

Ohio's State Tests

PRACTICE TEST ANSWER KEY & SCORING GUIDELINES

> INTEGRATED MATHEMATICS I

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Question No.	Item Type	Content Cluster	Content Standard	Answer Key	Points
1	Table Item	Understand that a function from one set (called the domain) to another set (called the range)Understand the concept of a function, and use functionassigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F.IF.1)			2 points
2	Multiple Choice	Interpret linear models.	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7)★	A	1 point
3	Graphic Response	Experiment with transformations in the plane.	ansformations tracing paper, or geometry		1 point
4	Hot Text Item	Prove geometric theorems both formally and informally using a variety of methods.	Prove and apply theorems about triangles. Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
5	Equation Item	Create equations that describe numbers or relationships.	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A.CED.2)★		1 point
6	Multiple Choice	Make geometric constructions.	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G.CO.12)	D	1 point
7	Multiple Choice	Experiment with transformations in the plane.	Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself (G.CO.3)	D	1 point
8	Multi- Select Item	Understand congruence in terms of rigid motions.	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G.CO.6)	C, D	1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
9	Equation Item	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements. Justify the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (G.GPE.5)			1 point
10	Equation Item	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (G.GPE.7)★		1 point
11	Equation Item	Solve equations and inequalities in one variable.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.3)		1 point
12	Graphic Response	Represent and solve equations and inequalities graphically.	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes. (A.REI.12)		1 point
13	Equation Item	Summarize, represent, and interpret data on two categorical and quantitative variables.	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S.ID.5)★		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
14	Multi- Select Item	Summarize, represent, and interpret data on a single count or measurement variable.	In the context or real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers). (S.ID.3)★	B, F	1 point
15	Multiple Choice	Summarize, represent, and interpret data on two categorical and quantitative variables.	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. c. Fit a linear function for a scatter plot that suggests a linear association. (S.ID.6c)★	В	1 point
16	Matching Item	Reason quantitatively and use units to solve problems.	Define appropriate quantities for the purpose of descriptive modeling. (N.Q.2)★		1 point
17	Equation Item	Interpret the structure of expressions.	Interpret expressions that represent a quantity in terms of its context. <i>a.</i> Interpret parts of an expression, such as terms, factors, and coefficients. (A.SSE.1a)★		1 point
18	Multiple Choice	Create equations that describe numbers or relationships.	Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. (A.CED.1)★	D	1 point

Question No.	ltem Type	Content Content Cluster Standard		Answer Key	Points
19	Equation Item	Create equations that describe numbers or relationships. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A.CED.3)★			1 point
20	Multi- Select Item	Represent and solve equations and inequalities graphically. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)		B, E	1 point
21	Equation Item	Build a function that models a relationship between two quantities. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms (F.BF.2)			1 point
22	Equation Item	Construct and compare linear, quadratic, and exponential models, and solve problems.	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input- output pairs (include reading these from a table). (F.LE.2)★		1 point
23	Multiple Choice	Interpret expressions for functions in terms of the situation they model.Interpret the parameters in a linear or exponential function in terms of a context. (F.LE.5)★		С	1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
24	Equation Item	Solve systems of equations. Solve systems of linear equations algebraically and graphically (A.REI.6)			1 point
25	Equation Item	Understand the concept of a function, and use function notation.	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F.IF.2)		1 point
26	Multiple Choice	Reason quantitatively and use units to solve problems.	Use units as a way to understand problems and to guide the solution of multi- step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (N.Q.1)★	A	1 point
27	Multiple Choice	Interpret functions that arise in applications in terms of the context.	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)★	С	1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
28	Hot Text Item	Understand solving equations as a process of reasoning and explain the reasoning.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A.REI.1)		1 point
29	Editing Task Choice Item	Represent and solve equations and inequalities graphically.			1 point

Question 1

Question and Scoring Guidelines

Question 1

Complete the first table so that f(x) is a function.

Complete the second table so that g(x) is not a function.

x	<i>f</i> (<i>x</i>)	x	g(x)
-1		-1	
	-8		-8
6		6	

Points Possible: 2

Content Cluster: Understand the concept of a function, and use function notation.

Content Standard: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then f(x) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation y = f(x). (*F.IF.1*)

Scoring Guidelines

Exemplar Response

x	<i>f</i> (<i>x</i>)	x	g(x)
-1	2	-1	2
3	-8	6	-8
6	64	6	64

Other Correct Responses

- For f(x), any input value that is not equal to -1 or 6, or equal to -1 or 6 with the same output value as the other instance in the table.
- For g(x), any input value that is equal to -1 or 6, as long as the output value is different than the other instance in the table.

For this item, a full-credit response includes:

• A correctly completed table for *f*(*x*) (1 point);

AND

• A correctly completed table for g(x) (1 point).

Question 1

Sample Responses

Sample Response: 2 points

Complet	Complete the first table so that $f(x)$ is a function.					
Complet	Complete the second table so that $g(x)$ is not a function.					
		1			1	
x	<i>f</i> (<i>x</i>)		x	g(x)		
-1	2		-1	2		
3	-8		6	-8		
6	64		6	64		

Notes on Scoring

This response earns full credit (2 points) for two correctly completed tables showing correct relations. For full credit, a response demonstrates an understanding that a function from one set (input) to another set (output) assigns to each input value exactly one output value.

In the first table, f(x) is a function if the input is any value that is not equal to -1 or 6, or equal to -1 or 6 with the same output value. In the second table, g(x) is not a function if the input value is equal to -1 or 6, as long as the output value is different than the other instance in the table.

Sample Response: 2 points

Complete the first table so that $f(x)$ is a function.						
Complete the second table so that $g(x)$ is not a function.						
x	<i>f</i> (<i>x</i>)		X	g(x)		
-1	-8		-1	2		
-1	-8		6	-8		
6	-8		6	14		

Notes on Scoring

This response earns full credit (2 points) for two correctly completed tables showing correct relations. For full credit, a response demonstrates an understanding that a function from one set (input) to another set (output) assigns to each input value exactly one output value.

In the first table, f(x) is a function. The response assigns x = -1 to f(-1) = -8 twice.

In the second table, g(x) is not a function. The response assigns x = 6 to the two different y values, g(6) = -8 and g(6) = 14.

Sample Response: 1 point

Complete the first table so that f(x) is a function.					
Complete the second table so that $g(x)$ is not a function.					
Y	<i>f</i> (<i>x</i>)		×	$q(\mathbf{x})$	
x	(x)		X	g(x)	
-1	2		-1	2	
-1	-8		6	-8	
6	2		6	14	

Notes on Scoring

This response earns partial credit (1 point) because it shows one incorrect table. While this response demonstrates a correctly completed table for g(x), the response cannot earn full credit because the table for f(x) shows a relation that is not a function. The same input value x = -1 assigns two different output values, f(-1) = 2 and f(-1) = -8.

Sample Response: 0 points

Complete the first table so that f(x) is a function. Complete the second table so that g(x) is not a function.

x	f(x)	x	g(x)
-1	2	-1	2
6	-8	3	-8
6	64	6	14

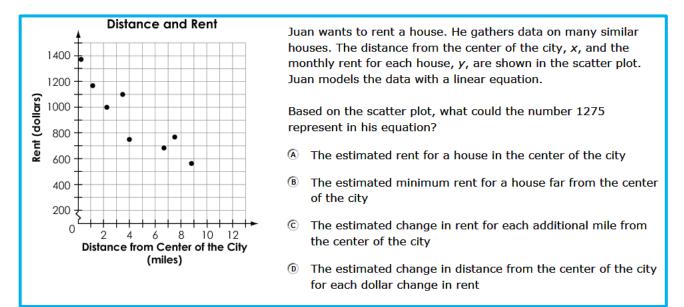
Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect tables.

Question 2

Question and Scoring Guidelines

Question 2



Points Possible: 1

Content Cluster: Interpret linear models.

Content Standard: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7) \star

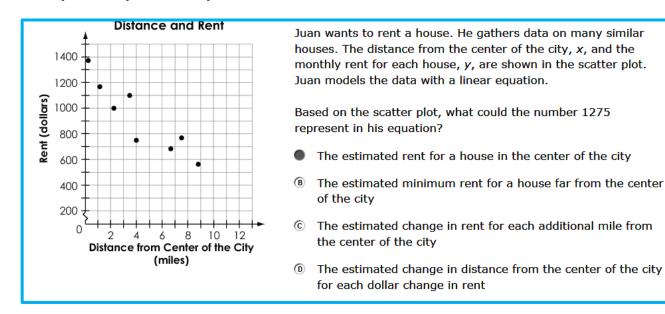
Scoring Guidelines

<u>Rationale for Option A:</u> **Key** – The student correctly interpreted 1275 as the *y*-intercept of the line of best fit. For each data point, the *x*-coordinate represents the distance from the center of the city to the house and the *y*-coordinate is an amount of rent for the house. For a *y*-intercept, (0, 1275) shows a rent of \$1275 for a house that is 0 miles away from the center of the city.

<u>Rationale for Option B</u>: This is incorrect. The student may have confused the rent of \$1275 with the distance of 1275 miles and thought that 1275 was the *x*-intercept.

<u>Rationale for Option C:</u> This is incorrect. The student may have associated the wording "estimated change in rent" with the slope and misinterpreted 1275 as the slope of the line of best fit.

<u>Rationale for Option D:</u> This is incorrect. The student may have associated the wording "estimated change in distance" with the slope and misinterpreted 1275 as the slope of the line of best fit.



Sample Response: 1 point

Question 3

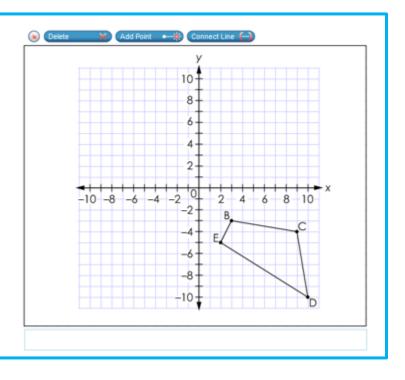
Question and Scoring Guidelines

Question 3

Quadrilateral BCDE is shown on the coordinate grid.

Keisha reflects the figure across the line y = x to create B'C'D'E'.

Use the Connect Line tool to draw quadrilateral B'C'D'E'.



