

# **Ohio's State Tests**

PRACTICE TEST ANSWER KEY & SCORING GUIDELINES

> INTEGRATED MATHEMATICS II

# Table of Contents

Questions 1 – 31: Content Summary and Answer Key	. iii
Question 1: Question and Scoring Guidelines Question 1: Sample Responses	
Question 2: Question and Scoring Guidelines	
Question 3: Question and Scoring Guidelines	
Question 4: Question and Scoring Guidelines	
Question 5: Question and Scoring Guidelines	
Question 6: Question and Scoring Guidelines	
Question 7: Question and Scoring Guidelines Question 7: Sample Response	
Question 8: Question and Scoring Guidelines	
Question 9: Question and Scoring Guidelines Question 9: Sample Responses	
Question 10: Question and Scoring Guidelines	
Question 11: Question and Scoring Guidelines	
Question 12: Question and Scoring Guidelines	
Question 13: Question and Scoring Guidelines	
Question 14: Question and Scoring Guidelines11 Question 14: Sample Responses11	
Question 15: Question and Scoring Guidelines	

Question 16: Question and Scoring Guidelines	
Question 16: Sample Responses	141
Question 17: Question and Scoring Guidelines	147
Question 17: Sample Response	149
Question 18: Question and Scoring Guidelines	
Question 18: Sample Responses	
Question 19: Question and Scoring Guidelines	
Question 19: Sample Responses	
Question 20: Question and Scoring Guidelines	
Question 20: Sample Response	
Question 21: Question and Scoring Guidelines	
Question 21: Sample Responses	
Question 22: Question and Scoring Guidelines	
Question 22: Sample Responses	
Question 23: Question and Scoring Guidelines	
Question 23: Sample Responses	
Question 24: Question and Scoring Guidelines	
Question 24: Sample Response	
Question 25: Question and Scoring Guidelines	
Question 25: Sample Responses	
Question 26: Question and Scoring Guidelines	
Question 26: Sample Responses	
Question 27: Question and Scoring Guidelines	
Question 27: Sample Responses	
Question 28: Question and Scoring Guidelines	
Question 28: Sample Response	
Question 29: Question and Scoring Guidelines	
Question 29: Sample Responses	
Question 30: Question and Scoring Guidelines	
Question 30: Sample Response	249
Question 31: Question and Scoring Guidelines	
Question 31: Sample Responses	

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
1	Equation Item	Build new functions from existing functions	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k$ $f(x)$ , f(kx), and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. <i>a</i> . Focus on transformations of graphs of quadratic functions, except for $f(kx)$ . ( <i>F.BF.3a</i> )		1 point
2	Graphic Response	Analyze functions using different representations	Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. <i>a.</i> Graph linear functions and indicate intercepts. ( <i>F.IF.7a</i> )★		1 point
3	Multiple Choice	Understand and apply theorems about circles	Prove that all circles are similar using transformational arguments. (G.C.1)	С	1 point
4	Graphic Response	Define trigonometric ratios, and solve problems involving right triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
5	Multi- Select Item	Understand independence and conditional probability, and use them to interpret data	Understand the conditional probability of A given B as P (A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (S.CP.3)★	A, D, F	1 point
6	Table Item	Solve systems of equations	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$ . (A.REI.7)		2 points
7	Multi- Select Item	Create equations that describe numbers or relationships	Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. (A.CED.1)★	C, D	1 point
8	Equation Item	Apply geometric concepts in modeling situations	Apply geometric methods to solve design problems e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. (G.MG.3)★		2 points
9	Equation Item	Explain volume formulas, and use them to solve problems	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)★		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
10	Equation Item	Translate between the geometric description and the equation for a conic section	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G.GPE.1)		1 point
11	Multiple Choice	Understand similarity in terms of similarity transformations	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G.SRT.2)	A	1 point
12	Equation Item	Understand similarity in terms of similarity transformations	Verify experimentally the properties of dilations given by a center and a scale factor. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G.SRT.1b)		1 point
13	Hot Text Item	Prove and apply theorems both formally and informally involving similarity using a variety of methods	Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)		1 point

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
14	Equation Item	Define trigonometric ratios, and solve problems involving right triangles	Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)		1 point
15	Equation Item	Find arc lengths and areas of sectors of circles	Find arc lengths and areas of sectors of circles. a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (G.C.5a)		1 point
16	Table Item	Understand independence and conditional probability, and use them to interpret data	Construct and interpret two- way frequency tables of data when two categories are associated with each object being classified. Use the two- way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)★		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
17	Multiple Choice	Understand independence and conditional probability, and use them to interpret data	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)★	D	1 point
18	Equation Item	Use the rules of probability to compute probabilities of compound events in a uniform probability model	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. (S.CP.7) $\star$		1 point
19	Equation Item	Create equations that describe numbers or relationships	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (A.CED.4)★		1 point
20	Multiple Choice	Perform arithmetic operations on polynomials	Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A.APR.1b)	D	1 point

Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
21	Multi- Select Item	Interpret the structure of expressions	Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$ , students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$ ; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ . (A.SSE.2)	B, C, E	1 point
22	Equation Item	Solve equations and inequalities in one variable	Solve quadratic equations in one variable. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero- Product Property after factoring. (A.REI.4b)		1 point
23	Equation Item	Interpret functions that arise in applications in terms of the context	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)★		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
24	Multiple Choice	Interpret functions that arise in applications in terms of the context.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)★	С	1 point
25	Equation Item	Analyze functions using different representations	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F.IF.9)		1 point
26	Equation Item	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements	Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
27	Equation Item	Write expressions in equivalent forms to solve problems	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A.SSE.3b)★		1 point
28	Multiple Choice Item	Interpret functions that arise in applications in terms of the context.	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person- hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)★	A	1 point
29	Equation Item	Analyze functions using different representations	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. <i>a.</i> Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. ( <i>F.IF.8a</i> )		1 point

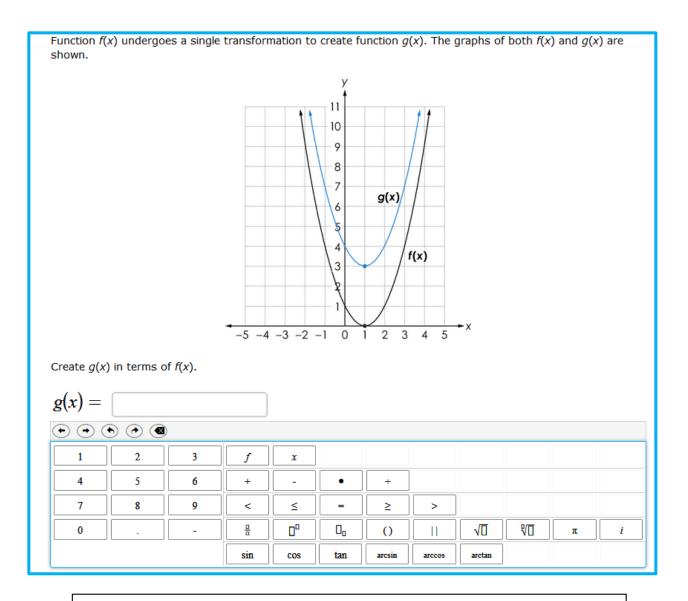
Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
30	Multiple Choice	Use the rules of probability to compute probabilities of compound events in a uniform probability model	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6) $\star$	A	1 point
31	Editing Task Choice Item	Understand and apply theorems about circles	Prove that all circles are similar using transformational arguments. (G.C.1)		1 point

Integrated Math II Practice Test

**Question 1** 

**Question and Scoring Guidelines** 

# **Question 1**



Points Possible: 1

Content Cluster: Build new functions from existing functions.

**Content Standard:** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

a. Focus on transformations of graphs of quadratic functions, except for f(kx) (F.BF.3a)

# **Scoring Guidelines**

#### Exemplar Response

• g(x) = f(x) + 3

#### Other Correct Responses

• Any equivalent equation.

For this item, a full-credit response includes:

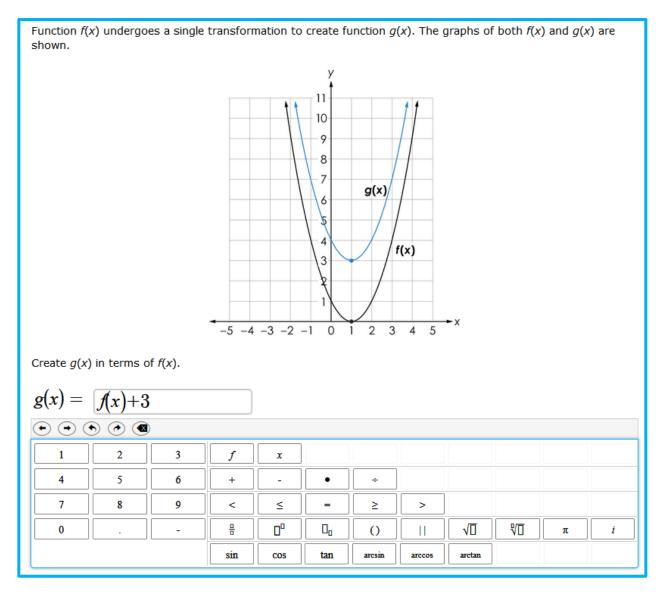
• A correct equation (1 point).

Integrated Math II Practice Test

**Question 1** 

Sample Responses

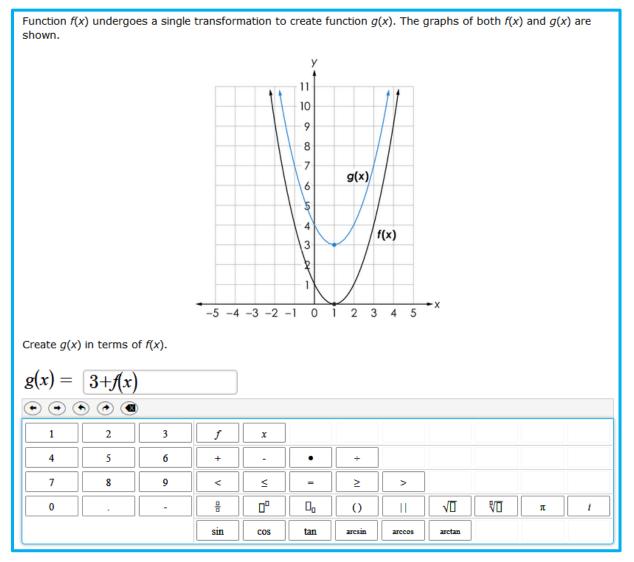
### Sample Response: 1 point



#### Notes on Scoring

This response earns full credit (1 point) because it shows a correct function g(x) in terms of f(x). The response correctly recognizes a single vertical transformation of 3 units up performed on a graph of f(x) to create the graph of g(x). The transformation translating a graph of f(x), by k units up if k is positive and down if k is negative, is represented by the equation g(x) = f(x) + k. When k = 3, g(x) = f(x) + 3.

## Sample Response: 1 point

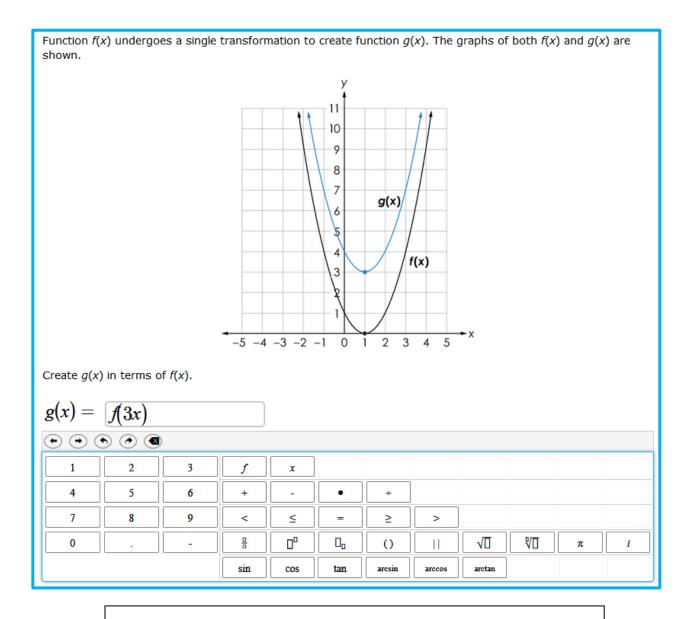


#### Notes on Scoring

This response earns full credit (1 point) because it shows a correct function g(x) in terms of f(x). The response correctly recognizes a single vertical transformation of 3 units up performed on a graph of f(x) to create the graph of g(x). The transformation translating a graph of f(x), by k units up if k is positive and down if k is negative, is represented by the equation g(x) = f(x) + k. When k = 3, g(x) = f(x) + 3.

The same equation using a commutative property over the addition can be written in the form of g(x) = k + f(x) or g(x) = 3 + f(x).

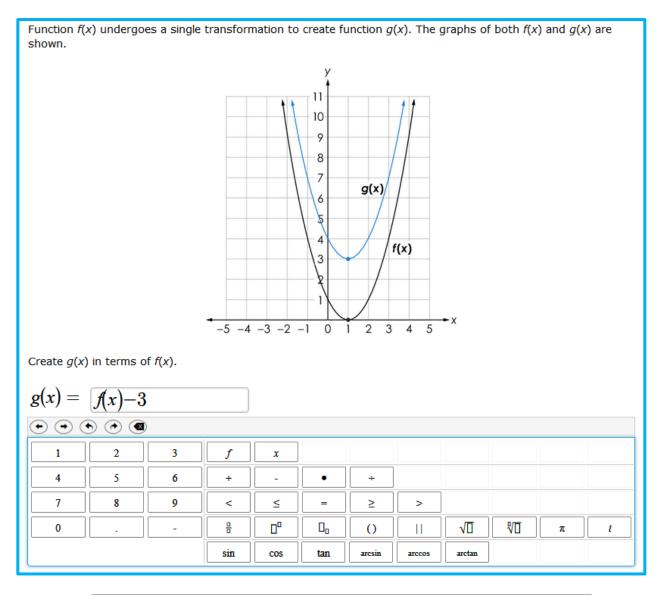
#### Sample Response: 0 points



#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an equation representing a transformation other than a vertical translation of the original graph f(x) up 3 units. The graph of g(x) = f(3x) will show a horizontal compression of the graph of f(x), not a vertical translation.

#### Sample Response: 0 points



#### Notes on Scoring

This response earns no credit (0 points) because it shows an equation representing a transformation other than a vertical translation of the original graph f(x) up 3 units. The graph of g(x) = f(x) - 3 will show a translation of the graph of f(x) down by 3 units.

Integrated Math II Practice Test

**Question 2** 

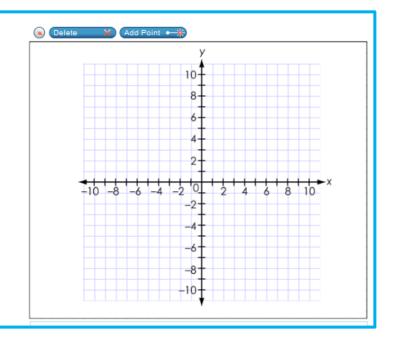
**Question and Scoring Guidelines** 

# **Question 2**

A function is shown.

$$f(x) = x^2 + 2x - 3$$

Use the Add Point tool to show the *x*-intercepts and maximum or minimum of the function.



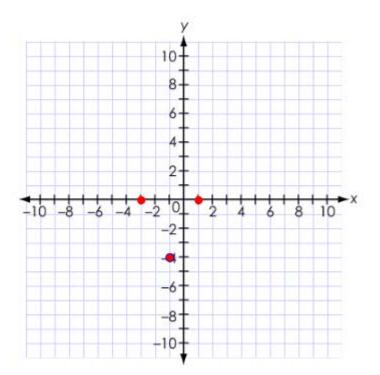
#### Points Possible: 1

Content Cluster: Analyze functions using different representations

**Content Standard:** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. *a.* Graph linear functions and indicate intercepts. (*F.IF.7a*)  $\star$ 

# **Scoring Guidelines**

#### Exemplar Response



#### Other Correct Responses

• N/A

For this item, a full-credit response includes:

• The correct *x*-intercepts and minimum of the function (1 point).

Integrated Math II Practice Test

**Question 2** 

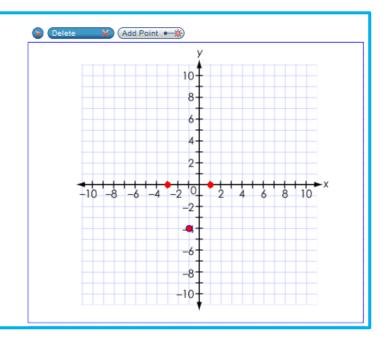
Sample Responses

#### Sample Response: 1 point

A function is shown.

$$f(x) = x^2 + 2x - 3$$

Use the Add Point tool to show the *x*-intercepts and maximum or minimum of the function.

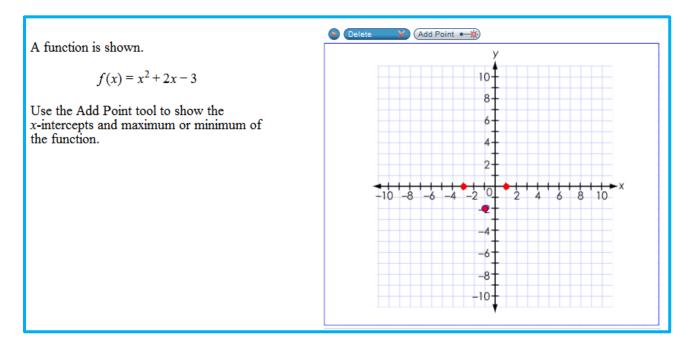


#### **Notes on Scoring**

This response earns full credit (1 point) for placing all three points, two *x*-intercepts and one minimum of the function, in the correct locations on a coordinate grid.

The x-intercepts and the minimum point of f(x) can be found by graphing a function on a calculator and locating the vertex (minimum) point at (-1, 4) and the two points of intersection with the x-axis at (-3, 0) and (1, 0).

