV.2.2.2
Inverse functions can be applied to logarithmic and exponential functions to solve problems.	Apply the inverse relationship between exponential and logarithmic functions to solve problems in mathematics and real-world contexts.	IMV.2.2.3
Domain is the set of all values of the independent variable for which the function is defined. **Range** is the set of all values assumed by the dependent variable (that is, the set of all function values).	Determine the domain and range of functions as represented by symbols and graphs, where appropriate.	IMVI.1.1.1
There are situations for which there are no elementary algorithms to find zeros, but for which logarithms may be used to solve.	Identify situations involving functions for which there is no elementary algorithm to find zeros (for example, $a^x = x^n$), and distinguishes them as such.	IMVI.1.1.5
Functions can be represented in multiple ways.	Compare and contrast characteristics of different families of functions and translate among verbal, tabular, graphical, and symbolic representations of functions.	IMVI.1.1.6

Unit 6: Assessment

EVIDENCE of LEARNING			
Understanding 2, 5	Standards **IMV.2.1.3** **IMV.2.1.4**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will solve a logarithmic inequality and give the solution in interval notation. Students will apply logarithms to make predictions for an exponential decay scenario. Unit 6 Assessment - Exponential and Logarithmic Functions **Teacher will assess:** Students need to know and understand the properties of exponents/logarithms to solve equations and inequalities as well as how to determine an appropriate domain and range based on restrictions. **Performance:** **Mastery:** Score of 75% or above **Scoring Guide:** See assessment	**R/R Quadrant 21 Century** C Critical Thinking

Unit 6: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1, 2, 3	IMV.2.2.2 IMVI.1.1.6	**1. Lesson Title:** Speed Dating with Logarithms and Exponentials **Learning Objective:** Students will solve exponential and logarithmic equations with their 'date' in this interactive and self-checking speed dating activity.	Cooperative Learning Providing	B Collaboration

	IMVI.1.1.5	**Activity:** Teachers will need to photocopy the Speed Dating Cards so that the problem and answer are copied back to back. Each student will need their own card. It is necessary to arrange the desks so that two rows are facing each other. Inside-Outside Circle format could be used in lieu of Speed dating, if necessary. Pass out the cards and give the students 10 minutes to become an expert on solving and explaining their problem to someone else. They may ask others for help, if needed. After 10 minutes, they will need to be the expert on the problem. The answers are copied on the back of the card so that students can check themselves. Once students are experts they will take a seat at a desk and will exchange problems with the person in front of them. They will then solve this problem and if needed, will get help from the expert on that problem. Once enough time has passed (you may want to set a timer, or just feel out the class) ask students to rotate. One row will stand up and move in the same direction. The person who is bumped off the edge will come around to the open seat on the other end. Remember to remind kids to get their original problem back before they move! This activity presents a great opportunity for students to talk about the problems and really help each other out.	Practice	Communication
2, 3, 5	**IMV.2.1.3** **IMV.2.1.4** IMV.2.2.3	**2. Lesson Title:** How Do We Use Logarithms and Exponentials **Learning Objective:** Students will learn about real-world uses of Exponential and Logarithmic functions and apply that understanding to practice problems. **Activity:** Students receive the first 4 pages of Problem Solving with Exponential and Logarithmic Functions worksheet. Ask students to work in pairs to read the sections and complete the *Now you try...* problems. Students may use their book or a phone if they need to look up a word. If they are still confused by a term after using these resources, they can ask teacher or another team for clarification. Students should rely on each other, and on available information resources, as they work through the activity. As students work, make sure students stay on task. Ask clarifying questions such as:	Cooperative Learning Cues & Questions Providing Practice	C/D Collaboration Communication Critical Thinking

		What is the equation for this type of problem? What do the parameters (letters) mean in the equation? What are you trying to find? What do you know? How can find the answer? If students cannot answer a question, help them refer back to an appropriate place in the reading. How is that used in solving the problem? What strategies did we talk about for documenting the information in a problem? The last 2 pages are assigned as in-class team work or homework.		
1, 2, 3	IMV.2.1.1	**3. Lesson Title:** Rational Exponents **Learning Objective:** Students will practice converting from expressions with rational exponents to corresponding radical expressions. **Activity:** Students will be given the worksheet Unit 6 Rational Exponents to complete with a partner in a Rally Coach format. Student A will work on the first problem while Student B coaches. Students will switch roles after each problem.	Cooperative Learning Providing Practice	B Communication
3, 4	IMV.2.1.2	**4. Lesson Title:** Solving Exponential Equations **Learning Objective:** Students will explore the algebraic methods of solving exponential equations using guided notes during a lesson. **Activity:** Students will use Unit 6 Solving Exponential Equations Guided Notes to take notes during the lesson. At the end of the lesson, students may reflect on what they have learned stating any specific details they would like to remember, frequent misconceptions, questions, etc. It is for their own use in practicing, studying, setting personal learning objectives, and so on. Teacher may introduce graphing calculators at his own discretion to explore estimating solutions, characteristics of graphs, etc.	Summarizing & Note Taking Advance Organizers Providing Practice	A/B Critical Thinking
4	IMV.2.1.5 IMVI.1.1.1	**5. Lesson Title:** Graph and analyze exponential functions **Learning Objective:** Students will graph exponential functions and compare the	Providing Practice	C Critical Thinking