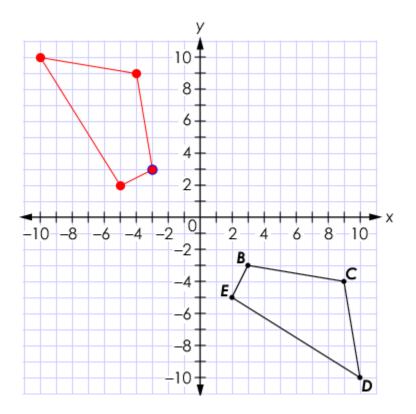
Points Possible: 1

Content Cluster: Experiment with transformations in the plane.

Content Standard: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (*G.CO.5*)

Scoring Guidelines

Exemplar Response



Other Correct Responses

• Additional lines and points are ignored.

For this item, a full-credit response includes:

• The correct quadrilateral (1 point).

Question 3

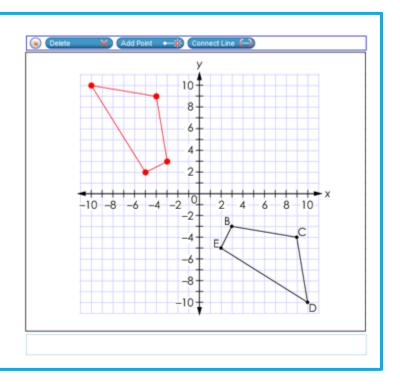
Sample Responses

Sample Response: 1 point

Quadrilateral BCDE is shown on the coordinate grid.

Keisha reflects the figure across the line y = x to create B'C'D'E'.

Use the Connect Line tool to draw quadrilateral B'C'D'E'.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct quadrilateral B'C'D'E' with the vertices B'(-3, 3), C'(-4, 9), D'(-10, 10) and E'(-5, 2).

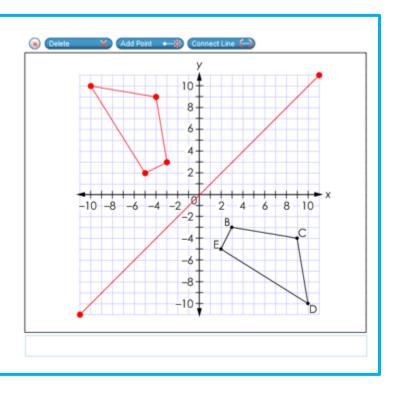
A reflection over a line y = x is a transformation in which each point of the original quadrilateral has an image that is the same distance from the line of reflection as the original point on the opposite side of the line. The reflection of the point (x, y) across the line y = x is the point (y, x).

Sample Response: 1 point

Quadrilateral BCDE is shown on the coordinate grid.

Keisha reflects the figure across the line y = x to create B'C'D'E'.

Use the Connect Line tool to draw quadrilateral B'C'D'E'.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct quadrilateral B'C'D'E' with the vertices B'(-3, 3), C'(-4, 9), D'(-10, 10) and E'(-5, 2) and a correct line segment belonging to the line of reflection y = x.

A reflection over a line y = x is a transformation in which each point of the original quadrilateral has an image that is the same distance from the line of reflection as the original point on the opposite side of the line. The reflection of the point (x, y) across the line y = x is the point (y, x).

Sample Response: 0 points

 Delete 🗙 🗚 Add Point 🛛 🔸 Connect Line 🕞 Quadrilateral BCDE is shown on the coordinate grid. 10 Keisha reflects the figure across 8 the line y = x to create B'C'D'E'. Use the Connect Line tool to draw guadrilateral B'C'D'E'. 0 -10 -8 -6 2 8 10 6 -42 4 B F

Notes on Scoring

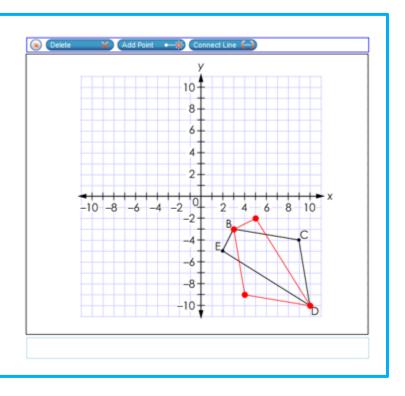
This response earns no credit (0 points) because it shows an incorrect quadrilateral due to an extra reflection. The quadrilateral BCDE is first reflected across the line y = x and then across the line y = -x.

Sample Response: 0 points

Quadrilateral BCDE is shown on the coordinate grid.

Keisha reflects the figure across the line y = x to create B'C'D'E'.

Use the Connect Line tool to draw quadrilateral B'C'D'E'.



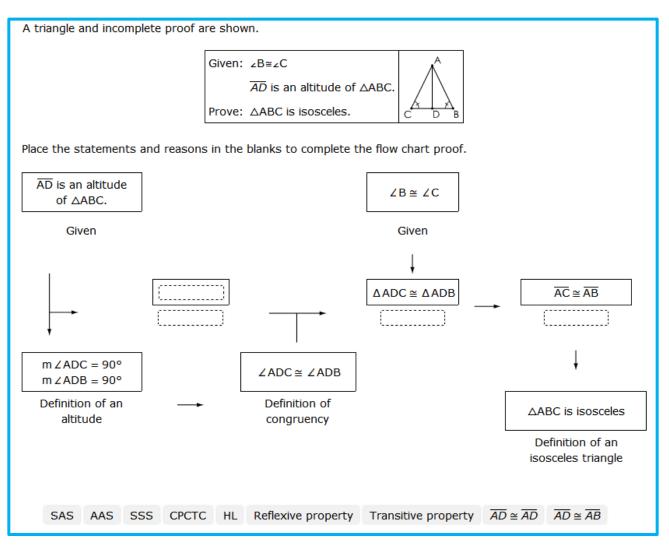
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect quadrilateral due to the wrong line of reflection being used. The quadrilateral BCDE is reflected across the line y = -x instead of y = x.

Question 4

Question and Scoring Guidelines

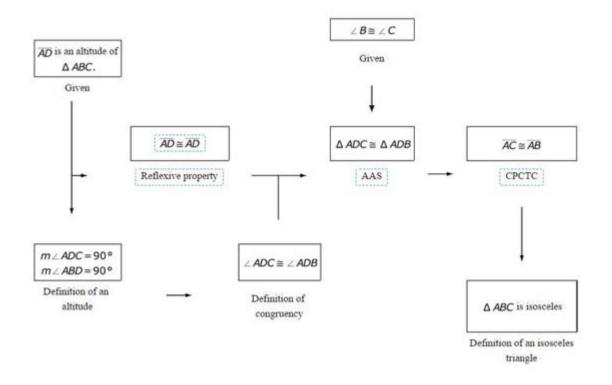
Question 4



Points Possible: 1 Content Cluster: Prove geometric theorems both formally and informally using a variety of methods. Content Standard: Prove and apply theorems about triangles. Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)

Scoring Guidelines

Exemplar Response



Other Correct Responses

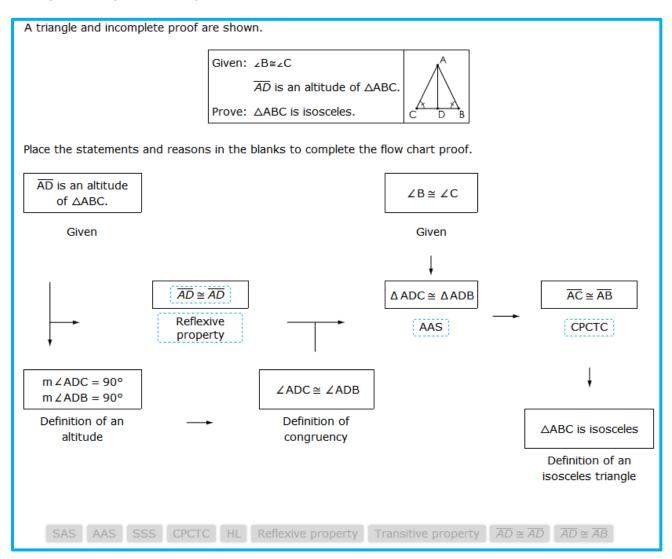
• In the first logical step, the statement and reason can be in any order and in any box as long as they are both there.

For this item, a full-credit response includes:

• The correct flow chart (1 point).

Question 4

Sample Responses

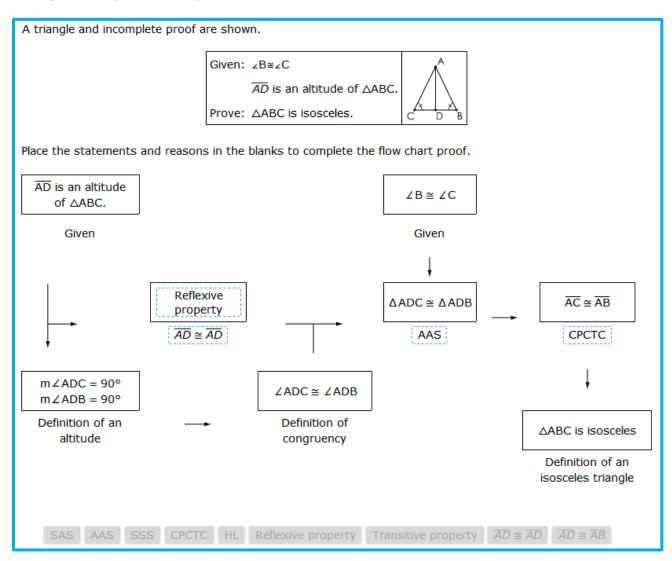


Notes on Scoring

This response receives full credit (1 point) because it shows correct statements and reasons needed to complete the flow chart.

In a flow chart, a correctly chosen piece of missing information should show where each statement is logically connected with the previous statement, which then allows for the next correct statement. The intent of this flow chart is to prove that if the triangle has congruent base angles, then the triangle is isosceles.

There are already two statements in the flow chart that show pairs of congruent angles: $\angle B \cong \angle C$ (Given) and $\angle ADC \cong \angle ADB$ (Definition of congruency). This enables the conclusion that $\triangle ADC$ will be congruent to $\triangle ABC$ as long as a pair of congruent sides is present, because of the Angle-Angle-Side (AAS) criterion. Looking at the diagram, $\overline{AD} \cong \overline{AD}$ by the Reflexive Property, so these statements can go in the left slots. This then allows AAS to be used in the middle slot. Following from that, it can be concluded that $\overline{AC} \cong \overline{AB}$ because of CPCTC (corresponding parts of congruent triangles are congruent). According to the definition, a triangle is an isosceles triangle if a pair of sides is congruent; therefore, $\triangle ABC$ is isosceles since \overline{AC} and \overline{AB} are congruent.

