Francis Howell School District Mission Statement

Francis Howell School District is a learning community where all students reach their full potential.

Vision Statement

Francis Howell School District is an educational leader that builds excellence through a collaborative culture that values students, parents, employees, and the community as partners in learning.

Values

Francis Howell School District is committed to:

- Providing a consistent and comprehensive education that fosters high levels of academic achievement for all
- Operating safe and well-maintained schools
- Promoting parent, community, student, and business involvement in support of the school district
- Ensuring fiscal responsibility
- Developing character and leadership

Francis Howell School District Graduate Goals

Upon completion of their academic study in the Francis Howell School District, students will be able to:

- 1. Gather, analyze and apply information and ideas.
- 2. Communicate effectively within and beyond the classroom.
- 3. Recognize and solve problems.
- 4. Make decisions and act as responsible members of society.

Mathematics Graduate Goals

Upon completion of their mathematics study in the Francis Howell School District, students will be able to:

- 1. Communicate mathematically
- 2. Reason mathematically
- 3. Make mathematical connections
- 4. Use mathematical representations to model and interpret practical situations

Mathematics Rationale for Math Analysis

Math Analysis is designed to prepare students for the study of Calculus. It will develop the students' mathematical concepts, improve their problem solving skills and enhance their ability to reason logically. The course will prepare students with a solid foundation in mathematics, especially those who intend to pursue a degree or job in a field that requires a solid mathematics background.

Course Description for Math Analysis

This course focuses on analyzing various functions through the integration of algebraic and geometric concepts with a strong emphasis on graphing. Functions include but are not limited to polynomials, rational, exponential and logarithmic. Math Analysis combined with Trigonometry builds a strong foundation for future study in Calculus.

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CURRICULUM MAP – CONCEPTS Math Analysis

| | Chapter/Concepts | Number of Days (including review and test) |
|------|------------------------------------|--|
| | SEMESTER | |
| | Functions and Graphs | 15 |
| 1-1 | Review of Basics | |
| 1-2 | Review of Basics | |
| 1-3 | Graphing | |
| 1-4 | Transformations | |
| 1-5 | Combinations and Compositions | |
| 1-6 | Inverse Functions | |
| | | |
| | Polynomial Functions | 10 |
| 2-1 | Quadratics | |
| 2-2 | Polynomial of Higher Degree | |
| 2-3 | Real Zeros of Polynomial Functions | |
| 2-5 | Fundamental Theorem of Algebra | |
| | | |
| | Limits | 15 |
| 8-1 | Sequences and Series | |
| 11-1 | Intro to Limits | |
| 11-2 | Evaluating Limits | |

| 11-4 | Limits of Sequences at Infinity | |
|-------------|--|----|
| 2-6 | Asymptotes of Rational Functions | |
| 2-7 | Graphs of Rational Functions | |
| | | |
| Solvin | g Equations | 10 |
| 7-3 | Partial Fraction Decomposition | |
| Appendix B3 | Solving Various Types of Equations (radical, rational and absolute value) | |
| Appendix B4 | Solving Various Types of Inequalities (radical, rational and absolute value) | |
| | | |
| Expor | nential and Logarithmic Functions | 10 |
| 3-1 | Exponential Functions and Graphs | |
| 3-2 | Logarithmic Functions and Graphs | |
| 3-3 | Properties of Logarithms | |
| 3-4 | Solving Equations | |
| 3-5 | Exponential Growth and Decay | |
| | | |
| Conic | S | 10 |
| 9-1 | Parabolas | |
| 9-2 | Ellipses | |
| 9-3 | Hyperbolas | |
| 9-4 | Systems of Quadratic Equations | |
| Appendix B1 | Circles | |
| | | |
| Deriva | 10 | |
| Supplement | Minimum/Maximum/Point of Inflection | |
| Supplement | Derivatives | |

| 11-3 | Tangent to a Curve | |
|------|--------------------|--|
| | | |
| | Optional | |
| | Linear Programming | |
| | Induction Proofs | |
| | Coordinate Proofs | |
| | Optimization | |
| | | |

| Strand:1 Algebraic Relationships 1, 2 Geometric and Spatial Relationships 3 |
|---|
| |

Learner Objectives: Students will identify and apply characteristics of a relation.

