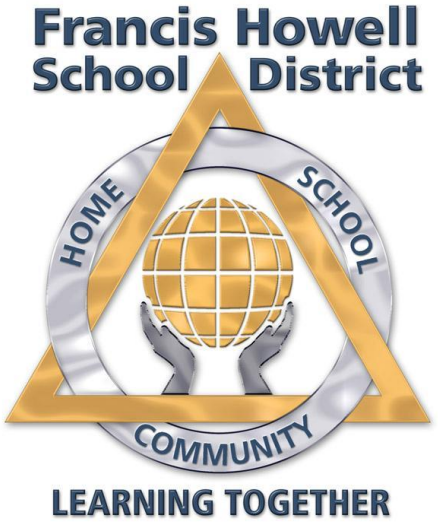
Algebra 2

Curriculum



Board Approved:

Francis Howell School District

Mission Statement

The mission of the Francis Howell School District is to prepare students today for success tomorrow.

Vision Statement

Every student will graduate with college and career readiness skills.

Values

Francis Howell School District is committed to:

- Providing a consistent and comprehensive education that fosters high levels of academic achievement
- Operating safe and well-maintained facilities
- Providing a safe learning environment for all students
- Promoting parent, community, student, and business involvement in support of the school district
- Ensuring fiscal responsibility
- Developing responsible citizens
- Operating as a professional learning community
- Making appropriate use of technology

Francis Howell School District Graduate Goals

Upon completion of their academic study in the Francis Howell School District, students will be able to:

1. Gather, analyze and apply information and ideas.
2. Communicate effectively within and beyond the classroom.
3. Recognize and solve problems.
4. Make decisions and act as responsible members of society.

Mathematics Graduate Goals

Upon completion of their Mathematics study in the Francis Howell School District, students will be able to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Course Rationale

In order to be effective citizens in the 21st century, students need to understand mathematics. Students often encounter problem situations that require reasoning, computation and communication. Students regularly study the most efficient methods for reaching solutions, but also realize that examining different solution methods help develop more flexible problem solving skills. The instruction and assessment is focused on instilling students with enduring understandings of mathematics. Algebra II improves students' ability to think analytically and is imperative for student success for our complex world. Algebraic reasoning is crucial for the development of problem solving, logical reasoning and technological skills. This development enhances opportunities for lifelong learning.

Course Description

This course is designed for the student who wishes to continue the study of mathematics beyond geometry and is essential for students planning to attend college. Investigation of real-world applications and trigonometry is incorporated throughout the course. Throughout this course, students are assessed on their ability to demonstrate the 8 Mathematical Practices.

CONTENT Curriculum Team

Curriculum Committee

Danna Tedder	Francis Howell Central
Christine Edwards	Francis Howell North
Jennifer Pointer	Francis Howell High

Mathematics Content Leader	Amy Ridling
Director of Student Learning	Dr. Chris Greiner
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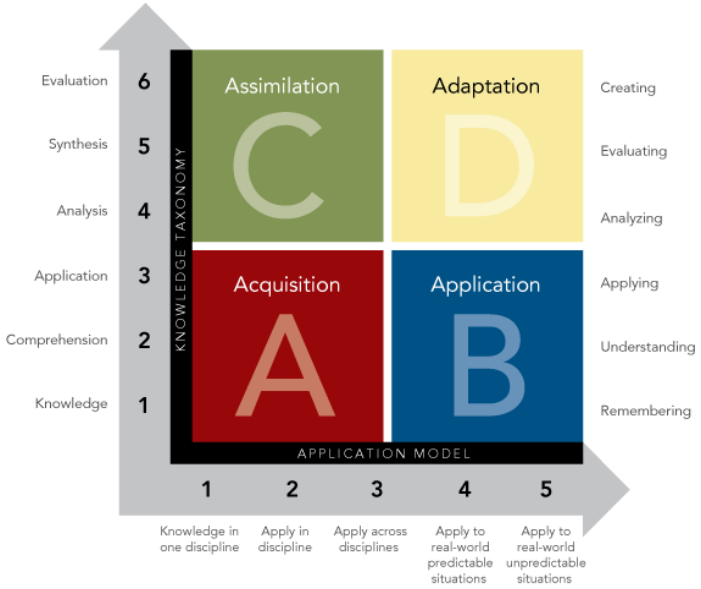
Curriculum Notes

All FHSD performance tasks and sample learning activities are aligned not only to understandings and standards, but also the Rigor and Relevance Framework and 21st Century Skills. Information on these two things is provided below or by clicking on the hyperlinks.

Rigor and Relevance Framework

The Rigor/Relevance Framework is a tool developed by the International Center to examine curriculum, instruction, and assessment along the two dimensions of higher standards and student achievement.



The Rigor/Relevance Framework has four quadrants.
Quadrant A represents simple recall and basic understanding of knowledge for its own sake. Examples of Quadrant A knowledge are knowing that the world is round and that Shakespeare wrote Hamlet.

Quadrant C represents more complex thinking but still knowledge for its own sake. Quadrant C embraces higher levels of knowledge, such as knowing how the U.S. political system works and analyzing the benefits and challenges of the cultural diversity of this nation versus other nations.

Quadrants B and D represent action or high degrees of application. Quadrant B would include knowing how to use math skills to make purchases and count change. The ability to access information in wide-area network systems and the ability to gather knowledge from a variety of sources to solve a complex problem in the workplace are types of Quadrant D knowledge.

21st Century Skills

A	B	C	D
Students gather and store bits of knowledge and information. Students are primarily expected to remember or understand this knowledge.	Students use acquired knowledge to solve problems, design solutions, and complete work. The highest level of application is to apply knowledge to new and unpredictable situations.	Students extend and refine their acquired knowledge to be able to use that knowledge automatically and routinely to analyze and solve problems and create solutions.	Students have the competence to think in complex ways.

These skills have been pared down from 18 skills to what are now called the 4Cs. The components include critical thinking, communication, collaboration, and creativity. Critical thinking is focused, careful analysis of something to better understand and includes skills such as arguing, classifying, comparing, and problem solving. Communication is the process of transferring a thought from one mind to others and receiving thoughts back and includes skills such as choosing a medium (and/or technology tool), speaking, listening, reading, writing, evaluating messages. Collaboration is working together with

others to achieve a common goal and includes skills such as delegating, goal setting, resolving conflicts, team building, decision-making, and managing time. Creativity is expansive, open-ended invention and discovery of possibilities and includes skills such as brainstorming, creating, designing, imagining, improvising, and problem-solving.

Standards

Standards aligned to this course can be found:

Revised Missouri Learning Standards
MO Department of Education 6-12 Mathematics

Common Core State Standards
Common Core State Mathematics Standards

National Educational Technology Standards
http://www.iste.org/STANDARDS

Units & Standards Overview
Semester 1 **Semester 2**

Unit 1: Linear Extensions	Unit 2: Quadratic Relationships	Unit 3: Polynomials	
A2.REI.B.3 A2.REI.A.1 F.IF.B.7 A2.IF.A.2 **A2.BF.A.3** ISTE 1c	**A2.IF.A.1** **A2.BF.A.3** **A2.FM.A.1** A2.NQ.B.5 A2.NQ.B.6 A2.IF.A.2	A2.APR.A.3 A2.APR.A.4 A2.APR.A.2 A2.APR.A.1 A2.IF.A.1 **A2.APR.A.5** A2.NQ.B.7 ISTE 1c ISTE 5b	
PE Assessment:Equations and Inequalities PE	PE Assessment: Quadratic Relationships PE	PE Assessment: Polynomials PE	
Unit 4: Rational and Radical Functions	**Unit 5: Exponential and Logarithmic Functions**	**Unit 6: Statistics**	**Unit 7: Trigonometry**
A2.REI.A.2 A2.BF.A.3 **A2.IF.A.1** A2.APR.A.4 A2.NQ.A.1 A2.NQ.A.2 A2.NQ.A.3 A2.NQ.A.4	A2.SSE.A.1 A2.SSE.A.2 **A2.SSE.A.3** A2.SSE.A.4 A2.BF.A.1 A2.BF.A.2 ISTE 5a A2.BF.A.3	A2.DS.A.1 **A2.DS.A.2** A2.DS.A.3 **A2.DS.A.4** **A2.DS.A.5** A2.DS.A.6 A2.DS.A.7 **A2.DS.B.8** **A2.DS.B.9** HSS.IC.B.5	**HSF-TF.A.1** **HSF-TF.A.2** **G-SRT.8** ISTE 1c
PE Assessment:Rational and Radical Functions PE	PE Assessment: Exponential and Logarithmic Functions PE	PE Assessment: Stats PE	PE Assessment: Trig PE

Course Map

	Unit Description	Unit Timeline	PE Summary	PE Standards
Semester 1 Unit 1: Linear Extensions	Students will extend their Algebra 1 understandings to model situations with linear equations, inequalities, and systems of linear equations and inequalities. Students will analyze if the solution is realistic or not.	4 weeks.	Students create and solve linear equations and inequalities given a real-world situation. Students create and solve systems of equations and inequalities to represent a real-world situation. Students graph, analyze, and relate characteristics of piecewise-defined functions to represent a situation.	A2.REI.A.1 **A2.REI.B.3** F.IF.7B A2.IF.A.2 **A2.BF.A.3**
Semester 1 Unit 2: Quadratic Relationships	Quadratic functions are used to model real life situations and data. Students will develop several ways to solve quadratic equations and graph quadratic functions. Students will describe characteristics of these functions in terms of real-life situations	6 weeks	Students will identify and interpret key characteristics of functions from a variety of representations. Students analyze a quadratic function to determine the best method to find a solution(s). Students describe the effects of transformations of quadratic functions and translate between equivalent forms of functions. Students create functions and apply them to model situations. Students represent and perform operations with complex numbers.Students solve systems of linear non-linear, quadratic-quadratic equations and inequalities.	***A2.IF.A.1*** ***A2.BF.A.3*** ***A2.FM.A.1*** *A2.NQ.B.5* *A2.NQ.B.6* *A2.IF.A.2* *A2.REI.B.3*
Semester 1 Unit 3: Polynomials	Students will be able to perform operations on polynomials, solve polynomials and analyze the graphs of polynomial functions.	5 weeks	Students find the least common multiple of two or more polynomials. Students understand the Remainder Theorem and use it to solve problems. Students extend the knowledge of factoring to include factors with complex coefficients. Students identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial. Students know and apply the Fundamental Theorem of Algebra.	A2.APR.A.3 A2. APR.A.2 A2.APR.A.1 **A2.APR.A.5** A2.NQ.B.7
Semester 2 Unit 4: Rational and Radical	Student will understand how to graph, analyze, and evaluate rational and radical expressions.	6 weeks	Students perform operations with rational expressions, create and solve rational and radical equations and inequalities, and graph, analyze and	A2.REI.A.2 A2.BF.A.3 A2.IF.A.2 **A2.IF.A.1**

Functions			relate characteristics of rational and radical functions to applicable situations.	A2.APR.A.4 A2.NQ.A.1 A2.NQ.A.3 A2.NQ.A.4
Semester 2 Unit 5: Exponential and Logarithmic Functions	Students define and investigate exponential and logarithmic functions. Students will explore these functions through graphing and solving equations and inequalities. Real life applications will include exponential growth and decay.	6 weeks	Students graph radical functions and state the domain and range, solve radical equations and note domain restrictions, describe the transformation of a rational function, graph and identify the key characteristics of a rational function, and solve word problems using rational equations.	**A2.IF.A.1** **A2.NQ.A.4** **A2.BF.A.3**
Semester 2 Unit 6: Statistics	Students will evaluate surveys, studies, and experiments. Create and use graphs of probability distributions. Use the Empirical Rule to find probabilities. Compare sample statistics and population statistics.	3 weeks	Students calculate the probability of a random variable occurring within a specified interval, distinguish between types of distributions, calculate margin of error and find confidence intervals, and calculate the mean and standard deviation from a sample of data.	**A2.DS.B.8** **A2.DS.A.4** **A2.DS.B.9**
Semester 2 Unit 7: Trigonometry	Students will explore the unit circle as it pertains to the coordinate plane, and use it to extend the domain of trigonometric functions to all real numbers.	2 weeks	Students will answer questions related to the unit circle and applied problems.	**HSF-TF.A.1** **HSF-TF.A.2** **G-SRT.8**

Unit 1: Linear Extensions

Content Area: Mathematics	**Course: Algebra 2**	**UNIT: Linear Extensions**
Unit Description: Students will explore how situations in their life can be modeled by linear equations, inequalities, and systems of linear equations and inequalities. This exploration will include graphing these equations and inequalities, as well as solving them for an exact or estimated value. Students will think about the meaning of each solution in terms of a specific real-world context and analyze if the solution is realistic or not.		**Unit Timeline:** 4 weeks

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Systems of equations are two or more equations with the same solution.
2. Systems of inequalities are two or more inequalities with a solution that satisfies both inequalities.
3. The three methods used to find solutions of systems are graphing, substitution and elimination.
4. Absolute value equations can be solve with multiple solutions.
5. Inequalities, compound inequalities and absolute value inequalities can be solved, and could present a set of solutions.
6. Piecewise defined functions are defined by multiple subfunctions, each subfunction applying to a certain interval of the main functions domain.
7. Transformations of functions compared to the parent function.