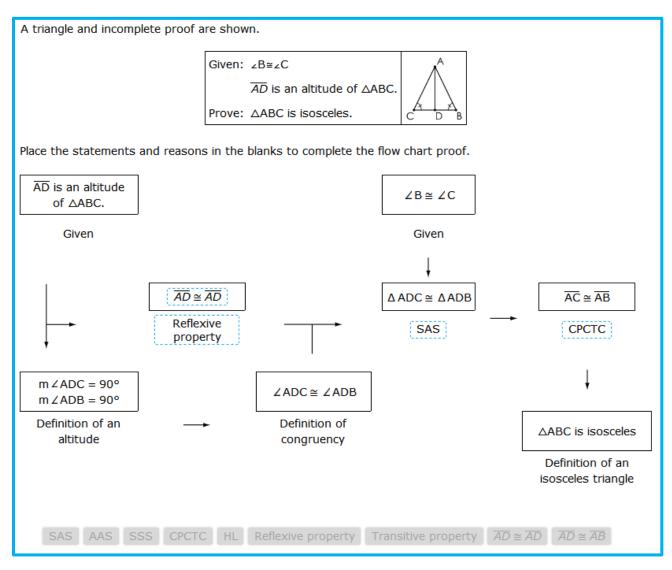


Notes on Scoring

This response receives full credit (1 point) because it shows correct statements and reasons needed to complete the flow chart.

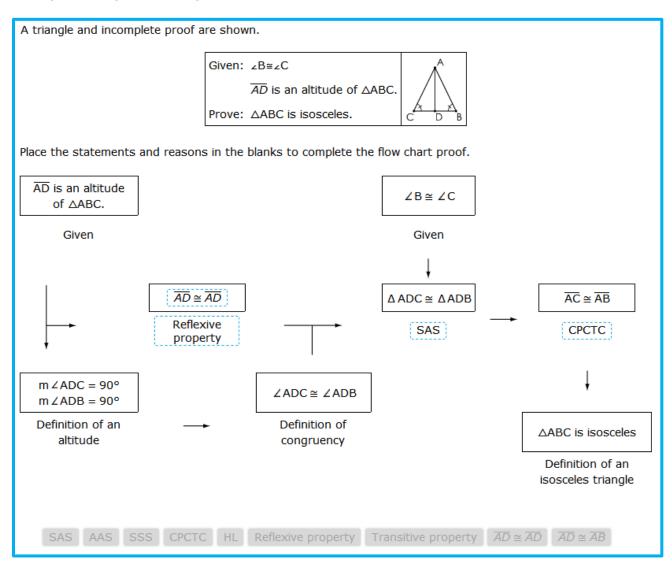
A correctly chosen missing piece of information should complete the flow chart to make each statement logically connected with the previous statement and allow for the next correct statement. The intent of this flow chart is to prove that if the triangle has congruent base angles, the triangle is isosceles.

There are already two statements in the flow chart that show pairs of congruent angles: $\angle B \cong \angle C$ (Given) and $\angle ADC \cong \angle ADB$ (Definition of congruency). This enables the conclusion that $\triangle ADC$ will be congruent to $\triangle ABC$ as long as a pair of congruent sides is present because of the Angle-Angle-Side (AAS) criterion. Looking at the diagram, $\overline{AD} \cong \overline{AD}$ by the Reflexive Property, so these statements can go in the left slots. This then allows AAS to be used in the middle slot. Following from that, it can be concluded that $\overline{AC} \cong \overline{AB}$ because of CPCTC (corresponding parts of congruent triangles are congruent). According to the definition, a triangle is an isosceles triangle if a pair of sides is congruent; therefore, $\triangle ABC$ is isosceles since \overline{AC} and \overline{AB} are congruent.

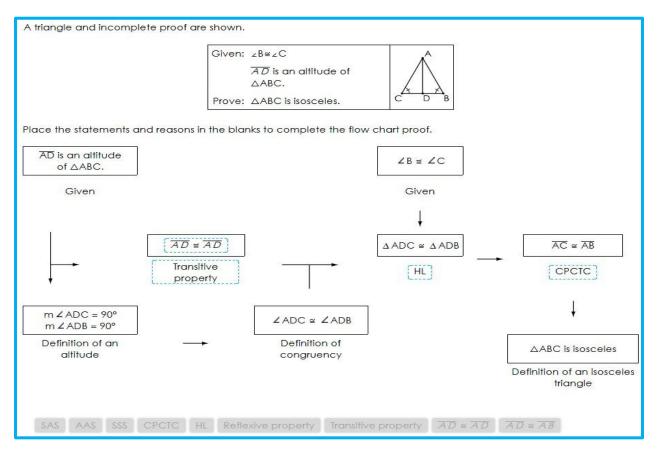


Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect reason in the middle slot (SAS) for the triangle congruence statement. Triangles that have only one pair of congruent sides, $\overline{AD} = \overline{AD}$, and two pairs of congruent angles cannot be associated with the SAS criteria.



Notes on Scoring This response receives no credit (0 points) because it shows an incorrect reason in the slot on the right.



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect reason in the lower-left slot.

Question 5

Question and Scoring Guidelines

Question 5

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

2	3	x	y							
5	6	+	-	•	÷					
8	9	<	≤	=	2	>				
	-				0		$\sqrt{\Box}$	V□	π	i
		sin	cos	tan	arcsin	arccos	arctan			
	2 5 8	2 3 5 6 8 9	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A.CED.2)*

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• $y = 270000(1.029)^{x}$

Other Correct Responses

• Any equivalent equation.

For this item, a full-credit response includes:

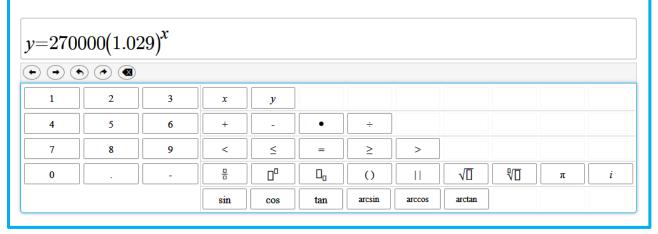
• The correct model (1 point).

Question 5

Sample Responses

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

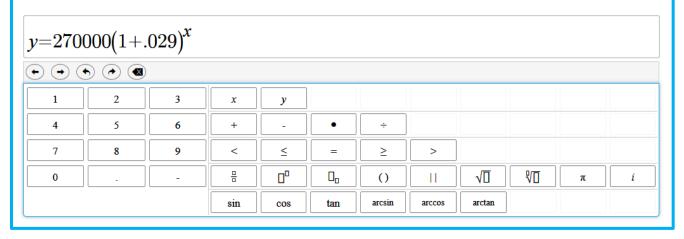


Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation that models a growing population of bats, *y*, after *x* years. The response correctly identifies the exponential model $P(1 + r)^x$, where P = 270,000 is the initial quantity of bats, $(1 + r)^x$ is a factor not depending on *P*, and r = 2.9% is the average annual percent rate, over the period of *x* years. If each year a population of bats grows by a constant factor of (1+.029), then after a period of *x* years, the population of bats, *y*, becomes $270,000(1 + .029)^x$, or $y = 270,000(1.029)^x$.

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, *y*, after *x* years.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation that models a growing population of bats, *y*, after *x* years. The response correctly identifies the exponential model $P(1 + r)^x$, where P = 270,000 is the initial quantity of bats, $(1 + r)^x$ is a factor not depending on *P*, and r = 2.9% is the average annual percent rate, over the period of *x* years. If each year a population of bats grows by a constant factor of (1+.029), then after a period of *x* years, the population of bats, *y*, becomes $270,000(1 + .029)^x$.

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

y=270	000(.029	$\left(\mathbf{P} \right)^{x}$									
	• • •	,									
1	2	3	x	y							
4	5	6	+	_	•	÷					
7	8	9	<	≤	=	2	>				
0	-	-	<u>-</u>			0		$\sqrt{\Box}$	V□	π	i
			sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns no credit (0 points). The response correctly identifies an exponential model, but uses r^x as a factor not depending on *P* instead of $(1 + r)^x$. A population modeled by the equation $y = 270,000(.029)^x$, where the factor independent from *P* is smaller than 1, represents decay, not growth.

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

270000	0(1.029x))									
$\bullet \bullet \bullet$	• • •										
1	2	3	x	y							
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	≥	>				
0	-	-				0		$\sqrt{\Box}$	₽□	π	i
			sin	cos	tan	arcsin	arccos	arctan			

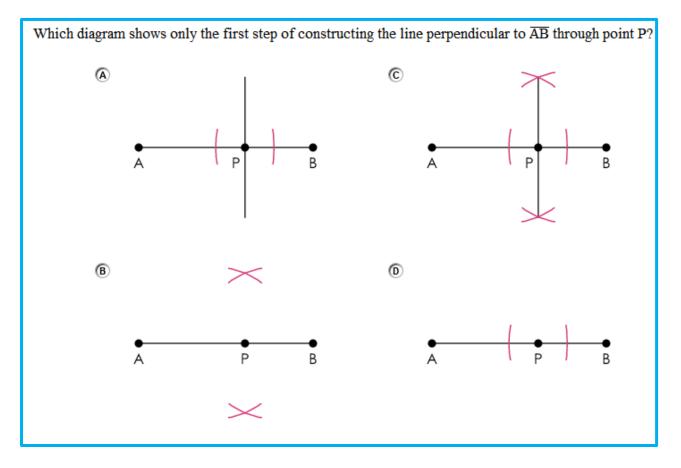
Notes on Scoring

This response earns no credit (0 points) because it confuses an exponential model with a linear model and uses (1 + r)xas a factor not depending on *P*, instead of $(1 + r)^x$. A population modeled by the equation y = 270,000(1.029)x has an average rate of change of 270,000 • 1.029 bats per year, instead of an average annual rate of 2.9 percent per year.

Question 6

Question and Scoring Guidelines

Question 6



Points Possible: 1

Content Cluster: Make geometric constructions.

Content Standard: Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G.CO.12)

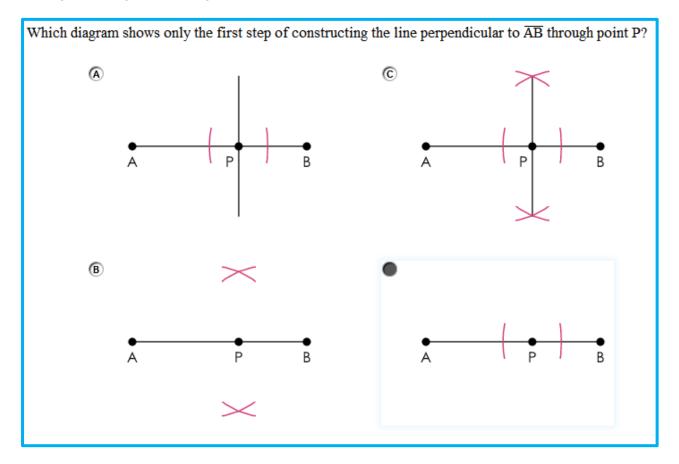
Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may not have realized that there are other steps between creating two points on a line segment AB that are equidistant from point P and drawing the line through point P perpendicular to the line segment AB.