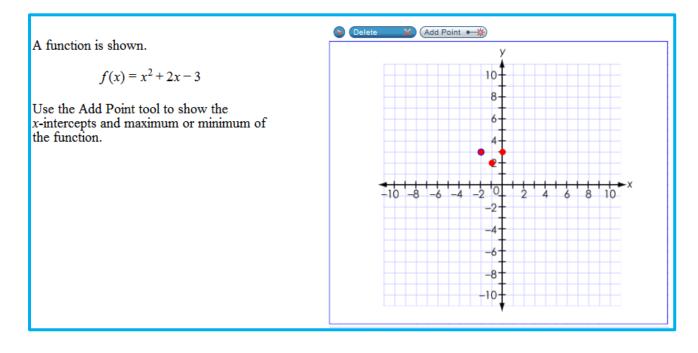
### Sample Response: 0 points



#### **Notes on Scoring**

This response earns no credit (0 points) because one out of three points is plotted incorrectly. The correct x-intercepts are at (-3, 0) and (1, 0). The minimum point (-1, -2) is incorrect.

#### Sample Response: 0 points



#### Notes on Scoring

This response earns no credit (0 points) because all three points are plotted incorrectly. The response shows points at (-2, 3), (-1, 2) and (0, 3), instead of (-3, 0), (1, 0) and (-1, -4).

Integrated Math II Practice Test

**Question 3** 

**Question and Scoring Guidelines** 

# **Question 3**

Circle J is located in the first quadrant with center (a, b) and radius s. Felipe transforms Circle J to prove that it is similar to any circle centered at the origin with radius t.

Which sequence of transformations did Felipe use?

- A Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{t}{c}$ .
- <sup>(B)</sup> Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{s}{t}$ .
- <sup>C</sup> Translate Circle J by (x-a, y-b) and dilate by a factor of  $\frac{t}{s}$ .
- Translate Circle J by (x-a, y-b) and dilate by a factor of  $\frac{s}{t}$ .

Points Possible: 1

Content Cluster: Understand and apply theorems about circles

**Content Standard:** Prove that all circles are similar using transformational arguments. (*G.C.1*)

# **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have noticed that Circle J is located to the right of Circle O, and may have thought that he or she needed to translate the center of Circle O to the right a units and up b units, and to use addition to represent this translation.

<u>Rationale for Option B:</u> This is incorrect. The student may have noticed that Circle J is located to the right of Circle O, and may have thought that he or she needed to translate the center of Circle O to the right a units and up b units and to use addition to represent this translation. The student may have also used the inverse of the correct scale factor. <u>Rationale for Option C:</u> **Key** – The student recognized that translating Circle J with the center at (a, b) to the origin (0, 0) involves a subtraction of a units from the x-coordinate and b units from the y-coordinate of the center of Circle J. Dilation by a scale factor  $\frac{t}{s}$  (radius of the image/radius of the pre-image) overlays Circle J on any other circle centered at the origin with radius t, proving a similarity.

<u>Rationale for Option D:</u> This is incorrect. The student recognized that translating Circle J with the center at (a, b) to the origin (0, 0) involves a subtraction of a units from the x-coordinate and b units from the y-coordinate of the center of Circle J but he or she used the inverse of the correct scale factor.

#### Sample Response: 1 point

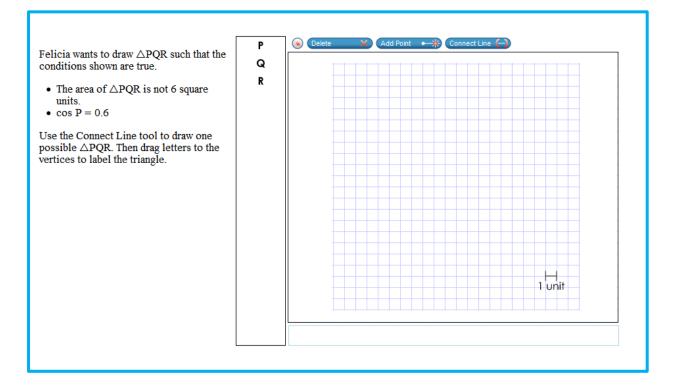
Circle J is located in the first quadrant with center (*a*, *b*) and radius *s*. Felipe transforms Circle J to prove that it is similar to any circle centered at the origin with radius *t*. Which sequence of transformations did Felipe use? (A) Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{t}{s}$ . (B) Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{s}{t}$ . (C) Translate Circle J by (x - a, y - b) and dilate by a factor of  $\frac{t}{s}$ . (C) Translate Circle J by (x - a, y - b) and dilate by a factor of  $\frac{t}{s}$ . (C) Translate Circle J by (x - a, y - b) and dilate by a factor of  $\frac{t}{s}$ .

Integrated Math II Practice Test

**Question 4** 

**Question and Scoring Guidelines** 

# **Question 4**



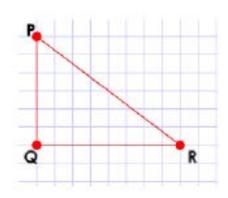
#### Points Possible: 1

**Content Cluster:** Define trigonometric ratios, and solve problems involving right triangles

**Content Standard:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (*G.SRT.6*)

# **Scoring Guidelines**

Exemplar Response



Other Correct Responses

• Any right triangle for which the relationship

# $\frac{\text{the length of the leg adjacent to angle P}}{\text{hypotenuse}} = 0.6$

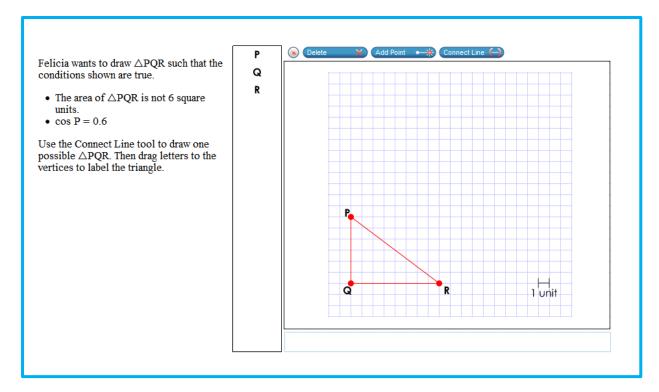
holds and whose area is not 6 square units.

For this item, a full-credit response includes:

• A correct triangle (1 point).

**Question 4** 

Sample Responses



#### Notes on Scoring

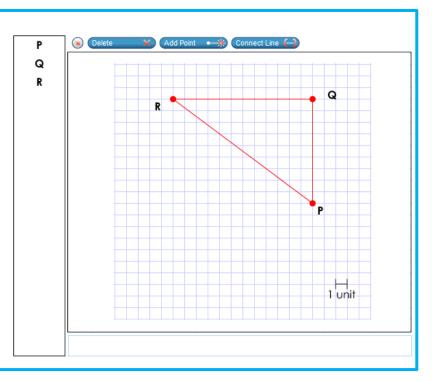
This response earns full credit (1 point) because it shows a correct triangle with  $\cos P = \frac{6}{10}$  or  $\frac{3}{5}$  and the area  $A = \frac{1}{2} \bullet 8 \bullet 6 = 24$  sq units.

This item asks students to draw and label a right triangle PQR that has an area that is not 6 sq units and where  $\cos P = 0.6$ . In right triangles, the cosine of an angle equals to the length of the adjacent leg over the length of the hypotenuse. Since  $\cos P = 0.6$ , the length of the adjacent leg/length of the hypotenuse is  $0.6 \text{ or } \frac{6}{10}$ . From here, the length of an adjacent leg can be 6 units, and the length of the hypotenuse is 10 units. Therefore, by the Pythagorean Theorem, the length of the leg that is opposite the angle P is 8 units. Drawing any right triangle with the relationship "the length of the leg adjacent to vertex P over the length of the hypotenuse equals 0.6", and with the leg adjacent to P not 3 units long, yields a correct response. A right triangle PQR with side lengths 3, 4 and 5 units long has a  $\cos P = 0.6$  and an area of 6 sq units ( $A = \frac{1}{2}bh$ ), which contradicts the given condition and, therefore, is not a correct response.

Felicia wants to draw  $\triangle$  PQR such that the conditions shown are true.

- The area of △PQR is not 6 square units.
- $\cos P = 0.6$

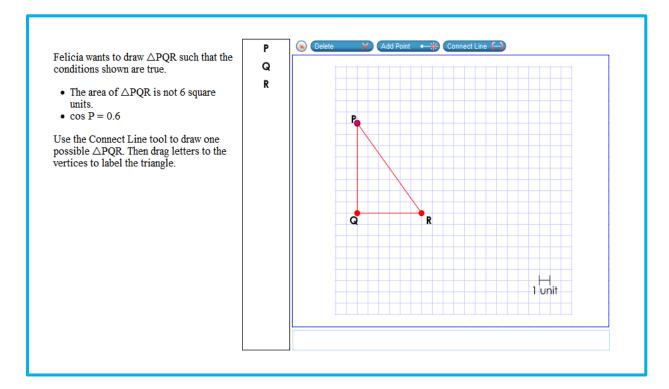
Use the Connect Line tool to draw one possible  $\triangle$ PQR. Then drag letters to the vertices to label the triangle.



#### Notes on Scoring

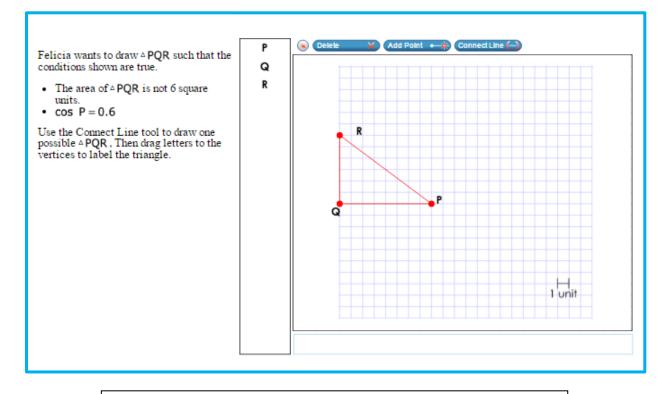
This response earns full credit (1 point) because it shows a correct triangle with  $\cos P = \frac{9}{15}$  or  $\frac{3}{5}$  and the area  $A = \frac{1}{2} \bullet 12 \bullet 9 = 54$  sq units.

This item asks students to draw and label a right triangle PQR that has an area that is not 6 sq units and where  $\cos P = 0.6$ . In right triangles, the cosine of an angle equals to the length of the adjacent leg over the length of the hypotenuse. Since  $\cos P = 0.6$ , the length of the adjacent leg/length of the hypotenuse is  $0.6 \text{ or } \frac{3}{5} \text{ or } \frac{9}{15}$ . From here, the length of an adjacent leg can be 9 units, and the length of the hypotenuse is 15 units. Therefore, by the Pythagorean Theorem, the length of the leg that is opposite to the angle P is 12 units. Drawing any right triangle with the relationship "the length of the leg adjacent to vertex P over the length of the hypotenuse equals 0.6", and with the leg adjacent to P not 3 units long, yields a correct response. A right triangle PQR with side lengths 3, 4 and 5 units has a  $\cos P = 0.6$  and an area of 6 sq units ( $A = \frac{1}{2}bh$ ), which contradicts the given condition and, therefore, is not a correct response.



#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect triangle with  $\cos P = \frac{8}{10}$  or  $\frac{4}{5}$  instead of  $\frac{6}{10}$ .



### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect triangle with sin P =  $\frac{6}{10}$ , but cos P =  $\frac{8}{10}$  or  $\frac{4}{5}$ , instead of cos P =  $\frac{6}{10}$ .

**Question 5** 

**Question and Scoring Guidelines** 

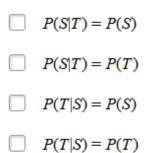
# **Question 5**

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.



 $P(S \cup T) = P(S) \bullet P(T)$ 

 $P(S \cap T) = P(S) \bullet P(T)$ 

Points Possible: 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data

**Content Standard:** Understand the conditional probability of A given B as P (A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (S.CP.3)

 $(\star)$  indicates that modeling should be incorporated into the standard.

# **Scoring Guidelines**

<u>Rationale for First Option</u>: **Key** – The student correctly identified that if the two events are independent, then the conditional probability of S given T, or P(S|T), equals to a probability of S, or P(S).

<u>Rationale for Second Option</u>: This is incorrect. The student may have thought that the probability of S given T, or P(S | T), is defined by the conditional event, P(T).

<u>Rationale for Third Option</u>: This is incorrect. The student may have thought that the probability of T given S, or  $P(T \mid S)$ , is defined by the probability of the conditional event S or P(S).

<u>Rationale for Fourth Option</u>: **Key** – The student correctly identified that if the two events are independent, then the conditional probability of T given S, or P(T|S), must be equal to a probability of T, or P(T).

<u>Rationale for Fifth Option</u>: This is incorrect. The student may have mistaken "union" for "intersection" of the probabilities, and concluded that the probability of a union of two events  $P(T \cup S)$  is the product of probabilities,  $P(S) \bullet P(T)$ .

<u>Rationale for Sixth Option:</u> **Key** – The student correctly identified that if two events S and T are independent, then the probability of the intersection of two events, or events occurring together,  $P(S \cap T)$ , is equal to the product of their probabilities,  $P(S) \bullet P(T)$ .

**Question 5** 

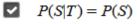
Sample Responses

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.



$$P(S|T) = P(T)$$

$$P(T|S) = P(S)$$

 $\square P(T|S) = P(T)$ 

$$\square \quad P(S \cup T) = P(S) \bullet P(T)$$

 $\square P(S \cap T) = P(S) \bullet P(T)$ 

### Notes on Scoring

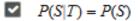
This response receives full credit (1 point) because it selects all three correct answer choices, A, D and F, and no incorrect answer choices.

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.



$$\square P(S|T) = P(T)$$

$$P(T|S) = P(S)$$

 $\square P(T|S) = P(T)$ 

$$P(S \cup T) = P(S) \bullet P(T)$$

$$\square P(S \cap T) = P(S) \bullet P(T)$$

#### **Notes on Scoring**

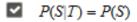
This response receives no credit (0 points) because it selects three correct and one incorrect answer choices.

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.



$$\square P(S|T) = P(T)$$

P(T|S) = P(S)

$$\square P(T|S) = P(T)$$

$$\square \quad P(S \cup T) = P(S) \bullet P(T)$$

 $\square P(S \cap T) = P(S) \bullet P(T)$ 

#### **Notes on Scoring**

This response receives no credit (0 points) because it selects only two correct answer choices.

**Question 6** 

**Question and Scoring Guidelines** 

# **Question 6**

A system of equations is shown.

y + 1 = 2x $x^2 - 4x = y - 4$ 

What are the solutions to the system of equations?

(	,	)
(	,	)

**Points Possible:** 2

Content Cluster: Solve systems of equations

Content Standard: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle  $x^2 + y^2 = 3$ . (A.REI.7)

# **Scoring Guidelines**

Exemplar Response

- (1, 1)
- (5,9)

### Other Correct Responses

• The order of the ordered pair solutions can be switched

For this item, a full-credit response includes:

- One correct solution (1 point)
   AND
- Another correct solution (1 point).

**Question 6** 

Sample Responses

A system of equations is shown. y + 1 = 2x  $x^2 - 4x = y - 4$ What are the solutions to the system of equations? (11, 1) (5, 9))

#### Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions (ordered pairs) to a system that consists of a linear equation and a simple guadratic equation in two variables. There are several methods to solve the system. One method is to rearrange the first equation to represent y in terms of x, or y = 2x - 1, and then substitute the expression (2x - 1) for y into the other equation to create a quadratic equation with only one variable, or  $x^2 - 4x = (2x - 1) - 4$ . Next, change the quadratic equation into standard form, or  $x^{2} - 6x + 5 = 0$ , and then factor it as (x - 1)(x - 5) = 0. By the Zero – Product Property, each factor in the product has to be set equal to zero, or x - 1 = 0 and x - 5 = 0. From here, x = 1and x = 5. By substituting each x-value back into the equation y = 2x - 1, the corresponding y-values are y = 2(1) - 1 = 1and y = 2(5) - 1 = 9. Since the solutions to the system are ordered pairs, the correct answers are (1, 1) and (5, 9).

A system of equations is shown. y + 1 = 2x  $x^2 - 4x = y - 4$ What are the solutions to the system of equations? (5, 9) (1, 1)

#### Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions (ordered pairs) to a system that consists of a linear equation and a simple quadratic equation in two variables. There are several methods to solve the system. One method is to rearrange the first equation to represent y in terms of x, or y = 2x - 1, and then substitute the expression (2x - 1) for y into the other equation to create a quadratic equation with only one variable, or  $x^2 - 4x = (2x - 1) - 4$ . Next, change the quadratic equation into standard form, or  $x^2 - 6x + 5$  and then factor it as (x - 1)(x - 5) = 0. By the Zero – Product Property, each factor in the product has to be set equal to zero, or x - 1 = 0 and x - 5 = 0. From here, x = 1and x = 5. By substituting each x-value back into the equation y = 2x - 1, the corresponding y-values are y = 2(1) - 1 = 1and y = 2(5) - 1 = 9. Since the solutions to the system are ordered pairs, and the order of these ordered pairs does not matter, the response (5, 9) and (1, 1) is also correct.

A system of equations is shown. y + 1 = 2x  $x^2 - 4x = y - 4$ What are the solutions to the system of equations? (11, 1) (5, 5)

#### Notes on Scoring

This response earns partial credit (1 point) because it shows only one correct solution (ordered pair) to a system of a linear equation and a simple quadratic equation in two variables. The correct response for the second ordered pair is (5, 9), not (5, 5).

A system of equations is shown. y + 1 = 2x  $x^2 - 4x = y - 4$ What are the solutions to the system of equations? (11, 1) (6, 11)

#### Notes on Scoring

This response earns partial credit (1 point) because it shows only one correct solution (ordered pair) to a system of a linear equation and a simple quadratic equation in two variables. The correct response for the second ordered pair is (5, 9), not (6, 11).

A system of equations is shown. y + 1 = 2x  $x^2 - 4x = y - 4$ What are the solutions to the system of equations? ( -1 , -1 ) ( -5 , -9 )

#### Notes on Scoring

This response earns no credit (0 points) because it shows no correct solutions (ordered pairs) to a system of a linear equation and a simple quadratic equation in two variables.

A system of equations is shown. y + 1 = 2x  $x^2 - 4x = y - 4$ What are the solutions to the system of equations? ( 0 , -1 ) ( 4 , 7 )

#### Notes on Scoring

This response earns no credit (0 points) because it shows no correct solutions (ordered pairs) to a system of a linear equation and a simple quadratic equation in two variables.