		similarities and differences of both the equations and graphs. Students will use exponential functions to produce a table of values, vice versa. **Activity:** Students will work in teams of 2 - 4 to complete the Unit 6 - Graphing Exponential Functions worksheet. Students should compare answers and discuss the similarities and differences. In the final few problems, students will use their findings and understanding to generate exponential functions to represent a given table of values. The teacher may review answers and emphasize the findings to preface upcoming lessons. Teacher may or may not encourage students to utilize graphing calculators as a supporting tool.	Identifying Similarities & Differences Generating and Testing Hypotheses	Communication
4	IMV.2.2.1 **IMV.2.1.4** IMV.2.2.3 ISTE 5.b	**6. Lesson Title:** Analyzing the Relationship between a Logarithmic Function and Its Inverse **Learning Objective:** Students will be able to demonstrate numerically and graphically that the inverse of a logarithmic function is an exponential function. Students will be able to solve logarithmic and exponential equations using inverse operations, and recognize real world applications that can be modeled with these functions. **Activity:** Students should work together in groups of 2 - 4 to complete this inquiry. This activity will give students the opportunity to investigate the inverse of the logarithmic function and explore several uses of the logarithmic function and its inverse in real world applications. Students will determine numerous ordered pairs that satisfy a given logarithmic function, reverse the x and y coordinates, and use regression to determine the function that represents the inverse of the given logarithmic function. Students will then use this relationship to solve problems. Unit 6 Inquiry - Analyzing the Relationship between a Logarithmic Functions and its Inverse	Summarizing Identifying Similarities & Differences	C Critical Thinking Collaboration Technology

Unit 6: Resources

UNIT RESOURCES

Teacher Resources:

- *Kuta software*
- *Precalculus with Limits, A Graphing Approach, Larson, 2005*
- *Trigonometry, Sullivan, 2009*
- *Desmos - Exponentials*

Student Resources:

- *Cliffs Notes - Logarithmic and Exponential Functions*
- *Khan Academy - Logarithmic and Exponential Functions*
- *Precalculus with Limits, A Graphing Approach, Larson, 2005*
- *Trigonometry, Sullivan, 2009*
- *Study Island*

Vocabulary:

Real Exponents - exponents that are real numbers

Exponential Function is $f(x) = a^x$ where a > 0, a $\neq 1$, and *x* is any real number.

Compound Interest - After *t* years, the balance *A* in an account with principal *P* and annual interest rate *r* (in decimal form) is given by the following formulas.

3. For *n* compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
4. For continuous compounding: $A = Pe^{rt}$

Exponential Growth - occurs when the growth rate of the value of a mathematical function is proportional to the function's current value, resulting in its growth with time being an exponential function, i.e., a function in which the time value is the exponent.

Exponential Decay - When a population or group of something is declining, and the amount that decreases is proportional to the size of the population. In exponential decay, the total value decreases but the proportion that leaves remains constant over time.

Logarithmic Functions - For x > 0, a > 0, and $a \neq 1$, $y = log_a x$ if and only if $x = a^y$.

Natural Logarithm - For x > 0, $y = ln\, x$ if and only if $x = e^y$.

Domain is the set of all values of the independent variable for which the function is defined.

Range is the set of all values assumed by the dependent variable (that is, the set of all function values).

Unit 7: Conics

Content Area: Mathematics	Course: Pre AP Calculus	UNIT: Conics
Unit Description: In this unit, students will analyze the characteristics and graphs of the various conic sections, including finding the intersection points of a system of conics.		**Unit Timeline:** 9 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Conic sections can be represented both graphically and algebraically.
2. Conic sections equations can be classified and converted from general form to standard form to show key characteristics.
3. Conic sections can be graphed using key characteristics.
4. Equations of conic sections can be written given key characteristics.
5. Systems of conic sections may be solved algebraically.
6. Conic sections may represent real-world problems and be solved graphically or algebraically.

Essential Questions: *Students will keep considering…*

- What is the intersection of a cone and a plane parallel to a line along the side of the cone?
- What do graphs of conics look like?

- What is the difference between the algebraic representations of ellipses and hyperbolas?
- How can conics be applied to a real-world situation?

Students will know/understand ...	Students Will Be Able to ...	Standard
Conic section is the intersection of a plane and a double-napped cone. **Circle** is the collection of all point *(x, y)* that are equidistant from a fixed point *(h,k)*. **Standard form of a circle:** $r^2 = (x - h)^2 + (y - k)^2$ **Parabola** is the set of all points (*x,y*) in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, not on the line. **Standard form of a parabola:** $y = a(x - h)^2 + k$ and $x = a(x - k)^2 + h$ **Ellipse** is the set of all points *(x, y)* in a plane, the sum of whose distances from two distinct fixed points (**foci**) is constant. **Standard form of a ellipse:** $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ and $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ **Hyperbola** is the set of all points *(x, y)* in a plane, the difference of whose distances from two distinct fixed points (**foci**) is positive constant. **Standard form of a hyperbola:** $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ and	Determine an equation representing each of the conic sections from its locus description.	IMVI.3.1.1

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ **General form:** $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ **Locus:** The set of all points that share a property. This usually results in a curve or surface.		
Center of a circle is the given point from which all points on a circle are the same distance. **Foci** is two fixed points on the interior of an ellipse used in the formal definition of the curve. Directrix **Asymptote of a hyperbola** are two lines that intersect at the center of the hyperbola. $y = k \pm \frac{b}{a}(x - h)$ and $y = k \pm \frac{a}{b}(x - h)$ **Major Axis** is the chord joining the major vertices. **Minor Axis** is the chord joining the minor vertices. **Transverse Axis** is the line segment connecting the vertices. **Conjugate Axis** of a hyperbola is the line segment of length 2*b*.	**Analyze a quadratic equation in x and y representing a conic with center at (h, k), recognize the type of conic section represented, express the equation in a form useful for graphing, and construct a graph of a conic.**	**IMVI. 3.1.2**
Conic sections can be used to represent real-world situations. ***Systems of conic sections can be solved algebraically.***	**Use conic sections to model and solve problems (including non-linear systems) from mathematics and other disciplines.**	**IMVI.3.1.3**