<u>Rationale for Option B:</u> This is incorrect. The student may not have realized that the arc marks above and below point P cannot be constructed before constructing points on line segment AB that are equidistant from point P.

<u>Rationale for Option C:</u> This is incorrect. The student may have identified the last step instead of the first.

<u>Rationale for Option D:</u> **Key** – The student correctly identified that the first step is to create two points on line segment AB that are equidistant from point P, to use as the centers for constructing arcs above and below point P.



Question 7

Question and Scoring Guidelines

Question 7

Parallelogram ABCD is shown. Point E is the midpoint of segment AB. Point F is the midpoint of segment CD. 8 4 -8 8 -40 4 -4 8 Which transformation carries the parallelogram onto itself? A reflection across line segment AC a reflection across line segment EF **B** C a rotation of 180 degrees clockwise about the origin a rotation of 180 degrees clockwise about the center of the parallelogram 0

Points Possible: 1

Content Cluster: Experiment with transformations in the plane.

Content Standard: Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself (*G.CO.3*)

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that if diagonal \overline{AC} divides ABCD into two congruent triangles, then the parallelogram would have a line of symmetry over diagonal \overline{AC} . However, since \overline{AC} is not perpendicular to \overline{BD} , vertex B will not be carried onto vertex D.

<u>Rationale for Option B</u>: This is incorrect. The student may have thought that since points E and F are midpoints of the sides \overline{AB} and \overline{CD} , the parallelogram has a horizontal line of symmetry. However, since \overline{EF} is not perpendicular to \overline{AB} and \overline{CD} , vertex A will not be carried onto vertex B, and vertex D will not be carried onto vertex C.

<u>Rationale for Option C:</u> This is incorrect. The student may have realized that a 180-degree rotation could carry the parallelogram onto itself, but did not take into account that this depends on where the center of rotation is. When the center of rotation is at the origin, the image of the parallelogram is in Quadrant III, meaning the image will not carry onto the pre-image.

<u>Rationale for Option D:</u> **Key** – The student noted that all parallelograms have 180-degree rotational symmetry about the center of the parallelogram (i.e., vertex A will be carried onto vertex C, vertex B will be carried onto vertex D, vertex C will be carried onto vertex A, and vertex D will be carried onto vertex B).

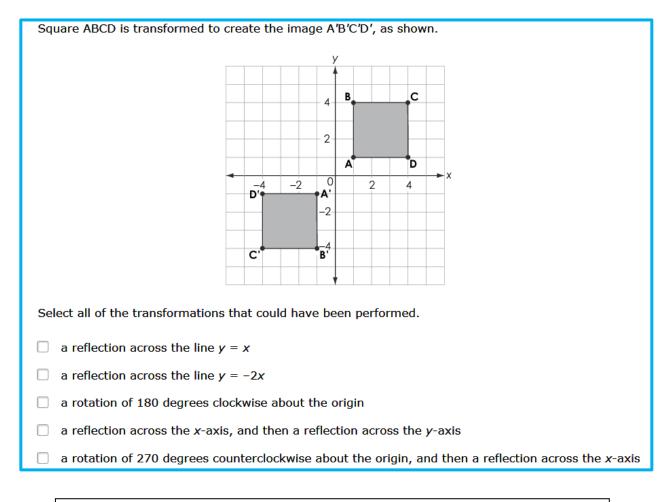
Parallelogram ABCD is shown. Point E is the midpoint of segment AB. Point F is the midpoint of segment CD. 8 4 8 -8 -4 0 4 -4 8 Which transformation carries the parallelogram onto itself? a reflection across line segment AC A B a reflection across line segment EF a rotation of 180 degrees clockwise about the origin C a rotation of 180 degrees clockwise about the center of the parallelogram

Sample Response: 1 point

Question 8

Question and Scoring Guidelines

Question 8



Points Possible: 1

Content Cluster: Understand congruence in terms of rigid motions.

Content Standard: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (*G.CO.6*)

Scoring Guidelines

<u>Rationale for First Option</u>: This is incorrect. The student may have thought that both given figures have to be carried onto themselves by reflecting across y = x, instead of carrying ABCD onto A'B'C'D'.

<u>Rationale for Second Option:</u> This is incorrect. The student may have seen that the line of reflection of y = -2x would create an image of square ABCD in Quadrant III, but did not confirm that the line of reflection is a perpendicular bisector of each line segment created by connecting corresponding vertices.

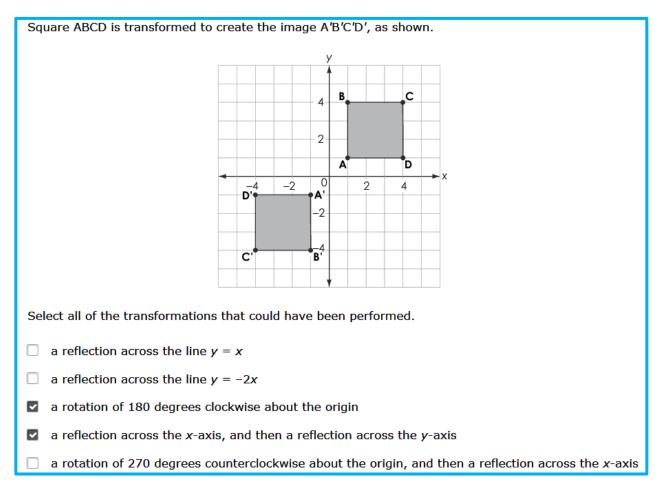
<u>Rationale for Third Option:</u> **Key** – The student correctly identified that with a 180-degree rotation, any point (x, y) will carry onto a point (-x, -y), so that a point A (1, 1) carries onto A'(-1, -1); B (1, 4) carries onto B' (-1, -4); C(4, 4) carries onto C'(-4, -4) and D(4, 1) carries onto D'(-4, -1).

<u>Rationale for Fourth Option:</u> **Key** – The student correctly identified that with a reflection across the *x*-axis, any point (x, y) will carry onto the point (x, -y), and then, the next reflection across the *y*-axis, will carry any point (x, -y) onto (-x, -y). Therefore, point A (1, 1) first carries onto (1, -1) and then onto A'(-1, -1); point B (1, 4) first carries onto (1, -4) and then onto B'(-1, -4); point C(4, 4) first carries onto (4, -4) and then onto C'(-4, -4); and point D(4, 1) first carries onto (4, -1) and then onto D'(-4, -1).

<u>Rationale for Fifth Option:</u> This is incorrect. The student may have seen that this set of transformations creates a final image in the same location as A'B'C'D' but did not see that this set of transformations does not carry the vertices in ABCD to their corresponding vertices in A'B'C'D'.

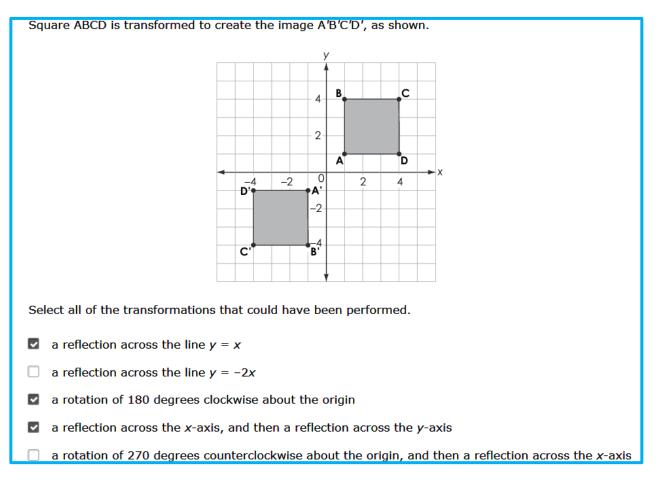
Question 8

Sample Responses



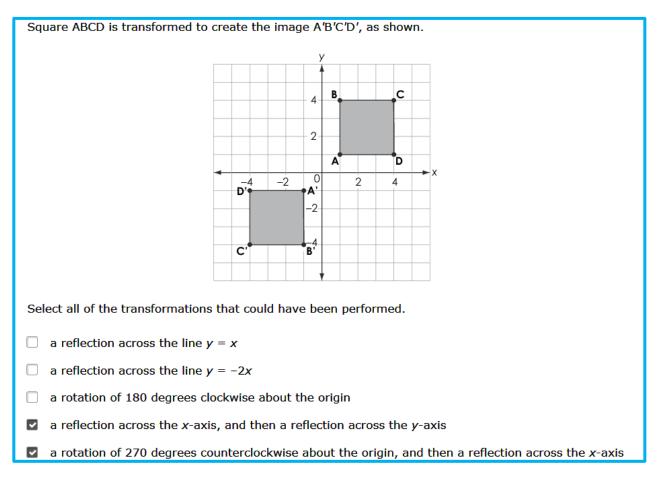
Notes on Scoring

This response earns full credit (1 point) because it selects both correct options, C and D, and no incorrect answer choices.



Notes on Scoring

This response earns no credit (0 points) because it selects both correct options, C and D, and one incorrect option, A.



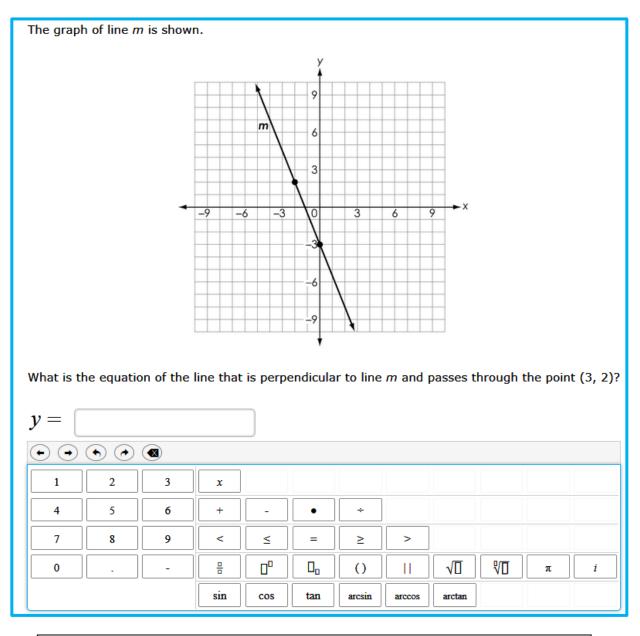
Notes on Scoring

This response earns no credit (0 points) because it selects one correct option, D, and one incorrect option, E.