**Question 7** 

**Question and Scoring Guidelines** 

# **Question 7**

An inequality is shown.

(x-5)(x+2) < 0

Select all of the numbers that belong to the solution set of this inequality.

-3
-2
-1
2
5

Points Possible: 1

**Content Cluster:** Create equations that describe numbers or relationships

**Content Standard:** Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. (A.CED.1)★

 $(\star)$  indicates that modeling should be incorporated into the standard.

# **Scoring Guidelines**

<u>Rationale for First Option</u>: The student may incorrectly evaluate the inequality (x-5)(x+2) < 0 for x = -3 as  $(-3-5)(-3+2) = -8 \cdot 1 = -8 < 0$  instead of 8 > 0.

<u>Rationale for Second Option</u>: The student may incorrectly substitute x = -2 into the weak inequality  $(x - 5)(x + 2) \le 0$  instead of the strict inequality (x - 5)(x + 2) < 0, realizing that when the weak inequality is evaluated for x = -2, the result is a true statement, 0 = 0, which makes -2 a solution.

<u>Rationale for Third Option</u>: **Key** – The student correctly evaluates the inequality (x - 5)(x + 2) < 0 for x = -1 to get  $(-1 - 5)(-1 + 2) = -6 \cdot 1 = -6$  and correctly notes that -6 < 0.

<u>Rationale for Fourth Option</u>: **Key** – The student correctly evaluates the inequality (x - 5)(x + 2) < 0 for x = 2 to get  $(2 - 5)(2 + 2) = -3 \cdot 4 = -12$  and correctly notes that -12 < 0.

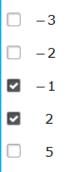
<u>Rationale for Fifth Option</u>: The student may incorrectly substitute x = 5 into the weak inequality  $(x - 5)(x + 2) \le 0$  instead of the strict inequality  $(x - 5)(x + 2) \le 0$ , realizing that when the weak inequality is evaluated for x = 5, the result is a true statement, 0 = 0, which makes 5 a solution.

### Sample Response: 1 point

An inequality is shown.

(x-5)(x+2) < 0

Select all of the numbers that belong to the solution set of this inequality.



**Question 8** 

**Question and Scoring Guidelines** 

# **Question 8**

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.												
The large size box must be designed to have a volume of 1,000 cubic centimeters.												
A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.												
B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?												
A												
B. Length of Base = centimeters												
B. Heig	ht of Tri	iangulai	· Face	=				centin	neters			
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Points Possible: 2

Content Cluster: Apply geometric concepts in modeling situations

**Content Standard:** Apply geometric methods to solve design problems e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.  $(G.MG.3) \star$ 

 $(\star)$  indicates that modeling should be incorporated into the standard.

# **Scoring Guidelines**

#### Exemplar Response

- A.  $1000 = 64y^3$
- B. Length of Base = 20
- B. Height of Triangular Face = 12.5

#### Other Correct Responses

- Any equivalent equation for Part A.
- Any equivalent values for Part B.

For this item, a full-credit response includes:

• A correct equation for Part A (1 point);

AND

• A correct set of values for Part B (1 point).

Note: Students receive 1 point if their answer for Part A is equivalent to  $1000 = \frac{1}{3} (8y)^2 \cdot 5y$ , and if their answer for Part B is correct based on this incorrect equation.

**Question 8** 

Sample Responses

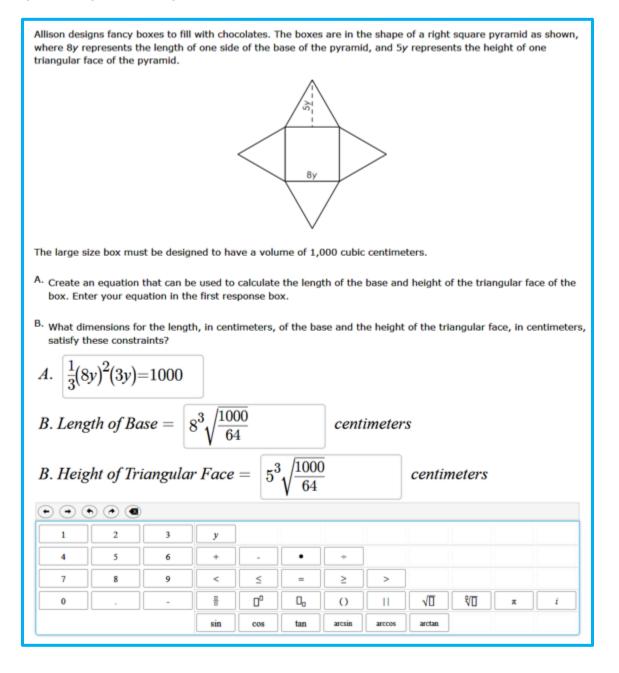
Allison desi where 8y n triangular f	epresents th	ne length o									
				<	8y		>				
The large s	ize box mus	t be desig	ned to ha	ve a volu	ime of 1,	000 cubio	c centime	ters.			
A. Create a box. Ent	n equation er your equ					th of the	base and	d height o	f the tria	ngular fa	ce of the
	mensions for hese constr $00=64y^3$	aints?	h, in cent	imeters,	of the ba	ise and th	he height	of the tri	angular f	ace, in co	entimeters,
B. Leng	th of Ba	se = 2	20			cent	imeter	5			
B. Heig	ht of Tri	angular	· Face	= 12	2.5	~		centin	neters		
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#### Notes on Scoring

This response earns full credit (2 points) because it shows a correct equation that can be used to calculate the volume of a square pyramid and the two correct values for the length of the base and a correct height of the triangular face.

The formula for the volume of a square pyramid is  $V = \frac{1}{3}Bh$ , where B is the area of the square base and h is the height of the pyramid. Since the length of the square base is 8y, the area of the square base is  $(8y)^2 = 64y^2$ . A cross-section of a pyramid that is created by a plane cut through the apex and that is perpendicular to the base forms an isosceles triangle. A half of this triangle is a right triangle with one leg being the height of a pyramid; another leg is half of a side of the square base, 4y, and the hypotenuse is the height of the triangular face, 5y. Dimensions of this right triangle are 3y, 4y and 5y (Pythagorean triple), where 3y is the height of the pyramid. Thus, an equation representing the volume of the pyramid is  $1000 = \frac{1}{2} \cdot 64y^3 \cdot 3y$  or  $64y^3 = 1000$ .

The correct solution to this equation is y = 2.5. Using this value, the length of the base is 8y or  $8 \cdot 2.5 = 20 \text{ cm}$ , and the height of the triangular face is 5y or 12.5 cm.



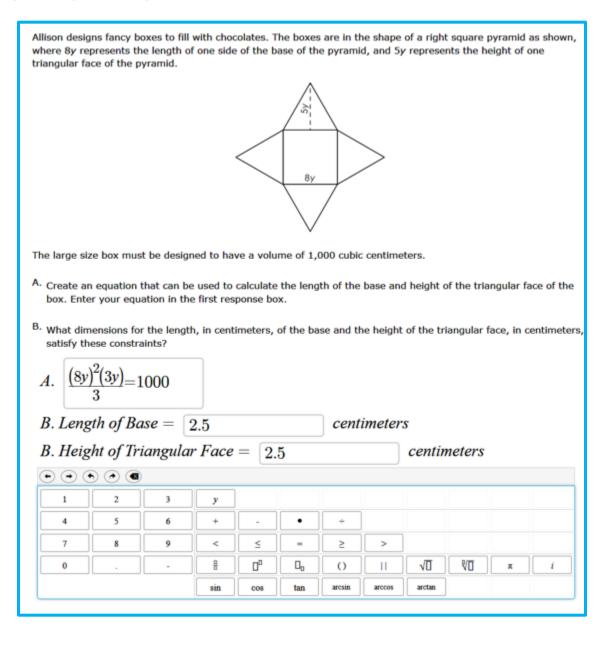
64

#### Notes on Scoring

This response earns full credit (2 points) because it shows a correct equation that can be used to calculate the volume of a square pyramid and the two correct values for the length of the base and a correct height of the triangular face.

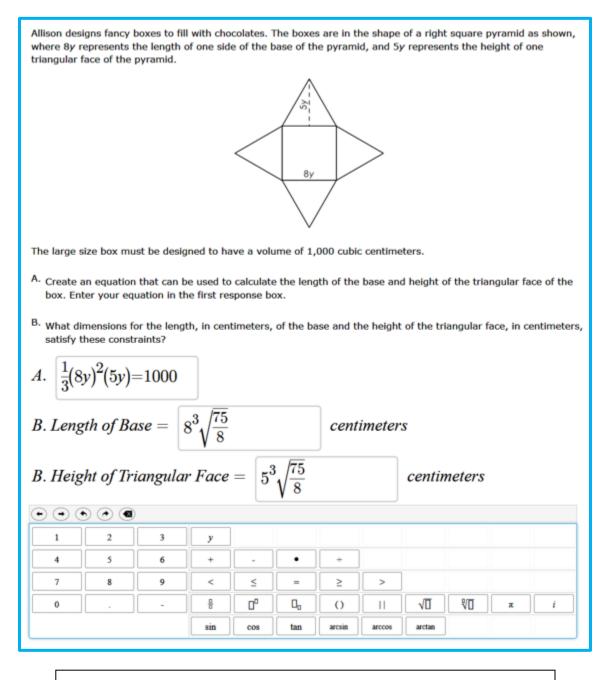
The formula for the volume of a square pyramid is  $V = \frac{1}{3}Bh$ , where B is the area of the square base and h is the height of the pyramid. Since the length of the square base is 8y, the area of the square base is  $(8y)^2 = 64y^2$ . A cross-section of a pyramid that is created by a plane cut through the apex and that is perpendicular to the base forms an isosceles triangle. A half of this triangle is a right triangle with one leg being the height of a pyramid; another leg is half of a side of the square base, 4y, and the hypotenuse is the height of the triangular face, 5y. Dimensions of this right triangle are 3y, 4y and 5y (Pythagorean triple), where 3y is the height of the pyramid. Thus, an equation representing the volume of the pyramid is  $1000 = \frac{1}{3} \cdot 64y^3 \cdot 3y$  or  $64y^3 = 1000$ .

The correct solution to this equation is y = 2.5. Using this value, the length of the base is 8y or  $8 \cdot 2.5 = 20 \text{ cm}$ , and the height of the triangular face is 5y or 12.5 cm.



#### Notes on Scoring

This response earns partial credit (1 point) because it shows an equation equivalent to the correct equation that can be used to calculate the volume of the square pyramid. The length of the base and the height of the triangular face are incorrect.



### Notes on Scoring

This response earns partial credit (1 point) because it shows an incorrect equation (the height of the triangular face is mistakenly used instead of the height of the pyramid) to calculate the volume of a square pyramid, but correctly shows the length of the base and the height of the triangular face, based on this incorrect equation.

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid. 8y The large size box must be designed to have a volume of 1,000 cubic centimeters. A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box. <sup>B.</sup> What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?  $\frac{1}{3}(8y)^2(5y)=1000$ A. B. Length of Base = 2.1centimeters *B. Height of Triangular Face* = 2.1centimeters • • • • • • 2 3 1 у 4 5 6 + . ÷ 7 8 9 < ≤ ≥ > -0 믕 00 00 0 Ш ЛŪ ۷Ū tan arcsin arccos arctan sin 008

#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation and incorrect values for the length of the base and the height of the triangular face.

Allison designs fancy bo where 8y represents the triangular face of the py	e length of o									
			<			>				
The large size box must	be designe	d to hav	e a volun	ne of 1,0	000 cubic	centime	ters.			
<ul> <li>A. Create an equation the box. Enter your equa</li> <li>B. What dimensions for</li> </ul>	tion in the f	irst resp	onse box	ς.						
satisfy these constrai	nts?		,							-,
A. $8y \cdot 5y = 1000$ B. Length of Bag					aant	imator				
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## Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation and incorrect values for the length of the base and the height of the triangular face.

**Question 9** 

**Question and Scoring Guidelines** 

# **Question 9**

The planets in our	solar system can b	e modeled using	spheres. The diameters for Jupiter and Saturn are shown in the diagram.
The volume of Sat What is Saturn's d			km d
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Points Possible: 1

**Content Cluster:** Explain volume formulas, and use them to solve problems

**Content Standard:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.  $(G.GMD.3) \star$ 

 $(\star)$  indicates that modeling should be incorporated into the standard.

# Scoring Guidelines

### Exemplar Response

• 120530.340

### Other Correct Responses

• Any value between 120530 and 120531.

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 9** 

Sample Responses

Jupiter Saturn Saturn Jupiter Jupiter Jupiter Jupiter Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn Saturn S	The planets in our	solar system can l	be modeled using sp	heres. The diameters for Jupiter and Saturn are shown in the diagram.
$\begin{array}{c} \bullet \bullet$			volume of Jupiter.	
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	$\bullet \bullet \bullet$	•		
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### Notes on Scoring

This response earns full credit (1 point) because it shows a correct answer for Saturn's diameter, in kilometers, rounded to the nearest thousandth.

In this situation, the correct solution process uses the formula for the volume of a sphere,  $V = \frac{4}{3} (\pi \cdot r^3)$ , and the formula for the radius of a sphere being half of the diameter. A solution process may consist of two parts. In the first part, the process identifies the radius of Jupiter being half of the diameter, then uses the formula for finding the volume of Jupiter. In the second part, the process is reversed. First, it applies 59.9% to find the volume of Saturn, then it uses the formula for the volume of a sphere to find the radius of Saturn, and then it doubles the radius to find the diameter of Saturn

The planets in ou	e planets in our solar system can be modeled using spheres. The diameters for Jupiter and Saturn are shown in the diagram.						
The volume of S	aturn is 59.9% the	Jupiter 142,984 kr	Saturn				
			r answer to the pagest they and th				
	diameter, <i>d</i> , in kilo		r answer to the nearest thousandth.				
What is Saturn's	diameter, <i>d</i> , in kilo	meters? Round you:	r answer to the nearest thousandth.				
What is Saturn's	diameter, <i>d</i> , in kilo	meters? Round you:	r answer to the nearest thousandth.				
What is Saturn's $120530$	diameter, <i>d</i> , in kilo	meters? Round you:	r answer to the nearest thousandth.				
What is Saturn's	diameter, <i>d</i> , in kilo	meters? Round you:	r answer to the nearest thousandth.				
What is Saturn's	diameter, <i>d</i> , in kilo .000 .000 .2 .2 .5 .	meters? Round you:	r answer to the nearest thousandth.				

#### Notes on Scoring

This response earns full credit (1 point) because it shows a correct answer for Saturn's diameter, in kilometers, rounded to an allowable value between 120530 and 120531.

In this situation, the correct solution process uses the formula for the volume of a sphere,  $V = \frac{4}{3}(\pi \cdot r^3)$ , and the formula for the radius of a sphere being half of the diameter. A solution process may consist of two parts. In the first part, the process identifies the radius of Jupiter being the half of the diameter, then uses the formula for finding the volume of Jupiter. In the second part, the process is reversed. First, it applies 59.9% to find the volume of Saturn, then it uses the formula for the volume of a sphere to find the radius of Saturn, and then it doubles the radius to find the diameter of Saturn.

The planets in ou	he planets in our solar system can be modeled using spheres. The diameters for Jupiter and Saturn are shown in the diagram.						
	uturn is 59.9% the v diameter, <i>d,</i> in kilo		Saturn				
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## Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer for Saturn's diameter, in kilometers, rounded to the nearest thousandth.

The planets in our	The planets in our solar system can be modeled using spheres. The diameters for Jupiter and Saturn are shown in the diagram.						
The volume of Sat What is Saturn's d		_	Saturn				
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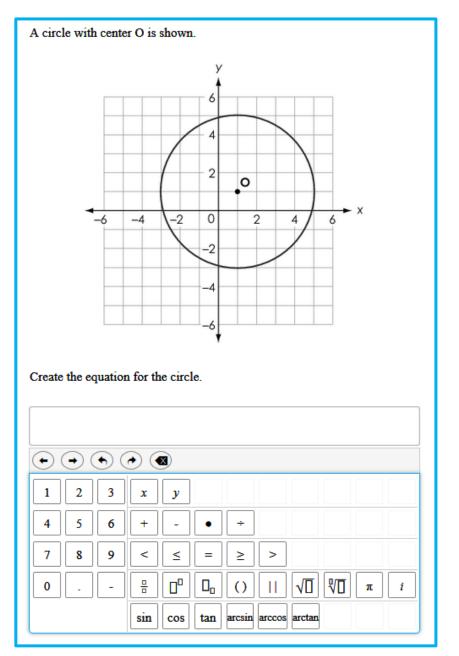
# Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer for Saturn's diameter, in kilometers, rounded to the nearest thousandth.

Question 10

**Question and Scoring Guidelines** 

# **Question 10**



#### Points Possible: 1

**Content Cluster:** Translate between the geometric description and the equation for a conic section

**Content Standard:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (*G.GPE.1*)

# **Scoring Guidelines**

Exemplar Response

•  $(x-1)^2 + (y-1)^2 = 4^2$ 

#### Other Correct Responses

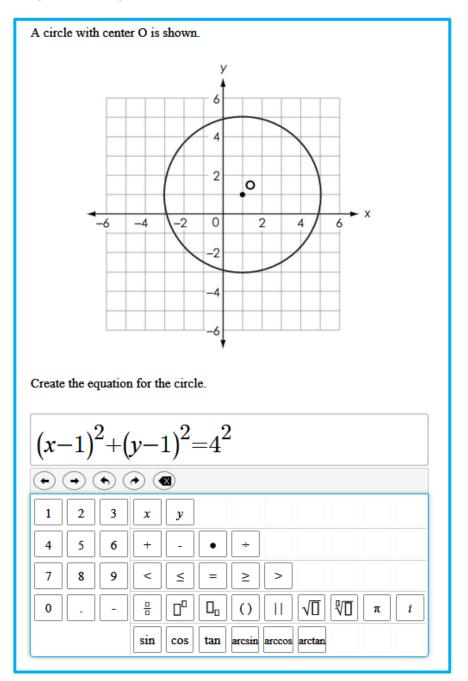
• Any equivalent equation.