Essential Questions: ***Students will keep considering…***
How to create a system to model given situations?
How to determine the solution to a system?
What are the characteristics of the parent functions?
How do the graphs compare to the parent functions?
How to solve absolute value equations and inequalities?
How to precisely translate between equivalent forms of functions?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
Systems of equations are two or more equations with the same solution. Systems of inequalities are two or more inequalities with a solution that satisfies both inequalities. ***The three methods used to find these solutions are graphing, substitution and elimination.***	**A2.REI.B.3**	***Create and solve systems of equations that may include non-linear equations and inequalities. Create and system of equations/inequalities to model a given situation.*** **Construct a system of linear equations and inequalities graphically to find a solution. (generating tables of values)** **Precisely solve a linear system of equations algebraically (substitution/elimination).** Look for and make sense of the solution of a system of linear equations/inequalities. Explain constraints and validity of solutions of a system of equations/inequalities. Use technology or other appropriate tools strategically to represent and solve systems of equations/inequalities. Model systems of equations algebraically (include systems of 3 variables) and graphically and determine viability of solutions. Model systems of inequalities graphically and	**A2.REI.B.3**

		determine viability of solutions.	
Equations and inequalities, including those that involve absolute value.	A2.REI.A.1	*Create and solve equations and inequalities, including those that involve absolute value.* Create equations and inequalities to model situations with one and more than one variable. Make sense of problems and persevere to solve and graph equations and inequalities with one and more than one variable. Represent constraints by equations and inequalities. (Domain and Range) Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	A2.REI.A.1
Piecewise defined functions graphs and key features. **Piecewise-defined Function-** A function defined by multiple subfunctions, each subfunction applying to a certain interval of the main functions domain.	F.IF.B.7	Analyze and relate characteristics of piecewise-defined functions to applicable situations. Construct and graph piecewise-defined function including step functions and absolute value functions.	F.IF.B.7
Equivalent forms of functions.	A2.IF.A.2	*Translate between equivalent forms of functions.* Precisely translate between equivalent forms (point-slope to slope intercept and vice versa)	A2.IF.A.2
Transformations of linear and absolute value functions.	**A2.BF.A.3**	***Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for linear and absolute value functions.*** **Identify function transformations from an equation or graph. (include linear and absolute value)**	**A2.BF.A.3**

		Construct a function given a description of transformations and the parent function. (include linear and absolute value) **Compare and contrast similarities and differences among graphs. (include linear and absolute value)**	

Unit 1: Linear Extensions

EVIDENCE of LEARNING			
Understanding	**Standards**	**Unit Performance Assessment**: **Description of Assessment Performance Event:** Unit 1: Linear Extensions PE **Teacher will assess:**	**R/R Quadrant 21 Century**
1, 2, 3, 4, 5, 6, 7	**A.REI.B.3** A2.REI.A.1 F.IF.B.7 A2.IF.A.2 **A2.BF.A.3**	Can students create and solve systems of equations that may include non-linear equations and inequalities? Can students create and solve equations and inequalities, including those that involve absolute value? Can students graph, analyze and relate characteristics of piecewise-defined functions to applicable situations? **Performance:** **Mastery** Students will show that they really understand when they… 1. Complete the assessment with 75% or greater **Scoring Guide:** Unit 1: Linear Extensions Scoring Guide	B B C Critical Thinking

Unit 1: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1, 2, 3	**A2.REI.B.3**	**1. Lesson:** Match My Answer (substitution and elimination) **Objective:** Students will be able to solve systems of equations algebraically. **Activity:** In pairs, students will use the methods of substitution and elimination to solve the same system. Students A will use substitution, student B will use elimination then they will compare their answers and provide feedback to one another. Students will alternate methods throughout the remainder of the activity.	Cooperative Learning Feedback Practice	B Communication Collaboration
1, 2, 3	**A2.REI.B.3**	**2. Lesson**: 3 x 3 Systems (Substitution and Elimination) **Objective**: Students will be able to solve 3 x 3 systems of equations algebraically. **Activity**: In pairs, students will use the methods of substitution and elimination to solve the 3 x 3 system. Students will collaborate to determine which variable should be eliminated in the system provide.	Cooperative Learning Feedback Practice	B Communication Collaboration Critical Thinking
6	A2.IF.A.2 F.IF.7b	**3. Lesson**: Piecewise Function Activity **Objective**: Students will be able to relate and graph piecewise functions to real-world applications. **Activity**: In cooperative learning groups, students will use the cooperative learning strategy mix and match to sort and match piecewise functions, tables, and graphs to real-world situations. The teacher will use cues and questions to assist the students as needed. Some of the cues and questions could be: How is _____ related to _____? What are the parts or features of ___? Why do you think ___? What ideas justify ___? What is the function of ___? What evidence can you find? How can you identify the given parts? What conclusions can you draw?	Cooperative Learning Cues & Questions Identifying Similarities & Differences	C Communication Collaboration Critical Thinking

4,5	A2.REI.A.1	**4. Lesson**: Fixing Multi-Step Equation & Inequality Errors - Sage & Scribe **Objective**: Students will be able to critique the reasoning of others to identify errors in solving equation process. **Activity**: In pairs, students will use the cooperative learning structure Sage & Scribe to identify errors in the process of solving equations. Sage & Scribe requires one student to be the writer while the other student is the thinker. The thinker will describe the error identification to the writer and the writer will record what the thinker describes. Students provide each other with feedback throughout the activity through the tip-tip-teach model.	Cooperative Learning Feedback Reinforcing Effort Providing Recognition Cues & Questions	C Communication Collaboration Critical Thinking
7	**A2.BF.A.3** ISTE 1c	**5. Lesson**: Transformations Bundle - Desmos **Objective**: Graphs of functions are objects of geometry that you can move, stretch, and reflect to create new versions of the originals. Some graphs cannot be transformed into other graphs by moving, stretching, or reflecting. Function notation gives us handy tools for describing, combining, and understanding these transformations. **Activity**: This bundle assumes that students are familiar with function notation (e.g. that they have expressed a quadratic function in the form f(x)=x^2). Students need not have familiarity with function transformations; the activities in this bundle introduce these ideas. The last two activities in the bundle extend the basic understanding built in the beginning by using transformations to identify symmetry of functions, and then considering transformations of the plane.	Feedback Reinforcing Effort Providing Recognition Identifying Similarities & Differences Practice	C Communication Collaboration Critical Thinking

UNIT RESOURCES
Teacher Resources: *KHAN Academy - Systems of Equations* *KHAN Academy - Absolute Value* *KHAN Academy - Functions* *HOLT McDougall Algebra 2 2007* *DESMOS activity - Systems* *KUTA Worksheets*
Student Resources: *KHAN Academy - Systems of Equations* *KHAN Academy - Absolute Value* *KHAN Academy - Functions* *Cliff's Notes - Algebra 2*
Vocabulary: **Piecewise-defined Function**- A function defined by multiple subfunctions, each subfunction applying to a certain interval of the main functions domain.

Unit 2: Quadratics

Content Area: Mathematics	**Course: Algebra 2**	**UNIT: Quadratics**
Unit Description: Quadratic functions are used to model real life situations and data. Students will develop several ways to solve quadratic equations and graph quadratic functions. Students will describe characteristics of these functions in terms of real life situations.		**Unit Timeline:** 6 weeks

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Quadratic functions can be used to describe some relationships between quantities.
2. There are a variety of methods of solving quadratic equations.
3. Quadratic functions have certain key features that can be displayed by graphs and/or tables.
4. All quadratics have complex solutions.

Essential Questions: ***Students will keep considering…***

How do you identify the characteristics of a quadratic function and model the characteristics using algebraic symbols and graphical representation?

How do the constants a, h, and k affect the graph of a quadratic function $g(x) = a(x - h)^2 + k$?

How can you solve quadratic equations using a variety of methods?

What are complex numbers and how do you find their conjugate?

How can you perform operations with complex numbers?

How can you translate between multiple forms of a quadratic function?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
There are key characteristics of a quadratic relationships. **Vertex** - the point at which the axis of symmetry intersects a parabola **Minimum Value** - the y-coordinate of a vertex of the quadratic function $f(x) = ax^2 + bx + c$, where $a > 0$ **Maximum Value** - the y-coordinate of a vertex of the quadratic function $f(x) = ax^2 + bx + c$, where $a < 0$ **Zero** - the x-intercepts of the graph of a quadratic equation; the points for which $f(x) = 0$ **y-intercept** - the y-coordinate of the point at which a graph crosses the y-axis **axis of symmetry** - a line about which a parabola is symmetric **parabola** - the graph of a quadratic function is called a parabola, u-shaped **interval notation** - A notation for representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included. **domain** - the set of all x-coordinates of the ordered pairs of the function (interval notation) **range** - the set of all y-coordinates of the ordered pairs of the function (interval notation) **standard form** - a quadratic equation written in the form $ax^2 + bx + c = 0$, where a, b, and c are integers and $a \neq 0$ **vertex form** - a quadratic function in the form $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is it's axis of symmetry. **orientation** - the direction a parabola opens, if $a > 0$, opens up; if $a < 0$, opens down.	**A2.IF.A.1**	***Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.*** **Identify the orientation, axis of symmetry, vertex, y-intercept, zero(s), maximum/minimum values, domain and range from a graph, from a table, and algebraically.** **Look for and make sense of the key characteristics as they apply to a real-life application.** **Model the key characteristics using algebraic symbolism and graphical representation.** Compare and contrast how to identify key characteristics when given a quadratic in standard or vertex form.	**A2.IF.A.1**