Question 9

Question and Scoring Guidelines

Question 9



Points Possible: 1

Content Cluster: Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

Content Standard: Justify the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (*G.GPE.5*)

Scoring Guidelines

Exemplar Response

• $y = \frac{2}{5} + \frac{4}{5}$

Other Correct Responses

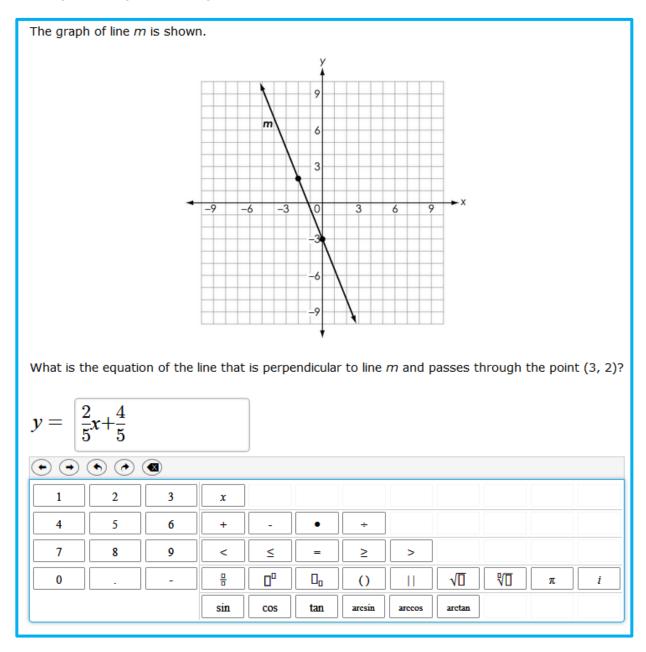
• Any equivalent equation.

For this item, a full-credit response includes:

• A correct equation (1 point).

Question 9

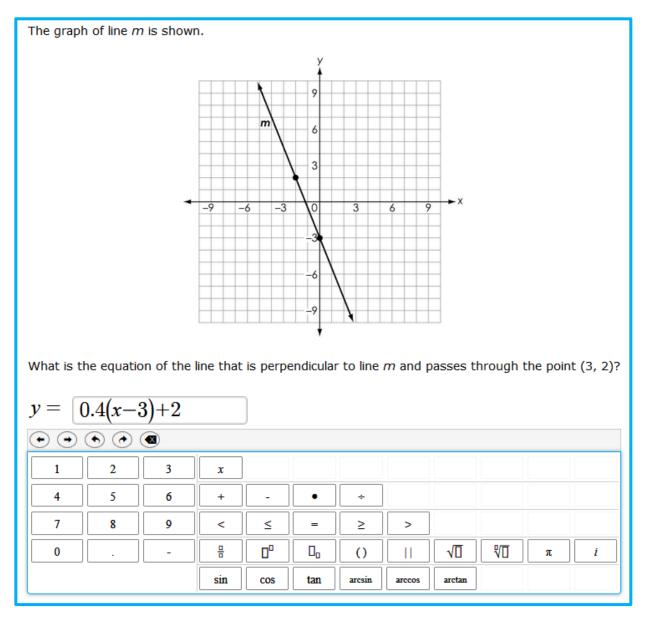
Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation of a line perpendicular to a given line that passes through a given point.

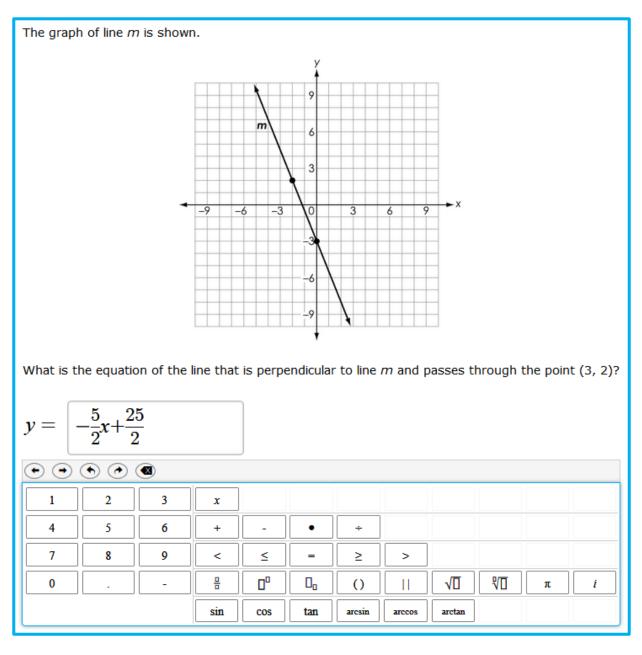
For this situation, the student can find the slope-intercept form of the equation of the line to get the correct answer. The slope of any line perpendicular to the given line is $\frac{2}{5}$ because it is the opposite reciprocal of the slope of line $m_1 - \frac{5}{2}$. If the slope of a perpendicular line, $\frac{2}{5}$, and the point it passes through, (3, 2), are substituted back into the slope-intercept form y = mx + b, the equation becomes $2 = \frac{2}{5} \cdot 3 + b$. From here, $b = \frac{4}{5}$, and the y-intercept of the perpendicular line is located at $(0, \frac{4}{5})$. The equation for the perpendicular line is then $y = \frac{2}{5} \cdot x + \frac{4}{5}$.



Notes on Scoring

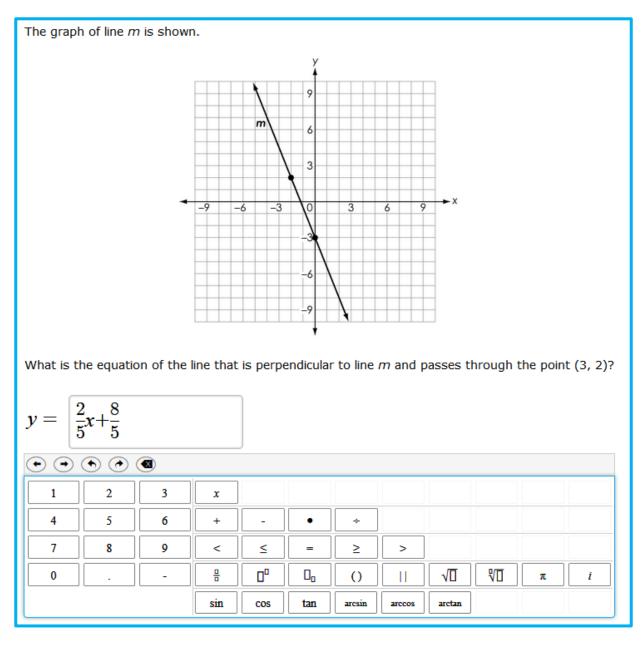
This response earns full credit (1 point) because it shows a correct equivalent equation of a line perpendicular to a given line that passes through a given point.

For this situation, the student can solve the point-slope form of the equation of the perpendicular line for y to get the correct answer. The slope of any line perpendicular to the given line is $\frac{2}{5}$, because it is the opposite reciprocal of the slope of line $m_1 - \frac{5}{2}$. If the slope of a perpendicular line, $\frac{2}{5}$ or 0.4, and the point it passes through, (3, 2), are substituted back into the slope-point form $y - y_1 = m(x - x_1)$, the form becomes y - 2 = 0.4(x - 3), and then y = 0.4(x - 3) + 2.



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation of the line perpendicular to a given line that passes through a given point.



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation of the line perpendicular to a given line that passes through a given point.

Question 10

Question and Scoring Guidelines

Question 10

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?



Points Possible: 1

Content Cluster: Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

Content Standard: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (G.GPE.7)*

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 21.8

Other Correct Responses

• Any number greater than or equal to 21.7 and less than or equal to 22.

For this item, a full-credit response includes:

• A correct value (1 point).

Question 10

Sample Responses

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?



Notes on Scoring

This response earns full credit (1 point) because it shows a correct value for the perimeter of triangle ABC, rounded to the nearest tenth.

The perimeter of triangle ABC is the sum of the three side lengths. The side lengths can be found by substituting the coordinates of the end points into the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, two at a time. The length of side AB is $\sqrt{(-4 + 1)^2 + (0 - 6)^2}$ or $\sqrt{45}$; the length of side BC is $\sqrt{(-1 - 3)^2 + (6 + 1)^2}$ or $\sqrt{65}$; and the length of side AC is $\sqrt{(-4 - 3)^2 + (0 + 1)^2}$ or $\sqrt{50}$.

The sum of the three side lengths is approximately 21.841529 or 21.8 rounded to the nearest tenth. Answers between 21.7 and 22 are accepted to allow for minor rounding errors.

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?



Notes on Scoring

This response earns full credit (1 point) because it shows a correct allowed value for the perimeter of triangle ABC, rounded to the nearest tenth that is greater than or equal to 21.7 and less than or equal to 22.

The perimeter of triangle ABC is the sum of the three side lengths. The side lengths can be found by substituting the coordinates of the end points into the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, two at a time. The length of side AB is $\sqrt{(-4 + 1)^2 + (0 - 6)^2}$ or $\sqrt{45}$; the length of side BC is $\sqrt{(-1 - 3)^2 + (6 + 1)^2}$ or $\sqrt{65}$; and the length of side AC is $\sqrt{(-4 - 3)^2 + (0 + 1)^2}$ or $\sqrt{50}$.

The sum of the three side lengths is approximately 21.841529 or 21.8 rounded to the nearest tenth. Answers between 21.7 and 22 are accepted to allow for minor rounding errors.

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?



Notes on Scoring

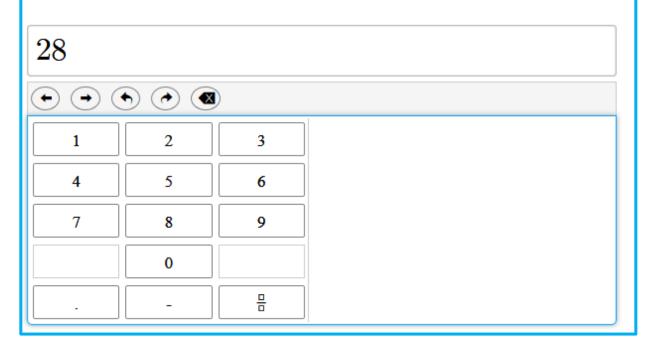
This response earns full credit (1 point) because it shows a correct allowed value for the perimeter of triangle ABC, rounded to the nearest tenth.