Unit 7: Assessment

EVIDENCE of LEARNING			
Understanding 2, 3, 5	Standards **IMVI.3.1.2** **IMVI.3.1.3**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will identify each conic section in a system of equations in general form and then graph the system. Students will solve a system containing a parabola and ellipse algebraically. Unit 7 Assessment - Conics **Teacher will assess:** Students should know the different conic sections both by graph and equations in standard/general form as well as the key characteristics for each. Students should know how to graph conic sections on the coordinate plane. Students should know how to solve systems of nonlinear equations using substitution or elimination. **Performance:** **Mastery:** Score of 75% or above **Scoring Guide:** See assessment	**R/R Quadrant 21 Century** B Critical Thinking

Unit 7: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**

6	**IMVI.3.1.3**	**1. Lesson Title:** Conic Pictures in the Real-world **Learning Objective:** The students will be able to take real-world pictures of each of the four conic sections: circle, ellipse, hyperbola, and parabola. **Activity:** The students will take real-world pictures of each conic section and post it to the teacher's website. The pictures need to be taken by the student and not pulled from the web.	Nonlinguistic	A Creativity
6	IMVI.3.1.1 **IMVI.3.1.2** **IMVI.3.1.3**	**2. Lesson Title:** Conic Applications **Learning Objective:** Students will be able to solve real-world problems using conic sections. **Activity:** Students will work in pairs to complete the worksheet of conic applications. This will give students an idea of how conics are used outside of the classroom. Conic Applications Worksheet	Problem-Based Cooperative Learning Providing Practice	B/C Critical Thinking
1, 2, 3, 4	IMVI.3.1.1 **IMVI.3.1.2**	**3. Lesson Title:** Conic Sections Sort and Match Activity **Learning Objective:** Students will be able to practice the vocabulary of conic sections while investigating similarities and differences in the properties. **Activity:** Students should be arranged in teams of 2 - 4 students. Each team will get a set of cards. Students should be instructed to sort the cards into categories that have a common characteristic. Students should record their findings on post-it notes and be prepared to share with other groups and/or the class. Have students summarize the similarities and differences between the conic sections. Conic Sections Sort and Match Activity	Summarizing Identifying Similarities & Differences Nonlinguistic Representation	B/C Critical Thinking Collaboration Communication
5	**IMVI.3.1.3**	**4. Lesson Title:** Solving Systems of Conics **Learning Objective:** Students will be able to solve systems of conic sections	Identifying Similarities & Differences	B/C Critical

		using substitution, elimination, and graphing. **Activity:** Students will work in pairs to complete the worksheet Solving Conic Systems. Students should solve each problem using all 3 methods, then compare and contrast which approach would be more efficient and when. The graph will provide a visual representation of the problem. There is a warm-up and Exit slip included to evaluate individual student mastery.	Advance Organizers	Thinking

Unit 7: Resources

UNIT RESOURCES
Teacher Resources: • *Kuta software* • *Precalculus with Limits, A Graphing Approach, Larson, 2005* • *Trigonometry, Sullivan, 2009* • *Desmos-Exploring Conic Sections*
Student Resources: • *Cliffs Notes* - The Four Conic Sections • *Khan Academy - Conic Sections* • *Precalculus with Limits, A Graphing Approach, Larson, 2005* • *Trigonometry, Sullivan, 2009* • *Study Island* • *SparkNotes: Conic Sections* • *iXL - PreCalculus, Conic Sections*
Conic section: intersection of a plane and a double-napped cone. **Circle:** the collection of all point *(x, y)* that are equidistant from a fixed point *(h,k)*. **Parabola:** the set of all points (*x,y*) in a plane that are equidistant from a fixed line and a fixed point not on the line.

Ellipse: the set of all points *(x, y)* in a plane, the sum of whose distances from two distinct fixed points (**foci**) is constant.

Hyperbola: the set of all points *(x, y)* in a plane, the difference of whose distances from two distinct fixed points (**foci**) is positive constant.

General form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Locus: The set of all points that share a property. This usually results in a curve or surface.

Center: the given point from which all points on a conic are the same distance.

Foci (Focus): two fixed points on the interior of a conic used to create the points of the curve.

Directrix: a fixed line that is a given distance from the vertex of a parabola that is used to create the points of the parabola.

Asymptote of a hyperbola: two lines that intersect at the center of the hyperbola. $y = k \pm \frac{b}{a}(x - h)$ and $y = k \pm \frac{a}{b}(x - h)$

Major Axis: the chord joining the major vertices of an ellipse

Minor Axis: the chord joining the minor vertices of an ellipse

Transverse Axis: the line segment connecting the vertices of a hyperbola

Conjugate Axis: the line segment connecting the co-vertices of a hyperbola

Unit 8: Identities and Trigonometric Equations

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Identities and Trigonometric Equations**
Unit Description: Students will use identity formulas to simplify trigonometric expressions and use the formulas to establish an identity. They will find exact values of non-special angles using sum/difference formulas, double angle formulas, half-angle formulas, Pythagorean identity formulas, and Fundamental identities. Students will solve trigonometric equations using algebra skills and trigonometry, including exact values and identity formulas.		**Unit Timeline:** 18 - 20 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Identity formulas and algebraic manipulations can be used to establish an identity.
2. Identity formulas can be used to find exact values.
3. Using given information, exact values and trigonometric formulas, students can find the sum or difference of two angles.
4. Using given information along with double angle and half angle formulas, students can find the exact value of given angles.
5. Trigonometric functions and algebraic manipulations can be used to solve trigonometric equations.