- **Concepts:** A1C: Classify objects and representations
 - A1D: Identify and compare functions
 - A1E: Describe the effects of parameter changes
 - A2B: Describe and use mathematical manipulation
 - G3B: Use transformations on functions
 - G3C: Use symmetry

| | Students Should Know | Students Should Be Able to |
|---|--|--|
| • | Recognize and understand basic function notation | • Find domain and range of a function and write in interval notation |
| • | Understand concept of domain and range | (A1D, A1E, A2B) |
| • | Recognize line and point symmetry | • Find combinations and compositions of functions (A2B, G3B) |
| • | Recognize graphs of basic equations | • Use proper function notation (A1C, A1D, A1E) |
| | | • Find even/odd functions/symmetry (A1C, A1D, A2B, G3B, G3C) |
| | | • Find inverse of a function (A1D, A2B, G3B, G3C) |

Instructional Support

| Student Essential Vocabulary | | | | | |
|------------------------------|---------------|------------|--------------|-------------------|--------------------|
| Relation | Function | Domain | Range | Interval Notation | Vertical Line Test |
| Reflection | Increasing | Decreasing | Constant | Even Function | Odd Function |
| Point Symmetry | Line Symmetry | Inverse | Combinations | Composition | |

| Sample Learning Activities | Sample Assessments | |
|--|--|--|
| Learning Activity #1 : | Assessment #1: | |
| Find the domain of each function. Write your answers in interval notation. a) $f(x) = -3x^2 + 4x + 5$ | Determine whether the following statement is true or false. Justify your answer: If $f(x) = x+1$ and $g(x) = 6x$, then $(f \Box g)(x) = (g \Box f)(x)$. | |
| $g(x) = \frac{3x}{x+5}$ | | |
| c) $h(x) = \sqrt{x+2}$ Solutions: a) $(-\infty, \infty)$ b) $(-\infty, -5)$ and $(-5, \infty)$ c) $[-2, \infty)$ | Solution: $(f \Box g)(x) = 6x + 1$ and $(g \Box f)(x) = 6(x+1) = 6x + 6$. False because $(f \Box g)(x) \neq (g \Box f)(x)$. | |
| Activity's Alignment | Assessment's Alignment | |
| CONTENT MA 4 Patterns and relationships | CONTENT MA 4 Patterns and relationships | |
| PROCESS 1.6 Discover/evaluate relationships | PROCESS 1.6 Discover/evaluate relationships | |
| DOK 2 | | |
| INSTRUCTIONAL Generating and testing an hypothesis | DOK 3 | |
| STRATEGIES | LEVEL OF Mastery Level – 90% | |
| | EXPECTATION | |
| Learning Activity #2: | Assessment #2: | |
| Find the inverse function of $f(x) = x^3 + 2$. Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same set of axes. | Multiple Choice: Find the inverse of the function $f(x) = (x+4)^3 - 3$. | |
| | (A) $f^{-1}(x) = \sqrt[3]{x-4} + 3$ (B) $f^{-1}(x) = \sqrt[3]{x-1}$ | |
| Solution: $f^{-1}(x) = \sqrt[3]{x-2}$ | (C) $f^{-1}(x) = \sqrt[3]{x+3} - 4$ (D) None of these | |

| | | Solution: C | |
|---------------|--|-------------|--|
| | | | Assessment's Alignment |
| | Activity's Alignment | CONTENT | MA 2 Geometric and spatial sense |
| CONTENT | MA 2 Geometric and spatial sense | PROCESS | 1.10 Apply information, ideas and skills |
| PROCESS | 1.10 Apply information, ideas and skills | | |
| | | DOK | 2 |
| DOK | 2 | LEVEL OF | Mastery Level – 90% |
| INSTRUCTIONAL | Nonlinguistic representation | EXPECTATION | |
| STRATEGIES | | LAILETATION | 1 |
| | | | |

| | Student Resources | Teacher Resources |
|----------------|--|--|
| Houghton Mif | flin; Precalculus with Limits: A Graphing Approach © 2005; | Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; |
| Larson, Hostet | ler, Edwards; ISBN #0-618-39480-X | Larson, Hostetler, Edwards |
| | | |
| Section 1.2 | Functions; | |
| Section 1.3 | Graphs of Functions; | |
| Section 1.4 | Shifting, Reflecting, and Sketching Graphs; | |
| Section 1.5 | Combinations of Functions; | |
| Section 1.6 | Inverse Functions | |

| Identity Equity and Readiness | | | |
|-------------------------------|----------------------|--|--|
| Gender Equity | Technology Skills | | |
| Racial/Ethnic Equity | Research/Information | | |
| Disability Equity | Workplace/Job Prep | | |

| Content Area: Mathematics | | Course: Math Analysis | Strand:2 Algebraic Relationships 1, 2, 4 Geometric and Spatial Relationships 2, 3 | |
|---|---------------------------------|-----------------------|---|--|
| Learner Objectives: Students will analyze and apply properties given a graph or equation. | | | | |
| Concepts: | A1D: Identify and compare funct | ions | | |