For this item, a full-credit response includes:

• A correct equation (1 point).

**Question 10** 

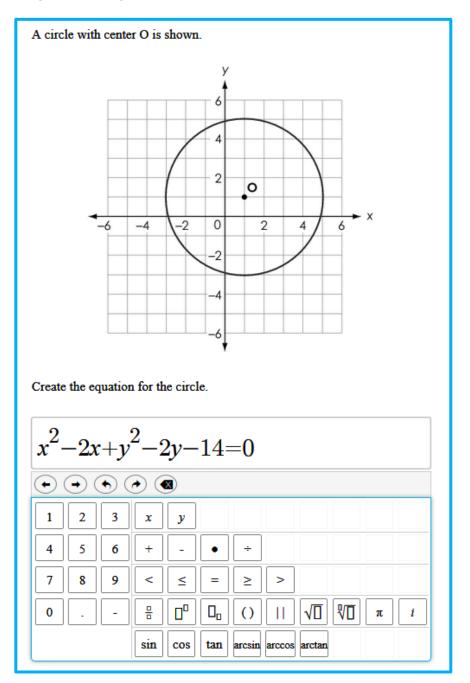
Sample Responses



#### Notes on Scoring

This response earns full credit (1 point) because it shows the correct center-radius form for the equation of the circle  $(x - 1)^2 + (y - 1)^2 = 4^2$ .

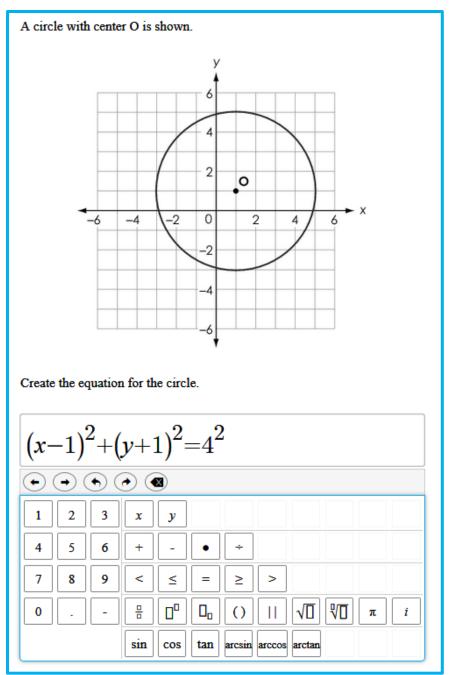
On the coordinate plane, the center-radius form for the equation of a circle with center (h, k) and radius r is  $(x - h)^2 + (y - k)^2 = r^2$ . The given circle has a center at (1, 1) and a radius of 4 units. By substituting h = 1, k = 1 and r = 4 in the center-radius form for h, k and r, respectively, the equation of the circle is  $(x - 1)^2 + (y - 1)^2 = 4^2$ , which is equivalent to the equation  $(x - 1)^2 + (y - 1)^2 = 16$ .



### Notes on Scoring

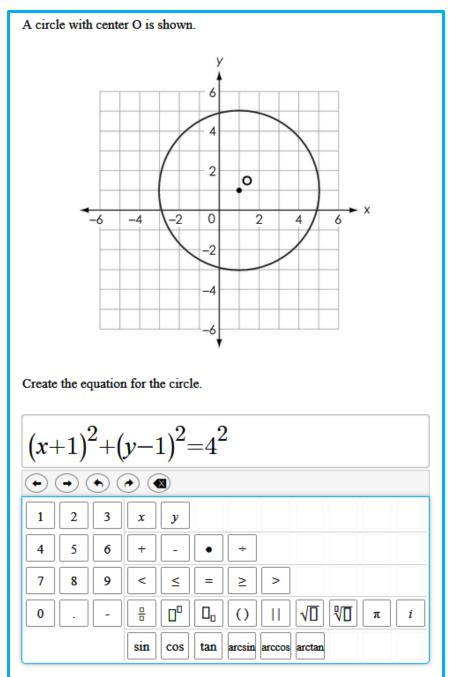
This response earns full credit (1 point) because it shows the correct general form for the equation of the circle  $(x - 1)^2 + (y - 1)^2 = 16$ .

On the coordinate plane, the center-radius form for the equation of a circle with center (h, k) and radius r is  $(x - h)^2 + (y - k)^2 = r^2$ . The given circle has a center at (1, 1) and a radius of 4 units. By substituting h = 1, k = 1 and r = 4 in the center-radius form for h, k and r, respectively, the equation of the circle is  $(x - 1)^2 + (y - 1)^2 = 16$ . When the equation is multiplied out and like terms are combined, the equation appears in general form,  $x^2 - 2x + y^2 - 2y - 14 = 0$ .



## Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect center-radius form for the equation of the circle. The correct equation in center-radius form is  $(x-1)^2 + (y-1)^2 = 4^2$ .



### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect center-radius form for the equation of the circle. The correct equation in center-radius form is  $(x-1)^2 + (y-1)^2 = 4^2$ .

**Question 11** 

**Question and Scoring Guidelines** 

# **Question 11**

Kyle performs a transformation on a triangle. The resulting triangle is similar but not congruent to the original triangle.

Which transformation did Kyle perform on the triangle?

- A dilation
- reflection
- © rotation
- translation

Points Possible: 1

**Content Cluster:** Understand similarity in terms of similarity transformations

**Content Standard:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (*G.SRT.2*)

# **Scoring Guidelines**

<u>Rationale for Option A:</u> **Key** – The student noted that dilation will preserve the shape and orientation (measures of all angles remain the same) but may change the side lengths proportionally, making the triangles not congruent.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that since a reflection can change orientation, that would make the two triangles not congruent, not remembering that orientation does not affect the congruence of two shapes.

<u>Rationale for Option C:</u> This is incorrect. The student may have thought that since a rotation can change the placement of an objection, that would make the two triangles not congruent, not remembering that placement does not affect the congruence of two triangles.

<u>Rationale for Option D:</u> This is incorrect. The student may have selected an option that definitely would produce congruence rather than one that would not produce congruence.

### Sample Response: 1 point

Kyle performs a transformation on a triangle. The resulting triangle is similar but not congruent to the original triangle.

Which transformation did Kyle perform on the triangle?

- dilation
- reflection
- © rotation
- translation

Question 12

**Question and Scoring Guidelines** 

# Question 12

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of  $\frac{1}{2}$  about the origin to create triangle A'B'C'.

What is the length, in units, of side  $\overline{B'C'}$ ?

1	2	3			
4	5	6			
7	8	9			
	0				
	-	<u>-</u>			

Points Possible: 1

**Content Cluster:** Understand similarity in terms of similarity transformations

**Content Standard:** Verify experimentally the properties of dilations given by a center and a scale factor.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G.SRT.1b)

# **Scoring Guidelines**

Exemplar Response

• 3.25

#### Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• The correct length (1 point).

**Question 12** 

Sample Responses

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of $\frac{1}{2}$ about the origin to create triangle A'B'C'.					
What is the leng	What is the length, in units, of side $\overline{B'C'}$ ?				
3.25	3.25				
$\bullet \bullet \bullet \bullet$	$\bullet \bullet \bullet \bullet \bullet \bullet$				
1	2	3			
4	5	6			
7	8	9			
	0				
	. <u>-</u> <u>-</u>				

### Notes on Scoring

This response earns full credit (1 point) because it shows the correct length of side  $\overline{B'C'}$ .

When a triangle is dilated by a positive scale factor of  $\frac{1}{2}$ , all side lengths change by this scale factor, regardless of the center of dilation. The distance formula and coordinates of points B (2.5, 3) and C (0, -3) can be used to calculate the length of a side  $\overline{BC}$  as  $\sqrt{(2.5-0)^2 + (3+3)^2} = 6.5$ . By applying the scale factor  $\frac{1}{2}$  to the length of BC, the length of a side B'C' =  $\frac{1}{2} \cdot$  BC = 3.25 units.

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of $\frac{1}{2}$ about the origin to create triangle A'B'C'. What is the length, in units, of side $\overline{B'C'}$ ?					
$\boxed{\frac{13}{4}}$					
	2	3			
4	5	6			
7	8	9			
	<b>0</b>				
·	_				

### Notes on Scoring

This response earns full credit (1 point) because it shows the correct length of side  $\overline{B'C'}$ .

When a triangle is dilated by a positive scale factor of  $\frac{1}{2}$ , all side lengths change by this scale factor, regardless of the center of dilation. The distance formula and coordinates of points B (2.5, 3) and C (0, -3) can be used to calculate the length of a side  $\overline{BC}$  as  $\sqrt{(2.5-0)^2 + (3+3)^2} = 6.5$ . By applying the scale factor  $\frac{1}{2}$  to the length of BC, the length of a side B'C' =  $\frac{1}{2} \cdot BC = 3.25 = \frac{13}{4}$  units.

	Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of $\frac{1}{2}$ about the origin to create triangle A'B'C'.				
What is the leng	th, in units, of s	ide <mark>B'C'</mark> ?			
6.5	6.5				
1	2	3			
4	5	6			
7	8	9			
· ·	-	<u><u></u></u>			

#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect length of side  $\overline{B'C}$ '.

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of  $\frac{1}{2}$  about the origin to create triangle A'B'C'. What is the length, in units, of side  $\overline{B'C'}$ ? 8.5  $\bullet \bullet \bullet \bullet \blacksquare$ 1 2 3 5 4 6 7 8 9 0 -\_

### Notes on Scoring

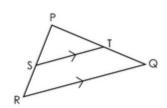
This response earns no credit (0 points) because it shows an incorrect length of side  $\overline{B'C}$ '.

Question 13

**Question and Scoring Guidelines** 

## **Question 13**

Triangle PQR is shown, where  $\overline{ST}$  is parallel to  $\overline{RQ}$ .



Marta wants to prove that  $\frac{SR}{PS} = \frac{TQ}{PT}$ .

Place a statement or reason in each blank box to complete Marta's proof.

Statements	Reasons
1. ST    RQ	1. Given
2. ∠PST ≅ ∠R and ∠PTS ≅ ∠Q	<ol> <li>If two parallel lines are cut by a transversal, then corresponding angles are congruent.</li> </ol>
3. △PQR ~ △PTS	3.
4.	4.
5. $PR = PS + SR, PQ = PT + TQ$	5. Segment addition postulate
$\frac{6}{PS} + \frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution
7. $\frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition
$\frac{8}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality

$\frac{PR}{PS} = \frac{PQ}{PT}$	$\frac{PS}{SR} = \frac{PT}{ST}$	∠P≅ ∠P
AA Similarity	ASA Similarity	SSS Similarity
Reflexive property	Segment addition postulate	Corresponding sides of similar triangles are proportional.
Corresponding sides of similar triangles are congruent.	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.	If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

#### Points Possible: 1

**Content Cluster:** Prove and apply theorems both formally and informally involving similarity using a variety of methods

**Content Standard:** Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)

# **Scoring Guidelines**

Exemplar Response

Statements	Reasons	
1. <u>ST</u> ∥ <u>RQ</u>	1. Given	
2. $\angle PST \cong \angle R$ and $\angle PTS \cong \angle Q$	<ol><li>If two parallel lines are cut by a transversal, then corresponding angles are congruent.</li></ol>	
3. $\triangle PQR \sim \triangle PTS$	3. AA Similarity	
$\frac{4}{PS} = \frac{PQ}{PT}$	<ol> <li>Corresponding sides of similar triangles are proportional.</li> </ol>	
5. $PR = PS + SR$ , $PQ = PT + TQ$	5. Segment addition postulate	
$6. \frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution	
7. $\frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition	
$\frac{8}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality	

#### Other Correct Responses

• N/A

For this item, a full-credit response includes:

• A correct proof (1 point).

**Question 13** 

Sample Responses

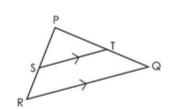
Triangle PQR is shown, where $\overline{ST}$ is parallel to $\overline{RQ}$ .			
Marta wants to prove th			2
Place a statement or reason in each blank box to complete Marta's proof.			
1. ST    RQ	its	1. Given	Reasons
<ol> <li>SI    RQ</li> <li>∠PST ≅ ∠R and ∠PTS ≅ ∠Q</li> </ol>		2. If two paralle	el lines are cut by a hen corresponding angles t.
<ol> <li>∆PQR ~ △PTS</li> </ol>		3. AA Similarity	
4. $\frac{PR}{PS} = \frac{PQ}{PT}$		<ol> <li>Corresponding are proportion</li> </ol>	ng sides of similar triangles nal.
5. PR = PS + SR, PQ	= PT + TQ	5. Segment addition postulate	
$\frac{\text{PS} + \text{SR}}{\text{PS}} = \frac{\text{PT} + \text{TQ}}{\text{PT}}$		6. Substitution	
7. $\frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$		7. Commutative property of addition	
8. $\frac{SR}{PS} = \frac{TQ}{PT}$		8. Subtraction p	property of equality
	$\frac{PS}{SR} = \frac{PT}{ST}$		∠P≅ ∠P
	ASA Similarity		SSS Similarity
Reflexive property	Segment addition postulate		
Corresponding sides of similar triangles	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.		If two parallel lines are cut by a transversal, then

#### Notes on Scoring

This response earns full credit (1 point) because it shows a correct proof of a theorem about a triangle where a line parallel to one side divides the other two proportionally.

In this situation, the existence of two pairs of congruent angles supports the statement about similarity of triangles PQR and PTS. Since a pair of parallel lines cut by two intersecting transversals form two triangles and two pairs of corresponding congruent angles, the triangles are similar by the Angle-Angle postulate. Having justified a similarity of the triangles, the next step is to select a statement showing a proportionality of sides,  $\frac{PR}{PS} = \frac{PQ}{PT}$ , along with correct reasoning (corresponding sides of similar figures are proportional).

Triangle PQR is shown, where  $\overline{ST}$  is parallel to  $\overline{RQ}$ .



Marta wants to prove that  $\frac{SR}{PS} = \frac{TQ}{PT}$ .

Place a statement or reason in each blank box to complete Marta's proof.

Statements	Reasons
1. ST    RQ	1. Given
2. ∠PST ≅ ∠R and ∠PTS ≅ ∠Q	<ol> <li>If two parallel lines are cut by a transversal, then corresponding angles are congruent.</li> </ol>
<ol> <li>△PQR ~ △PTS</li> </ol>	3. AA Similarity
4.	4.
5. $PR = PS + SR, PQ = PT + TQ$	5. Segment addition postulate
$\frac{6}{PS} + \frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution
7. $\frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition
$\frac{8.}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality

$\frac{PR}{PS} = \frac{PQ}{PT}$	$\frac{PS}{SR} = \frac{PT}{ST}$	∠P≅ ∠P
Corresponding sides of similar triangles are proportional.	ASA Similarity	SSS Similarity
Reflexive property	Segment addition postulate	
Corresponding sides of similar triangles are congruent.	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.	If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

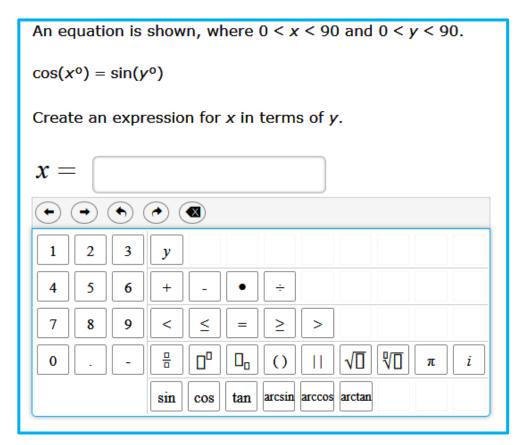
### Notes on Scoring

This response earns no credit (0 points) because it shows an incomplete proof (step 4 is missing) of a theorem about a triangle where a line parallel to one side divides the other two proportionally.

Question 14

**Question and Scoring Guidelines** 

## **Question 14**



#### Points Possible: 1

**Content Cluster:** Define trigonometric ratios, and solve problems involving right triangles

**Content Standard:** Explain and use the relationship between the sine and cosine of complementary angles. (*G.SRT.7*)

# **Scoring Guidelines**

Exemplar Response

• 90 – y

### Other Correct Responses

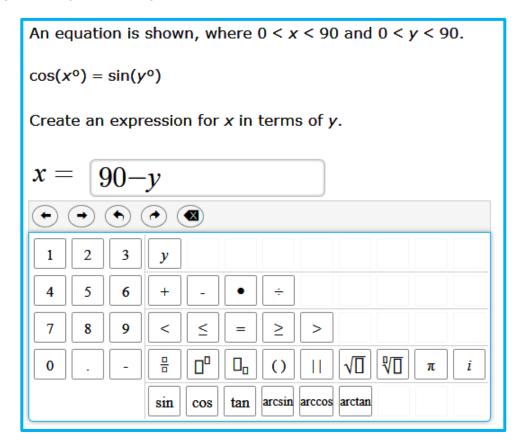
• Any equivalent expression.

For this item, a full-credit response includes:

• A correct expression (1 point).

**Question 14** 

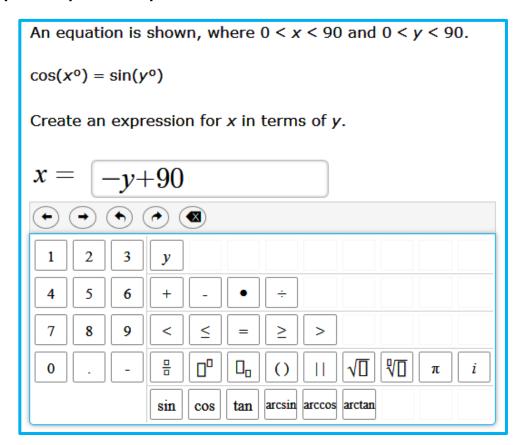
Sample Responses



### Notes on Scoring

This response earns full credit (1 point) because it shows a correct expression for *x* in terms of *y*.