Essential Questions: ***Students will keep considering…***

- What process will be used to correctly establish an identity?
- How do we use formulas to find exact values using of non-unit circle angles?
- How do we use the sum and difference formulas to find exact values using given information?
- How can double and half angle formulas to find exact values of specific trigonometric functions?
- What inθformation and processes are needed to solve trigonometric equations and find all the solutions between 0 and 2π?

Students will know/understand ...	Students Will Be Able to ...	Standard
Pythagorean Identities: $\sin^2\theta+\cos^2\theta=1$ $1 + \tan^2\theta=\sec^2\theta$ $\cot^2\theta+1=\csc^2\theta$ **Reciprocal Identities:** $\csc\theta= 1/\sin\theta$ $\sec\theta = 1/\cos\theta$ $\cot\theta = 1/\tan\theta$ **Quotient Identities**: $\tan\theta = (\sin\theta/\cos\theta)$ $\cot\theta = (\cos\theta/\sin\theta)$	**Prove and apply trigonometric identities.**	**F.TF.9**
Sum and difference formulas: sin(a + b) = sin(a)cos(b) + cos(a)sin(b) cos(a + b) = cos(a)cos(b) – sin(a)sin(b) $\tan(a+b)=\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$ sin(a – b) = sin(a)cos(b) – cos(a)sin(b) cos(a – b) = cos(a)cos(b) + sin(a)sin(b) $\tan(a-b)=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$ **Double Angle formulas:**	**Apply sum and difference formulas, double-angle formulas, and half-angle formulas to simplify trigonometric expressions.** **Apply the sum and difference formulas, double-angle formulas, and half-angle formulas to find the exact value of non-unit circle angles.**	**IMVI.2.1.7**

$\sin(2a) = 2\sin(a)\cos(a)$ $\cos(2a) = \cos^2(a) - \sin^2(a)$ $\cos(2a) = 2\cos^2(a) - 1$ $\cos(2a) = 1 - 2\sin^2(a)$ $\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$ **Half angle formulas:** $\sin(\frac{a}{2}) = \pm\sqrt{\frac{(1-\cos a)}{2}}$ $\cos(\frac{a}{2}) = \pm\sqrt{\frac{(1+\cos a)}{2}}$ $\tan(\alpha/2) = \pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}}$ $\tan(\alpha/2) = \frac{1-\cos(\alpha)}{\sin(\alpha)}$ $\tan(\alpha/2) = \frac{\sin(\alpha)}{1+\cos(\alpha)}$		
trigonometric equation - an equation containing a trigonometric function.	**Solve trigonometric equations,including quadratic, noting the periodic nature of solutions when applicable, and interpret the solutions graphically.**	**IMVI.2.2.3**

Unit 8: Assessment

EVIDENCE of LEARNING			
Understanding 1 3 5	Standards **F.TF.9** **IMVI.2.1.7** **IMVI.2.2.3**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Students will be able to find exact values using identity formulas, establish an identity statement and solve a trigonometric equation. Unit 8 Identities and Equations PE **Teacher will assess:** Students' knowledge of trigonometric identities and their use of formulas to find exact values and students' use of algebraic methods to solve trigonometric equations. **Performance:** **Mastery:** A score of 75% or above **Scoring Guide:** Unit 8 Identities and Equations PE	**R/R Quadrant 21 Century** C Critical Thinking

Unit 8: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**

1	**IMVI.2.1.7**	**1. Lesson Title:** Rally Coach: Introduction to Identities **Learning Objective:** Students will simplify trigonometric expressions and establish identities. **Activity:** After teacher/student guided notes (Lesson 1, Lesson 2, Lesson 3), partner A works a problem while Partner B coaches. Partner B works a problem while Partner A coaches.	Feedback & Reinforcing Effort Cues & Questions Providing Practice Cooperative Learning	C Collaboration Communication Critical Thinking
1	**IMVI.2.1.7**	**2. Lesson Title:** Speed Dating: Establish the Identity **Learning Objective:** Students will establish identities. **Activity:** After guided notes over double angle formulas, in pairs, students will work together (coaching and encouraging) to establish an identity from one of their cards as directed by the teacher. They will write the problem on their own answer sheet. The teacher will direct the partner A to move to the right and work another problem with a new partner. Students will continue for 4-5 more problems.	Feedback & Reinforcing Effort Cues & Questions Providing Practice Cooperative Learning	C Collaboration Communication Critical Thinking
5	**IMVI.2.2.**	**3. Lesson Title:** Trig Equations-Match My Answer **Learning Objective:** Students will be able to solve trigonometric equations. **Activity:** Students will be given either trig equations worksheet A or B. Each student will work the problems on their worksheet individually. When finished students will match up with another student who has the different lettered worksheet. The answers to the corresponding problems will be the same (though problems are different). Students will compare answers and discuss and rework any problems in which the answers are different. When finished, students will	Cooperative Learning	C Communication Critical Thinking

		report to the teacher with their completed sheets. Equations Match my Answer		
3, 4	**IMVI.2.1.7** ISTE 5.d	**4. Lesson Title:** Using trig identities to find exact values **Learning Objective:** Students will use the trig identities to find the exact value of a given angle. **Activity:** Kahoot! is a game-based classroom response system. Games are displayed on a shared screen – for example a smart TV, a laptop or an interactive whiteboard. Players join in using their own device – whether that is a smartphone, iPad, laptop, or desktop doesn't matter, as long as they have a browser and good internet connection. Players do NOT need a kahoot account to play. Trig identities and exact values	Providing Practice Feedback	Critical Thinking

Unit 8: Resources

UNIT RESOURCES
Teacher Resources: ● *Kuta software* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos*
Student Resources: ● *Khan Academy* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos* ● *KHAN Academy - Trig Identities and Equations*

- *Spark Notes - Trig Equations*

Vocabulary:

Trigonometric equation: - an equation containing a trigonometric function.
Trigonometric identity equation: An equation which is true for every value of the variable..
Establishing an identity: Using algebraic manipulations and trigonometric identity formulas to show that an identity equation is true for all values of the variable.