- A1E: Describe the effects of parameter changes
- A2C: Utilize equivalent forms
- A2D: Utilize systems
- A4A: Analyze changes
- G2A: Use coordinate systems
- G3A: Use transformations on objects
- G3B: Use transformations on functions
- G3C: Use symmetry

| Students Should Know | Students Should Be Able to |
|---|---|
| Apply Order of Operations | • Recognize the transformation of a parent graph from the function |
| • Complete the square | (A1D, A1E, A4A, G2A, G3B, G3C) |
| • Graph basic functions by creating a table | • Identify, describe and graph the transformation |
| | • Graph segmented functions (piecewise) (A2C, A4A, G2A, G3B, |
| | G3C) |
| | Identify types of discontinuity |
| | • Graph conic sections (A1D, A1E, A2D, A4A, G2A, G3A, G3B, |
| | G3C) |
| | • Describe characteristics of functions including even/odd, symmetry, |
| | end behavior, continuity, and domain/range (A1D, A1E, A2C, G2A, |
| | G3B, G3C) |

| Student Essential Vocabulary | | | | | |
|------------------------------|-------------------|------------|----------------------------|-----------------------------|----------------------------|
| Parent Function | Domain/Range | Symmetry | Degree of Polynomial | Even/Odd Function | Leading Co-efficient |
| End Behavior | Piecewise-defined | Continuous | Discontinuous | Jump Discontinuity | Infinite Discontinuity |
| Point Discontinuity | Transformations | Reflection | Horizontal/Vertical Shifts | Horizontal/Vertical Stretch | Horizontal/Vertical Shrink |



| Activity's Alignment | | | |
|----------------------|----------------------------------|--|--|
| CONTENT | MA 2 Geometric and spatial sense | | |
| PROCESS | 3.3 Apply one's own strategies | | |
| DOK | 2 | | |
| INSTRUCTIONAL | Nonlinguistic representation | | |
| STRATEGIES | | | |

Learning Activity #2:

Sketch the graph of the piecewise-defined function by hand.

$$h(x) = \begin{cases} 3+x, & x < 0 \\ x^2 + 1, & x \ge 0 \end{cases}$$

Is it continuous or discontinuous? If it is discontinuous, then state whether it has point discontinuity, jump discontinuity, or infinite discontinuity. Determine the intervals over which the function is increasing, decreasing or constant.

Solution: It has jump discontinuity. Note: Graph has open circle at (0,3) and closed circle at (0,1)



Increasing (- ∞ , 0) U (0, ∞)

| Assessment's Alignment | | | |
|------------------------|--|--|--|
| CONTENT | MA 2 Geometric and spatial sense | | |
| PROCESS | 1.10 Apply information, ideas and skills | | |
| DOK | 2 | | |
| LEVEL OF | Mastery Level – 85% | | |
| EXPECTATION | | | |

Assessment #2:

1.) Algebraically determine whether the function $g(x) = -3x^3 + 5x - 2$ is even, odd, or neither.

Solution: $g(-x) = -3(-x)^3 + 5(-x) - 2 \implies g(-x) = 3x^3 - 5x - 2$ Neither even or odd

2.) Use the Leading Coefficient Test to determine the end behavior of the graph of the polynomial function $f(x) = -3x^7 + 4x - 1$.

Solution:
$$As x \to \infty$$
, $f(x) \to -\infty$; $As x \to -\infty$, $f(x) \to \infty$

| | A ativity?a Alignmont | | Assessment's Alignment |
|---------------|-------------------------------------|-------------|-------------------------------------|
| | Activity's Angliment | | Assessment s Angnment |
| CONTENT | MA 4 Patterns and relationships | CONTENT | MA 4 Patterns and relationships |
| PROCESS | 1.6 Discover/evaluate relationships | PROCESS | 1.6 Discover/evaluate relationships |
| DOK | 2 | DOK | 2 |
| INSTRUCTIONAL | Similarities and differences | LEVEL OF | Mastery Level – 80% |
| STRATEGIES | | EXPECTATION | |
| | | | · |

| | Student Resources | Teacher Resources |
|----------------|---|--|
| Houghton Miff | lin; Precalculus with Limits: A Graphing Approach © 2005; | Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; |
| Larson, Hostet | ler, Edwards; ISBN #0-618-39480-X | Larson, Hostetler, Edwards |
| | | |
| Section 1.2 | Functions; | Appendix: Discontinuity |
| Section 1.3 | Graphs of Functions; | |
| Section 1.4 | Shifting, Reflecting, and Sketching Graphs; | |
| Section 2.2 | Polynomial Functions of Higher Degree; | |
| Appendix B.1 | The Cartesian Plane; | |
| Section 9.1 | Introduction to Conics: Parabolas; | |
| Section 9.2 | Ellipses; | |
| Section 9.3 | Hyperbolas; | |
| Section 9.4 | Rotation and Systems of Quadratic Equations | |

| Identity Equity and Readiness | | | |
|-------------------------------|----------------------|--|--|
| Gender Equity | Technology Skills | | |
| Racial/Ethnic Equity | Research/Information | | |
| Disability Equity | Workplace/Job Prep | | |

| ∂ \mathbf{I} \mathcal{I} | Content Area: Mathematics | Course: Math Analysis | Strand:3 Numbers and Operations 1, 2 Algebraic Relationships 1, 2, 4 |
|---------------------------------------|----------------------------------|-----------------------|--|
|---------------------------------------|----------------------------------|-----------------------|--|

Learner Objectives: Students will solve polynomial, rational, radical and absolute value equations and inequalities.