Transformations **Parent function** - the simplest, most general function in a family of functions (quadratic parent function $y = x^2$) **Vertex form** - a quadratic function in the form $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and x = h is it's axis of symmetry.	**A2.BF.A.3**	***Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for quadratic functions.*** **Precisely describe the transformations from a graph or equation in vertex form.** **Construct a quadratic equation from a description of the transformations.**	**A2.BF.A.3**
Quadratic functions model real world situations. ***An quadratic equation can be derived from the roots.*** ***The discriminant can be used to determine the number and type of roots of a quadratic equation.*** ***Solving Applications of Quadratic Functions*** **Completing the Square** - a process used to make a quadratic expression into a perfect square trinomial **Quadratic Formula** - the formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ which gives the solution of a quadratic equation of the form $ax^2 + bx + c$, where $a \neq 0$ **Discriminant** - in the quadratic formula, the expression $b^2 - 4ac$ **Square Root Property** - for any real number n, if $x^2 = n$, then $x = \pm \sqrt{n}$	**A2.FM.A.1**	***Create functions and use them to solve applications of quadratic modeling problems.*** **Look for and make use of a variety of methods to solve a quadratic equation including, but not limited to factoring, completing the square, quadratic formula, square root property, and graphing.** Attend to precision when solving a quadratic equations. Construct viable arguments to justify steps in solving quadratic equations. Make sense of a real-life quadratic application to create and solve an quadratic equation. Model real-world situations using quadratic functions.	**A2.FM.A.1** ISTE.5a ISTE.5b
Complex Numbers imaginary unit (i) - the principal square root of -1 complex numbers - any number that can be written in the	A2.NQ.B.5	*Represent complex numbers.* Attend to precision when simplifying expressions involving complex numbers.	A2.NQ.B.5

form a + bi, where a and b are real numbers and i is the imaginary unit pure imaginary number - a complex number with no real part, ie) a + bi where a = 0 conjugates - binomials of the form a + bi and a - bi, where a and b are real numbers		Identify the conjugate of a complex number.	
Operations of Complex Numbers	A2.NQ.B.6	*Add, subtract, multiply and divide complex numbers.* Attend to precision when adding, subtracting, multiplying and dividing complex numbers. Construct a quadratic equation given complex solutions. Look for and make use of structure to solve for variables within equivalent forms of complex numbers. ie) a + bi = c + di, if and only if a = c and b = d	A2.NQ.B.6 ISTE.1c
Quadratic functions can be translated between standard and vertex form	A2.IF.A.2	*Translate between equivalent forms of functions.* Precisely convert a quadratic function in standard form to vertex form and vice versa. Find equivalent forms of functions to highlight key characteristics.	A2.IF.A.2

Unit 2: Assessment

EVIDENCE of LEARNING			
Understanding	Standards	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Unit 2: Quadratics PE **Teacher will assess:**	**R/R Quadrant 21 Century**
1, 2, 3, 4	**A2.IF.A.1**	***Can students identify and interpret key characteristics of quadratic functions represented graphically, with tables and algebraic symbolism to solve problems?***	**C**
	A2.BF.A.3	***Can students analyze a quadratic function to determine the best method to find the solution(s)?***	**C**
	A2.FM.A.1	***Can students describe the effects of transformations algebraically and graphically, create vertical and horizontal translations, vertical and horizontal reflections and dilations of quadratic functions?***	**B**
		Can students create quadratic functions and use them to solve applications of quadratic modeling problems?	**C**
	A2.NQ.B.5	*Can students represent complex numbers?*	A
		Can students add, subtract, multiply and divide complex numbers?	B
	A2.NQ.B.6	*Can students translate between equivalent forms of functions?*	B
	A2.IF.A.2	**Performance:** **Mastery:** *Students will show that they really understand when they…* 1. Complete the assessment with 75% or greater **Scoring Guide:** Unit 2: Quadratics PE Scoring Guide	Critical Thinking

Unit 2: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1	**A2.BF.A.3** **A2.IF.A.1**	**1. Lesson:** Transformations **Objective:** Students will be able to describe the transformations from a graph or equation in vertex form and identify axis of symmetry and vertex with precision. **Activity:** In pairs, students will receive 4 sheets of color-coded task cards that include graphs, equations, axis of symmetry, and the vertex of quadratic functions.. Students will discuss similarities and differences to match the four appropriate function cards together. Students will justify their reasoning based on the different representations of the quadratic relationships modeled in different representations. Task Cards: quad eq-vertex-aos-graph activity	Cooperative Learning Identifying Similarities & Differences	B Communication Collaboration
4	A2.NQ.B.5 **A2.IF.A.1**	**2. Lesson:** Complex Numbers **Objective:** Attend to precision when adding, subtracting, multiplying and dividing complex numbers. **Activity:** In pairs, students will receive a set of puzzle pieces. Have students work together to match equivalent expressions to form a new square. Students should be in agreement on placement of each piece using justification as needed for their decision. Complex Number Puzzle	Cooperative Learning Assigning Practice	B Communication Critical Thinking
1, 2, 3	**A2.FM.A.1** ISTE.5a ISTE.5b	**3. Lesson:** Create Quadratic Functions **Objective:** Students will be able to find the angle of the ramp that maximizes the (horizontal) length of the jump. **Activity:** This activity begins by presenting a scenario in which a motorcycle rider jumps off a ramp and travels along a quadratic path through the air. In	Problem-based	B Technology Critical Thinking

		Problem 1, students use a graphical model to explore the effect of setting the ramp at different angles to discover that the relationship between the angle of the ramp and the horizontal distance of the jump can also be described by a quadratic function. Students use this function to find the angle that maximizes the horizontal distance of the jump. In Problem 2, students relate the angle of the ramp and the airtime of the jump, and then they use a similar process to discover that the airtime of the jump increases without bound as the angle of the ramp approaches 90°. Finally, they use their results to make recommendations for the rider. Download this zip file:Motorcycle Jump Activity		
1, 3	**A2.IF.A.1**	**4. Lesson:** Identifying Key Characteristics **Objective:** Students will be able to identify the orientation, axis of symmetry, vertex, y-intercept, zero(s), maximum/minimum values, domain and range from a graph, from a table, and algebraically. **Activity:** Pass out a card to each student. Answers may be added to the backside after printing them off. Students will take their card, a pencil, and some scrap paper and they will get up and move about the room. Every student will find one other student to match with (if there is an odd number the teacher could participate, or you can instruct the one student out to wait for an available partner.) Essentially students will find a partner, and they will take turns reading the questions on the card to the other (in this case finding characteristics of a quadratic function). The student asking has the answers on the back facing them, and so can correct and coach the student answering as needed. Once both have had a chance to ask, they trade cards (hence the activity name) and go find a new partner to quiz. This is a great cooperative learning activity that gets students up and moving in the middle of a long lesson (which is very important). It also allows students to coach peers and review material in a way that is safe and comfortable, without the pressure of other assessment techniques. Teachers can circulate and see how students are doing, take a whole-class formative approach after such as thumbs up, OR an exit slip to see how students feel about their ability to answer the questions. Found at http://mrwannerz.weebly.com/teaching-blog/quiz-quiz-quadratics-trade-kagan-activity	Feedback Cooperative Learning	A Collaboration Communication

4	A2.NQ.B.6 ISTE.1c	**5. Lesson:** Operations on Complex Numbers **Objective:** Students will attend to precision when adding, subtracting, multiplying and dividing complex numbers. **Activity:** Kahoot! is a game-based classroom response system. Games are displayed on a shared screen – for example a smart TV, a laptop or an interactive whiteboard. Players join in using their own device – whether that is a smartphone, iPad, laptop, or desktop doesn't matter, as long as they have a browser and good internet connection. Players do NOT need a kahoot account to play. Operations on Complex Numbers Kahoot	Feedback	B Technology
1, 2	A2.FM.A.1	**6. Lesson:** Solve quadratic applications **Objective:** Students will make sense of a real-life quadratic application to create and solve an quadratic equation. **Activity:** Students work together in pairs to complete the Quadratics Application Activity. This activity requires student to extend their understanding of quadratics and use critical thinking to make the appropriate changed to the equation based on the real-life problem. Students should discuss and be able to justify the changes made to the equation and the process used to solve the problem. A graphing calculator may be used as a resource, if needed.	Cooperative Learning Problem-Based	C Critical Thinking Collaboration
1, 2, 3, 4	A2.IF.A.2	**7. Lesson:** 3 Forms of a Quadratic Equation **Objective:** Students will be able to form generalizations between the three forms of a quadratic function: general, vertex, and factored. **Activity:** Through an inquiry format, the student will use a graphing calculator to aid them in writing equations and switching between the different forms. Students work together in pairs or individually to complete the 3 Forms of a Quadratic Equation Activity. This activity requires student to extend their understanding of quadratics and use critical thinking to make the appropriate changed to the equation based on the real-life problem. There are questions to guide the students through the 3 parts of the activity. Students should discuss and be able to justify the changes made to the equation and the process used to solve the problem. A graphing calculator will be used as a resource.	Identifying Similarities & Differences Problem Based Feedback Providing Practice	C Critical Thinking Collaboration Technology

1,2,3	**A2.IF.A.1** **A2.BF.A.3** **A2.FM.A.1** ISTE 5.c	**8. Lesson:** Weightless Wonder PBL **Objective:** Students will use this PBL exploring parabolic flights of NASA's Weightless Wonder jets to solve quadratic equations and evaluate and graph quadratic functions. Students will find the maximum, y-intercept, x-intercepts and interpret their significance. Students will determine the effects of parameter changes on the graph of a quadratic equation. **Activity:** Weightless Wonder PBL This PBL contains a background article and video to set up the problem. Students should be arranged in teams of 3-4 to work through the stages of the problem and a series of worksheets for each stage. At the end of each stage, the teacher should lead out a class discussion comparing similarities and differences as well as the solutions. All of the worksheets with solutions are contained at the aforementioned website "Weightless Wonder PBL."	Identifying Similarities & Differences Problem-Based Feedback	C Critical Thinking Collaboration Communication Technology

Unit 2: Resources

UNIT RESOURCES
Teacher Resources: *KHAN Academy - Quadratics* *HOLT McDougall Algebra 2 2007* *DESMOS activity - Quadratics* *KUTA Worksheets*
Student Resources: *KHAN Academy - Quadratics* *HOLT McDougall Algebra 2 2007* *Cliff's Notes - Algebra 2*
Vocabulary: **Vertex** - the point at which the axis of symmetry intersects a parabola **Minimum Value** - the y-coordinate of a vertex of the quadratic function $f(x) = ax^2 + bx + c$, where $a > 0$ **Maximum Value** - the y-coordinate of a vertex of the quadratic function $f(x) = ax^2 + bx + c$, where $a < 0$ **Zero** - the x-intercepts of the graph of a quadratic equation; the points for which $f(x) = 0$ **Y-intercept** - the y-coordinate of the point at which a graph crosses the y-axis **Axis of symmetry** - a line about which a parabola is symmetric

Parabola - the graph of a quadratic function is called a parabola, u-shaped
Interval Notation - A notation for representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parenthese and/or brackets are used to show whether the endpoints or excluded or included.
Domain - the set of all x-coordinates of the ordered pairs of the function (interval notation)
Range - the set of all y-coordinates of the ordered pairs of the function (interval notation)
Standard form - a quadratic equation written in the form $ax^2 + bx + c = 0$, where a, b, and c are integers and $a \neq 0$
Vertex form - a quadratic function in the form $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and x = h is it's axis of symmetry.
Orientation - the direction a parabola opens, if $a > 0$, opens up; if $a < 0$, opens down
Parent function - the simplest, most general function in a family of functions (quadratic parent function $y = x^2$)
Vertex form - a quadratic function in the form $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and x = h is it's axis of symmetry.
Completing the Square - a process used to make a quadratic expression into a perfect square trinomial
Quadratic Formula - the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ which gives the solution of a quadratic equation of the form $ax^2 + bx + c$, where $a \neq 0$
Discriminant - in the quadratic formula, the expression $b^2 - 4ac$
Square Root Property - for any real number n, if $x^2 = n$, then $x = \pm \sqrt{n}$
Imaginary unit (i) - the principal square root of -1
Complex numbers - any number that can be written in the form a + bi, where a and b are real numbers and i is the imaginary unit
Pure imaginary number - a complex number with no real part, ie) a + bi where a = 0
Conjugates - binomials of the form a + bi and a - bi, where a and b are real numbers
Quadratic inequality - *a quadratic equation in the form* $y > ax^2 + bx + c$ *(including* $<, \leq, \geq$*)*

Unit 3: Polynomials

Content Area: Mathematics	Course: Algebra 2	UNIT: Polynomials
Unit Description: Students will be able to perform operations on polynomials, solve polynomials and analyze the graphs of polynomial functions.		**Unit Timeline:** 5 weeks

DESIRED Results
Transfer Goal - ***Students will be able to independently use their learning to……*** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. A polynomial function has distinguishing “behaviors”. You can look at its algebraic form and know something about its graph. You can look at its graph and know something about its algebraic form.
2. Knowing the zeros of a polynomial functions can help you understand the behavior of its graph.
3. If (x-a) is a factor of a polynomials, then the polynomial has value 0 when x=a. If a is a real number, then the graph of the polynomial has (a,0) as an x-intercept.
4. You can divide polynomials using steps that are similar to the long division steps that you use to divide whole numbers.
5. The degree of the polynomial will determine the shape of its graph, the maximum number of turning points, its end behavior, and the number of roots (including multiple and complex roots) so that real world data can be analyzed in terms of its maximum, minimum, and break-even values.
6. Polynomials can be added, subtracted, multiplied and divided.
7. A variety of methods can be used to factor and find all the zeros of a polynomial function, which include real and imaginary zeros.