Unit 9: Polar, Complex, and Vectors

Content Area: Mathematics	**Course: Pre AP Calculus**	**UNIT: Polar Coordinates, Complex Plane, and Vectors**
Unit Description: Students will extend their concept of coordinates to include graphing on the polar coordinate plane. Students graph polar coordinates and convert between polar and rectangular coordinates. They will graph polar equations and polar form of linear equations and circles. Students will write complex numbers in polar form and find products, quotients, powers and roots of complex numbers in polar form. Students will add and subtract vectors graphically and algebraically. They will find a position vector, unit vector, the magnitude of a vector. Students will apply properties of vectors to real-world situations.		**Unit Timeline:** 15 days

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Points and curves in a plane can be expressed in both rectangular and polar forms.
2. Operations can be performed on complex numbers written in rectangular and polar form with equivalent representations.
3. Vectors can be represented graphically using magnitude and direction.
4. Vectors can be added algebraically to represent real world problems, such as force needed to move an object.

Essential Questions: ***Students will keep considering…***

- How can points and curves in a plane be expressed in rectangular and polar forms and what formulas will be used?
- What processes are followed when converting complex numbers from rectangular to polar form and vice versa?
- How do we draw vectors graphically using the tail to tip and/or parallelogram methods?
- Why would we need to add vectors algebraically in real world situations such as force and navigation?

Students will know/understand ...	Students Will Be Able to ...	Standard
Rectangular form of a coordinate - (x, y) **Polar form of a coordinate** - (r, θ)	Express points in the plane in both rectangular and polar forms.	IMVI.3.2.1
Conversion formulas can be used to convert from polar to rectangular, vice versa. **polar to rectangular:** x = r$\cos\theta$, y = r$\sin\theta$ **rectangular to polar:** $\tan\theta$ = y/x, $r = \sqrt{x^2 + y^2}$	**Find equivalent representations for points and curves, including the conics, in both rectangular and polar forms.**	**IMVI.3.2.2**
rectangular form of a complex number: $x + yi$ **polar form of a complex number:** r($\cos\theta + i \sin\theta$) **DeMoivre's theorem:** $z^n = r^n(cos(n\theta) + isin(n\theta))$	**Relates and uses rectangular and polar representations of complex numbers, and uses DeMoivre's theorem.**	**IMVI.2.2.2**
vector - a quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another.	Defines vectors in two dimensions as objects having magnitude and direction, and represents them geometrically.	IMVI.4.1.1
Vector addition geometrically - the process of finding one	Illustrates and applies the properties of vector	IMVI.4.1.2

vector that is equivalent to the result of the successive application of two or more given vectors**.** Vector addition may may be done geometrically by using the tail to tip method of drawing one vector, then drawing the second vector starting at the tip of the first and drawing the resultant vector from the tail of the first vector to the tip of the second vector. **Resultant vector** - sum of two vectors	addition and scalar multiplication to represent, investigate, and solve problems.	
Vector addition algebraically - the process of finding one vector that is equivalent to the result of the successive application of two or more given vectors. Vector addition may be done algebraically by finding the sum of the corresponding x and y coordinates and then finding the magnitude and direction of the resultant vector. **Magnitude:** length of a vector found by using the formula, $r = \sqrt{x^2 + y^2}$	Models geometric translations with vector addition to solve problems.	IMVI.4.1.4

Unit 9: Assessment

EVIDENCE of LEARNING

Understanding	Standards	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):**	**R/R Quadrant 21 Century**
1, 2	**IMVI.3.2.2** **IMVI.2.2.2**	Students will graph polar equations by checking for symmetry and use the graphing calculator to plot points. Students will use De Moivre's Theorem to simplify an expression. Polar, Complex, Vector PT **Teacher will assess:**	C Critical Thinking

		Students' knowledge of symmetry to graph a polar equation and students' use of DeMoivre's Theorem to raise a complex number to a power and write the result in standard form. **Performance:** **Mastery:** *Students will score a 75% or above.* **Scoring Guide:** Polar, Complex, Vector PT	

Unit 9: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:** 3
1	IMVI.3.2.1	**1. Lesson Title:** Unit 9 Polar Coordinates - Rally Coach **Learning Objective:** Students will be able to convert from Polar Coordinates to Rectangular Coordinates and from Rectangular Coordinates to Polar Coordinates. Unit 9 Polar Coordinates - Rally Coach **Activity:** After guided notes (**Polar Coordinates Lesson 1**, **Conversions Lesson 2**), partner A works a problem while Partner B coaches. Partner B works a problem while Partner A coaches.	Feedback & Reinforcing Effort Cues & Questions Providing Practice Cooperative Learning	C Collaboration Communication Critical Thinking
1	**IMVI.3.2.2**	**2. Lesson Title:** Unit 9 Polar Equations: Circles and Lines **Learning Objective:** Students will be able to transform a polar equation to an	Advance organizer	A Collabora