Concepts: N1B: Represent and use rational numbers

- N1D: Classify and describe numeric relationships
- N2D: Apply operations on real and complex numbers
- A1D: Identify and compare functions
- A2A: Represent mathematical situations
- A2B: Describe and use mathematical manipulations
- A2C: Utilize equivalent forms
- A2D: Utilize systems
- A4A: Analyze change

| Students Should Know | Students Should Be Able to |
|--|---|
| • Identify the degree of the function | Polynomial functions |
| • Divide using synthetic and long division | • Use the degree of a function to identify the number and nature of |
| • Solve quadratics | roots including multiplicity (A1D) |
| Multiply polynomials | • Write the polynomial function given its roots (N2D) |
| • Solve systems of equations | • Use Factor Theorem/Rational Root Theorem to determine the |
| • Factor polynomials including sum and difference of cubes | roots of the polynomial (N2D) |
| | • Approximate real roots using upper and lower bounds (A2C) |
| | • Use critical points, roots and end behavior to graph (A4A) |
| | Radical/Rational/Absolute Value functions |
| | • Solve equations and inequalities (A2C) |
| | • Determine the domain and extraneous roots of the function |
| | (N1D) |
| | Decompose into partial fractions (A2D) |
| | • Graph functions (A1C) |
| | • Identify symmetry, points of discontinuity, asymptotes and end |
| | behavior (A4A) |

| Student Essential Vocabulary | | | | | |
|------------------------------|-------------------|------------------------|------------------|--------------------------|------------------|
| Roots | Zeros | Complex Numbers | Radicals | Common Denominators | Extraneous Roots |
| Degree of the Function | Multiplicity | Asymptotes | End Behavior | Limits | Decompose |
| Irrational/Rational | Interval Notation | Point of Discontinuity | Partial Fraction | Relative Maximum/Minimum | Extrema |
| Upper Bound | Lower Bound | Critical Points | | | |



| PROCESS | 1.6 Discover/evaluate relationships | PROCESS | 1.6 Discover/evaluate relationships |
|--|---|----------------------------------|--|
| DOK | 3 | DOK | 2 |
| INSTRUCTIONAL | Similarities and differences | LEVEL OF | Z Mastery Level 75% |
| STRATEGIES | | EXPECTATION | Mastery Lever - 7570 |
| Learning Activity #2 | : | Assessment #2. | <u> </u> |
| Solve the radical e | equation/inequality. | Solve the radio | cal equation/inequality. |
| (Write answer for | inequality in interval notation) | (Write answer | for inequality in interval notation) |
| 1. $\sqrt{k+9} - \frac{1}{2}$ | $\sqrt{k} = \sqrt{3}$ 2. $\sqrt[4]{5x-9} < 2$ | 1. $\sqrt{x+2}-7$ | $>\sqrt{x+9}$ 2. $5-\sqrt{x+4}>2$ |
| Solution: 1. | k = 3 2. [9/5, 5) | Solution: 1. 1 | no solution 2. [-4, 5) |
| | Activity's Alignment | | Assessment's Alignment |
| CONTENT | MA 1 Number Sense | CONTENT | MA 1 Number Sense |
| PROCESS | 1.10 Apply information, ideas and skills | PROCESS | 1.10 Apply information, ideas and skills |
| DOK | 2 | | |
| INSTRUCTIONAL | Nonlinguistic Representation | DOK | 2 |
| STRATEGIES | | LEVEL OF | Mastery Level – 80% |
| T | | EXPECTATION | |
| Learning Activity #3 | | A an an an an A #2 . | |
| Decompose into partia | l fractions. | Assessment #5: | |
| Decompose into partia | | Decompose into pa | rtial fractions. |
| 5x + 26 | | Decompose into pu | |
| $\frac{3x^2+20}{3x^2+7x-6}$ | | 7m - 7 | |
| 3x + 7x = 0 | | $\frac{1}{m^2 + m - 12}$ | |
| | | | |
| Solution: | | Solution: | |
| 5x + 26 8 | -1 | | |
| $\frac{1}{3x^2+7x-6} = \frac{1}{3x-2} + \frac{1}{x+3}$ | | 7m - 7 | 5 2 |
| | | $\frac{1}{m^2+m-12}=\frac{1}{m}$ | $\frac{1}{m+4} + \frac{1}{m-3}$ |
| Activity's Alignment | | | - |
| CONTENT | MA 5 Mathematical Systems | | Assessment's Alignment |
| | · | CONTENT | MA 5 Mathematical Systems |

| PROCESS | 1.6 Discover/evaluate relationships | PROCESS | 1.6 Discover/evaluate relationships |
|------------------------------------|-------------------------------------|--------------------------------|-------------------------------------|
| DOK INSTRUCTIONAL STRATEGIES | 2 Homework and practice | DOK LEVEL OF EXPECTATION | 2 Mastery Level – 80% |