The perimeter of triangle ABC is the sum of the three side lengths. The side lengths can be found by substituting the coordinates of the end points into the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, two at a time. The length of side AB is $\sqrt{(-4 + 1)^2 + (0 - 6)^2}$ or $\sqrt{45}$; the length of side BC is $\sqrt{(-1 - 3)^2 + (6 + 1)^2}$ or $\sqrt{65}$; and the length of side AC is $\sqrt{(-4 - 3)^2 + (0 + 1)^2}$ or $\sqrt{50}$.

The sum of the three side lengths is approximately 21.841529 or 21.8 rounded to the nearest tenth. Answers between 21.7 and 22 are accepted to allow for minor rounding errors.

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?

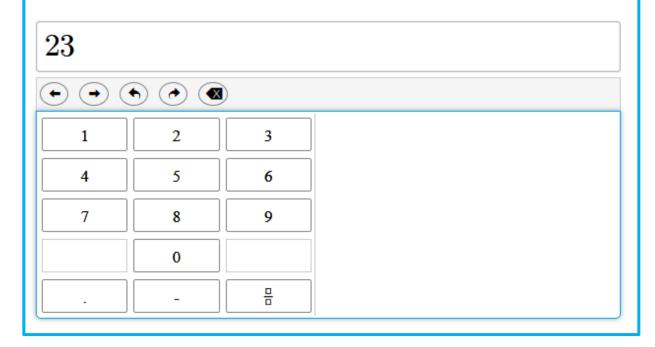


Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for a perimeter of a triangle ABC that falls outside of the allowable range of values.

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for a perimeter of a triangle ABC that falls outside of the allowable range of values.

Question 11

Question and Scoring Guidelines

Question 11

An equation is shown. $3x + \frac{4}{5} = 7 - 2x$ What is the solution to the equation? x =ک (م) 🔿 (→ 2 3 1 4 5 6 9 7 8 0 묘 _

Points Possible: 1

Content Cluster: Solve equations and inequalities in one variable.

Content Standard: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (*A.REI.3*)

Scoring Guidelines

Exemplar Response

 $\bullet \quad \frac{31}{25}$

Other Correct Responses

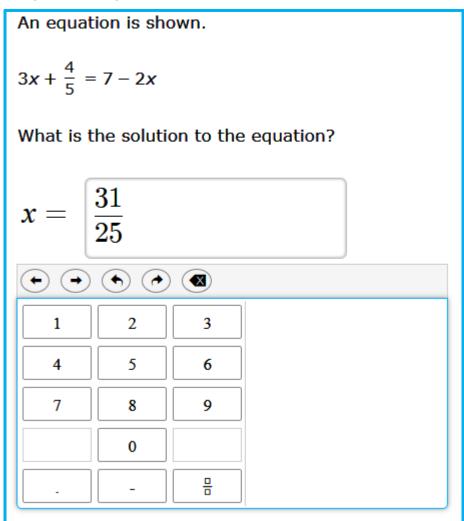
• Any equivalent values

For this item, a full-credit response includes:

• A correct value (1 point)

Question 11

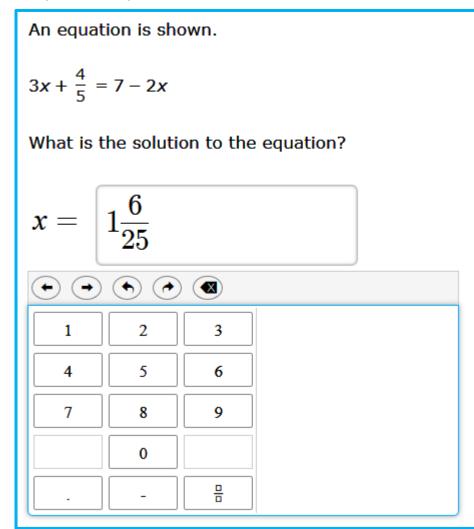
Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows the correct solution when solving linear equations in one variable.

To find a solution, the linear equation must be rearranged to isolate x. It can be solved in many ways. One way to solve the equation $3x + \frac{4}{5} = 7 - 2x$ is to add 2x to both sides to get $3x + (2x) + \frac{4}{5} = 7 - 2x + (2x)$, which simplifies to $5x + \frac{4}{5} = 7$. Then, $-\frac{4}{5}$ could be added to both sides of the equation to get $5x + \frac{4}{5} + (-\frac{4}{5}) = 7 + (-\frac{4}{5})$, which simplifies to $5x = 6\frac{1}{5}$ or $5x = \frac{31}{5}$. The next step is to multiply both sides by $\frac{1}{5}$ to get $(\frac{1}{5})(5x) = (\frac{31}{5})(\frac{1}{5})$, which simplifies to $x = \frac{31}{25}$.



Notes on Scoring

This response earns full credit (1 point) because it shows the correct solution when solving linear equations in one variable.

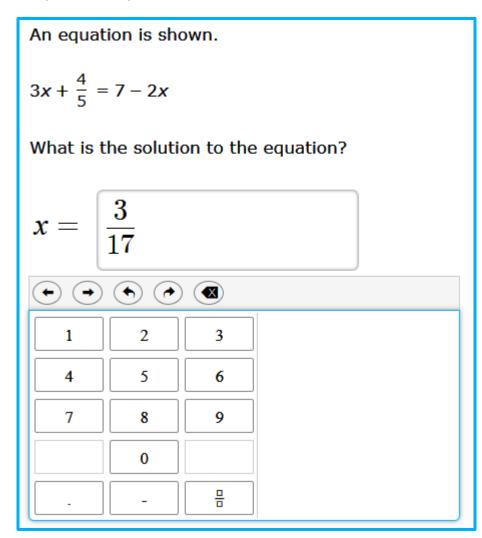
To find a solution, the linear equation must be rearranged to isolate x. It can be solved in many ways. One way to solve the equation $3x + \frac{4}{5} = 7 - 2x$ is to add 2x to both sides to get $3x + (2x) + \frac{4}{5} = 7 - 2x + (2x)$, which simplifies to $5x + \frac{4}{5} = 7$. Then, $-\frac{4}{5}$ could be added to both sides of the equation to get $5x + \frac{4}{5} + (-\frac{4}{5}) = 7 + (-\frac{4}{5})$, which simplifies to $5x = 6\frac{1}{5}$ or $5x = \frac{31}{5}$. The next step is to multiply both sides by $\frac{1}{5}$ to get $(\frac{1}{5})(5x) = (\frac{31}{5})(\frac{1}{5})$, which simplifies to $x = \frac{31}{25}$, which is equivalent to $1\frac{6}{25}$.

An equation is shown. $3x + \frac{4}{5} = 7 - 2x$ What is the solution to the equation? x = $(\blacklozenge$ **e** (💌) 2 3 1 4 5 6 7 8 9 0 믐

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect solution when solving linear equations in one variable.

To find a solution, the linear equation must be rearranged to isolate x. It can be solved in many ways. One way to solve the equation $3x + \frac{4}{5} = 7 - 2x$ is to add 2x to both sides to get $3x + (2x) + \frac{4}{5} = 7 - 2x + (2x)$, which simplifies to $5x + \frac{4}{5} = 7$. Then, $-\frac{4}{5}$ could be added to both sides of the equation to get $5x + \frac{4}{5} + (-\frac{4}{5}) = 7 + (-\frac{4}{5})$, which simplifies to $5x = 6\frac{1}{5}$ or $5x = \frac{31}{5}$. The student may have ended his or her solution without multiplying each side by $\frac{1}{5}$.



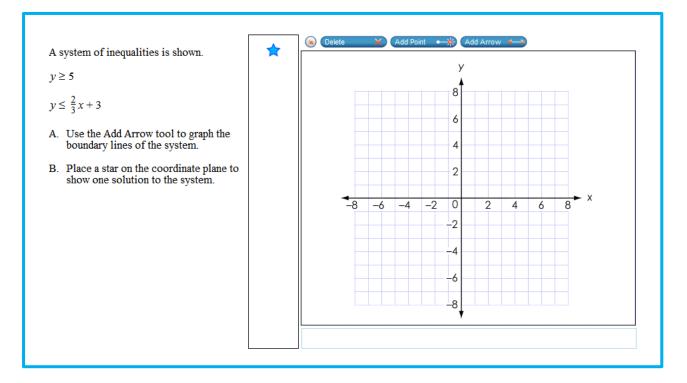
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect solution when solving linear equations in one variable.

Question 12

Question and Scoring Guidelines

Question 12



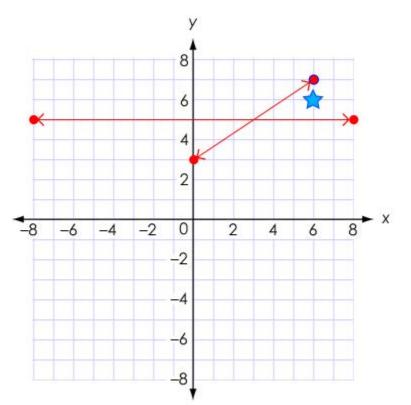
Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

Content Standard: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (*A.REI.12*)

Scoring Guidelines

Exemplar Response



Other Correct Responses

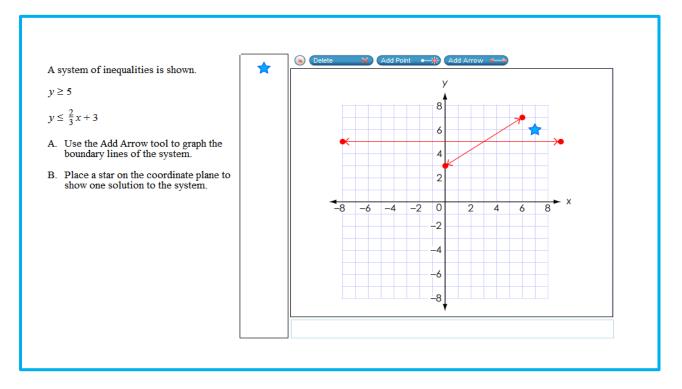
- The star can be placed anywhere in the correct region.
- Multiple stars may be placed on the coordinate plane as long as they are all in the correct region.

For this item, a full-credit response includes:

• Two correct lines and at least one correct placement of a star (1 point).