		equation in rectangular coordinates in order to identify and graph the equation. **Unit 9 Polar Equations: Circles and Lines - Frayer Model** **Activity:** Students will work with a partner to complete a Frayer model to compare and contrast polar equations of circles and lines.	Nonlinguistic representation Cooperative Learning	tion Communi cation Critical Thinking
3	IMVI.4.1.2	**3. Lesson Title:** Unit 9 Vectors - Rally Coach **Learning Objective:** Students will be able to graph a resultant vector in order to use in applied problems of physics and engineering. Unit 9 Vectors - Rally Coach **Activity:** Partner A works a problem while Partner B coaches. Partner B works a problem while Partner A coaches.	Feedback & Reinforcing Effort Cues & Questions Providing Practice Cooperative Learning	C Collabora tion Communi cation Critical Thinking
2	**IMVI.2.2.2**	**4. Lesson Title:** Unit 9 Complex Plane **Learning Objective:** Students will be able to convert a complex number from rectangular form to polar form in order to find products and quotients of complex numbers in polar form and to apply DeMoivre's Theorem. Unit 9 Complex Plane - Match My Answer **Activity:** Students will be given either complex number worksheet A or B. Each student will work the problems on their worksheet individually. When finished students will match up with another student who has the different lettered worksheet. The answers to the corresponding problems will be the same (though problems are different). Students will compare answers and discuss and rework any problems in which the answers are different. When finished, students will report to the teacher with their completed sheets.	Cooperative Learning	C Communi cation Critical Thinking

4	IMVI.4.1.4	**5. Lesson Title:** Unit 9 Vectors Applications - Practice **Learning Objective:** Students will be able to solve real-world problems involving vectors. Unit 9 Vectors Applications - Practice **Activity:** Students will practice solving application problems involving vectors.	Assigning Homework & Providing Practice	C Critical Thinking

Unit 9: Resources

UNIT RESOURCES
Teacher Resources: ● *Kuta software* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos*
Student Resources: ● *Khan Academy* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Desmos* ● *SparkNotes - Polar* ● *KHAN Academy - Vectors* ● *KHAN Academy - Polar and Complex*
Vocabulary: **Rectangular form of a coordinate**: (x, y) **Polar form of a coordinate:** (r, θ) **Rectangular form of a complex number:** $x + yi$ **Polar form of a complex number**: r(cosθ + i sinθ)

Vector: a quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another
Vector addition geometrically: the process of finding one vector that is equivalent to the result of the successive application of two or more given vectors**.** Vector addition may may be done geometrically by using the tail to tip method of drawing one vector, then drawing the second vector starting at the tip of the first and drawing the resultant vector from the tail of the first vector to the tip of the second vector.
Resultant vector: sum of two vectors
Vector addition algebraically: the process of finding one vector that is equivalent to the result of the successive application of two or more given vectors. Vector addition may be done algebraically by finding the sum of the corresponding x and y coordinates and then finding the magnitude and direction of the resultant vector.

Magnitude: length of a vector found by using the formula, $r = \sqrt{x^2 + y^2}$

Unit 10: Limits and Derivatives

Content Area: Mathematics	Course: Pre AP Calculus	UNIT: Limits and Derivatives
Unit Description: Students will explore the concepts of limits and derivatives, which are the foundations for Calculus. Students will learn about the meanings of, and the properties for determining limits and derivatives, and connect them to graphing rational and polynomial functions. Students will find limits and derivatives algebraically, and with the aid of technology.		**Unit Timeline:** 14 days

DESIRED Results
Transfer Goal - ***Students will be able to independently use their learning to……*** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. The concept of a limit is one of the foundations of calculus.
2. The limit of a function is the value approached by f (x) as x approaches a given value or infinity.
3. The limit of a function can be evaluated graphically, numerically, algebraically or on a graphing calculator.
4. The slope of a line in algebra is the average rate of change while the slope of the tangent to a curve at a point in calculus is the instantaneous rate of change (the derivative of a functions).
5. Critical values including points of inflection and intervals of concavity and extreme values of functions can be found using 1st and 2nd derivative tests.
6. Finding the derivative of a function may require the use of several rules, including rules for: sums, products, quotients, powers.
7. The physics concepts of position, velocity and acceleration are related mathematically by the derivative.
8. Sequences and series are a direct result of finding patterns.

Essential Questions: ***Students will keep considering…***

- What strategies can you use for finding limits?
- How do you find the slope of the tangent line to a curve at a point?
- How do you use the limit definition to find the derivative of a function?
- How do you determine continuity at a point and continuity on an open interval?
- How do you determine one-sided limits and continuity on a closed interval?
- How do you analyze and sketch the graph of a function?
- How do you solve applied minimum and maximum problems?
- What information do the first and second derivatives of a function give one about the function itself?

Students will know/understand ...	Students Will Be Able to ...	Standard
Limit - If *f(x)* becomes arbitrarily close to a unique number *L* as *x* approaches *c* from either side, the **limit** of *f(x)* as *x* approaches *c* is *L*. This is written as $\lim_{x \to c} f(x) = L$	Express limits symbolically using correct notation. ● Given a function *f*, the limit of *f(x)* as *x* approaches *c* is a real number *L* is *f(x)* can be made arbitrarily close to *L* by taking *x* sufficiently close to *c* (but not equal to *c*). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = L$.	EK 1.1A1
One-sided limit - either of the two limits of a function f(x) of a real variable x as x approaches a specified point either from below or from above (from the left or right) **Limit to infinity** - limits that have a value of ±∞, where the function grows without bound as it approaches some value a.	**The concepts of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.**	**EK 1.A12**
Indeterminate form - the form of a limit when the limit is not possible to determine because it is in the form of $\frac{0}{0}$.	A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.	EK 1.A13
Removable Discontinuity - also known as a hole in the	Numerical and graphical information can be used to	EK 1.1B1