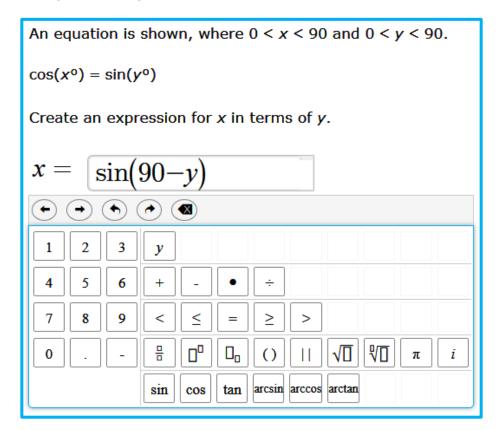
This situation recognizes a relationship between angle x and angle y. If the sine value of one angle equals the cosine value of another angle, or sin  $x = \cos y$ , where 0 < y < 90, then angles are complementary. Angles are complementary if the sum of their measures is 90°, or x + y = 90. Therefore, x = 90 - y.



#### **Notes on Scoring**

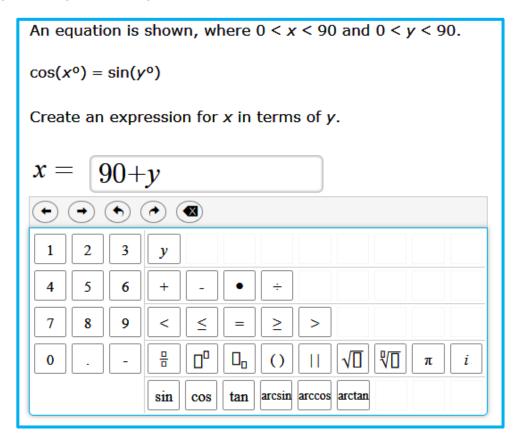
This response earns full credit (1 point) because it is equivalent to the expression 90 - y.

This situation recognizes a relationship between angle x and angle y. If the sine value of one angle equals the cosine value of another angle, or sin  $x = \cos y$ , where 0 < y < 90, then the angles are complementary. Angles are complementary if the sum of their measures is 90°, or x + y = 90. Therefore, x = 90 - y.



#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect expression for x in terms of y.



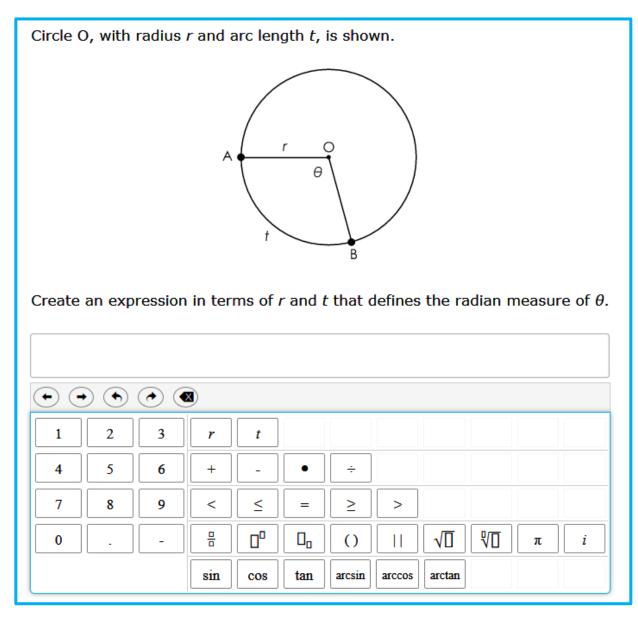
### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect expression for *x* in terms of *y*.

Question 15

**Question and Scoring Guidelines** 

# Question 15



### Points Possible: 1

Content Cluster: Find arc lengths and areas of sectors of circles

**Content Standard:** Find arc lengths and areas of sectors of circles. a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (*G.C.5a*)

# Scoring Guidelines

### Exemplar Response

•  $\frac{t}{r}$ 

### Other Correct Responses

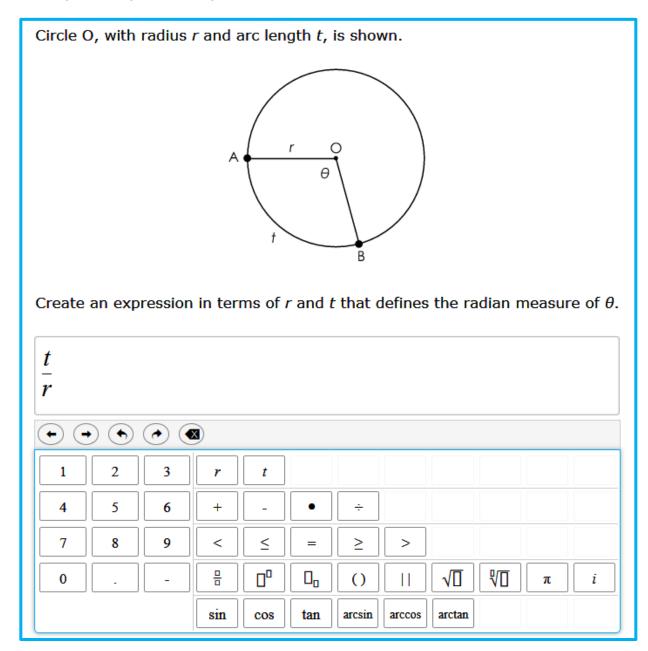
• Any equivalent expression.

For this item, a full-credit response includes:

• A correct expression (1 point).

**Question 15** 

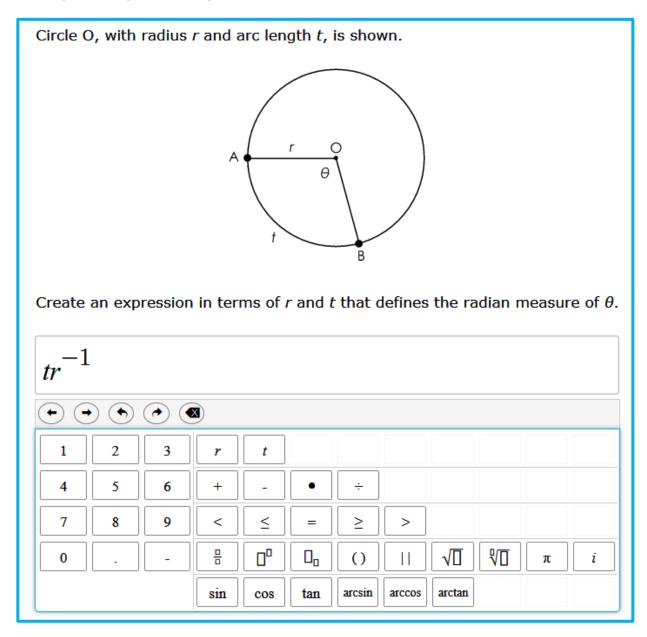
Sample Responses



### Notes on Scoring

This response earns full credit (1 point) because it shows a correct expression that defines the radian measure of a central angle  $\theta$ .

In this situation, an expression that defines the radian measure of angle  $\theta$  can be obtained from the formula for the arc length or the distance between points A and B along the arc, *t*, which equals the product of a central angle (in radians),  $\theta$ , and the length of a radius, *r*. By solving a formula  $t = r\theta$  for  $\theta$ , the radian measure of an angle  $\theta$  is  $=\frac{t}{r}$ 

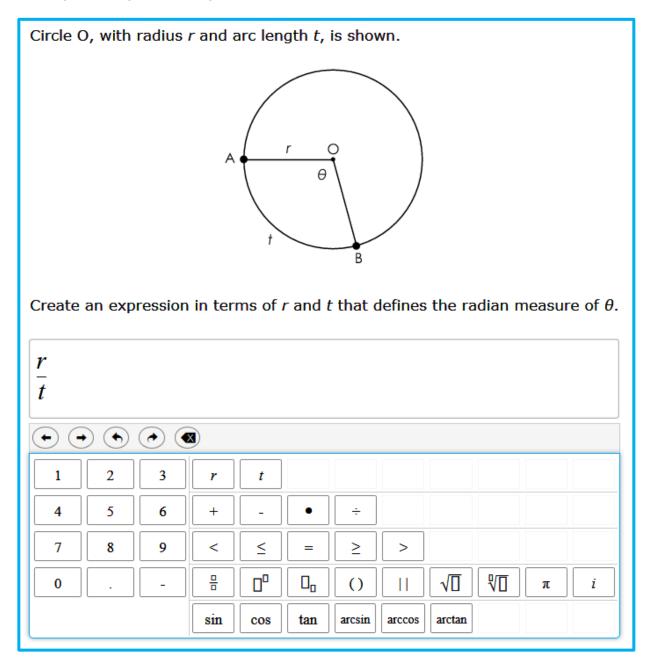


#### **Notes on Scoring**

This response earns full credit (1 point) because it shows an equivalent form for a correct expression that defines the radian measure of a central angle  $\theta$ .

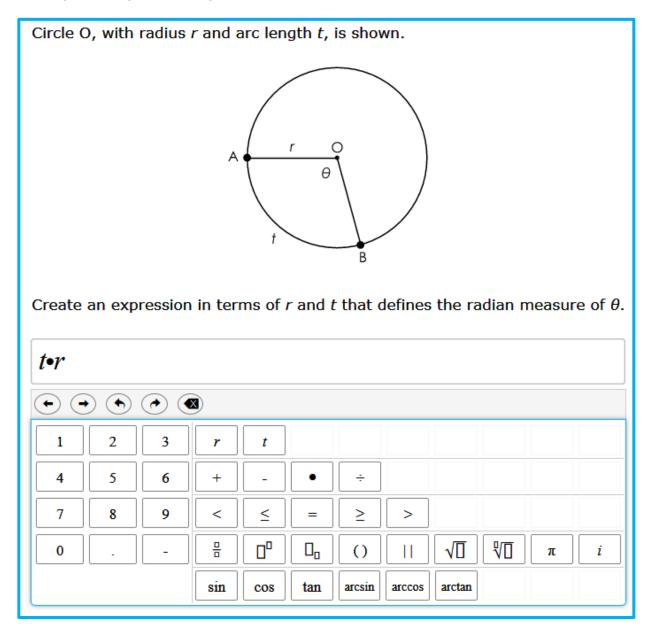
In this situation, an expression that defines the radian measure of angle  $\theta$  can be obtained from the formula for the arc length or the distance between points A and B along the arc, *t*, which equals the product of a central angle (in radians),  $\theta$ , and the length of a radius, *r*. By solving a formula *t* = *r* $\theta$  for,  $\theta$  the radian

measure of an angle is  $\theta = \frac{t}{r}$  or  $\theta = t \cdot \frac{1}{r}$  or  $\theta = t \cdot r^{-1}$ .



#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect expression that defines the radian measure of a central angle  $\theta$ .



#### Notes on Scoring

This response earns no credit (0 points) because it shows an equivalent of an incorrect expression that defines the radian measure of a central angle  $\theta$ .

# Question 16

# **Question and Scoring Guidelines**

### **Question 16**

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male			80
Female			120
Total	150	50	200

Points Possible: 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data

**Content Standard:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English.

Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4) $\star$ 

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

Exemplar Response

	Adults	Children	Total
Male	60	20	80
Female	90	30	120
Total	150	50	200

Other Correct Responses

• A table with equivalent values.

For this item, a full-credit response includes:

• A correct table (1 point).

**Question 16** 

Sample Responses

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male	60	20	80
Female	90	30	120
Total	150	50	200

#### Notes on Scoring

This response earns full credit (1 point) because it shows a completed table with correct values.

This situation requires a correct construction of a two-way frequency table of data. Since the probability of selecting a child, given it is a female, is 0.25, the number of female children is 120 • .25 or 30 (Row 2/Column 2). The number of female adults is 120 – 30 or 90 (Row 2/Column 1). Since the probability of selecting a male, given it is a child, is 0.4, the number of male children is 50 • 0.4 or 20 (Row 1/Column 2). The number of .

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male	20	60	80
Female	30	90	120
Total	150	50	200

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows a table with the correct values in the wrong cells.

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male	30	50	80
Female	120	0	120
Total	150	50	200

#### Notes on Scoring

This response earns no credit (0 points) because it shows a table with incorrect values.

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male	80	0	80
Female	70	50	120
Total	150	50	200

#### Notes on Scoring

This response earns no credit (0 points) because it shows a table with incorrect values.

Question 17

**Question and Scoring Guidelines** 

### **Question 17**

Sam is picking fruit from a basket that contains many different kinds of fruit.

Which set of events is independent?

- Event 1: He picks a kiwi and eats it. Event 2: He picks an apple and eats it.
- Event 1: He picks an apple and eats it. Event 2: He picks an apple and eats it.
- © Event 1: He picks a kiwi and eats it. Event 2: He picks a kiwi and puts it back.
- Event 1: He picks a kiwi and puts it back. Event 2: He picks an apple and puts it back.

Points Possible: 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data

**Content Standard:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5) $\star$ 

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that because the fruits were different, the two events must be independent, but missed that picking and eating a fruit without replacing it with the same fruit will affect the likelihood of picking a different fruit from the basket.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that, knowing that an apple was picked and eaten, P(E | E)=1 yields certainty for an apple to be picked and eaten.

<u>Rationale for Option C:</u> This is incorrect. The student may have switched the order of events and thought that because a kiwi was picked and put back, it did not affect the likelihood of picking another kiwi, which makes the two events independent.

<u>Rationale for Option D:</u> **Key** – The student correctly identified that picking a fruit from the basket and putting it back does not affect the likelihood of picking a different fruit and putting it back, which makes the two events independent.

#### Sample Response: 1 point

Sam is picl fruit.	king fruit from a basket that	conta	ains many different kinds of
Which set o	of events is independent?		
۲	Event 1: He picks a kiwi and eats it. Event 2: He picks an apple and eats it.	©	Event 1: He picks a kiwi and eats it. Event 2: He picks a kiwi and puts it back.
B	Event 1: He picks an apple and eats it. Event 2: He picks an apple and eats it.	•	Event 1: He picks a kiwi and puts it back. Event 2: He picks an apple and puts it back.

# **Question 18**

# **Question and Scoring Guidelines**

## Question 18

The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5.

What is the probability of flipping heads or rolling an odd number?

$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
	-		

Points Possible: 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model

**Content Standard:** Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. (S.CP.7)\*

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

Exemplar Response

• 0.75

Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 18** 

Sample Responses

The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5. What is the probability of flipping heads or rolling an odd number? 0.75← → ♠ ⊘ 🛛 2 1 3 5 4 6 7 8 9 0 믐

#### Notes on Scoring

This response earns full credit (1 point) because it shows the correct probability.

There are several strategies that can be used to solve problems with compound events. In this situation, the two compound events (i.e., events happening at the same time) are flipping heads (A) and rolling an odd number (B).

The Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), can be used to calculate the probability of flipping heads or rolling an odd number. If the probability of flipping heads is 0.5 and the probability of rolling an odd number is 0.5, then the probability of flipping heads and rolling an odd number is 0.5 • 0.5 = 0.25, since these two events are independent. By substituting these values in the formula, the probability of flipping heads or rolling an odd number is P(A or B) = 0.5 + 0.5 - 0.25 = 0.75.

The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5.

What is the probability of flipping heads or rolling an odd number?

$\frac{3}{4}$			
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
	-	<u> </u>	

#### Notes on Scoring

This response earns full credit (1 point) because it shows the equivalent value of a correct probability.

There are several strategies that can be used to solve problems with compound events. In this situation, the two compound events (i.e., events happening at the same time) are flipping heads (A) and rolling an odd number (B).

The Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), can be used to calculate the probability of flipping heads or rolling an odd number. If the probability of flipping heads is 0.5 and the probability of rolling an odd number is 0.5, then the probability of flipping heads and rolling an odd number is 0.5 • 0.5 = 0.25, since these two events are independent. By substituting these values in the formula, the probability of flipping heads or rolling an odd number is

 $P(A \text{ or } B) = 0.5 + 0.5 - 0.25 = 0.75 \text{ or } \frac{3}{4}$ 

through 6, and an oc	The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5. What is the probability of flipping heads or rolling an odd number?				
1	<u>1</u>				
6					
1	2	3			
4	5	6			
7	8	9			
	0				
	-				

### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	The probability of fli through 6, and an odd What is the probabili	d number landing fa	ce up is 0.5.	up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 mber?
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0.5			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\bullet \bullet \bullet \bullet$			
	1	2	3	
	4	5	6	
	7	8	9	
		0		
		-	<b>-</b>	

### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability.

Question 19

**Question and Scoring Guidelines** 

## **Question 19**

An engineer is designing a conical container. It needs to hold a specific volume and have a specific height. She needs to know the radius of the container, r, in terms of its volume, V, and height, h.

Create an equation that the engineer can use to determine the radius.

<i>r</i> =	
$\bullet \bullet \bullet \bullet \bullet \blacksquare$	
1 2 3	V h
4 5 6	+ - • ÷
7 8 9	
0	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
	sin cos tan arcsin arccos arctan

Points Possible: 1

**Content Cluster:** Create equations that describe numbers or relationships

**Content Standard:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (A.CED.4)★

 $(\star)$  indicates that modeling should be incorporated into the standard.

### **Scoring Guidelines**

Exemplar Response

• 
$$r = \sqrt{\frac{3V}{nh}}$$

Other Correct Responses

• Any equivalent equation.

For this item, a full-credit response includes:

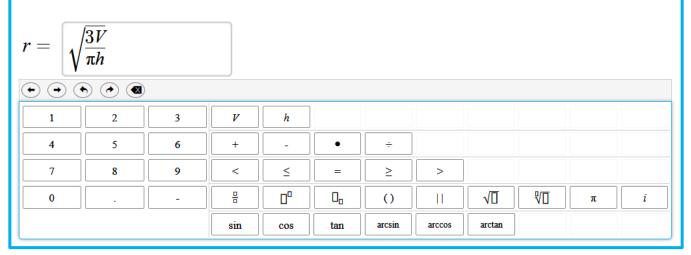
• A correct equation (1 point).

**Question 19** 

Sample Responses

An engineer is designing a conical container. It needs to hold a specific volume and have a specific height. She needs to know the radius of the container, r, in terms of its volume, V, and height, h.

Create an equation that the engineer can use to determine the radius.



#### Notes on Scoring

This response receives full credit (1 point) because it shows a correct equation to determine *r* in terms of *V* and *h*. This situation requires students to create an equation that can be used to determine the radius, *r*, of a conical container in terms of its volume, *V*, and height, *h*. The formula for the volume of a conical container,  $V = \frac{1}{3}\pi r^2 h$ , relates all three variables. To determine the radius, the formula has to be solved for *r*. This can be obtained by multiplying both sides by 3 to get  $3V = \pi r^2 h$ . Next, the student must divide both sides by  $(\pi * h)$  to get  $\frac{3V}{\pi * h} = r^2$ , and then take the square root on both sides to get  $r = \sqrt{\frac{3V}{(\pi * h)}}$ .