Essential Questions: ***Students will keep considering…***
How can you tell when/if a polynomial expression can be simplified?
How can a polynomial be expressed graphically and what does each part of the graph represent?
How are all the different representations of a polynomial function related?
Why do polynomials have special rules for operations?
What is the best way to solve a polynomial equation?
Why do we factor polynomials?
What do complex numbers mean as solutions of polynomials?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
Polynomials can be added, subtracted, multiplied and divided. *Factoring techniques and the Zero Product Property are used to solve polynomial equations.*	A2.APR.A.3	*Find the least common multiple of two or more polynomials.* Perform operations on polynomials, including addition, subtraction, multiplication, and factoring. Solve polynomial equations using factoring techniques and the Zero Product Property.	A2.APR.A.3
Polynomials can be divided by polynomials Synthetic Division - a shorthand method of dividing a polynomial by a linear binomial by using only the coefficients.	A2.APR.A.4	*Add, subtract, multiply and divide rational expressions.* Use long division and synthetic division to divide polynomials.	A2.APR.A.4
The Rational Root Theorem may be used to find all zeros of a polynomial function Rational Root Theorem - a method of generating possible zeros of a polynomial function. Multiplicity - the multiplicity of root *r* is the number of times that *x - r* is a factor of *P(x)*.	A2.APR.A.2	*Understand the Remainder Theorem and use it to solve problems.* Solve polynomial equations using factoring techniques and the Zero Product Property and polynomial division. Use factoring techniques to solve general polynomial equations, which could include complex solutions. Extend operations on polynomial expressions to	A2.APR.A.2

		include long division of a polynomial of degree 2 or higher by a binomial. Express the result as a quotient with a remainder. Understand the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division of *p(x)* by *(x-a)* is *p(a)*, so *p(a) = 0* if and only if *(x-a)* is a factor of *p(x).*	
Some factors of polynomials will have complex coefficients	A2.APR.A.1	*Extend the knowledge of factoring to include factors with complex coefficients.*	A2.APR.A.1
Polynomials have reasonable domain and range *Polynomials have local maximum and/or minimums and turning points.* Polynomial Function- a function whose rule is a polynomial. Turning point - where a graph changes from increasing to decreasing or from decreasing to increasing. Local Maximum -a point on the graph of a polynomial where no other nearby points have a greater *y* coordinate. Local Minimum - a point on the graph of a polynomial where no other nearby points have a greater *y* coordinate	A2.IF.A.1	*Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.* Identify the domain and range of a function. Find the local maximum and minimums and turning points.	A2.IF.A.1
Graphs of polynomial functions have key characteristics which include end behavior, zeros, maximums and minimums ***Key characteristics can be used to sketch a graph of a polynomial function*** **Extrema** - The maximum and minimum values of a	**A2.APR.A.5**	***Identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function defined by the polynomial.*** **Analyze graphs of polynomial functions using degree, end behavior, and zeros for odd/even functions.**	**A2.APR.A.5**

function. **Relative Maximum** - A point on the graph of a function where no other nearby points have a greater y - coordinate. **Relative Minimum** - A point on the graph of a function where no other nearby points have a lesser y - coordinate. **Turning Points** - Point at which a graph turns - the location of the relative maxima and minimum **End Behavior** - a description of the values of a function as *x* approaches positive infinity or negative infinity.		**Sketch the graphs of polynomial functions using end behavior, zeros, local maximum, and minimum.**	
Polynomial functions have roots and zeros and may be found through a variety of methods Fundamental Theorem of Algebra - Every polynomial function of degree n > 1 has at least one zero, where zero may be a complex number. Remainder Theorem - if a polynomial *p(x)* is divided by *x -r*, the remainder is a constant *p(r)*. Synthetic Substitution - the use of synthetic division to evaluate a function. Depressed Polynomial - the quotient when a polynomial is divided by one of its binomial factors. Descartes Rule of Signs **-** a method of determining the number of positive, negative,and imaginary zeros. Factor Theorem - the binomial *x - r* is a factor of the polynomial *p(x)* if and only if *p(r)* = 0 Complex Conjugate Theorem - if one factor of the form, *a + bi* is a zero, then *a - bi* is also a zero.	A2.NQ.B.7	*Know and apply the Fundamental Theorem of Algebra.* Use the Rational Root Theorem and Factor Theorem to find all the roots of a polynomial function. Find roots and zeros of a polynomial function and write equations, including complex and real roots and the Fundamental Theorem of Algebra. Apply the Remainder Theorem to evaluate functions. Apply and use Descartes Rules of Signs to determine the number of possible positive, negative and imaginary zeros.	A2.NQ.B.7

Unit 3: Assessment

EVIDENCE of LEARNING			
Understanding	Standards	**Unit Performance Assessment**: **Description of Assessment Performance Event:** Unit 3: Polynomials PE	**R/R Quadrant**
2, 3, 4, 6, 7	A2.APR.A.3 A.APR.A.4 A2.APR.A.2 A2.IF.A.1 **A2.APR.A.5** A2.NQ.B.7	**Teacher will assess:** *Can students find the least common multiple of two or more polynomials?* *Can students perform operations on polynomials, including addition, subtraction, multiplication, division, and factoring?* *Can students extend their knowledge of factoring to include factors with complex coefficients?* *Do students know and can they apply the Fundamental Theorem of Algebra?* *Can students understand the Remainder Theorem and use it to solve problems?* **Can students identify zeros of polynomials when suitable factorizations are available, and use the zeros to sketch the function?** **Performance:** **Mastery:** *Students will show that they really understand when they…* 1. Complete the assessment with 75% or greater. **Scoring Guide:** *Unit 3: Polynomials Scoring Guide*	B B C B C B

Unit 3: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
3	A2.NQ.B.7	**1. Lesson:** Finding Rational Roots **Objective:** Students will be able to find all the roots of a polynomial function. **Activity**: In groups of 4, students will complete the cooperative learning structure Simultaneous Round table using Rational Roots Roundtable. Students will work the problem in Box 1 on their paper. As they finish the first problem, each student passes the paper to the left. Each student then checks the problem that was just passed to them and puts their initials on the given line. Each student then works the problem in Box 2 and when finished passes the paper to the left. Working and passing continues in this fashion until all problems have been completed and checked.	Cooperative Learning	C Collaboration Critical Thinking
7	A2.APR.A.3 A2.APR.A.2	**2. Lesson:** Classify Factoring Methods **Objective:** Students will be able to determine the most efficient method of factoring given polynomials. **Activity**: Students will work independently to complete the Classifying Factoring. Students will classify each polynomial by the most efficient method to factor using a column graphic organizer. After completing the organizer, students will then compare their classifications with a partner. Each pair of students will discuss and reach consensus on each problem upon which they disagreed. After students have shared, the teacher will then ask students to identify the similarities and differences between the various polynomials in each column. After classifying each problem, students will factor each problem using the appropriate method as independent practice.	Cooperative Learning Independent Practice Non-linguistic Representations Similarities and Differences	B Critical Thinking Collaboration

1, 5	**A2.APR.A.5** ISTE 5b	**3. Lesson:** Discovering polynomial functions **Objective:** Students will be able to explain how graphs behave in relation to their equations. **Activity:** Students will follow instructions on the Graphs of polynomial functions discovery worksheet graph polynomial functions using their graphing calculators and answer questions relating to each equation and graph. This activity guides students through the discovery of how graphs behave in relation to key features in their equations.	Identifying Similarities and Differences Cues and Questions Non-Linguistic Representations	C Critical Thinking
3, 7	A2.NQ.B.7 ISTE 1c	**4. Lesson**: The Remainder and Factor Theorems **Objective:** Students will be able to use the factor and remainder theorem to solve problems. **Activity:** Using Kahoot Remainder and Factor Theorem students will play Kahoot! Kahoot! is a game-based classroom response system. Games are displayed on a shared screen – for example a smart TV, a laptop or an interactive whiteboard. Players join in using their own device – whether that is a smartphone, iPad, laptop, or desktop, as long as they have a browser and good internet connection. Players do NOT need a kahoot account to play. Players may work with a partner. Students will record their work on their own piece of paper. After each problem, the teacher will review if necessary.	Providing practice	A Technology
3, 6	A2.APR.A.3	**5. Lesson:** Fixing Polynomial and Exponential Errors **Objective:** Students will be able to analyze problems, find the error, explain why the error occurred and then simplify correctly. **Activity:** In pairs, students will use the cooperative learning structure Sage & Scribe to identify errors in the process of solving equations. Students will use Error Identification Sage & Scribe to complete the activity. Sage & Scribe requires one student to be the writer while the other is the thinker. The thinker will describe the error identification to the writer and the writer will record what the thinker describes. Students provide each other with feedback throughout the activity through the tip-tip-teach model.	Cooperative Learning	C Communication Collaboration Critical Thinking

1, 2, 5	A2.IF.A.1	**6. Lesson:** Analyzing Graphs of Polynomial Functions **Objective:** Students will be able to analyze graphs of polynomial functions to make generalizations about key-features. **Activity:** Students will complete the cooperative learning structure Quiz-Quiz-Trade to analyze graphs. Students will use the Quiz-Quiz Trade Polynomial Graphs to complete the activity. Students will begin with a problem card and a partner. Students will quiz each other, stating the correct answer and providing feedback. After quizzing each other, students will trade cards. On the teacher's signal, students will walk around the room and choose a different partner when teacher says "stop". Again students will quiz each other and continue for several rounds in this fashion.	Cooperative Learning Feedback	A Collaboration Communication Critical Thinking
4, 6	A2. APR.A.4	**7. Lesson:** Dividing Polynomials **Objective:** Students will be able to divide polynomials using both long and synthetic division to assist them in finding all the zeros of a function. **Activity:** Students will complete the cooperative learning structure of match my answer to practice using both long and synthetic division to determine whether a given factor is a zero of a polynomial function. Each student will be given an A or B half of the Polynomial Long and Synthetic Division worksheet. The teacher may decide how to pair students, but each pair should contain an A and B worksheet with which to compare their answers. Students will work on each problem simultaneously, one student doing long division and the other synthetic division. When the problem is complete, they should compare their answer to their partner's. If the answers match, they can continue on to the next problem. If the answers don't match, then they should review each other's work to find and correct any errors made. Students will repeat this process on each question until the worksheets are complete. The teacher should have the answer key available and observe students working to provide any necessary prompts to guide student work.	Providing Practice Cooperative Learning	A Collaboration Communication
4, 6, 7	A2.APR.A.3 A2.APR.A.4 A2.APR.A.2	**8. Lesson:** Polynomial Farm **Objective:** Students will be able to add, subtract, multiply, and divide polynomials. Students will be able to factor completely first- and second degree binomials and trinomials in one variable.	Problem-Based Cooperative Learning	C/D Critical Thinking

		Activity: Students will investigate the relationship between adding, subtracting, multiplying, dividing, and factoring polynomials in real-world scenarios. In this activity, students are asked to help a farmer calculate the perimeter and area of his produce fields. Place students in pairs and give them the Polynomial Farm worksheet to complete together. The teacher should monitor their work and offer cues and questions, when necessary. Upon completion of the task, students will participate in self-reflection and a class-wide discussion. Students may be asked what challenges they faced, what problem-solving skills they used and developed, and what they enjoyed most about the task. The teacher may also ask what other real-world scenarios students can think of that may require these skills. Polynomial Farm PBL Activity	Nonlinguistic Representation	Collaboration Communication