Question 12

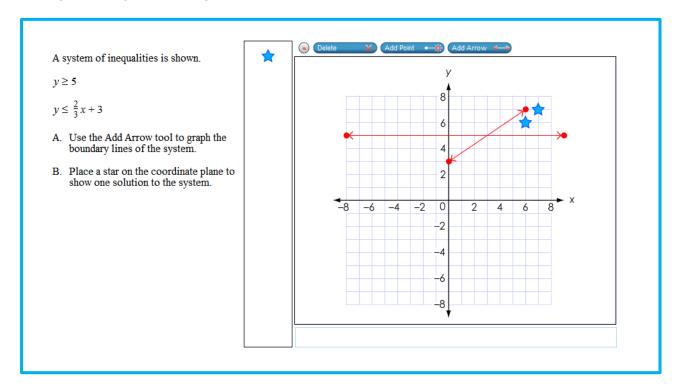
Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows two correct lines and one correct placement of a star, indicating a correct portion of a plane.

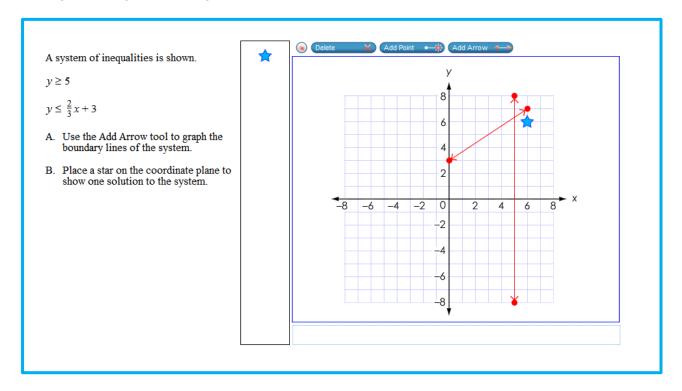
The graph of $y \ge 5$ includes a horizontal line with the y-intercept at (0, 5) and the half plane above the line. The graph of $y \le 2/3x + 3$ is a line with the y-intercept at (0, 3), a slope of 2/3 and the half plane below the line. The solution set for a system of the two linear inequalities is the intersection of the solution sets (corresponding half planes) for the individual inequalities.



Notes on Scoring

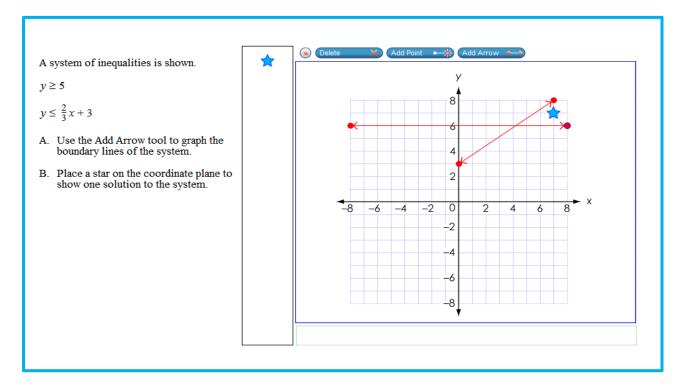
This response earns full credit (1 point) because it shows two correct lines and a correct placement of two stars, indicating a correct portion of a plane.

The graph of $y \ge 5$ includes a horizontal line with the y-intercept at (0, 5) and the half plane above the line. The graph of $y \le 2/3x + 3$ is a line with the y-intercept at (0, 3), a slope of 2/3 and the half plane below the line. The solution set for a system of the two linear inequalities is the intersection of the solution sets (corresponding half planes) for the individual inequalities.



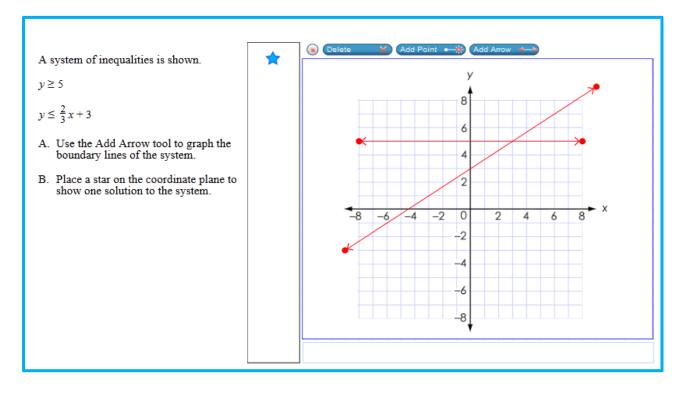
Notes on Scoring

This response earns no credit (0 points) because it shows one correct and one incorrect line, and a correct placement of a star based on the lines graphed.



Notes on Scoring

This response earns no credit (0 points) because it shows one correct and one incorrect line, and a correct placement of a star based on the lines graphed.



Notes on Scoring

This response earns no credit (0 points) because it shows two correct lines, but omits the placement of a star.

Question 13

Question and Scoring Guidelines

Question 13

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source. Nuclear Yes No 16 20 Yes Coal 2 12 No Given that a state does not use nuclear power, what percentage of those states use coal? % • • • • • • 1 2 3 4 5 6 7 8 9 0 -_

Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

Content Standard: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S.ID.5) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 62.5

Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• The correct probability (1 point).

Question 13

Sample Responses

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

					No	1
						lear
					Yes	No
			Coal	Yes	16	20
			COal	No	2	12
Given that a state do	es not use nuclear p	ower, what per	centage	of those s	tates use	coal?
62.5		%				
$\bullet \bullet \bullet \bullet$						
1	2	3				
4	5	6				
7	8	9				
	0					
	-	-				

Notes on Scoring

This response earns full credit (1 point) because it shows a correct percentage of states that use coal, given that a state does not use nuclear power, $\frac{20}{(20+12)}$ = .625 or 62.5%, as a result of a correct interpretation of relative frequencies for two categories presented in a two-way table

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuclear	
		Yes	No
Coal	Yes	16	20
	No	2	12

Given that a state does not use nuclear power, what percentage of those states use coal?

$\frac{2000}{32}$		%		
$\bullet \bullet \bullet \bullet$				
1	2	3		
4	5	6		
7	8	9		
	0			
	-	-		

Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent form, $\frac{2000}{32} = 62.5$, of a correct percentage of states that use coal, given that a state does not use nuclear power, $\frac{20}{(20+12)} = .625$ or 62.5%, as a result of a correct interpretation of relative frequencies for two categories presented in a two-way table

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

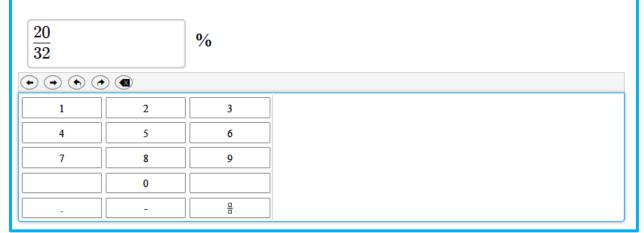
 Nuclear

 Yes
 No

 Coal
 Yes
 16
 20

 No
 2
 12

Given that a state does not use nuclear power, what percentage of those states use coal?



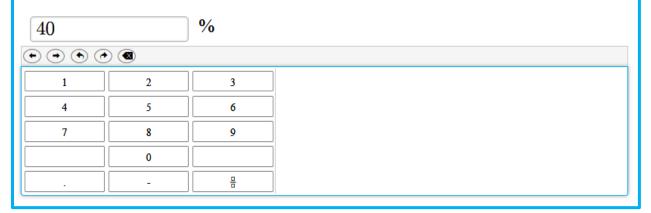
Notes on Scoring

This response earns no credit (0 points) because it provides an incorrect percentage of states that use coal, given that a state does not use nuclear power.

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuclear		
		Yes	No	
Coal	Yes	16	20	
	No	2	12	

Given that a state does not use nuclear power, what percentage of those states use coal?



Notes on Scoring

This response earns no credit (0 points) because it includes an incorrect percentage of states that use coal, given that a state does not use nuclear power.

Question 14

Question and Scoring Guidelines

Question 14

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

Content Standard: In the context or real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers). (*S.ID.3*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for First Option</u>: This is incorrect. The student may not have realized that since the time value (10) for the outlier point is equal to the mean of the time values (10) in the data set, the mean of the time will not change.

<u>Rationale for Second Option:</u> **Key** – The student correctly calculated the mean of the original data set as 4.06, and the mean of the data set as 3.37 with the outlier removed, and realized that the mean of the distance changed.

<u>Rationale for Third Option:</u> This is incorrect. The student may not have realized that in the original data set, the removed data point was the median point and equaled the average of the fifth and seventh time values, when ordered, so the median of the time will not change.

<u>Rationale for Fourth Option</u>: This is incorrect. The student may have thought that the median of the distances with the outlier included is 3.7. The median of the distances with the outlier omitted is [(3.7 + 3.7) / 2] = 3.7, so there is no change in the medians, because the two middles are both 3.7.

<u>Rationale for Fifth Option:</u> This is incorrect. The student may not have realized that since the time value for the outlier point is within the range of the time of the entire data set, the range of the time will not change.

<u>Rationale for Sixth Option:</u> **Key** – The student correctly recognized that the distance value for the outlier point is much greater than the distance values for other points. Range of the distances of the original data set is 11.0 - 1.0 = 10. Range of the distances when the outlier is removed is 4.9 - 1.0 = 3.9. The range of the distances will change.

Question 14

Sample Responses

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Notes on Scoring

This response earns full credit (1 point) because it shows correct choices B and F.

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Notes on Scoring

This response earns no credit (0 points) because it shows incorrect answer choices A and C, in addition to the correct choices B and F.

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Notes on Scoring

This response earns no credit (0 points) because it shows incorrect answer choices A and C, in addition to the correct choice B.

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

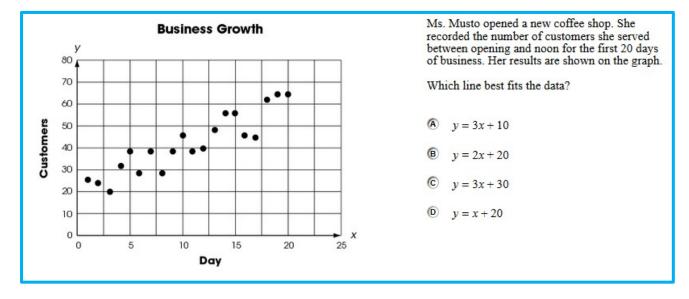
Notes on Scoring

This response earns no credit (0 points) because it shows incorrect answer choice D, in addition to the correct choices B and F.