An engineer is designing a conical container. It needs to hold a specific volume and have a specific height. She needs to know the radius of the container, r, in terms of its volume, V, and height, h.

 $((3V)\pi^{-1}h^{-1})^{\bar{2}}$ + + + + @ V 1 2 3 h 5 + • 4 6 ÷ \_ 7 8 9 < ≤ = ≥ > 8 ᄳ 0 0. 0 П  $\sqrt{0}$ π i sin cos tan arcsin arccos arctan

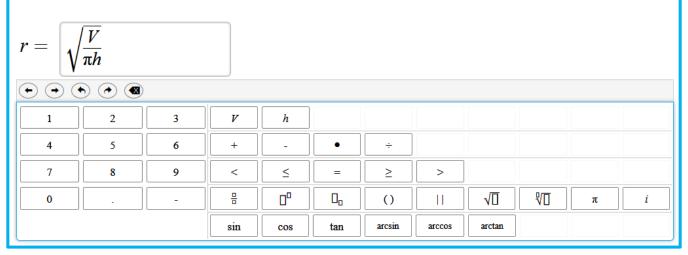
Create an equation that the engineer can use to determine the radius.

#### Notes on Scoring

This response receives full credit (1 point) because it shows a correct equation to determine *r* in terms of *V* and *h*. This situation requires students to create an equation that can be used to determine the radius, *r*, of a conical container in terms of its volume, *V*, and height, *h*. The formula for the volume of a conical container,  $V = \frac{1}{3}\pi r^2 h$ , relates all three variables. To determine the radius, the formula has to be solved for *r*. This can be obtained by multiplying both sides by 3 to get  $3V = \pi r^2 h$ . Next, the student must divide both sides by  $(\pi * h)$  to get  $\frac{3V}{(\pi \cdot h)} = r^2$ , and then take the square root on both sides to get  $r = \sqrt{\frac{3V}{(\pi \cdot h)}}$ . Or  $r = (3V \cdot (\frac{1}{\pi \cdot h})^{(\frac{1}{2})})$  or its equivalent form  $r = (3V\pi^{-1}h^{-1})^{(\frac{1}{2})}$ .

An engineer is designing a conical container. It needs to hold a specific volume and have a specific height. She needs to know the radius of the container, r, in terms of its volume, V, and height, h.

Create an equation that the engineer can use to determine the radius.



#### Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect equation to determine *r* in terms of V and *h*.

An engineer is designing a conical container. It needs to hold a specific volume and have a specific height. She needs to know the radius of the container, r, in terms of its volume, V, and height, h.

Create an equation that the engineer can use to determine the radius.



#### Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect equation to determine *r* in terms of *V* and *h*.

# Question 20

# **Question and Scoring Guidelines**

### **Question 20**

Which expression is equivalent to  $(2x^2 + 3)(x + 4)$ ?

- (A)  $2x^3 + 12$
- (a)  $2x^2 + 11x + 12$ (c)  $2x^3 + 6x^2 + 4x + 12$
- (b)  $2x^3 + 8x^2 + 3x + 12$

#### **Points Possible:** 1

Content Cluster: Perform arithmetic operations on polynomials

**Content Standard:** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. b. Extend to polynomial expressions beyond those

expressions that simplify to forms that are linear or quadratic. (A.APR.1b)

### **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that only variable terms,  $2x^2$  and x, could be multiplied together and that only constant terms, 3 and 4, could be multiplied together; as a result, the student may have multiplied the first terms of each binomial and the last terms of each binomial.

<u>Rationale for Option B</u>: This is incorrect. The student may have disregarded the power of 2 on the first term, thus finding the product of 2x + 3 and x + 4 to get  $2x^2 + 8x + 3x + 12$  or  $2x^2 + 11x + 12$ .

<u>Rationale for Option C</u>: This is incorrect. The student may have incorrectly multiplied the constants by the variable terms,  $(2x^2) \cdot 3 = 6x^2$  and  $x \cdot 4 = 4x$ , within the same parentheses.

<u>Rationale for Option D</u>: **Key** – The student used a distributive property to multiply two binomials to get  $2x^2 \cdot x = 2x^3$  and  $2x^2 \cdot 4 = 8x^2$  and  $3 \cdot x = 3x$  and  $3 \cdot 4 = 12$ . The student realized that the expression did not have like terms and that the product was  $2x^3 + 8x^2 + 3x + 12$ .

#### Sample Response: 1 point

Which expression is equivalent to $(2x^2 + 3)(x + 4)$ ?
(a) $2x^3 + 12$
(B) $2x^2 + 11x + 12$
© $2x^3 + 6x^2 + 4x + 12$
• $2x^3 + 8x^2 + 3x + 12$

## **Question 21**

# **Question and Scoring Guidelines**

### **Question 21**

Select all of the expressions that are equivalent to  $9x^4 - y^2$ .  $(3x^2 - y)^2$   $(3x^2)^2 - (y)^2$   $9(x^2)^2 - (y)^2$   $(9x^2)^2 - (y)^2$  $(3x^2 + y)(3x^2 - y)$ 

Points Possible: 1

Content Cluster: Interpret the structure of expressions

**Content Standard:** Use the structure of an expression to identify ways to rewrite it. For example, to factor 3x(x - 5) + 2(x - 5), students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to (3x + 2)(x - 5); or see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ . (A.SSE.2)

### **Scoring Guidelines**

<u>Rationale for First Option</u>: This is incorrect. The student may have taken the square root of each term, but confused the difference of squares with the square of a difference.

<u>Rationale for Second Option:</u> **Key** – The student may have taken the square root of each monomial, creating an equivalent expression, or the student may have correctly applied the second power to both terms in the option,  $(3x^2)^2 = 9x^4$  and  $(y)^2 = y^2$ , to see if it equals the given expression,  $9x^4 - y^2$ .

<u>Rationale for Third Option</u>: **Key** – The student noticed that each variable could be written as a squared term,  $(3x^2)^2$  and  $y^2$ , leaving the coefficient unfactored.

<u>Rationale for Fourth Option</u>: This is incorrect. The student may have recognized that each term was a perfect square, but neglected to take the square root of the coefficient of the  $9x^4$  term.

<u>Rationale for Fifth Option:</u> **Key** – The student recognized the difference of squared terms, took the square root of each and wrote it as the product of two binomials, one using addition and one using subtraction.

**Question 21** 

Sample Responses

Select all of the expressions that are equivalent to  $9x^4 - y^2$ . ( $3x^2 - y$ )<sup>2</sup> ( $3x^2$ )<sup>2</sup> - (y)<sup>2</sup> 9( $x^2$ )<sup>2</sup> - (y)<sup>2</sup> ( $9x^2$ )<sup>2</sup> - (y)<sup>2</sup> ( $3x^2 + y$ )( $3x^2 - y$ )

#### Notes on Scoring

This response earns full credit (1 point) because it shows all correct options, B, C and E.

Select all of the expressions that are equivalent to  $9x^4 - y^2$ .  $(3x^2 - y)^2$   $(3x^2)^2 - (y)^2$   $9(x^2)^2 - (y)^2$   $(9x^2)^2 - (y)^2$  $(3x^2 + y)(3x^2 - y)$ 

#### Notes on Scoring

This response earns no credit (0 points) because it shows only two correct options, B and C, instead of three correct options, B, C and E.

Select all of the expressions that are equivalent to  $9x^4 - y^2$ . ( $3x^2 - y$ )<sup>2</sup> ( $3x^2$ )<sup>2</sup> - (y)<sup>2</sup> 9(x<sup>2</sup>)<sup>2</sup> - (y)<sup>2</sup> ( $9x^2$ )<sup>2</sup> - (y)<sup>2</sup> ( $3x^2 + y$ )( $3x^2 - y$ )

#### Notes on Scoring

This response earns no credit (0 points) because it shows only two correct options, B and E, and one incorrect option, A, instead of three correct options, B, C and E.

## Question 22

# **Question and Scoring Guidelines**

### **Question 22**

An equation is shown.  $2x^2 - 5x - 3 = 0$ What values of x make the equation true? x =x =1 (ہ + 1 2 3 + ÷ • \_ 5  $\leq$ 4 6 < =  $\geq$ > ᄳ 믑  $\sqrt{1}$ 9 0 7 8 i π arcsin arccos arctan 0  $\sin$ cos tan

Points Possible: 1

Content Cluster: Solve equations and inequalities in one variable

**Content Standard:** Solve quadratic equations in one variable. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for  $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (A.REI.4b)

### Scoring Guidelines

#### Exemplar Response

•  $x = -\frac{1}{2}$  and x = 3

#### Other Correct Responses

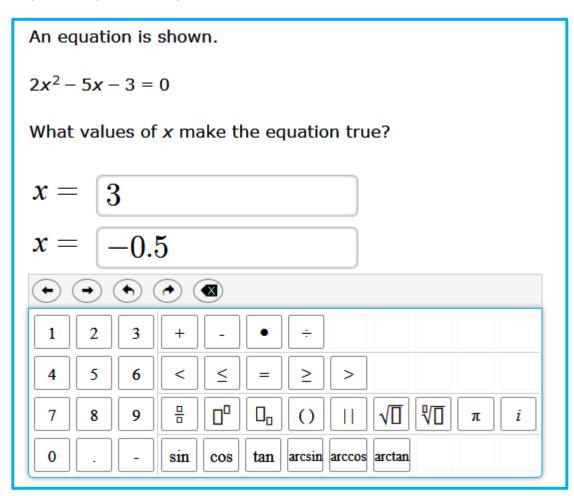
• Any pair of equivalent values in either order is accepted.

For this item, a full-credit response includes:

• A correct pair of values (1 point).

**Question 22** 

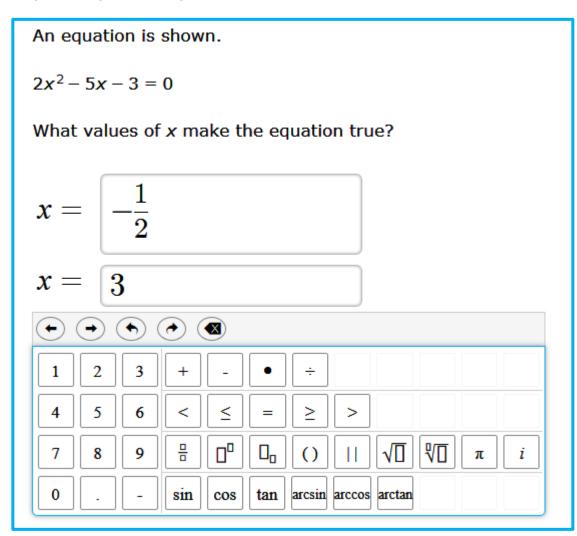
Sample Responses



#### Notes on Scoring

This response earns full credit (1 point) because it shows two correct values, x = 3 and = -0.5 or -1/2, that make the quadratic equation true.

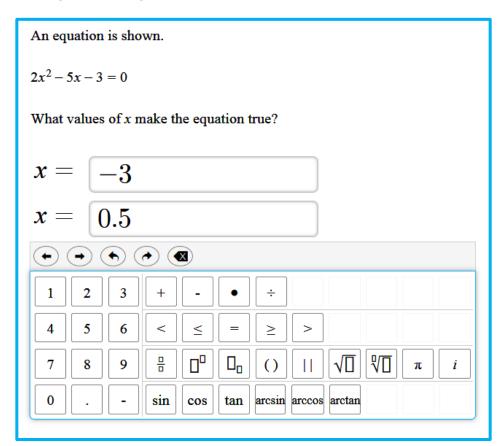
While there are several methods for solving a quadratic equation in the form of  $ax^2 + bx + c = 0$ , the use of a graphing utility may be a way to find x-intercepts or x values that make the equation true.



#### Notes on Scoring

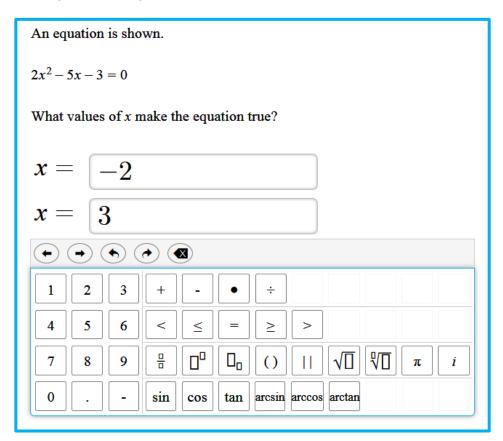
This response earns full credit (1 point) because it shows two correct values,  $x = -\frac{1}{2}$  and x = 3, that make the quadratic equation true.

While there are several methods for solving a quadratic equation in the form of  $ax^2 + bx + c = 0$ , the use of a graphing utility may be a way to find *x*-intercepts or *x* values that make the equation true.



#### Notes on Scoring

This response receives no credit (0 points) because it shows values opposite of the two correct *x* values. The sign switch makes both values incorrect.



#### Notes on Scoring

This response receives no credit (0 points) because it shows one correct *x* value and one incorrect *x* value.

## Question 23

# **Question and Scoring Guidelines**

### **Question 23**

A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after t seconds is modeled by the function shown.

#### $h(t) = -t^2 + \frac{4}{3}t + \frac{1}{4}$

 $(\bullet, \bullet) (\bullet) (\blacksquare)$ 

믐

2 3

5 6

7 8 9

1

4

After how many seconds will the grasshopper land on the ground?

Points	Possible:	1
i Onito	1 0351010.	

**Content Cluster:** Interpret functions that arise in applications in terms of the context

**Content Standard:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)★

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

Exemplar Response

• 1.5

### Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 23** 

Sample Responses

A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after t seconds is modeled by the function shown.



 $( \rightarrow) ( \rightarrow) ( \rightarrow) ( \blacksquare)$ 

 $\frac{3}{2}$ 

1 2 3

4 7 8 9

5 6

0

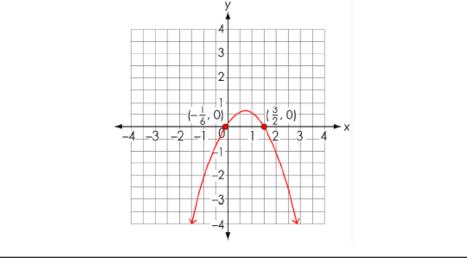
믐

After how many seconds will the grasshopper land on the ground?

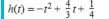
#### Notes on Scoring

This response receives full credit (1 point) because it shows a correct time in seconds,  $t = \frac{3}{2}$ , that is equivalent to t = 1.5.

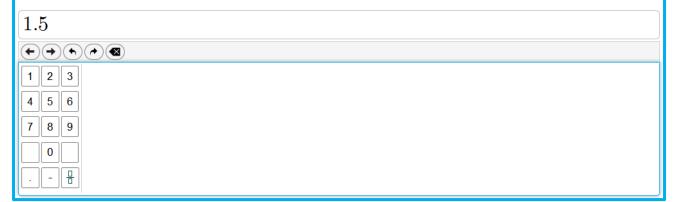
The height of the grasshopper, in feet, above the ground after t seconds is modeled by the quadratic function  $h(t) = -t^2 + (\frac{4}{2})t + (\frac{1}{4})$ . When the grasshopper lands on the ground, its height above the ground is 0 feet. By letting h(t) = 0 and solving the equation  $0 = -t^2 + (\frac{4}{2})t + (\frac{1}{4})$ , the time the grasshopper is on the ground is t = 1.5 sec. While there are several methods for solving a guadratic equation, the use of a graphing utility may be the quickest for finding solutions represented by the horizontal intercepts ("x-intercepts"). Because a variable t represents time, the correct solution is only a positive horizontal intercept of the graph of h(t).



A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after t seconds is modeled by the function shown.



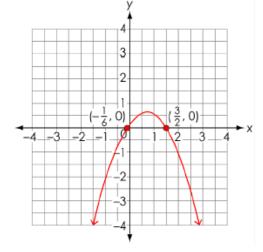
After how many seconds will the grasshopper land on the ground?



#### Notes on Scoring

This response receives full credit (1 point) because it shows a correct time in seconds, t = 1.5.

The height of the grasshopper, in feet, above the ground after t seconds is modeled by the quadratic function  $h(t) = -t^2 + (\frac{4}{3})t + (\frac{1}{4})$ . When the grasshopper lands on the ground, its height above the ground is 0 feet. By letting h(t) = 0 and solving the equation  $0 = -t^2 + (\frac{4}{3})t + (\frac{1}{4})$ , the time the grasshopper is on the ground is t = 1.5 sec. While there are several methods for solving a quadratic equation, the use of a graphing utility may be the quickest for finding solutions represented by the horizontal intercepts ("x-intercepts"). Because a variable t represents time, the correct solution is only a positive horizontal intercept of the graph of h(t).



A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after *t* seconds is modeled by the function shown.

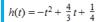


After how many seconds will the grasshopper land on the ground?

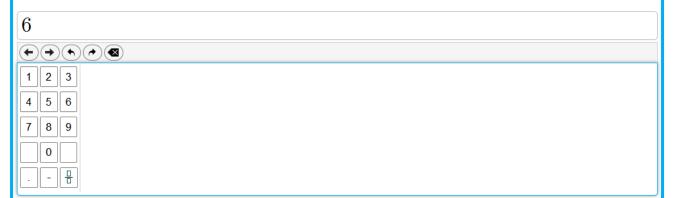
$-\frac{1}{6}$	
$\bullet \bullet \bullet \bullet \bullet \bullet \bullet$	
1 2 3	
4 5 6	
7 8 9	
Ο	
🗄	

This response receives no credit (0 points) because it shows an incorrect negative value for the time, t = -1/6.

A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after t seconds is modeled by the function shown.



After how many seconds will the grasshopper land on the ground?