UNIT RESOURCES

Teacher Resources:

KHAN Academy - Polynomials
HOLT McDougall Algebra 2 2007
DESMOS activity - Polynomials
KUTA Worksheets

Student Resources:

KHAN Academy - Polynomials
Cliff's Notes - Algebra 2

Vocabulary:

Cubic Function - Degree 3
Quartic Function - Degree 4
Quintic Function - Degree 5
Synthetic Division - a shorthand method of dividing a polynomial by a linear binomial by using only the coefficients.
Rational Root Theorem - a method of generating possible zeros of a polynomial function.
Multiplicity - the multiplicity of root *r* is the number of times that *x - r* is a factor of *P(x)*.
Polynomial Function- a function whose rule is a polynomial.
Turning point - where a graph changes from increasing to decreasing or from decreasing to increasing.
Location Principle - the function has at least one real zero between *a* and *b*.
Extrema - The maximum and minimum values of a function.
Relative Maximum - A point on the graph of a function where no other nearby points have a greater y - coordinate.
Relative Minimum - A point on the graph of a function where no other nearby points have a lesser y - coordinate.
Turning Points - Point at which a graph turns - the location of the relative maxima and minimum.
Fundamental Theorem of Algebra - Every polynomial function of degree n > 1 has at least one zero, where zero may be a complex number.
Remainder Theorem - if a polynomial *p(x)* is divided by *x -r*, the remainder is a constant *p(r)*.
Synthetic Substitution - the use of synthetic division to evaluate a function.
Depressed Polynomial - the quotient when a polynomial is divided by one of its binomial factors.
Descartes Rule of Signs - a method of determining the number of positive, negative,and imaginary zeros.
Factor Theorem - the binomial *x - r* is a factor of the polynomial *p(x)* if and only if *p(r)* = 0
Complex Conjugate Theorem - if one factor of the form, *a + bi* is a zero, then *a - bi* is also a zero.

Unit 4: Rational and Radical Functions

Content Area: Mathematics	**Course: Algebra 2**	**UNIT: Rational and Radical Functions**
Unit Description: Student will understand how to graph, analyze, and evaluate rational and radical expressions.		**Unit Timeline:** 6 weeks

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Basic properties and operations with fractions must be used to simplify, multiply, divide, add and subtract rational expressions.
2. When solving an equation involving rational expressions, multiplying by the common denominator can result in extraneous solutions.
3. Real world problems that involve work can often be solved using rational equations.
4. A rational function is a ratio of polynomial functions. If the simplified form of the function has a non-constant denominator, the resulting graph features asymptotic behavior which can be transformed from the parent function.
5. Radical expressions can be written and simplified in an equivalent form using rational exponents.
6. Radical equations can be solved by isolating the radical and squaring both sides of the equation. This process may introduce extraneous solutions.
7. The same techniques used to transform the graphs other functions can be applied to the graphs of square root functions.

Essential Questions: ***Students will keep considering…***

Why is it important to simplify rational expressions? Are a rational expression and its simplified form equivalent?
How can radicals be simplified and combined?

How can real number operations be extended to radical expressions and equations?
How do you use transformations to help graph functions?
What kinds of asymptotes are possible for a rational function?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
Rational Functions can be added, subtracted, multiplied and divided *Rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.* **Rational functions:** A quotient of two polynomials **Rational expression:** A fraction whose numerators and denominators are nonzero polynomials **Simplified form** of a rational expression: When its numerator and denominator have no common factors other than +/- 1 **Complex Fraction:** A fraction that contains a fraction in the numerator or denominator	A2.APR.A.4	*Add, subtract, multiply and divide rational expressions.* Precisely simplify rational expressions. Look for and make sense of the excluded values in the product or quotient of multiple rational expressions. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x)+r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials, and the degree of r(x) less than the degree of b(x), using long division or other methods. Attend to precision and simplify complex fractions. Make sense of problems and persevere to infer the domain of the sum or difference of two rational expressions.	A2.APR.A.4
Rational Functions can be graphed to show key features *Key features of a rational function may include: vertical asymptotes, x and y intercepts, horizontal asymptotes, end behavior, relative maxima/minima, intervals of increase/decrease.* ***Square root functions can be graphed to show key features***	**A2.IF.A.1**	***Identify and interpret key characteristics of functions represented graphically, with tables and with algebraic symbolism to solve problems.*** Precisely identify asymptotes, intercepts, maximum and minimum values, intervals of increase and decrease, end behavior, and points of discontinuity of rational functions. Construct and graph simple and complex rational functions using key features above.	A2.IF.A.1

Square root inequalities can be graphed to show key features ***Continuity:*** *Able to graph a function without picking up pencil.* ***Asymptotes:*** *A line that continually approaches a given curve but does not meet it at any finite distance.*		Use technology or other appropriate tools strategically to represent rational and square root functions. **Graph Square root functions and inequalities** **Identify and interpret key characteristics of square root functions and inequalities represented graphically, with tables and with algebraic symbolism to solve problems.**	
Rational functions can be transformed by changing the parameters of the equation.	A2.BF.A.3	*Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for cubic, square and cube root functions.* Look for and make sense of simple translations with rational functions by changing parameters.	A2. BF.A.3
Rational equations can be solved to produce no solution, 1 solution, multiple solutions and /or extraneous solutions. *Rational inequalities can be solved to produce an interval of possible solutions.* ***Rational Equation****: Equation that contains one or more rational expressions.* ***Extraneous solution****: a solution of an equation derived from an original equation that is not a solution of the original equation.* ***Rational Inequality****: An inequality that contains one or more rational expression.*	A2.REI.A.2	*Solve rational equations where numerators and denominators are polynomials and where extraneous solutions may result.* Solve rational and radical equations and check for extraneous solutions. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	A2.REI.A.2
Finding the square root of a number and squaring a number are inverse operations. To find the square root of a number x, you must find a number with a square of	A2.NQ.A.1	*Extend the system of powers and roots to include rational exponents.*	A2.NQ.A.1

x. Similarly, the inverse of raising a number to the nth power is finding the nth root of a number. *Radical expressions may be rewritten using rational exponents:* **nth roots:** an **nth root** of a number x, where n is a positive integer, is any of the n real or complex numbers r whose **nth** power is x		Rewrite radical expressions by using rational exponents.	
The different parts of an expression, such as terms, factors, and coefficients.	A2.NQ.A.2	*Create and recognize equivalent expressions involving radical and exponential forms of expressions.* Interpret parts of an expression, such as terms, factors, and coefficients. Simplify and evaluate radical expressions and expressions containing rational exponents.	A2.NQ.A.2
Radical Expressions can be added, subtracted, multiplied and divided.	A2.NQ.A.3	*Add, subtract, multiply and divide radical expressions.* Perform operations on radical and rational expressions including complex fractions and fractional exponents. Use the structure of an expression to identify ways to rewrite it. i.e. $x^4-y^4=(x^2)^2-(y^2)^2$	A2.NQ.A.3
Radical Equations can be solved. *Radical Inequalities can be solved.*	A2.NQ.A.4	*Solve equations involving rational exponents and/or radicals, and identify situations where extraneous solutions may result.* Evaluate expressions with basic radicals. Make sense of problems and persevere to solve equations with extraneous solutions.	A2.NQ.A.4

		Create radical and rational equations to model situations. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	

Unit 4: Assessment

EVIDENCE of LEARNING			
Understanding	**Standards**	**Unit Performance Assessment**: **Description of Assessment Performance Event:** Unit 5: Rational and Radical Functions PE **Teacher will assess:**	**R/R Quadrant**
1, 2, 3, 4, 5, 6, 7	A2.REI.A.2 A2.BF.A.3 **A2.IF.A.1** A2.APR.A.4 A2.NQ.A.1 A2.NQ.A.3 A2.NQ.A.4	*Can students perform operations with rational expressions?* *Can students create and solve rational and radical equations and inequalities?* *Can students graph, analyze and relate characteristics of rational and radical functions to applicable situations?* **Performance:** **Mastery** Students will show that they really understand when they… 1. Complete the assessment with 75% or greater **Scoring Guide:** Unit 5: Rational and Radical Functions Scoring Guide	B C B

Unit 4: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1, 2, 4, 6	A2.APR.A.4	**Lesson:** Match My Answer (Rationals) **Objective:** Students will be able to perform operations with rational expressions. **Activity:** In pairs, students will use the methods of substitution and elimination to solve the same system. Students A will use substitution, student B will use elimination then they will compare their answers and provide feedback to one another. Students will alternate methods throughout the remainder of the activity.	Cooperative Learning Feedback Reinforcing Effort	C Communication Collaboration
1, 2, 3, 4, 5, 6	A2.REI.A.2 **A2.IF.A.1**	**Lesson:** Light It Up (Rational Functions) **Objective:** Students will be able to state the domain, range and end behavior of rational functions. Write rational functions that model problem situations. Use rational functions to solve problems **Activity:** In this activity, students are presented with a real-world problem: Given a mirror and laser pointer, determine the position where one should stand so that a reflected light image will hit a designated target. This investigation allows students to develop several rational functions that models three specific forms of a rational function. Students explore the relationship between the graph, the equation, and problem context.	Feedback Reinforcing Effort Generating & Testing Hypothesis	C Communication Collaboration
1, 2, 3, 4, 5, 6, 7	A2.REI.A.2 A2.BF.A.3 **A2.IF.A.1** A2.APR.A.4 A2.NQ.A.1 A2.NQ.A.3 A2.NQ.A.4	**Lesson:** QR Review (Rational and Radical Functions) **Objective:** Students will be able to state the domain, range and end behavior of rational functions. Write rational functions that model problem situations. Use the knowledge from the unit. **Activity:** In this cooperative learning activity, students are presented with a real-world problem: They need to use the knowledge learned from the unit and apply it. The can get a quick answer by scanning the qr code.	Cooperative Learning Feedback Reinforcing Effort	C Communication Collaboration

UNIT RESOURCES
Teacher Resources: *KHAN Academy - Radical Equations and Functions* *KHAN Academy - Rational Equations and Functions* *HOLT McDougall Algebra 2 2007* *DESMOS activity - Rational Functions* *KUTA Worksheets*
Student Resources: *KHAN Academy - Radical Equations and Functions* *KHAN Academy - Rational Equations and Functions* *HOLT McDougall Algebra 2 2007* *Cliffs Notes - Algebra 2*
Vocabulary: **Rational functions:** A quotient of two polynomials **Rational expression:** A fraction whose numerators and denominators are nonzero polynomials **Simplified form** of a rational expression: When its numerator and denominator have no common factors other than +/- 1 **Complex Fraction:** A fraction that contains a fraction in the numerator or denominator ***Continuity:*** *Able to graph a function without picking up pencil.* ***Asymptotes:*** *A line that continually approaches a given curve but does not meet it at any finite distance.* ***Rational Equation****: Equation that contains one or more rational expressions.* ***Extraneous solution****: a solution of an equation derived from an original equation that is not a solution of the original equation.* ***Rational Inequality****: An inequality that contains one or more rational expression.* **nth roots:** an **nth root** of a number x, where n is a positive integer, is any of the n real or complex numbers r whose **nth** power is x

Unit 5: Exponential and Logarithmic Functions

Content Area: Mathematics	**Course: Algebra 2**	**UNIT: Exponential and Logarithmic Functions**
Unit Description: This unit defines and investigates exponential and logarithmic functions. Students will explore these functions through graphing and solving equations and inequalities. Real life applications will include exponential growth and decay.		**Unit Timeline:** 6 weeks

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. Properties of exponents are used to define logarithms.
2. The inverse relationships between exponential expressions and logarithms can be used to solve exponential and logarithmic equations.
3. Properties of logarithms can be used to solve logarithmic equations.
4. Logarithmic scales are used to solve real world problems.
5. Functions can be created using arithmetic operations and composition of functions.
6. Inverse functions can be derived from the original function and composition of functions can be used to show that two functions are inverses.

Essential Questions: *Students will keep considering…*

How do properties of exponents define logarithms?