| | Student Resources | Teacher Resources |
|----------------|---|--|
| Houghton Miff | lin; Precalculus with Limits: A Graphing Approach © 2005; | Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; |
| Larson, Hostet | ler, Edwards; ISBN #0-618-39480-X | Larson, Hostetler |
| | | |
| Section 2.1 | Quadratic Functions | |
| Section 2.2 | Polynomial Functions of Higher Degree | |
| Section 2.3 | Real Zeros of Polynomial Functions | |
| Section 2.4 | Complex numbers | |
| Section 2.5 | Fundamental Theorem of Algebra | |
| Section 2.6 | Rational Functions and Asymptotes | |
| Section 2.7 | Graphs of Rational Functions | |
| Section 7.3 | Multivariable Linear Systems | |
| Appendix B.3 | Solving Equations Algebraically and Graphically | |
| Appendix B.4 | Solving Inequalities Algebraically and Graphically | |

| Identity Equity and Readiness | | | |
|-------------------------------|--|----------------------|--|
| Gender Equity | | Technology Skills | |
| Racial/Ethnic Equity | | Research/Information | |
| Disability Equity | | Workplace/Job Prep | |

| Content Area: Mathematics | Course: Math Analysis | Strand:4 Number and Operations 2, 3 Algebraic Relationships 1, 2, 3, 4 Geometric and Spatial Relationships 3 |
|----------------------------------|-----------------------|---|
|----------------------------------|-----------------------|---|

Learner Objectives: Students will apply the laws of exponents and the laws of logarithms to simplify and solve equations.

- Concepts: N2C: Apply properties of operations
 - N2D: Apply operations on real and complex numbers
 - N3C: Compute problems
 - A1D: Identify and compare functions
 - A1E: Describe the effects of parameter changes
 - A2A; Represent mathematical situations
 - A2B: Describe and use mathematical manipulation
 - A2C: Utilize equivalent forms
 - A3A: Use mathematical models
 - A4A: Analyze change
 - G3B: Use transformations on functions

Students Should Know

Students Should Be Able to

| Understand rational exponents | Exponential functions |
|--|--|
| • Know and apply laws of exponents | • Evaluate, simplify and solve (N2D) |
| • Apply Order of Operations | • Graph using a table and/or transformations and identify asymptote |
| • Add, subtract, multiply and divide fractions | (A1D) |
| • Know and apply laws of logarithms | • Define the number e (N2D) |
| | • Solve real-life applications of exponential equations (A2A) |
| | Logarithmic functions |
| | • Change logarithmic expressions to exponential and vice-versa |
| | (A2B) |
| | \circ Use the laws of logarithms to simplify, evaluate and solve (A2C) |
| | o Graph (A2A) |
| | • Define common log and natural log (A2B) |
| | • Apply the change of base formula (A2C) |
| | • Solve real-life applications of logarithmic equations (A3A) |

| Student Essential Vocabulary | | | | | |
|------------------------------|------------------|-------------------|-------------------|-------------|----------|
| Logarithm | Exponential | Asymptote | Common Log | Natural Log | Base |
| Transformation | Y-intercept | Rational Exponent | Negative Exponent | Evaluate | Simplify |
| Log Form | Exponential Form | Antilog | Antiln | Expand | Condense |

| Sample Learning Activities | Sample Assessments |
|--|--|
| Learning Activity #1 : | Assessment #1: |
| Solve. Round answers to the nearest hundredth. | |
| $\log_x 4 = \frac{2}{2}$ | Solve. Round answers to the nearest hundredth. |
| 1. $2^{x+y} = 27$ 2. $\log_5 51 - 5x$ 3. 3 | 1. $6^{2x+1} = 12$ 2. $\log_3 21 = 2x+5$ 3. $\log_x 25 = \frac{-1}{2}$ |
| 4. $n^{\log_n 3} = x$ 5. $\log_8(x-3) - \log_8(x+4) = 1$ | |
| | 18 |

Solutions: 1. x = 1.75 2. x = 0.71 3. x = 84. x = 3 5. no solution

| Activity's Alignment | | |
|----------------------|--|--|
| CONTENT | MA 1 Number sense | |
| PROCESS | 1.10 Apply information, ideas and skills | |
| DOK | 2 | |
| INSTRUCTIONAL | Nonlinguistic Representation | |
| STRATEGIES | | |

Learning Activity #2:

The electric grid for St. Peters can handle 250,000 homes. Ten years ago there were 106,000 homes, now it is up to 175,000. If growth stays constant, when will the grid need to be modified? At that time, how many houses should it be able to handle so that it will be adequate for an additional 20 years?