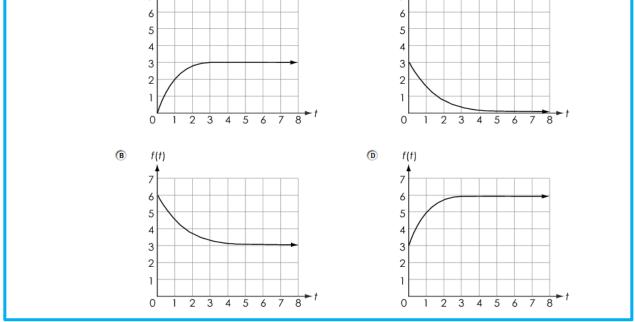
Notes on Scoring
This response receives no credit (0 points) because it shows an incorrect value for the time, $t = 6$ .

## Question 24

# **Question and Scoring Guidelines**

### **Question 24**

A radioactive substance decays exponentially. The initial amount of the substance is 3 grams, and over time, t, the amount, S(t), in grams, approaches 0. Which graph could be a model for f(t)? (a) f(t)(b) f(t)(c) f(t)



#### Points Possible: 1

**Content Cluster:** Interpret functions that arise in applications in terms of the context

**Content Standard:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)\*

 $(\star)$  indicates that modeling should be incorporated into the standard.

### **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have confused an exponential decay function with an exponential growth function that increases instead of decreases. He or she may have also incorrectly thought that the initial amount of the substance is the amount the substance approaches over time.

<u>Rationale for Option B:</u> This is incorrect. The student may have correctly understood that an exponentially decaying function decreases, but incorrectly thought that the initial amount of the substance is the amount the substance approaches over time.

<u>Rationale for Option C:</u> **Key** – The student correctly understood that the initial amount of 3 is represented by the *y*-intercept, (0, 3), and that the graph of an exponential decay function decreases as it approaches the *x*-axis.

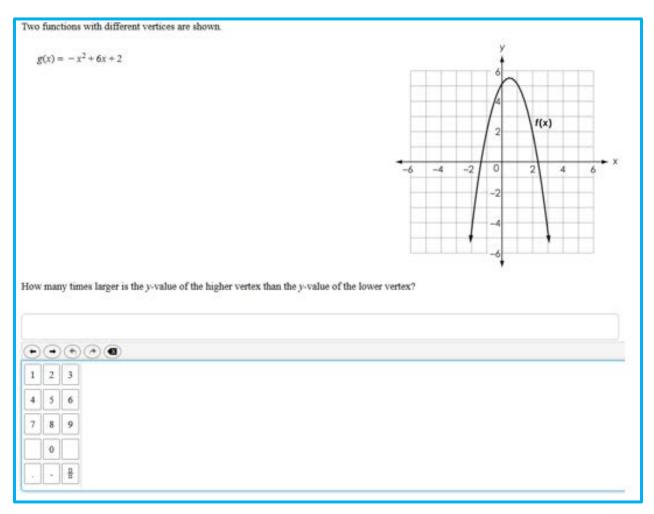
<u>Rationale for Option D:</u> This is incorrect. The student may have understood that the initial amount of 3 is represented by the y-intercept (0, 3), but the student may have confused a growth function with a decay function and thought that the decaying function increases and also incorrectly concluded that the graph approaches an x-value of 0 going from right to left.

A radioactive substance decays exponentially. The initial amount of the substance is 3 grams, and over time, t, the amount, S(t), in grams, approaches 0. Which graph could be a model for f(t)? A f(†) f(†) 5 6 В O f(†) f(†) 8 1 2 3 4 5 6 7 8 2 3 4 5 6 

## **Question 25**

# **Question and Scoring Guidelines**

### **Question 25**



Points Possible: 1

**Content Cluster**: Analyze functions using different representations

**Content Standard:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F.IF.9)

## **Scoring Guidelines**

Exemplar Response

• 2

### Other Correct Responses

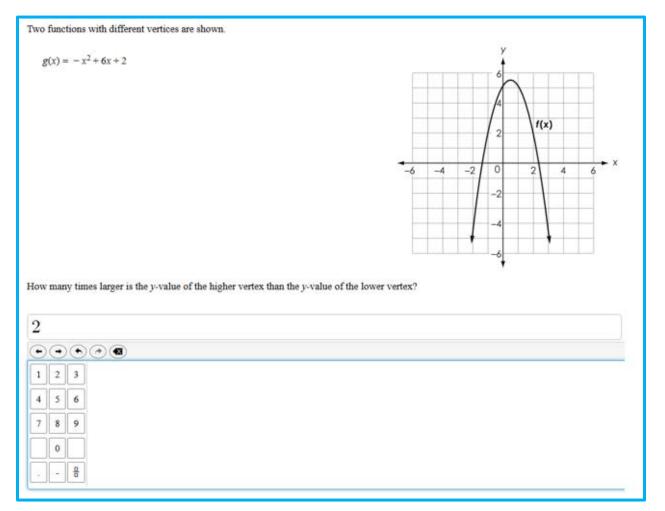
• Any equivalent value

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 25** 

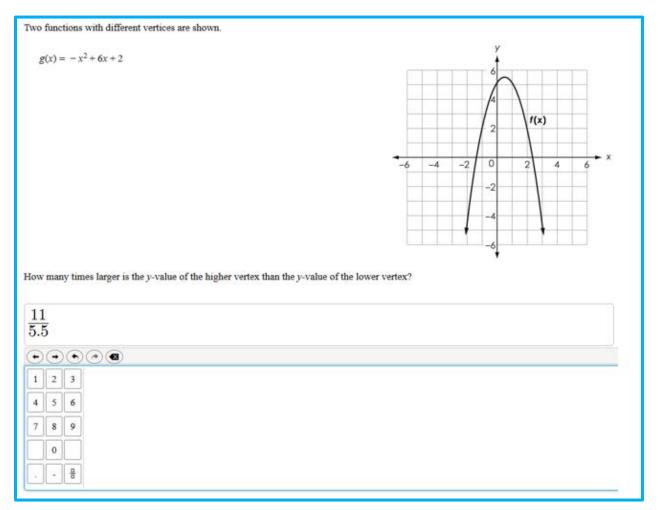
Sample Responses



#### Notes on Scoring

This response earns full credit (1 point) because it correctly compares the y-values of the vertices of two quadratic functions represented in different forms.

The vertex of the function f(x), which is shown on the graph, is (0.5, 5.5) where the yvalue is 5.5. The vertex of the function g(x) can be found using a variety of methods. One method is to use a graphing utility to graph  $g(x) = -x^2 + 6x + 2$ , and then use the graph to find the vertex, thereby finding that the vertex of g(x) is (3, 11) where the y-value is 11. Since 11 is the larger of the y-values and 5.5 is the smaller of the yvalues, 11 is 2 times larger than 5.5 because  $\frac{11}{5.5} = 2$ .



### Notes on Scoring

This response earns full credit (1 point) because it correctly compares the y-values of the vertices of two quadratic functions represented in different forms.

The vertex of the function f(x), which is shown on the graph, is (0.5, 5.5) where the y-value is 5.5. The vertex of the function g(x) can be found using a variety of methods. One method is to use a graphing utility to graph  $g(x) = -x^2 + 6x + 2$ , and then use the graph to find the vertex, thereby finding that the vertex of g(x) is (3, 11) where the y-value is 11.

Since 11 is the larger of the y-values and 5.5 is the smaller of the y-values,  $\frac{11}{5.5}$  represents how many times larger one y-value is than the other.

$g(x) = -x^2 + 6x + 2$	Ŷ
$g(x) = -x^2 + 6x + 2$	6
	$\sim$
	/ /(x)
	-6 -4 -2 0 2 4 6 x
	-2
	-4
	-6
	-6
How many times larger is the <i>y</i> -value of the higher verte	
11	
1 2 3	
11 • • • • • • • • • • • • • • • • • • •	
11 ••••• 1 2 3	
11 • • • • • • • • • • • • • • • • • • •	

### Notes on Scoring

This response earns no credit (0 points) because it incorrectly compares the y-values of vertices of two quadratic functions represented in different ways. The student may have calculated the y-value of the vertex of function g(x) and did not compare it to the y-value of the vertex of function f(x).

wo fu	nctions with di	fferent vertio	es are shown	1.									
g(x	$) = -x^2 + 6x + 6x$	2								Y			
								TT		6			
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							_				1		
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low m	any times large	er is the y-va	ue of the hig	ther vertex t	than the y-value	of the lower ve	ertex?		ł				
		er is the y-va	ue of the hig	gher vertex ti	than the y-value	of the lower ve	ertex?		ł				Η
		er is the y-va	ue of the hig	yher vertex ti	than the y-value	of the lower ve	ertex?						
5.5			ue of the hig	ther vertex t	than the y-value	of the lower ve	ertex?		4				
5.5 ⊙⊙			ue of the hig	gher vertex t	than the y-value	of the lower ve	ertex?		4				
5.5 •••			ue of the hig	ther vertex t	than the y-value	of the lower ve	ertex?		4				
5.5 •••• 1	• • • • •		ue of the hig	ther vertex t	than the y-value	of the lower ve	ertex?		4				
5.5 ••••• 1 : 4 : 7 1	• • • • • •		ue of the hig	ther vertex t	than the y-value	of the lower ve	ertex?		4				
5.5 ••••• 1 : 4 : 7 1	• • • • • • • • • • • • • • • • • • •		ue of the hig	ther vertex t	than the y-value	of the lower ve	ertex?						

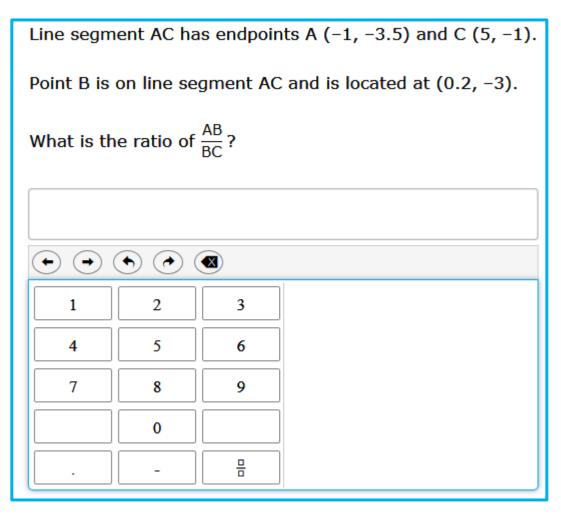
### Notes on Scoring

This response earns no credit (0 points) because it incorrectly compares the y-values of vertices of two quadratic functions represented in different ways. The student may have identified the y-value of the vertex of function f(x) and did not compare it to the y-value of the vertex of function g(x).

## Question 26

# **Question and Scoring Guidelines**

### **Question 26**



Points Possible: 1

**Content Cluster:** Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements

**Content Standard:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (*G.GPE.6*)

## **Scoring Guidelines**

Exemplar Response

 $\bullet \quad \frac{1}{4}$ 

Other Correct Responses

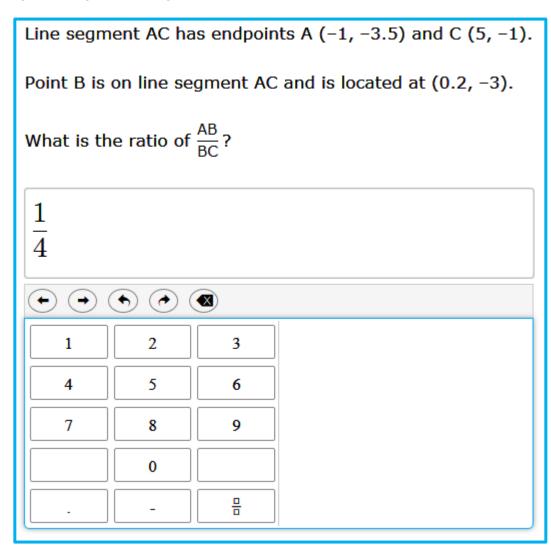
• Any equivalent value.

For this item, a full-credit response includes:

• A correct ratio (1 point).

**Question 26** 

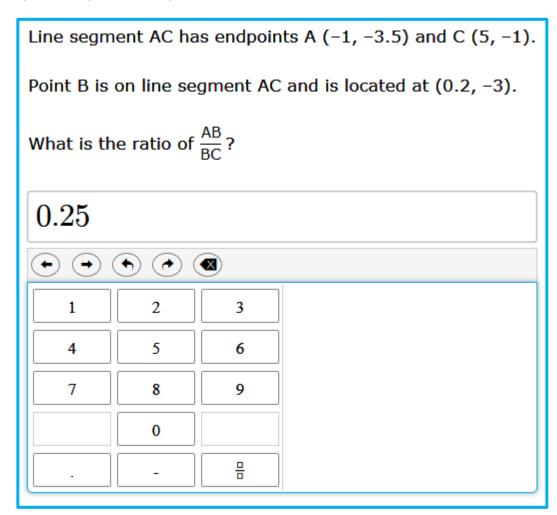
Sample Responses



### Notes on Scoring

This response earns full credit (1 point) because it shows a correct ratio of  $\frac{AB}{BC}$  or  $\frac{1}{4}$ .

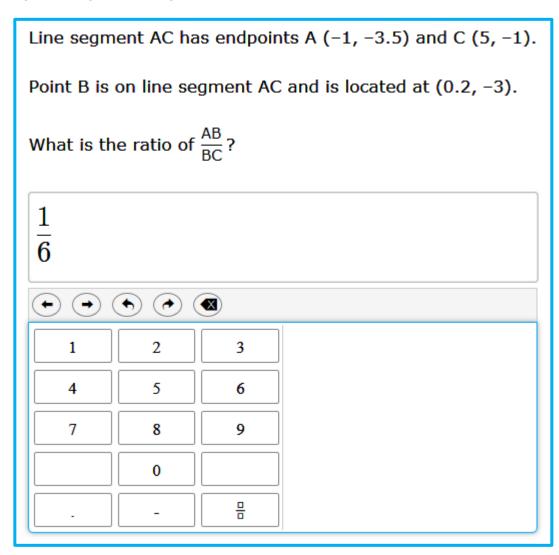
One of several ways to approach this situation is to use a distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , by substituting coordinates of points to find the length of a line segment AB and the length a line segment BC. Since AB = 1.3 and BC = 5.2, the ratio of  $\frac{AB}{BC}$  is  $\frac{1.3}{5.2}$  or  $\frac{1}{4}$ .



### Notes on Scoring

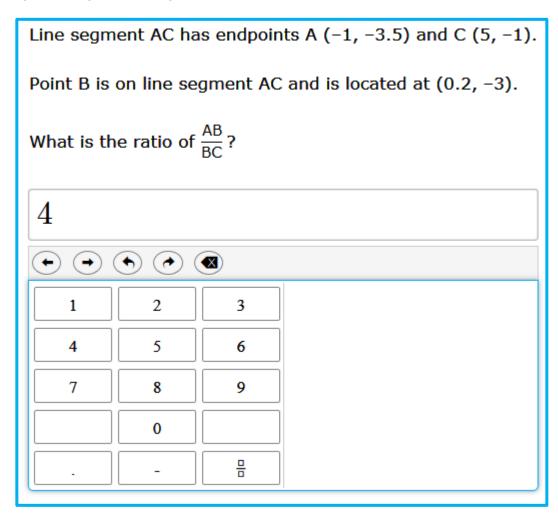
This response earns full credit (1 point) because it shows a correct ratio of  $\frac{AB}{BC}$  or  $\frac{1}{4}$  or .25.

One of several ways to approach this situation is to use a distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , by substituting coordinates of points to find the length of a line segment AB and the length of a line segment BC. Since AB = 1.3 and BC = 5.2, the ratio of  $\frac{AB}{BC}$  is  $\frac{1.3}{5.2}$  or  $\frac{1}{4}$ .



### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect ratio of  $\frac{AB}{BC}$  as  $\frac{1}{6}$ .



### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect ratio of  $\frac{AB}{BC}$  as 4.

## Question 27

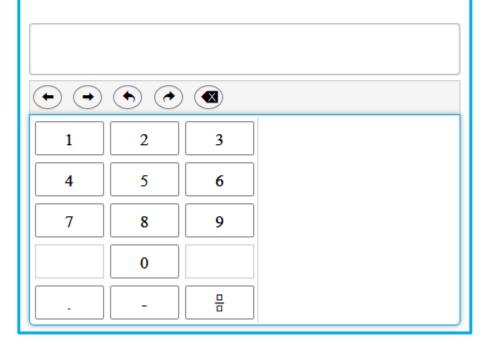
# **Question and Scoring Guidelines**

## **Question 27**

A function is shown.

 $f(x) = 5(x-2)^2 + 3$ 

What is the minimum value of the function?



Points Possible: 1

**Content Cluster:** Write expressions in equivalent forms to solve problems

**Content Standard:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  $(A.SSE.3b) \star$ 

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

Exemplar Response

• 3

### Other Correct Responses

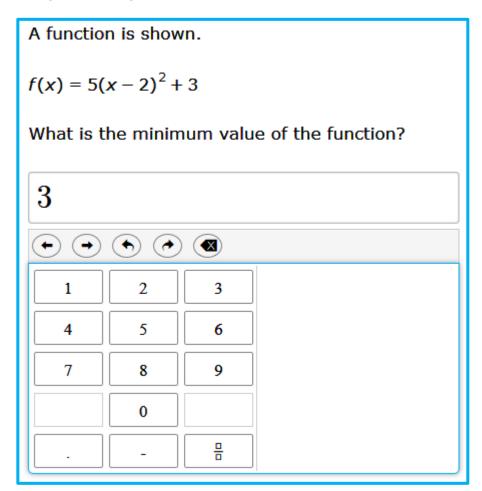
• Any equivalent value.

For this item, a full-credit response includes:

• The correct value (1 point).

**Question 27** 

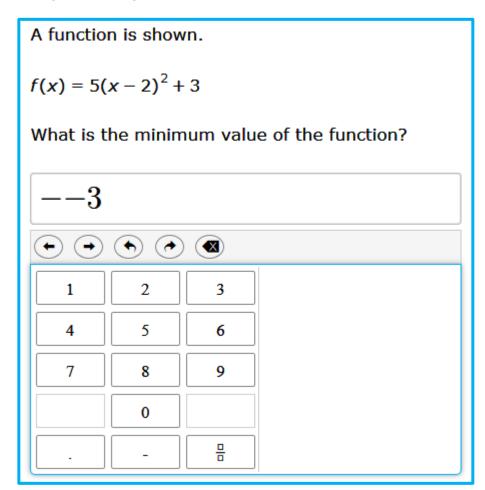
Sample Responses



#### Notes on Scoring

This response receives full credit (1 point) because it shows a correct minimum value, 3, for the function f(x).