How do we use exponential and logarithmic relationships to solve exponential and logarithmic equations?

How are logarithmic properties used to solve logarithmic equations?
How are logarithmic scales used to solve real world problems?
How can functions be created using operations and compositions of functions?
How can inverse functions be derived and how can composition of functions be used to show that two functions are inverses?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
A **logarithm** is the exponent to which a specified base is raised to obtain a given value.In the function $x = b^y$, y is called the logarithm, base b, of x. Usually written as $y = \log_b x$ and is read "y equals log base b of x." A **common logarithm** is a logarithmic with a base 10. A **logarithmic function** is the inverse of an exponential function. An **Exponential Function** is a function of the form $y=ab^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$. **Natural Base**,*e* is an irrational number approximately equal to 2.71828… **Natural Base Exponential Function** is an exponential function with base e, $y = e^x$ **A Natural Logarithm** is a logarithms with base e, written ln *x*.	A2.SSE.A.1	*Develop the definition of logarithms based on properties of exponents*. Define a logarithm of a given base b of a quantity to be the exponent to which you raise the base to get that quantity *(e.g., $\log_b(x) = y$ if and only if $b^y = x$)*.	A2.SSE.A.1
Exponential equations can be written in logarithmic form. *Logarithmic equations can be written in exponential form.* An **Exponential Equation** is an equation in which the variables occur as exponents. A **Logarithmic Equation** is an equation that contains one or more logarithms.	A2.SSE.A.2	*Use the inverse relationship between exponents and logarithms to solve exponential and logarithmic equations.*	A2.SSE.A.2
Product Property of Logarithms:	**A2.SSE.A.3**	***Use properties of logarithms to solve equations or find equivalent expressions.***	**A2.SSE.A.3**

$\log_b mn = \log_b m + \log_b n$ **<u>Quotient Property of Logarithms:</u>** $\log_b m/n = \log_b m - \log_b n$ **<u>Power Property of Logarithms:</u>** $\log_b a^p = p \cdot \log_b a$ **<u>Change of Base Formula</u>** - For all positive numbers a, b, and n, where $a \neq 1$ and $b \neq 1$, $\log_a n = \log_b n / \log_b a$		***Convert an exponent into a multiplier. (factor)*** ***Convert between a logarithm of factors and the sum of the logarithms of the individual factors.*** ***Convert between a logarithm of a quotient and the difference of the logarithms of the dividend and divisor.***	
In exponential growth, the base of the exponential expression, (1 + r) is the ***<u>Growth Factor</u>.*** *In exponential decay, the base of the exponential expression, (1 - r) is the* ***<u>Decay Factor</u>.*****<u>Exponential</u>** **<u>Growth</u>** occurs when a quantity increases exponentially over time. **<u>Exponential Decay</u>** occurs when a quantity decreases exponentially over time. **<u>Compound Interest</u>** is the interest paid on the principal of an investment and any previously earned interest. Compound Interest Formula: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ *A* represents the balance after *t* years. *P* represents the principal, or original amount. *r* represents the annual interest rate expressed as a decimal. *n* represents the number of times interest is compounded per year. *t* represents time in years. **Rate of continuous growth** is the rate at which something grows continuously. The value of *k* in the	A2.SSE.A.4	*Understand why logarithmic scales are used, and use them to solve problems.* Use logarithmic scales to compare quantities and solve problems involving logarithms. (e.g., pH scale, earthquake intensity, light intensity and sound intensity)	A2.SSE.A.4

exponential growth function, $f(x) = ae^{kt}$, where *a* is the initial value, and *t* is time in years. **Rate of continuous decay** is the rate at which something decays continuously. Represented by a constant *k* in the exponential decay function $f(x) = ae^{-kt}$, where *a* is the initial value, and *t* is time in years. **Logistic Growth Model** is a growth model that represents growth that has a limiting factor. Logistic models are the most accurate models for representing population growth.			
Functions can be added, subtracted, multiplied and divided. *In a composition of functions, the results of one function are used to evaluate a second function.* **Composition of Functions** - A function is performed, and then a second function is performed on the result of the first function. The composition of f and g is denoted by f•g, and [f•g](x)=f[g(x)].	A2.BF.A.1	*Create new functions by applying the four arithmetic operations and composition of functions (modifying the domain and range as necessary).* Create functions by performing operations on functions, including addition, subtraction, multiplication, division and composition of functions. Modify the domain and range if necessary. (*e.g., to restrict a domain in order to avoid a zero denominator in a quotient of functions*)	A2.BF.A.1
The inverse relation is the set of ordered pairs obtained by exchanging the coordinates of each ordered pair. The domain of the relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse. *Whether two functions are inverses can be determined by finding both of their compositions. If both compositions equal the identity function I(x) = x, then the functions are inverse functions.* **Inverse Relation** - Two relations are inverse relations if and only if whenever one relation contains the element (a,b) the other relation contains the element (b,a). **Inverse Function** - Two functions *f* and *g* are inverse	A2.BF.A.2	*Derive inverses of functions, and compose the inverse with the original function to show that the functions are inverses.*	A2.BF.A.2

functions if and only if both of their compositions are the identity function.			
Exponential and logarithmic functions can be transformed by changing the parameters of the function.	A2.BF.A.3	*Describe the effects of transformations algebraically and graphically, creating vertical and horizontal translations, vertical and horizontal reflections and dilations (expansions/compressions) for, exponential and logarithmic functions.*	A2.BF.A.3

Unit 6: Assessment

EVIDENCE of LEARNING			
Understanding 1 2 3 4 5 6	 A2.SSE.A.1 A2.SSE.A.2 **A2.SSE.A.3** A2.SSE.A.4 A2.BF.A.1 A2.BF.A.2	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Unit 6 Performance Task **Teacher will assess:** *Can students solve real world problems using logarithmic models?* *Can students solve logarithmic equations using exponential form and properties of logarithms?* *Can students rewrite logarithmic expressions in exponential form?* *Can students perform operations on functions including finding the inverse function?* **Performance:** **Mastery:** *Students will show that they really understand…* 1. when they score a 75% or above. **Scoring Guide:** Unit 6 Performance Task	**R/R Quadrant 21 Century** *B, C* *critical thinking*

Unit 5: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1	A2.SSE.A.1	1. **Activity:** Logarithm War **Objective**: Students will be able to evaluate logarithms. **Activity:** For this activity students will play war using the log cards: Log War. Students work in pairs or groups. Students divide cards evenly among players. Students will each flip over one card and as a group evaluate the logarithms. The player that laid down the highest value card will win all the cards on the table. Play will continue until one person has all the cards.	Cues & Questions, Cooperative Learning	A Critical Thinking Collaboration
2, 3	**A2.SSE.A.3** A2.SSE.A.2	**2. Lesson**: Logarithm Speed Dating **Objective:** Students will be able to solve exponential equations and inequalities using common logarithms. Solve logarithmic equations using the properties of logarithms. **Activity:** Using the Logarithms Speed Dating, students will complete a modified Quiz-Quiz-Trade cooperative learning structure. Students will sit across from each other in pairs. Teacher will set a timer for 2 minutes. Each student will have a task card with a problem. Students will work out Partner A's problem together and provide feedback to each other. If one partner is confused, the other student will tip, tip, teach to provide support. When the timer goes off, students trade cards and Partner A rotates to a new partner. The process is repeated with the new task card problem.	Cooperative Learning	C Critical Thinking Collaboration
2, 3	**A2.SSE.A.3** A2.SSE.A.2	3. **Lesson:** Logarithms Match My Answer **Objective:** Students will be able to solve logarithmic equations using the properties of logarithms. **Activity:** Using *Match Mine Logarithm Equations 1*, *Match Mine Logarithm Equations 2* students will complete the match mine cooperative learning structure. In pairs, students will use logarithms to solve equations. Student A	Cooperative Learning	A Critical Thinking Collaboration

		will have a different problem than student B, the answers to the problems pairs will match. Students must discuss the problems they do not agree on to come to a consensus of what the correct answer is for each problem.		
6	A2.BF.A.2 A2.BF.A.1	4. **Lesson:** Inverse and Composite Function kahoot **Objective:** Students will be able to find the inverse of a function, perform operations on functions, and simplify composite functions. **Activity:** Kahoot! is a game-based classroom response system. Games are displayed on a shared screen – for example a smart TV, a laptop or an interactive whiteboard. Players join in using their own device – whether that is a smartphone, iPad, laptop, or desktop, as long as they have a browser and good internet connection. Players do NOT need a kahoot account to play. Feedback is provided throughout the game. Students may play individually or with a partner.	Cues & Questions	B Critical Thinking
2, 3, 4	A2.SSE.A.2 **A2.SSE.A.3** A2.SSE.A.4 ISTE 5a	5. **Lesson:** Xbox Xponential PBL Activity **Objective:** Students will be able to use the properties of exponents to interpret expressions for exponential functions, determine an explicit expression from a context, construct exponential functions, interpret an exponential function in a context, represent data on a scatter plot. **Activity:** Xbox Xponential PBL In this launch activity, students write an exponential function based on the Atari 2600 and Moore's Law. They research other consoles and create a scatter plot of speed over time. They write and interpret an equation of the best-fit curve of this data to determine whether video game processors have followed Moore's Law. Students explore this even further in the project tasks, and can choose between three topics. In one task, students research the growth in capacity of other technologies like digital cameras or wireless bandwidth. In another, they investigate alternatives to processor speed, like memory or graphics, to measure the power of a console. In still one more task, they investigate why Nintendo's processors are always slower than the competition, and by how much.	Cues & Questions Cooperative Learning Generating & Testing Hypothesis Problem-based	C/D Critical thinking Collaboration Communication Technology

UNIT RESOURCES

Teacher Resources:

KHAN Academy - Logarithms and Functions
HOLT McDougall Algebra 2 2007
KUTA Worksheets
Study Island

Student Resources:

Study Island
Holt McDougal Algebra 2 2007
Virtual Nerd online website
Cliffs Notes Algebra 2 study guides
KHAN Academy - Logarithms and Functions

Vocabulary:

Change of Base Formula - For all positive numbers a, b, and n, where $a \neq 1$ and $b \neq 1$, $\log_a n = \log_b n / \log_b a$
Compound Interest : the interest paid on the principal of an investment and any previously earned interest.
Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A represents the balance after *t* years.
P represents the principal, or original amount.
r represents the annual interest rate expressed as a decimal.
n represents the number of times interest is compounded per year.
t represents time in years.

Composition of Functions - A function is performed, and then a second function is performed on the result of the first function. The composition of f and g is denoted by f•g, and [f•g](x)=f[g(x)].
Common logarithm: a logarithmic with a base 10.
Decay Factor.: the base of the exponential expression, (1 - *r)* In exponential decay problems.
Exponential Decay: when a quantity decreases exponentially over time.

Exponential Equation: an equation in which the variables occur as exponents.
Exponential Function: a function of the form $y=ab^x$,where $a \neq 0$, $b > 0$, and $b \neq 1$.
Exponential Growth: when a quantity increases exponentially over time.
Growth Factor: the base of the exponential expression, $(1 + r)$ in exponential growth problems.
Inverse Function :Two functions *f* and *g* are inverse functions if and only if both of their compositions are the identity function.
Inverse Relation:: Two relations are inverse relations if and only if whenever one relation contains the element (a,b) the other relation contains the element (b,a).
Logarithm: the exponent to which a specified base is raised to obtain a given value.
Logarithmic Equation: an equation that contains one or more logarithms.
Logarithmic function: the inverse of an exponential function.
Logistic Growth Model: a growth model that represents growth that has a limiting factor. Logistic models are the most accurate models for representing population growth.
Natural Base,e ::an irrational number approximately equal to 2.71828…
Natural Base Exponential Function: an exponential function with base e, $y = e^x$
Natural Logarithm:a logarithms with base e, written ln *x*.
Power Property of Logarithms: $\log_b a^p = p \cdot \log_b a$
Product Property of Logarithms: $\log_b mn = \log_b m + \log_b n$
Quotient Property of Logarithms: $\log_b m/n = \log_b m - \log_b n$
Rate of continuous decay: the rate at which something decays continuously. Represented by a constant *k* in the exponential decay function $f(x) = ae^{-kt}$, where *a* is the initial value, and *t* is time in years.
Rate of continuous growth: the rate at which something grows continuously. The value of *k* in the exponential growth function, $f(x) = ae^{kt}$, where *a* is the initial value, and *t* is time in years.