Growth formulas: $y = ne^{kt}$ or $y = y_0 c^{\frac{t}{T}}$

Solution: approximately 7 years, 681340 homes

| Activity's Alignment | | |
|----------------------|------|---------------------------------|
| CONTENT | MA 4 | Patterns and relationships |
| PROCESS | 1.6 | Discover/evaluate relationships |
| DOK | 2 | |
| | | |

4.
$$14^{\log_{14} 6} = x$$
 5. $\log_4(x-3) + \log_4(x+3) = 2$

Solutions: 1. x = 0.19 2. x = -1.11 3. $x = \frac{1}{625} = 0.0016$ 4. x = 6 5. x = 5

| Assessment's Alignment | | |
|------------------------|--|--|
| CONTENT | MA 1 Number sense | |
| PROCESS | 1.10 Apply information, ideas and skills | |
| DOK | 2 | |
| LEVEL OF | Mastery Level – 75% | |
| EXPECTATION | | |

Assessment #2:

You wish to purchase a new car. You will need to borrow \$15,000. Rates are currently 5.5% for 4 years or 6% for 5 years. Calculate the monthly payments for both loans. What will your total interest be on each of these loans if you make monthly payments? Why might you choose the 4 year loan? Why might you choose the 5 year loan? Finance formulas:

$$A = Pe^{rt} \qquad A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad P_n = P\left[\frac{1 - (1 + i)^{-n}}{i}\right]$$
$$F_n = P\left[\frac{(1 + i)^n - 1}{i}\right]$$

Solution:

4 year loan: monthly payment = \$348.85 interest = \$1744.80 pay less interest 5 year loan: monthly payment = \$289.99 interest = \$2399.40 lower monthly payments

| INSTRUCTIONAL Similarities and Differences | Assessment's Alignment | |
|--|------------------------|-------------------------------------|
| STRATEGIES | CONTENT | MA 4 Patterns and relationships |
| | PROCESS | 1.6 Discover/evaluate relationships |
| | | |
| | DOK | 3 |
| | LEVEL OF | Mastery Level – 70% |
| | EXPECTATION | |
| | | |

| | Student Resources | Teacher Resources |
|----------------|--|--|
| Houghton Miff | flin; Precalculus with Limits: A Graphing Approach © 2005; | Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; |
| Larson, Hostet | ler, Edwards; ISBN #0-618-39480-X | Larson, Hostetler, Edwards |
| | | |
| Section 3.1 | Exponential Functions and Their Graphs | |
| Section 3.2 | Logarithmic Functions and Their Graphs | |
| Section 3.3 | Properties of Logarithms | |
| Section 3.4 | Solving Exponential and Logarithmic Equations | |
| Section 3.5 | Exponential and Logarithmic Models | |

| Identity Equity and Readiness | | | |
|-------------------------------|----------------------|--|--|
| Gender Equity | Technology Skills | | |
| Racial/Ethnic Equity | Research/Information | | |
| Disability Equity | Workplace/Job Prep | | |

| Content Area: Mathematics | Course: Math Analysis | Strand: 5 Numbers and Operations 1, 2, 3 Algebraic Relationships 1, 2, 4 |
|---------------------------|-----------------------|--|
| | | |

Learner Objectives: Students will recognize and evaluate the limit of a function.

Concepts: N1D: Classify and describe numeric relationships

- N2B: Describe effects of operations
- N2C: Apply properties of operations

- N2D: Apply operations on real and complex numbers
- N3D: Estimate and justify solutions
- A1A: Recognize and extend patterns
- A1B: Create and analyze patterns
- A1D: Identify and compare functions
- A2B: Describe and use mathematical manipulation
- A2C: Utilize equivalent forms
- A4A: Analyze change

| Students Should Know | Students Should Be Able to |
|---|--|
| • Understand sigma notation | • Find the limit of functions as it approaches a constant or infinity |
| Identify excluded values | \circ graphically, numerically and algebraically (N1D) |
| • Identify asymptotes and points of discontinuity | \circ left handed/right handed limits (A1A) |
| Simplify rational expressions | \circ as it applies to types of discontinuity and end behavior (A2C) |
| Graph functions | • Identify series and sequences |
| | • Expand using sigma notation and vice versa (A1B) |
| | \circ Find the limit of the series (N2D) |

| Student Essential Vocabulary | | | | | |
|------------------------------|---------------------------|--------------|--------------|---------------|-------------------|
| Limit | Numerically | Left Handed | Right Handed | Discontinuity | Sigma Notation |
| Series | Sequence | End Behavior | Approaches | Infinity | Rational Exponent |
| Asymptotes | Piecewise Function | DNE | | | |

| Sample Learning Activities | Sample Assessments |
|--|---|
| Learning Activity #1 : | Assessment #1: |
| Find each limit algebraically. | Find each limit algebraically. |
| a) $\lim_{x \to 3} \frac{x-4}{x^2-7}$ b) $\lim_{x \to 4} \frac{x-4}{x^2-x-12}$ | a) $\lim_{x \to 5} \frac{x+4}{5x+2}$ b) $\lim_{x \to 2} \frac{x^2-4}{x^2-5x+6}$ |