graph **Nonremovable Discontinuity** - a type of discontinuity that exists when there is jump discontinuity	estimate limits.	
Direct substitution when $\lim_{x \to c} f(x) = f(c)$ **Dividing out technique** is used when direct substitution produces 0 in both the numerator and denominator. **Rationalizing technique** is used when direct substitution produces 0 in both numerator and denominator. Find the limit by rationalizing the function.	**The limit of a function can be found using algebraic manipulation.**	**EK 1.1C2**
Some functions may have no limits.	Asymptotic and unbounded behavior of functions can be explained and described using limits.	EK 1.1D1
There are a variety of notations for derivatives.	For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, $and\ y'$	EK 2.1A4
There are certain rules used to calculate derivatives for different function types.	**Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power functions**	**EK 2.1C2**
Power rule: $\frac{d}{dx}u^n = nu^{n-1}$ **Product rule**: $\frac{d}{dx}(fg) = f'g + fg'$ **Quotient rule**: $\frac{d}{dx}(\frac{f}{g}) = \frac{f'g - fg'}{g^2}$	Sums, differences, products, and quotients of functions can be differentiated using derivative rules.	EK 2.1C3
Chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x))(g')$	The chain rule provides a way to differentiate composite functions.	EK 2.1C4
Repeating the differentiating process can produce higher order derivatives, if they exist.	Differentiating *f'* produces the second derivative *f"*, provided the derivative of *f'* exists; repeating this	EK 2.1D1

	process produces higher order derivatives of *f*.	
There are a variety of notations used to represent higher-order derivatives.	Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be noted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.	EK 2.1D2
Critical points - points on a function representing local extrema and points of inflection. Local extrema are found using first derivative and inflection points are found using second derivative. **Concavity** - direction of the curvature of a graph (concave up or down) **Points of inflection** - a point on a curve at which a change in the direction of curvature occurs.	**First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local extrema, intervals of upward or downward concavity, and points of inflection.**	**EK 2.2A1**
The first derivative represents the slope of the tangent at a given point on the graph.	The derivative at a point is the slope of the line tangent to a graph at that point on the graph.	EK 2.3B1
Arithmetic series - $a + kd$ **Geometric series** - ar^k	Find the sum of an arithmetic or geometric series.	IMV.3.1.2
The rate of change in sequences can be used to classify sequences.	Investigate the rate of change found in sequences, and use it to characterize sequences as arithmetic, geometric, or neither.	IMV.3.1.1

Unit 10: Assessment

EVIDENCE of LEARNING			
Understanding	Standards	**Unit Performance Assessment**:	**R/R Quadrant**

2, 3, 5, 6	**EK 1.A12** **EK 1.1C2** **EK 2.2A1** **EK 2.1C2**	**Description of Assessment Performance Task(s):** Students will analyze a piece-wise function and identify various limits, including one-sided limits and limits to infinity. Students will also find a limit of a function algebraically, using the rationalizing technique. Students will use the first and second derivative to find the critical points and point of inflection of a polynomial function. Unit 10 Assessment Limits and Derivatives **Teacher will assess:** Student understanding of a limit and the skill of finding limits using various algebraic methods and the graphing calculator. Student use of the first and second derivative to determine relative extrema and the inflection points, if they exist. **Performance:** **Mastery:** Score of 75% or above **Scoring Guide:** See assessment	**21 Century** C Critical Thinking

Unit 10: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**

2, 3	EK 1.1A1 **EK 1.A12** EK 1.A13	**1. Lesson Title:** Limits of a Piecewise Function **Learning Objective:** Students will be able to find the limits of a piecewise function. **Activity:** Simultaneous Jot Thoughts - The teacher will present the graph to the students. Each pod will get a stack of post-its. The teacher will present one question at a time for the students. The students will write their answer on a post-it and once they are ready the teacher will ask the students to place their answer in the middle of the pod. The students will then discuss their answers and come to an agreement on the correct answer. Unit 10 Limits - Kagan Simultaneous Jot Thought	Cooperative Learning	B Collaboration Communication Critical Thinking
1, 6	**EK 2.1C2** EK 2.1C3	**2. Lesson Title:** The Limit Definition of a Derivative: Sum It Up **Learning Objective:** Students will be able to use the formal definition of a derivative to find the derivative of a function at a point. **Activity:** Students find the answer to each of the 10 questions then add or subtract as they go along to "Sum It Up" for a grand total at the end. A giant QR code is included if you like to use technology. This activity can be done independently, as pair and share, or many other strategies, including a race. The Limit Definition of a Derivative: Sum It Up	Providing Practice	B Collaboration
3. 5	EK 1.1B1	**3. Lesson Title:** Understanding Continuity and Derivatives **Learning Objective:** Students will be able to analyze graphs to describe continuity and its intervals. Students will be able to find 1st and 2nd derivatives and evaluate algebraically and graphically. **Activity:** Students will work together in pairs to practice the concept of continuity including both removable and nonremovable discontinuity. This worksheet could also be utilized as a Sage and Scribe activity on which each student would take turns either writing the problem as described by their partner and coaching when needed or working out the problem while explaining it to their partner.	Assigning Homework & Providing Practice	B Collaboration