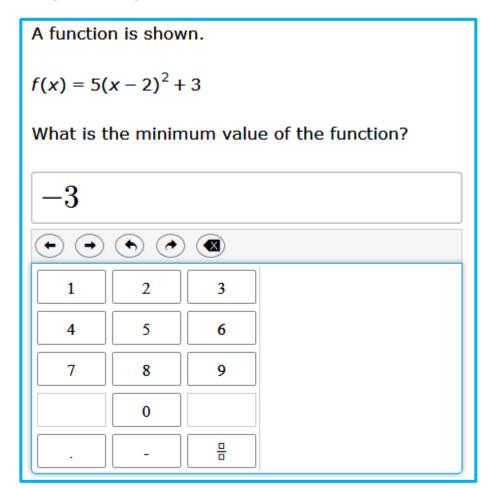
While there are several equivalent forms of the quadratic function, the vertex form  $f(x) = a(x - h)^2 + k$  reveals coordinates (h, k) of the vertex point, where k is the minimum value of the function, if a is positive. In this situation, a function  $f(x) = 5(x - 2)^2 + 3$  opens up, and has a vertex at (2, 3) with a minimum value at 3.



### Notes on Scoring

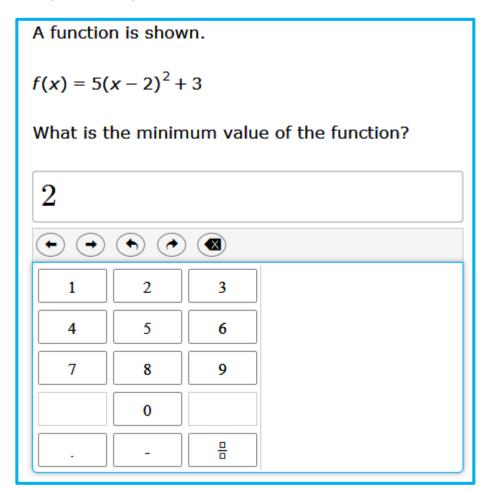
This response receives full credit (1 point) because it shows a correct minimum value, -3 or 3, for the function f(x).

While there are several equivalent forms of the quadratic function, the vertex form  $f(x) = a(x - h)^2 + k$  reveals coordinates (h, k) of the vertex point, where k is the minimum value of the function, if a is positive. In this situation, a function  $f(x) = 5(x - 2)^2 + 3$  opens up, and has a vertex at (2, 3) with a minimum value at 3.



### Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect minimum value for the function f(x).



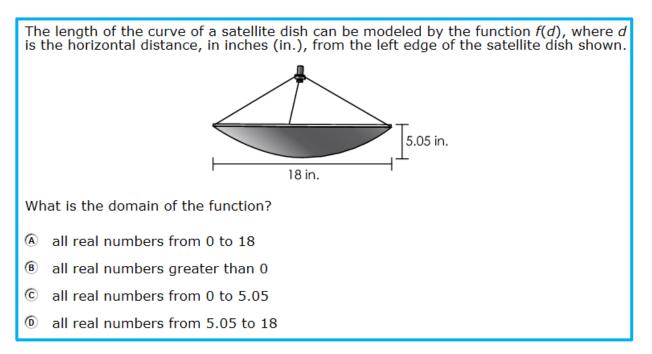
### Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect minimum value for the function f(x).

## Question 28

# **Question and Scoring Guidelines**

## **Question 28**



#### Points Possible: 1

**Content Cluster:** Interpret functions that arise in applications in terms of the context

**Content Standard:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)\*

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

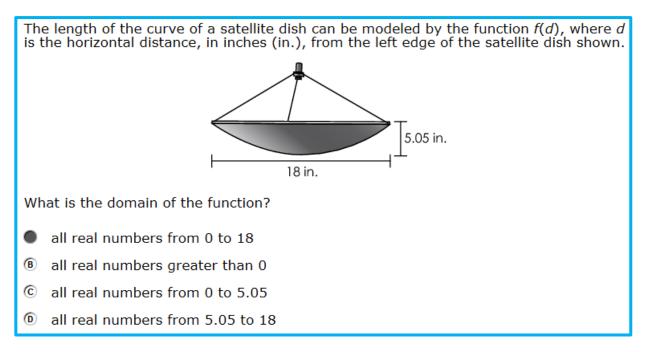
<u>Rationale for Option A:</u> **Key** – The student correctly identifies that the domain for this function describes the horizontal measurement of the dish and is the set of real values bounded by 0 and 18 inches.

<u>Rationale for Option B:</u> The student may correctly identify that the domain for this function must include values that are greater than 0 but does not realize that the horizontal distance from the left side of the dish cannot exceed 18 inches and selects all real numbers greater than zero.

<u>Rationale for Option C:</u> The student may confuse the domain and range of a function that would model the curve. The range is the set of values for the depth of the satellite dish. The domain is the set of values that describes a horizontal distance from the left edge of the satellite dish to a point on the satellite dish.

<u>Rationale for Option D:</u> The student may use the two given numbers in the graphic to represent the domain.

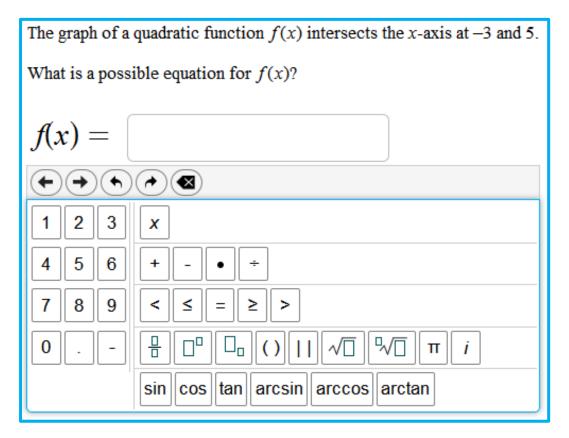
### Sample Response: 1 point



## Question 29

# **Question and Scoring Guidelines**

## **Question 29**



Points Possible: 1

**Content Cluster:** Analyze functions using different representations

**Content Standard:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

*a.* Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (*F.IF.8a*)

## Scoring Guidelines

Exemplar Response

•  $f(x) = x^2 - 2x - 15$ 

### Other Correct Responses

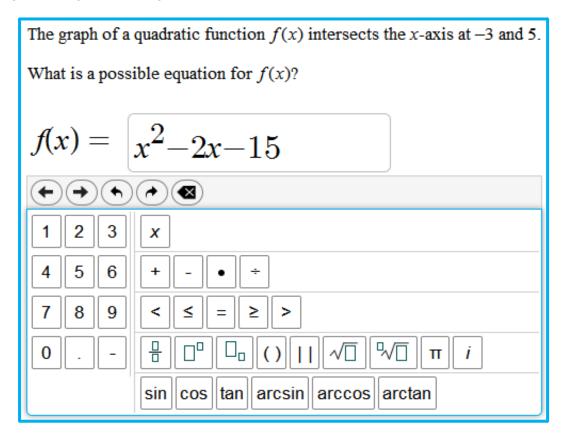
• Any function equivalent to f(x) = b(x+3)(x-5), where b is a real number.

For this item, a full-credit response includes:

• A correct function (1 point).

**Question 29** 

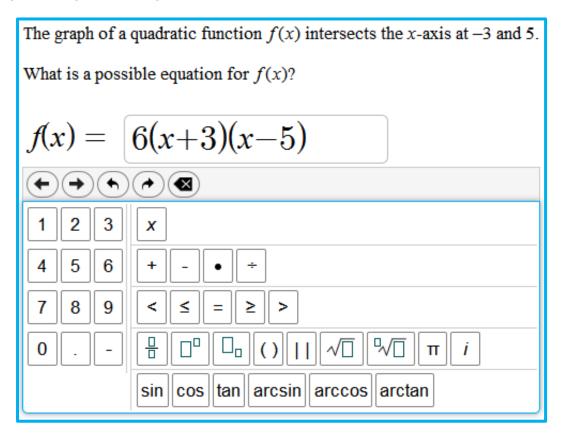
Sample Responses



### Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent form,  $y = x^2 - 2x - 15$ , for a correct quadratic function y = b(x + 3)(x - 5), where b = 1.

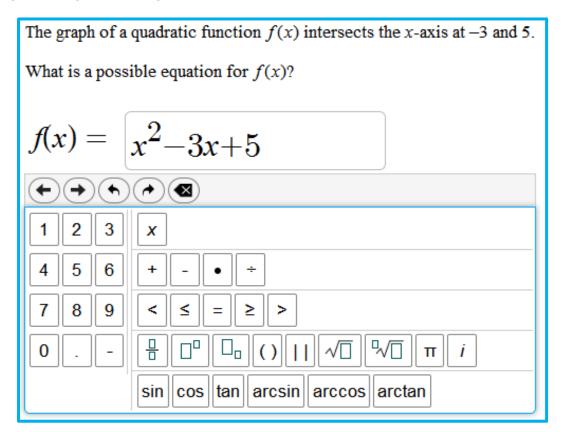
The intercept form for a quadratic function is y = b(x - r)(x - s), where r and s are the points where the function intersects the x-axis, and b is a non-zero real number. Working backwards by substituting both x-intercepts in the intercept form for a quadratic function creates an equation representing a family of quadratic functions with the same x-intercepts. In this situation, for x = -3 and x = 5, working backwards creates a function that is y = b(x + 3)(x - 5), where b is any non-zero real number.



### Notes on Scoring

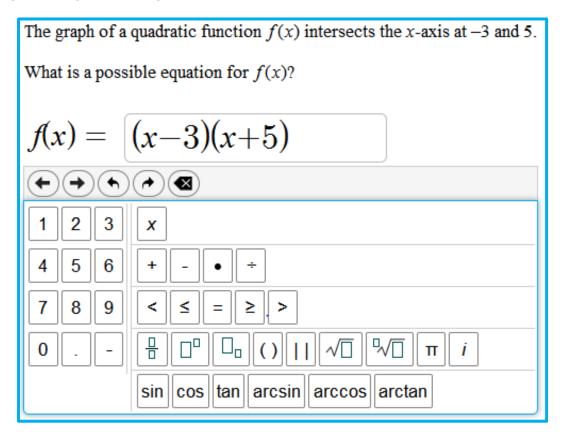
This response earns full credit (1 point) because it shows a correct equation, y = 6(x + 3)(x - 5), in the form of y = b(x + 3)(x - 5), where b = 6.

The intercept form for a quadratic function is y = b(x - r)(x - s), where *r* and *s* are the points where the function intersects the *x*-axis and b is a non-zero real number. Working backwards by substituting both *x*-intercepts in the intercept form for a quadratic function creates an equation representing a family of quadratic functions with the same *x*-intercepts. In this situation, for x = -3 and x = 5, working backwards creates a function that is y = b(x + 3)(x - 5), where *b* is any non-zero real number.



### Notes on Scoring

This response earns no credit (0 points) because it shows a quadratic function that is not equivalent to y = 1(x + 3)(x - 5).



### Notes on Scoring

This response receives no credit (0 points) because it shows a quadratic function that is not equivalent to y = 1(x + 3)(x - 5).

**Question 30** 

**Question and Scoring Guidelines** 

## Question 30

An online retailer conducts a random survey of its customers. The survey shows that 80% of the customers receive their purchases within four days, and 95% of those customers are satisfied with the quality of their purchases.

What percent of all customers receive their purchases within four days and are not satisfied with the quality of their purchases?

A 4%

- B 5%
- © 19%
- D 24%

Points Possible: 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model

**Content Standard:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.  $(S.CP.6) \star$ 

 $(\star)$  indicates that modeling should be incorporated into the standard.

## **Scoring Guidelines**

<u>Rationale for Option A:</u> **Key** – The student correctly concluded that 5% of the customers who received their purchases within four days were not satisfied (100 - 95 = 5%), and 5% of 80% of all customers who received their purchases within four days is  $0.05 \bullet 80 = 4\%$ .

<u>Rationale for Option B:</u> This is incorrect. The student may have found that only 5% of the customers who received their purchases within four days were not satisfied, and did not multiply 5% by 80% to find the percentage of all customers.

<u>Rationale for Option C:</u> This is incorrect. The student may have reversed the conditional probability, found the complement of the customers who received their purchases within four days is 20% and then found 95% of 20% as  $0.95 \cdot 0.20 = 0.19$  or 19%.

<u>Rationale for Option D:</u> This is incorrect. The student may have found the percentage of all customers who received their purchases within four days and were satisfied with the purchases by multiplying 95% by 80%, or  $0.95 \cdot 0.80 = 0.76$  or 76%, and then subtracted that from 100% to get 24%.

## Sample Response: 1 point

An online retailer conducts a random survey of its customers. The survey shows that 80% of the customers receive their purchases within four days, and 95% of those customers are satisfied with the quality of their purchases.

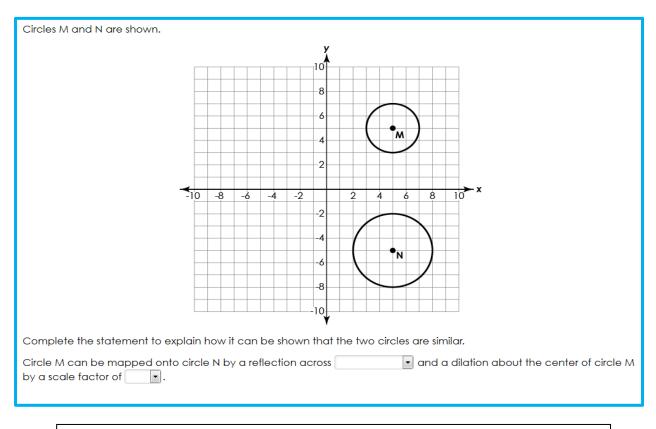
What percent of all customers receive their purchases within four days and are not satisfied with the quality of their purchases?

4%
5%
19%
24%

**Question 31** 

**Question and Scoring Guidelines** 

# **Question 31**



Points Possible: 1

Content Cluster: Understand and apply theorems about circles

**Content Standard:** Prove that all circles are similar using transformational arguments. (*G.C.1*)

# **Scoring Guidelines**

### Exemplar Response

• Circle M can be mapped onto circle N by a reflection across the x-axis and a dilation about its center by a scale factor of 1.5.

Other Correct Responses

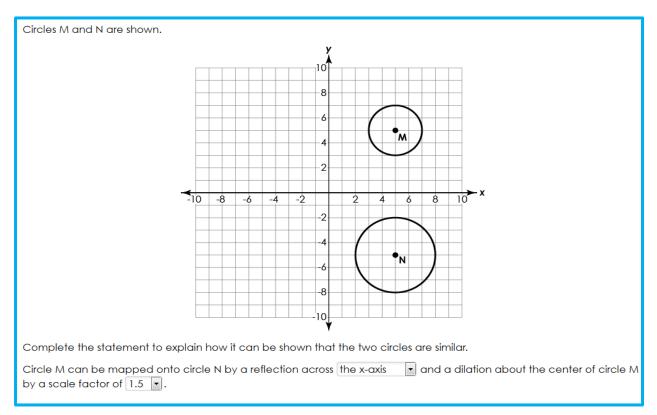
• N/A

For this item, a full-credit response includes:

• A correctly completed sentence (1 point).

**Question 31** 

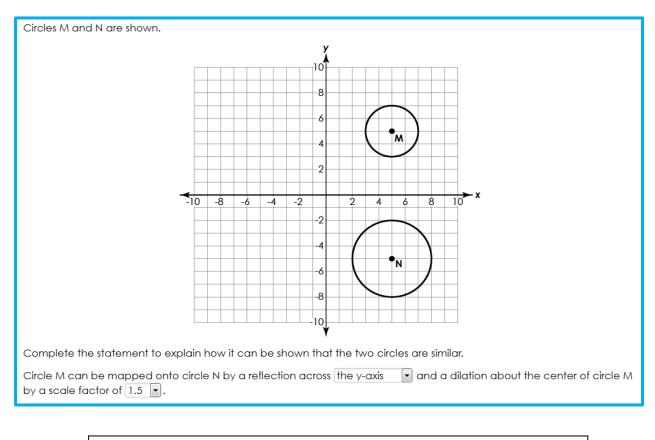
Sample Responses



### **Notes on Scoring**

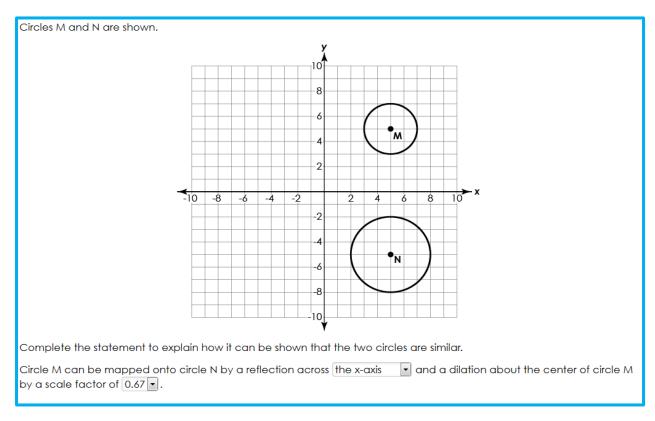
This response receives full credit (1 point) because it correctly completes the statement to explain that the two circles are similar.

Two circles are similar if one or more transformations (reflections, translations, rotations, dilation) can be found that map circle M onto circle N. A circle, by definition, is the set of points equidistant from a given point. Consequently, a circle is defined by the center and the length of the radius. Since the centers of the circles are equidistant from the *x*-axis, then after the reflection over the *x*-axis, the centers will coincide. Therefore, the correct option in the first drop-down menu is a reflection across the *x*-axis. A dilation about the center of a circle M is needed to increase the size of circle M. The scale factor of the dilation is equal to the ratio of the radius of the image Circle N to the original circle M, or  $\frac{3}{2}$  or 1.5.



### Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect selection of the reflection axis.



### Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect selection of the scale factor. The student may have calculated the scale factor of the dilation as the ratio of the radius of the image, circle M, to the radius of the image circle N, or  $\frac{2}{3}$  or 0.67.

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