Unit 6: Statistics

Content Area: Mathematics	Course: Algebra 2	UNIT: Statistics
Unit Description: Students will evaluate surveys, studies and experiments. Create and use graphs of probability distributions. Use the Empirical Rule to find probabilities. Compare sample statistics and population statistics.		**Unit Timeline:** 3 weeks

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. A parameter is a measure that describes a characteristic of a population and a statistic is a measure that describes a characteristic of a sample.
2. A distribution of data shows the frequency of each possible data value.
3. A random variable can be either discrete or continuous.
4. The normal distribution is a continuous, symmetric, bell-shaped distribution of a random variable.
5. A confidence interval is an estimate of a parameter stated as a range with a specific degree of certainty.

Essential Questions: *Students will keep considering…*

How can sample statistics be used to make inferences about a population?
How does the shape of a distribution determine which statistics are used to describe the distribution?
What is the difference between a theoretical probability distribution and an experimental probability distribution?

How can z-values be used to compare different distributions of data?
How can confidence intervals be used to make predictions about an entire population?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
Sample information can be collected using the following study types: surveys, experiments, and observational studies. **Parameter -** A measure that describes a characteristic of a population. **Statistic** - A measure that describes a characteristic of a sample. **Random sample -** A sample in which every member of the population has equal chance of being selected. **Survey** - Used to collect information about a population. **Experiment** - Something that is intentionally done to people, animals, or objects, and then the response is observed. **Observational study** - Individuals are observed and no attempt is made to influence the results.	A2.DS.A.3	*Describe and explain the purposes, relationship to randomization and differences, among sample surveys, experiments and observational studies.* Classify study types as a survey, an experiment or an observational study.	A2.DS.A.3
The shape of a distribution can be determined by looking at its histogram or box-and-whisker plot. *When distribution is symmetric, use the mean and standard deviation to describe.* *When distribution is skewed, use the five-number summary to describe.* **Distribution** - Shows the observed or theoretical frequency of each possible data value. **Negatively (left) skewed distribution** - The majority of the data are on the right of the mean. **Symmetric distribution** - The data are evenly distributed	A2.DS.A.1	*Analyze how random sampling could be used to make inferences about population parameters.* Use the shapes of distributions to select appropriate statistics. Use the shapes of distributions to compare data. Find confidence intervals for normally distributed data.	A2.DS.A.1

on both sides of the mean. **Positively (right) skewed distribution** - The majority of the data are on the left of the mean. **Inferential statistics** - Statistics like predictions and hypothesis testing are used to draw conclusions about a population by using a sample. **Statistical inferences** - Use information from a sample to draw conclusions about a population. **Confidence interval** - An estimate of a population parameter stated as a range with a specific degree of certainty.			
Survey questions can introduce bias if they are confusing, cause a strong reaction, encourage a certain response, or address more than one issue. *Experiments can also introduce bias if the sample is not randomly selected, or if the control and experimental groups are not similar.* **Bias -** An error that results in a misrepresentation of members of a population.	HSS.IC.B.5	Design statistical studies by choosing unbiased questions for surveys and developing procedures experiments.	HSS.IC.B.5
A random variable can be either discrete or continuous. *A probability distribution for a random variable has the following properties: can be created using theoretical or experimental probabilities, can be discrete or continuous, probability of each value must be between 0 and 1, sum of all the probabilities must equal 1.* *In a binomial distribution, the probability of X successes in n independent trials can be found using the Binomial Probability Formula.* $P(X) = {}_nC_X p^X q^{n-X}$ **Random variable** - The outcome of a random process that has a numerical value. **Discrete random variable** - The numerical outcome of a	A2.DS.A.6	*Analyze decisions and strategies using probability concepts.* Construct a probability distribution using the sample space. Analyze a probability distribution using expected value and the standard deviation. Identify and conduct a binomial experiment. Find probabilities using binomial distributions.	A2.DS.A.6

random event that takes on countable values. **Continuous random variable** - The numerical outcome of a random event that can take on any value. **Probability distribution** - A function that maps the sample space to the probabilities of the outcomes in the sample space for a particular random variable. **Theoretical probability distribution** - A distribution of probabilities based on what is expected to happen. **Experimental probability distribution** - A distribution of probabilities estimated from experiments. **Law of Large Numbers** - The variation in a data set decreases as the sample size increases. **Expected value** - The expected value of a discrete random variable is the weighted average of the values of the variable. **Binomial experiment** - An experiment in which there are exactly two possible outcomes for each trial, a fixed number of independent trials, and the probabilities for each trial are the same. **Binomial distribution** - A distribution that shows the probabilities of the outcomes of a binomial experiment			
The normal distribution is a continuous, symmetric, bell-shaped distribution of a random variable. ***The mean, median, and mode are equal and located at the center of the curve.*** ***The curve approaches, but never touches, the x-axis.*** ***The total area under the curve is equal to 1.*** **Normal distribution -** A continuous, symmetric, bell-shaped distribution of a random variable. **Empirical Rule -** Can be used to determine the area under the normal curve at specific intervals.	**A2.DS.B.8**	***Know and use the characteristics of normally distributed data sets; predict what percentage of the data will be above or below a given value that is a multiple of standard deviations above or below the mean.*** **Use the Empirical Rule to analyze normally distributed variables.**	**A2.DS.B.8**

Data are standardized by converting them to z-values. ***Once data are standardized, they can be compared using the standard normal distribution.*** **Z-value (z-score) -** The number of standard deviations that a given data value is from the mean. **Standard Normal distribution -** A normal distribution with a mean of 0 and a standard deviation of 1.	**A2.DS.B.9**	***Fit a data set to a distribution using its mean and standard deviation to determine whether the data is approximately normally distributed.*** **Apply the standard normal distribution and z-values.**	**A2.DS.B.9**
How well the sample represents the population is gauged by two important statistics – the survey's margin of error and confidence level. **Margin of error** - The limit on the difference between how a sample responds and how the total population would respond.	**A2.DS.A.5**	***Describe and explain how the relative sizes of a sample and the population affect margin of error predictions.***	**A2.DS.A.5**
Using repeated sampling can help determine if model is consistent with a data set.	**A2.DS.A.2**	***Determine whether a specified model is consistent with a given data set.***	**A2.DS.A.2**
The intent, meaning, and significance of a report or data presented can be deciphered.	A2.DS.A.7	*Evaluate reports based on data.*	A2.DS.A.7
A hypothesis test is used to assess a specific claim about the mean. ***The claim is either the null hypothesis or the alternative hypothesis.*** **Hypothesis test -** A test used to assess a specific claim about the mean. **Null hypothesis -** A specific hypothesis to be tested. It is expressed as an equality using =, ≤, or ≥ and is considered true until evidence indicates otherwise. **Alternative hypothesis -** Mutually exclusive to the null hypothesis. It is stated as an inequality using ≠, < , or >. **Maximum error of estimate -** The maximum difference	**A2.DS.A.4**	***Use data from a sample to estimate characteristics of the population and recognize the meaning of the margin of error in these estimates.*** **Perform hypothesis tests on normally distributed data.**	

between the estimate of the population mean and its actual value. **Critical region** - The range of values that suggests a significant enough difference to reject the null hypothesis. **Left-tailed test** - Test of significance to determine if you should reject or fail to reject the null hypothesis. **Two-tailed test** - Test of significance to determine if you should reject or fail to reject the null hypothesis. **Right-tailed test** - Test of significance to determine if you should reject or fail to reject the null hypothesis.			

Unit 6: Assessment

EVIDENCE of LEARNING			
Understanding 1 2 3 4 5	Standards **A2.DS.B.8** **A2.DS.B.9**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Unit 7 Statistics Assessment **Teacher will assess:** *Can students calculate the probability of a random variable occurring within a specified interval?* *Can students distinguish between types of distributions?* *Can students calculate margin of error and find confidence intervals?* *Can students calculate the mean and standard deviation from a sample of data?* **Performance:** **Mastery** Students will show that they really understand when they… 1. Complete the assessment with 80% or greater. **Scoring Guide:** Unit 7: Statistics Scoring Guide	**R/R Quadrant 21 Century** B, C critical thinking

Unit 6: Sample Activities

SAMPLE LEARNING PLAN				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
5	**A2.DS.A.2** **A2.DS.A.5**	**1. Lesson**:Simulations and Margin of Error **Objective**: Students will be able to perform a simulation and find margin of error. **Activity**: Using the Simulations and Margin of Error activity, students will explore margin of error and sample size in this real-life application: The Pew Research Center conducted a survey of a random sample of teens and concluded that 43% of all teens who take their cell phones to school text in class on a daily basis. How accurately did their random sample represent all teens? Students will enter the program into the calculator and run 10 trials of the program, recording their results as they go. Students will calculate margin of error as well as other prompts about the situation. Please see copy of guided notes for the lesson as well as the practice worksheet for homework in the appendix documents. Appendix Documents: Simulations and Margin of Error Sample Key Here; Simulations and Margin of Error Guided Notes; Simulations and Margin of Error Homework	Cooperative Learning	B, C, D collaboration, communication, critical thinking
4 2 1	A2.DS.A.1 A2.DS.A.7	**2.Lesson:** Comparing Distributions Without Graphing **Objective:** Students will be able to use a data set to make comparisons and draw conclusions about the data. **Activity:** Using the Best Class - Student copy students will create and explore a box plot and histogram for a data set. Then, they compare the two data displays by viewing them together and use the comparison to draw conclusions about the data. Appendix Documents: Best Class - Teacher copy TI-84 files: period 4, period 5	Providing Practice	B, C Communication Critical Thinking

1	A2.DS.A.3	**3. Lesson**: Estimating True Mean **Objective**: Students will be able to estimate the true mean of a population when the standard deviation is known. **Activity:** Students will use the Means with Confidence - Student to estimate the true mean of a population when the standard deviation is known by finding the sample mean, margin of error and confidence interval. Appendix Documents: Means with Confidence - Teacher TI-84 files: YAC, LONG, AVE	Cooperative Learning	B, C collaboration, communication, critical thinking
1 2 3 4	A2.DS.A.6 **A2.DS.B.8** **A2.DS.B.9**	**4. Lesson:** Percentiles and Z-Scores **Objective**: Students will be able to calculate percentiles, *z*-scores, and probabilities using normal distributions **Activity:** Using Percentiles and Z-Scores - Student, students will be able to calculate percentiles, z-scores, and probabilities using normal distributions. Appendix Documents: Percentiles and Z-Scores - Teacher	Providing Practice	B, C Communication Critical Thinking

Unit 6: Resources

UNIT RESOURCES
Teacher Resources: Textbook TI Education Khan Academy - Statistics
Student Resources: Textbook Cliffs Notes - Statistics TI Education Khan Academy - Statistics
Vocabulary: **Bias** - An error that results in a misrepresentation of members of a population.