		Understanding Continuity and Derivatives Worksheet		
1, 2, 3	**EK 1.A12** EK 1.A13 EK 1.1B1 **EK 1.1C2** EK 1.1D1 ISTE 5.a	**4. Lesson Title:** Match My Answer - Limits **Learning Objective:** Students will evaluate limits using algebraic methods, graphs and/or technology including one-sided and two-sided limits (infinity, kinks and jumps, removable and nonremovable discontinuities). **Activity:** Students will work together in pairs to complete a Match My Answer worksheet on evaluating limits. Students will complete their own individual problems and then compare their answers with their partner. The answers should match. If they do not match, then students should explore their own problem as well as their partner's problem to discern which is right. If neither answer seems to be correct after they have pleaded their cases, then they should meet with another pair to discuss the problem. The teacher will have all of the answers if students struggle to coming to a resolution. This activity provides students the opportunity to discuss problem-solving strategies as well as provide immediate feedback as to how they are progressing.	Cooperative Learning Feedback Providing Practice	B Collaboration Communication Critical Thinking Technology
4, 5, 6, 7	EK 2.1A4 **EK 2.1C2** EK 2.1C4 EK 2.1D1 EK 2.1D2 **EK 2.2A1** EK 2.3B1	**5. Lesson Title:** Derivatives and Critical Numbers with Anti-differentiation exploration challenge **Learning Objective:** Students will apply derivative rules to a variety of functions. Students will calculate the 1st and 2nd derivatives to answer a real-life application problem regarding slopes, max/min values, points of inflections, and instantaneous velocity. Students will be challenged to use their knowledge of derivatives to guess the original function when given the 1st derivative. **Activity:** Students will work in teams of 2-4 students to complete the Derivatives and Critical Numbers activity. The activity uses a variety of function and derivative notation and requires students to know when and which derivative rule per the problem. Students will apply their knowledge of derivatives and their meaning to answer questions for a real-life application. At the end, students will use the patterns observed in finding the derivative of a function to guess what the original	Cooperative Learning Feedback Providing Practice Generating & Testing Hypotheses	B/C/D Collaboration Communication Critical Thinking

		function was given its derivative. Students can check each other's work by differentiating the new function to see if it matches what they were given.		
8	IMV.3.1.2 ISTE 5.b	**6. Lesson Title:** Sum of Infinite Geometric Series TI-Inspire Activity **Objective:** Students will understand how a unit square can be divided into an infinite number of pieces. Students will understand a justification for the following theorem: The sum of the infinite geometric series 1/2 + 1/4 + 1/8 + 1/16 + ... = 1. Students will be able to explain why the sum of an infinite geometric series is a finite number if and only if $\|r\| < 1$. **Activity:** Students will use the TI-Inspire calculator to complete this activity. This lesson involves clicking on a slider to see that the area of a square that has been systematically divided into an infinite number of pieces approaches 1. As a result, students will: ● Connect the area of a square with the sum of the series 1/2 + 1/4 + 1/8 + 1/16... and realize that the sum is 1. ● Examine several infinite geometric series with various ratios to determine that the sum of an infinite geometric series is a finite number if and only if $\|r\|<1$. Sum of Infinite Geometric Series TI-Inspire Activity	Providing Practice Advance Organizer	B Critical Thinking Technology
8	IMV.3.1.2 IMV.3.1.1	**7. Lesson Title:** Sequences and Series **Objective:** Students will be able to calculate the sum of a sequence which is a series. Students will be able to determine the difference between an algebraic and geometric sequence. **Activity:** This is a powerpoint lesson accompanied by guided notes for the students to summarize and take notes. At the end of the lesson, students can continue to practice their understanding of sequences and series by completing the Simultaneous Round Table worksheet. During the simultaneous roundtable activity, students will be put into teams of 4. Each student will get a different worksheet to start with. To begin the activity, each student will do #1 on their worksheet. When	Summarizing & Notetaking Advance Organizer Identifying Similarities & Differences Cooperative Learning	B Critical Thinking Collaboration Communication

		complete, students should pass their papers to the next person in the circle. That person will check the student's work, coach the student if there are any errors, initial that the problem was checked and corrected and then proceed to complete #2 on the new worksheet. This process will continue until all 5 problems are complete. Sequences and Series PowerPoint Sequences and Series Guided Notes Sequences and Series Simultaneous RoundTable		

Unit 10: Resources

UNIT RESOURCES
Teacher Resources: ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Kuta Software*
Student Resources: ● *Khan Academy - Series and Sequences* ● *Khan Academy - Derivatives* ● *Khan Academy - Limits* ● *Precalculus with Limits, A Graphing Approach, Larson, 2005* ● *Trigonometry, Sullivan, 2009* ● *Cliff's Notes - Series and Sequences* ● *Cliff's Notes - Limits and Derivatives* ● *SparkNotes - Limits* ● *SparkNotes - Sequences and Series* ● *SparkNotes - Derivatives*

Vocabulary:

Limit - If *f(x)* becomes arbitrarily close to a unique number *L* as *x* approaches *c* from either side, the **limit** of *f(x)* as *x* approaches *c* is *L*. This is written as $\lim_{x \to c} f(x) = L$

One-sided limit - either of the two limits of a function f(x) of a real variable x as x approaches a specified point either from below or from above (from the left or right)

Limit to infinity - limits that have a value of ±∞, where the function grows without bound as it approaches some value a.

Indeterminate form - the form of a limit when the limit is not possible to determine because it is in the form of $\frac{0}{0}$.

Removable Discontinuity - also known as a hole in the graph

Nonremovable Discontinuity - a type of discontinuity that exists when there is jump discontinuity

Direct substitution when $\lim_{x \to c} f(x) = f(c)$

Dividing out technique is used when direct substitution produces 0 in both the numerator and denominator.

Rationalizing technique is used when direct substitution produces 0 in both numerator and denominator. Find the limit by rationalizing the function.

Power rule: $\frac{d}{dx} u^n = nu^{n-1}$

Product rule: $\frac{d}{dx}(fg) = f'g + fg'$

Quotient rule: $\frac{d}{dx}(\frac{f}{g}) = \frac{f'g - fg'}{g^2}$

Chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x))(g')$

Critical points - points on a function representing local extrema and points of inflection. Local extrema are found using first derivative and inflection points are found using second derivative.

Concavity - direction of the curvature of a graph (concave up or down)

Points of inflection - a point on a curve at which a change in the direction of curvature occurs.

Arithmetic series - $a + kd$

Geometric series - ar^k