| $\lim_{x \to \infty} \frac{x^3 - 7x + 3x^4 - 2x}{3x^4 - 2x}$ | $\frac{1}{2} = \frac{1}{2} = \frac{1}$ | $\lim_{x \to \infty} 3x + 3x +$ | $\frac{-7}{x+1} \qquad \qquad \lim_{x \to \infty} \frac{2x^3 - 3x + 1}{5x^3 + 2x^2 + 3}$ $\frac{1}{3} \qquad \qquad b) -4 \qquad \qquad c) 0 \qquad \qquad d) \frac{2}{5}$ |
|--|--|---|--|
| , | | , | |
| | Activity's Alignment | | Assessment's Alignment |
| CONTENT | MA 4 Patterns and relationships | CONTENT | MA 4 Patterns and relationships |
| PROCESS | 3.2 Apply others' strategies | PROCESS | 3.2 Apply others' strategies |
| DOK | 2 | DOK | 2 |
| INSTRUCTIONAL | Similarities and differences | I EVEL OF | Z Mastery Laval 80% |
| STRATEGIES | Homework and practice | EXPECTATION | Mastery Level – 80% |
| Learning Activity #2 Determine the limit NOT and give the ty a) $\lim_{x\to 6} \frac{4}{x-6}$ | : of each function. If the limit DNE, explain WHY ype of discontinuity involved. $\lim_{x \to 2} \begin{cases} x^2 + 3, & x > 2\\ -2x + 1, & x \le 2 \end{cases}$ b) | Assessment #2: Determine the limit and give the type of a) $\lim_{x \to -3} \frac{x}{x+3}$ | t of each function. If the limit DNE, explain WHY NOT f discontinuity involved. $\lim_{x \to 4} \begin{pmatrix} 2x-3, & x > 4 \\ x^2-6, & x \le 4 \end{pmatrix}$ |
| Solutions: a) Limit D the denominator is und graph has a vertical as | NE. When evaluated for $x = 6$, the result is $\frac{4}{0}$. Since defined at zero (and the numerator is not zero), the ymptote at $x = 6$ resulting in an infinite discontinuity. | Solutions: a) Limit Since the denomina | DNE. When evaluated for $x = -3$, the result is $-\frac{3}{0}$. tor is undefined at zero (and the numerator is not zero), |

| b) Limit D -3. When evaluated f handed limits do not a jump discontinuity wh | NE. When evaluated from the left, the limit is rom the right, the limit is 7. Since the left and right gree, the limit as x approaches 2 DNE. The graph has a ere $x = 2$. | the graph has a vert discontinuity. b) Li: When evaluated fro limits do not agree, discontinuity where | ical asymptote at $x = -3$ resulting in an infinite mit DNE. When evaluated from the left, the limit is 5. om the right, the limit is 10. Since the left and right handed the limit as x approaches -3 DNE. The graph has a jump e x = -3. |
|---|--|---|--|
| | Activity's Alignment | | |
| CONTENT | MA 4 Patterns and relationships | | Assessment's Alignment |
| PROCESS | 1.6 Discover/evaluate relationships | CONTENT | MA 4 Patterns and relationships |
| DOK | 2 | PROCESS | 1.6 Discover/evaluate relationships |
| INSTRUCTIONAL | Nonlinguistic representation | DOV | 2 |
| STRATEGIES | Homework and practice | DOK | |
| | · · · · · · · · · · · · · · · · · · · | LEVEL OF | Mastery Level – 75% |
| | | EXPECTATION | |
| | | | |

| | Student Resources | Teacher Resources |
|---------------|---|--|
| Houghton Mi | fflin; Precalculus with Limits: A Graphing Approach © 2005; | Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; |
| Larson, Hoste | etler, Edwards; ISBN #0-618-39480-X | Larson, Hostetler, Edwards |
| | | |
| Section 8.1 | Sequences and Series | |
| Section 11.1 | Introduction to Limits (Supplemented) | |
| Section 11.2 | Techniques for Evaluating Limits (Supplemented) | |
| Section 11.3 | The Tangent Line Problem (Supplemented) | |
| Section 11.4 | Limits at Infinity and Limits of Sequences (Supplemented) | |

| Identity Equity and Readiness | | | |
|-------------------------------|----------------------|--|--|
| Gender Equity | Technology Skills | | |
| Racial/Ethnic Equity | Research/Information | | |
| Disability Equity | Workplace/Job Prep | | |

| | | Strand: 6 |
|----------------------------------|-----------------------|--|
| Content Area: Mathematics | Course: Math Analysis | Algebraic Relationships 1, 2, 4 |
| | | Geometric and Spatial Relationships 2, 3 |

Learner Objectives: Students will determine and apply basic derivatives.