Binomial distribution - A distribution that shows the probabilities of the outcomes of a binomial experiment.
Binomial experiment - An experiment in which there are exactly two possible outcomes for each trial, a fixed number of independent trials, and the probabilities for each trial are the same.
Confidence interval - An estimate of a population parameter stated as a range with a specific degree of certainty.
Continuous random variable - The numerical outcome of a random event that can take on any value.
Discrete random variable - The numerical outcome of a random event that takes on countable values.
Distribution - Shows the observed or theoretical frequency of each possible data value.
Empirical Rule - Can be used to determine the area under the normal curve at specific intervals.
Expected value - The expected value of a discrete random variable is the weighted average of the values of the variable.
Experiment - Something that is intentionally done to people, animals, or objects, and then the response is observed.
Experimental probability distribution - A distribution of probabilities estimated from experiments.
Inferential statistics - Statistics like predictions and hypothesis testing are used to draw conclusions about a population by using a sample.
Law of Large Numbers - The variation in a data set decreases as the sample size increases.
Maximum error of estimate - The maximum difference between the estimate of the population mean and its actual value.
Negatively (left) skewed distribution - The majority of the data are on the right of the mean.
Normal distribution - A continuous, symmetric, bell-shaped distribution of a random variable.
Observational study - Individuals are observed and no attempt is made to influence the results.
Parameter - A measure that describes a characteristic of a population.
Positively (right) skewed distribution - The majority of the data are on the left of the mean.
Probability distribution - A function that maps the sample space to the probabilities of the outcomes in the sample space for a particular random variable.
Random sample - A sample in which every member of the population has equal chance of being selected.
Random variable - The outcome of a random process that has a numerical value.
Standard normal distribution - A normal distribution with a mean of 0 and a standard deviation of 1.
Statistic - A measure that describes a characteristic of a sample.
Statistical inferences - Use information from a sample to draw conclusions about a population.
Survey - Used to collect information about a population.
Symmetric distribution - The data are evenly distributed on both sides of the mean.
Theoretical probability distribution - A distribution of probabilities based on what is expected to happen.
Z-value (z-score) - The number of standard deviations that a given data value is from the mean.

Unit 7: Trigonometry

Content Area: Mathematics	Course: Algebra 2	UNIT: Trigonometry
Unit Description: Students will explore the unit circle as it pertains to the coordinate plane, and use it to extend the domain of trigonometric functions to all real numbers.		**Unit Timeline:** 2 weeks

DESIRED Results

Transfer Goal - ***Students will be able to independently use their learning to……***

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understandings – *Students will understand that… (Big Ideas)*

1. A ratio that compares the lengths of the sides of a right triangle is called a trigonometric function.
2. An angle on a coordinate plane is in standard position if one ray of the angle is placed on the positive x-axis and the other ray rotates about the origin.
3. Angles can be measured in degrees or radians.
4. When a point P(x, y) on the terminal side of the angle θ is known, the value of the six trigonometric functions can be found.
5. Using special right triangles builds understanding of the relationships in the unit circle.
6. The equation for the circumference of a circle makes the relationship between degrees and radians clear.

Essential Questions: ***Students will keep considering…***

- How is trigonometry used to find unknown values?
- Why are certain values undefined for certain functions?
- What types of real-world problems can be modeled and solved using trigonometry?

- Why is it important to know what quadrant the terminal side of an angle in standard position lies in?

Students will know/understand ...	Standard	Students Will Be Able to ...	Standard
A rule given by a ratio that compares the lengths of the sides of a right triangle is called a trigonometric function. **Trigonometric function:** a function whose rule is given by a trigonometric ratio. **Basic trigonometric ratios:** **sine** - ratio of the length of the opposite leg to the length of the hypotenuse. **cosine** - ratio of the length of the adjacent leg to the length of the hypotenuse. **tangent** - ratio of the length of the opposite leg to the length of the adjacent leg. **cotangent** - ratio of the length of the adjacent leg to the length of the opposite leg. **secant** - ratio of the length of the hypotenuse to the length of the adjacent leg. **cosecant** - ratio of the length of the hypotenuse to the length of the opposite leg	**G-SRT.8**	**Understand and use trigonometric relationships of acute angles in triangles.** **Determine the side lengths of right triangles using trigonometric functions.** **Use trigonometric ratios to solve right triangles in applied problems.**	**G-SRT.8**
An angle on a coordinate plane is in standard position if one ray of the angle is placed on the positive x-axis and the other ray rotates about the origin. **Standard position of an angle:** an angle is in standard position when its vertex is at the origin and one ray is on the positive x-axis. **Initial side of an angle:** The initial side of an angle is the ray on the x-axis **Angle of rotation:** the angle formed by rotating the terminal side and keeping the initial side in place.	**HSF-TF.A.1**	**Draw angles in standard position.** **Determine the values of the trigonometric functions for an angle in standard position.** **Use reference angles to find the exact value of special angles using special right triangles.** **Find coterminal angles and their exact values.**	**HSF-TF.A.1**

Coterminal angles: angles in standard position with the same terminal side. **Reference angle:** the positive acute angle formed by the terminal side of an angle and the x-axis. **Quadrantal Angles:** Angles whose terminal side does not lie in a quadrant.			
Angles can be measured in degrees or in radians. **Unit circle:** a circle with a radius of one unit. **Radian measure:** the measure of the central angle of a circle when the subtended arc is the same measure as the radius.	**HSF-TF.A.2**	**Find the values of trigonometric functions on the unit circle.** **Convert between radian and degree measure of an angle.** **Compute trigonometric ratios for all real numbers.**	**HSF-TF.A.2**

Unit 7: Assessment

EVIDENCE of LEARNING			
Understanding 1 2 3 4	Standards **HSF-TF.A.1** **HSF-TF.A.2** **G-SRT.8**	**Unit Performance Assessment**: **Description of Assessment Performance Task(s):** Performance Assessment **Teacher will assess:** *Students will answer questions related to the unit circle and applied problems.* **Performance:** **Mastery:** *Students will show that they really understand when they…* 1. achieve mastery on the performance assessment of 4 out of 6 possible points. **Scoring Guide:**Performance Assessment Scoring Guide	**R/R Quadrant 21 Century** C critical thinking

Unit 7: Sample Activities

SAMPLE LEARNING PLAN				
Pre-assessment: *Students will answer the following questions:* 1. Find the exact value of sin $5\pi/6$ 2. The angle of elevation between a 6 foot tall tree and a child on the ground is 40 degrees, how far away from the tree is the child?				
Understanding	**Standards**	**Major Learning Activities:**	**Instructional Strategy Category:**	**R/R Quadrant: 21C:**
1 2 3 4	**G-SRT.8**	**1. Lesson:** Trig Ratios Activity - **Objective:** Students will be able to use right triangle trigonometry to find the height of a building or other tall object. **Description:** Students will use a clinometer (protractor, straw, paper clip), or transit and a tape measure along with their knowledge of right triangle	Generating Testing and Hypothesis Cooperative Learning	C Critical Thinking, Collaboration

		trigonometry to find the height of objects. Trigonometric Ratios Activity How does your height affect the measurement? What units of measure should you use? Does your answer make sense? Given the information that you have, which trig ratio should you use? Is this an example of an angle of elevation or angle of depression? Could you change the process to use the opposite?	Cues and Questions	
3	**HSF-TF.A.1**	**2. Lesson:** Degrees and Radians **Objective:** Students will be able to convert between degrees and radians. **Description:** Students will use the cooperative learning structure Inside-Outside Circle to practice converting between degrees and radians. Each student will be given a problem card from Exact Values Inside Outside Circle. Students will form two circles with each person facing a partner. Students will quiz each other using their card. When the teacher prompts, students will trade cards and upon instruction from teacher, students on the inside circle will rotate a given direction. This will repeat for several rounds. How do you convert from radians to degrees, degrees to radians? What is a radian?	Cooperative Learning Providing Practice Cues & Questions	B
1 4 5 6	**HSF-TF.A.2** ISTE 1c	**3. Lesson:** Finding Exact Values of Trig Functions **Objective:** Students will be able to find the exact values of trig functions using a Kahoot activity. **Description:** Using Exact Values Kahoot, students will play Kahoot! Kahoot! is a game-based classroom response system. Games are displayed on a shared screen – for example a smart TV, a laptop or an interactive whiteboard. Players join in using their own device – whether that is a smartphone, iPad, laptop, or desktop, as long as they have a browser and good internet connection. Players do NOT need a kahoot account to play. Players may work with a partner. Students will record their work on their own piece of paper.	Feedback Providing Practice Cooperative Learning	C Model with mathematics Collaboration Use appropriate tools Technology
1, 4	**G-SRT.8** **HSF-TF.A.2**	**4. Lesson:** Solving right triangles **Objective:** Students will be able to solve a right triangle and find the three trigonometric ratios given a point on the terminal side.	Providing Practice	B Collaboration

		Description: Students will complete this Trig Simultaneous Round Table Activity #1 by completing 4 problems focused on solving a right triangle and trig ratios when given a point on the terminal side. Students should be separated into groups of 4. Each student in the group will start with 1 of 4 different worksheets. All students will work on and complete the first problem. When complete, students will pass the paper to the student on the right. Taking the next worksheet passed to them, students should check the problem and discuss any issues with the student who completed the problem before initialling that it is complete and correct. Once that is done, students will start the second problem and follow the same directions until all 4 problems are complete. The teacher should be walking around to check and provide prompts as students are working.	Cooperative Learning	
2, 3	**HSF-TF.A.1** **HSF-TF.A.2**	**5. Lesson:** Applying trigonometry to the Unit Circle **Objective:** Students will be able to draw angles in standard position, find reference angles, convert from radians to degrees and solving problems of elevation or depression. **Activity:** Students will complete this Trig Simultaneous Round Table Activity #2 by completing 4 problems focused on drawing an angle in standard position, finding the reference angle, finding the exact value of a trig function, converting from radians to degrees, and solving a problem of elevation or depression. Students should be separated into groups of 4. Each student in the group will start with 1 of 4 different worksheets. All students will work on and complete the first problem. When complete, students will pass the paper to the student on the right. Taking the next worksheet passed to them, students should check the problem and discuss any issues with the student who completed the problem before initialling that it is complete and correct. Once that is done, students will start the second problem and follow the same directions until all 4 problems are complete. The teacher should be walking around to check and provide prompts as students are working.	Providing Practice Cooperative Learning	B Collaboration

Unit 7: Resources

UNIT RESOURCES

Teacher Resources:
Algebra 2 Holt textbook, my.hrw.com
Kuta software
Cliffs Notes - Trigonometry
KHAN Academy - Trigonometry

Student Resources:
Algebra 2 Holt textbook, my.hrw.com
Cliffs Notes - Trigonometry
KHAN Academy - Trigonometry

Trigonometric function: a function whose rule is given by a trigonometric ratio.
Basic trigonometric ratios:
sine - ratio of the length of the opposite leg to the length of the hypotenuse.
cosine-ratio of the length of the adjacent leg to the length of the hypotenuse.
tangent-ratio of the length of the opposite leg to the length of the adjacent leg.
cotangent-ratio of the length of the adjacent leg to the length of the opposite leg.
secant-ratio of the length of the hypotenuse to the length of the adjacent leg.
cosecant-ratio of the length of the hypotenuse to the length of the opposite leg
Standard position of an angle: an angle is in standard position when its vertex is at the origin and one ray is on the positive x-axis.
Initial side of an angle: The initial side of an angle is the ray on the x-axis
Angle of rotation: the angle formed by rotating the terminal side and keeping the initial side in place.
Coterminal angles: angles in standard position with the same terminal side.
Reference angle: the positive acute angle formed by the terminal side of an angle and the x-axis.
Quadrantal Angles: Angles whose terminal side does not lie in a quadrant.
Law of Sines: The *law of sines* states that the ratio between the length of the side opposite an angle and the sine of that angle is the same for all interior angles in the same triangle.
Law of Cosines: For any △ABC, where *a* is the length of the side opposite angle A, *b* is the length of the side opposite angle B, and *c* is the length of the side opposite angle C, $a^2 = b^2 + c^2 - \cos(\text{angle A})$
Periodic function: functions that repeat exactly in regular intervals called cycles.
Period: the length of a cycle of a graph
Amplitude: Distance from x-axis to maximum height on the graph.
Frequency: number of cycles in a given unit of time.
phase shift: horizontal translation of a periodic function.
Pythagorean Identities: $\sin^2\theta+\cos^2\theta=1$

Reciprocal Identities: $\csc\theta = 1/\sin\theta$
$\sec\theta = 1/\cos\theta$
$\cot\theta = 1/\tan\theta$

Quotient Identities: $\tan\theta = (\sin\theta/\cos\theta)$
$\cot\theta = (\cos\theta/\sin\theta)$