Concepts: N2C: Apply properties of operations

- A1D: Identify and compare functions
- A2A: Represent mathematical situations
- A2B: Describe and use mathematical manipulation
- A4A: Analyze change
- G2A: Use coordinate systems
- G3C: Use symmetry

| Students Should Know | Students Should Be Able to |
|---|--|
| • Write an equation of a line given a point and slope | • Find the first and second derivative of a function by applying power |
| • Graph functions | rule, chain rule, product rule, and quotient rule (N2C, A1D) |
| • Evaluate a function for a given value | • Use derivatives to find instantaneous rate of change (slope of the |
| | tangent line) and critical points (A2B) |
| | • Determine the equation of a tangent line to the function at a given |
| | point and graph (A4A, G2A) |
| | • Sketch a graph of a function using critical points (A2A, G2A, G3C) |

| Student Essential Vocabulary | | | | | |
|------------------------------|---|------------------|-------------------|------------|--|
| Derivatives | Derivatives Power Rule Product Rule Quotient Rule Point of Inflection Tangent | | | | |
| Critical Point | Instantaneous Rate of Change | First Derivative | Second Derivative | Chain Rule | |

| Sample Learning Activities | | | Sample Assessments |
|---|--|--|---|
| Learning Activity #1 Find the 1 st derivat | : ive of each function. DO NOT SIMPLIFY | Assessment #1 Find the 1 st der | : ivative of each function. DO NOT SIMPLIFY. |
| a) $f(x) = 3x^5 - 4x^2$ | + 7x - 1 b) $g(x) = (5x^4 - 3x + 1)^6$ | a) $r(x) = 5x^6 +$ | $3x^4 - 2x^3 + 3x$ b) $w(x) = (7x^2 + 3x + 2)^5$ |
| c) $m(x) = (7x - 4)$ Solutions: a) $f'(x) = 15x^4 - 4$ | $\frac{5x-3}{(x^2-7)^3}$ -8x+7 b) g'(x) = $6(5x^4-3x+1)^5(20x^3-3)$ | c) $n(x) = (4x^5 - 5x^5)^{-1}$ Solutions: a) $r'(x) = 30x^5$ | $f(x) = \frac{8x+7}{(3x^4-5)^4}$ + 1) ³ (4x-1) d) d) + 12x ³ - 6x ² + 3 b)w'(x) = 5(7x ² + 3x + 2) ⁴ (14x + 3) |
| c) $m'(x) = 3(7x - 1)^{-1}$ | $(4)^{2}(7)(2x-3) + (7x-4)^{3}(2)$ | c) $n'(x) = 3(4x)$ | $(x^{5} + 1)^{2}(20x^{4})(4x - 1) + (4x^{5} + 1)^{3}(4)$ |
| h'(x) = $\frac{(x^2 - 7)}{d}$ | $\frac{x^{3}(5) - (5x - 3)(3)(x^{2} - 7)^{2}(2x)}{(x^{2} - 7)^{6}}$ | d) $f'(x) = \frac{(3x)}{(3x)^2}$ | $\frac{(x^4-5)^4(8)-(8x+7)(4)(3x^4-5)^3(12x^3)}{(3x^4-5)^8}$ |
| | A adjustanta A lignon and | | Assessment's Alignment |
| CONTENT | Activity's Alignment | CONTENT | MA 4 Patterns and relationships |
| PROCESS | 1.6Discover/evaluate relationships | PROCESS | 1.6 Discover/evaluate relationships |
| DOK | 2 | DOK | 2 |
| INSTRUCTIONAL | Similarities and Differences | LEVEL OF | Mastery Level – 80% |
| STRATEGIES | Homework and Practice | EXPECTATION | |
| Learning Activity #2: | | Assessment #2: | |
| Determine the equation given. | n of the line tangent to the function given at the point | Determine the equa given. | ation of the line tangent to the function given at the point |
| $f(x) = 5x^3 - 7x + 4; (1, $ | 2) | | |

| Solution: $y = 8x - 6$ | | $f(x) = 2x^3 + 5x^2 + 2$ Solution: $y = 4x + 2$ | 1; (-2, 5) 13 |
|------------------------------------|---------------------------------|--|---------------------------------|
| | Activity's Alignment | | Assessment's Alignment |
| CONTENT | MA 4 Patterns and relationships | CONTENT | MA 4 Patterns and relationships |
| PROCESS | 3.3 Apply one's own strategies | PROCESS | 3.3 Apply one's own strategies |
| DOK INSTRUCTIONAL STRATEGIES | 2 Homework and practice | DOK LEVEL OF EXPECTATION | 2 Mastery Level – 80% |

| Student Resources | Teacher Resources | |
|---|---|--|
| Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; Larson, Hostetler, Edwards; ISBN #0-618-39480-X | Houghton Mifflin; Precalculus with Limits: A Graphing Approach © 2005; Larson, Hostetler, Edwards | |
| Section 11.2 Techniques for evaluating limits (using difference quotient) Supplemental Resources | Appendix: Critical Points/Derivatives | |

| Identity Equity and Readiness | | | |
|-------------------------------|--|----------------------|--|
| Gender Equity | | Technology Skills | |
| Racial/Ethnic Equity | | Research/Information | |
| Disability Equity | | Workplace/Job Prep | |