Question 15

Question and Scoring Guidelines

Question 15



Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

Content Standard: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. c. Fit a linear function for a scatter plot that suggests a linear association. (S.ID.6c) \star

 (\star) indicates that modeling should be incorporated into the standard.

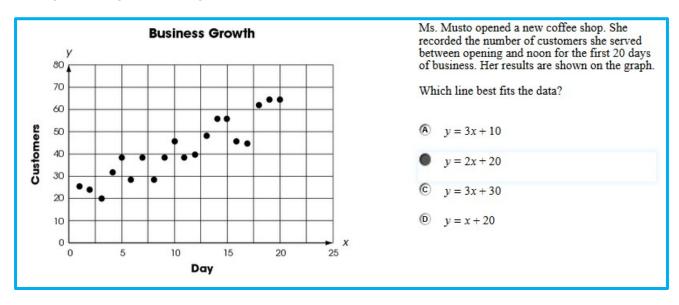
Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have set a line of best fit beneath all points and used the point with the highest y-value (20, 65) and the point with the lowest y-value (3, 20) to estimate the slope.

<u>Rationale for Option B:</u> **Key** – The student used the data on the scatter plot to set a line fitting the most points and determined that its slope must be greater than 1 and that the y-intercept is approximately 20.

<u>Rationale for Option C:</u> This is incorrect. The student may have set a line of best fit above all data points and used the point with the highest y-value (20, 65) and the point with the lowest y-value (3, 20) to estimate the slope.

<u>Rationale for Option D:</u> This is incorrect. The student may have chosen a line of best fit with the correct intercept and used the points (0.25, 2.5) and (4, 6.5) to estimate the slope without taking into account the scale used on the graph.



Sample Response: 1 point

Question 16

Question and Scoring Guidelines

Question 16

Select the most appropriate unit for each situation. square feet cubic feet feet minute minute minute Rate of walking to school Rate of painting a bedroom wall Rate of filling a bucket with water Rate of mopping the kitchen floor

Points Possible: 1

Content Cluster: Reason quantitatively and use units to solve problems.

Content Standard: Define appropriate quantities for the purpose of descriptive modeling. (*N.Q.2*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

Select the most appropriate unit for each situation.

	<u>feet</u> minute	square feet minute	cubic feet minute
Rate of walking to school			
Rate of painting a bedroom wall		V	
Rate of filling a bucket with water			~
Rate of mopping the kitchen floor			

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• A correct table (1 point).

Question 16

Sample Responses

Select the most appropriate unit for each situation.						
<u>feet</u> square feet <u>cubic feet</u> minute minute						
Rate of walking to school						
Rate of painting a bedroom wall						
Rate of filling a bucket with water						
Rate of mopping the kitchen floor						

Notes on Scoring

This response earns full credit (1 point) because it shows four correctly selected units for each situation.

The rate of walking to school is the distance to school divided by the time spent on walking to school. Since in this case the distance is measured in linear units, feet, the correct selection is the first box in the top row.

The rate of painting a bedroom wall is the area of the wall divided by the time spent on painting the wall. Since in this case the area is measured in square feet, the correct selection is the second box in the second row from the top. The rate of filling a bucket with water is measured by the volume of water divided by the time spent on filling the bucket. Since volume is measured in cubic units, or in this case in cubic feet, the correct selection is the third box in the third row from the top.

The rate of mopping the kitchen floor is the area of the floor divided by the time spent on mopping the floor. Since area is measured in square units, or in this case in square feet, the correct selection is the middle box of the bottom row.

Select the most appropriate unit for each situation.

	<u>feet</u> minute	square feet minute	cubic feet minute
Rate of walking to school			
Rate of painting a bedroom wall		V	
Rate of filling a bucket with water			
Rate of mopping the kitchen floor			

Notes on Scoring

This response earns no credit (0 points) because out of four responses it shows one incorrectly selected unit for Rate of mopping the kitchen floor. The student may think that because mopping is done with water, and the amount of water is measured in units of volume, the correct response is $\frac{\text{cubic feet}}{\text{minute}}$ instead of $\frac{\text{square feet}}{\text{minute}}$.

Select the most appropriate unit for each situation.											
feetsquare feetcubic feetminuteminuteminute											
Rate of walking to school	~										
Rate of painting a bedroom wall											
Rate of filling a bucket with water											
Rate of mopping the kitchen floor		V									

Notes on Scoring

This response earns no credit (0 points) because it shows one
incorrectly selected unit for Rate of painting a bedroom wall.
The student may think about the volume of the entire
bedroom and use the units of volume, $\frac{\text{cubic feet}}{\text{minute}}$, instead of
square feet
minute

Question 17

Question and Scoring Guidelines

Question 17

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

		pounds	
$\bullet \bullet \bullet$	•		
1	2	3	
4	5	6	
7	8	9	
	0		
	-	<u>_</u>	

Points Possible: 1

Content Cluster: Interpret the structure of expressions.

Content Standard: Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients. (A.SSE.1a) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 2.5

Other Correct Responses

• The correct value in any equivalent form.

For this item, a full-credit response includes:

• The correct value (1 point).

Question 17

Sample Responses

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

2.5		pounds	
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
	-	<u><u></u></u>	

Notes on Scoring

This response receives full credit (1 point) because it shows a correct total number of pounds of peaches and grapefruit, 2.5, that Juan buys. In the context of a situation, the first equation models the weight, p + g, of peaches, p pounds, and, grapefruit, g pounds, totaling 2.5 pounds of fruit.

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

$\frac{5}{2}$		pounds	
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
•	-		

Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent form, $\frac{5}{2} = 2.5$, of a correct total number of pounds of peaches and grapefruit, that Juan buys.

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

2.67		pounds	
$\bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
	-	<u>-</u>	

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect total number of pounds of peaches and grapefruit that Juan buys.

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

5.96		pounds	
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
	-	=	

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect total number of pounds of peaches and grapefruit that Juan buys.

Question 18

Question and Scoring Guidelines

Question 18

The population of rabbits on a large island doubles every year. On January 1, the population is 150 rabbits.

Which equation can be used to find the number of years, x, it will take for the population to reach 4,800?

- (A) 4,800 = 2x + 150
- (a) $4,800 = 2 \cdot 150^{x}$
- \bigcirc 4,800 = 2^x + 150
- (b) $4,800 = 150 \cdot 2^{x}$

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. (A.CED.1) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have recognized that the initial population of 150 rabbits doubles each year, but confused an exponential model where a different population size doubles each year with a linear model where a constant population size doubles each year.

<u>Rationale for Option B:</u> This is incorrect. The student may have correctly recognized that the situation should be modeled by an exponential model, but confused the initial population, 150, with the growth rate, 2.

<u>Rationale for Option C:</u> This is incorrect. The student may have correctly recognized that the situation should be modeled by an exponential model and recognized that the initial population is 150 rabbits, but incorrectly used addition instead of multiplication between factors of the exponential model $a \cdot b^x$.

<u>Rationale for Option D:</u> **Key** – The student correctly recognized that if the initial population of rabbits, 150, doubles at the end of the first year, the number of rabbits is $150 \cdot 2$. At the end of the second year, it is $150 \cdot 2 \cdot 2$ or $150 \cdot 2^2$. At the end of x years, the population of rabbits is $150 \cdot 2^x$. When the population reaches 4800 rabbits, the situation can be modeled by the exponential equation $4800 = 150 \cdot 2^x$.

Sample Response: 1 point

The population of rabbits on a large island doubles every year. On January 1, the population is 150 rabbits.

Which equation can be used to find the number of years, x, it will take for the population to reach 4,800?

- (A) 4,800 = 2x + 150
- (a) $4,800 = 2 \cdot 150^{x}$
- (c) 4,800 = 2^x + 150

4,800 = $150 \cdot 2^{x}$

Question 19

Question and Scoring Guidelines

Question 19

Kenji has at most \$30 to spend on lily bulbs and tulip bulbs at his local flower store. Lily bulbs cost \$4 each, and tulip bulbs cost \$2 each. Tax is included in the prices of the bulbs.

Create a constraint that can be used to represent all possible numbers of lily bulbs, *x*, and tulip bulbs, *y*, Kenji can buy.

$\bullet \bullet \bullet$	• • •										
1	2	3	x	y							
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0	-	-			0	0		$\sqrt{\Box}$	₽□	π	i
			sin	cos	tan	arcsin	arccos	arctan			

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A.CED.3)*

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• $4x + 2y \le 30$

Other Correct Responses

• Any equivalent inequality.

For this item, a full-credit response includes:

• A correct inequality (1 point).

Question 19

Sample Responses

Kenji has at most \$30 to spend on lily bulbs and tulip bulbs at his local flower store. Lily bulbs cost \$4 each, and tulip bulbs cost \$2 each. Tax is included in the prices of the bulbs.

Create a constraint that can be used to represent all possible numbers of lily bulbs, *x*, and tulip bulbs, *y*, Kenji can buy.

4x+2y	$4x + 2y \leq 30$											
$\bullet \bullet \bullet \bullet \bullet$												
1	2	3	x	y								
4	5	6	+	-	•	÷						
7	8	9	<	≤	=	2	>					
0		-	<u>-</u>			0		$\sqrt{\Box}$	₽Ū	π	i	
			sin	cos	tan	arcsin	arccos	arctan				

Notes on Scoring

This response earns full credit (1 point) because it shows a correct constraint, represented by a linear inequality.

In this situation, the constraint that can be used to represent all possible numbers of lily bulbs, x, and tulip bulbs, y, is a linear inequality describing the cost, \$30, of the entire purchase. The cost of x lily bulbs at \$4.00 each is \$4x. The cost of y tulip bulbs at \$2.00 each is \$2y. The total cost, (4x + 2y), cannot exceed \$30. Therefore, the constraint is $4x + 2y \le 30$.

Kenji has at most \$30 to spend on lily bulbs and tulip bulbs at his local flower store. Lily bulbs cost \$4 each, and tulip bulbs cost \$2 each. Tax is included in the prices of the bulbs.

Create a constraint that can be used to represent all possible numbers of lily bulbs, x, and tulip bulbs, y, Kenji can buy.

$30 \ge 2 \cdot y + 4 \cdot x$										
1 2 3										
4 5 6	+ - • ÷									
7 8 9										
0		i								
	sin cos tan arcsin arccos arctan									

Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent form of the correct constraint, represented by a linear inequality.

In this situation, the constraint that can be used to represent all possible numbers of lily bulbs, x, and tulip bulbs, y, is a linear inequality describing the cost, \$30, of the entire purchase. The cost of x lily bulbs at \$4.00 each is \$4x. The cost of y tulip bulbs at \$2.00 each is \$2y. The total cost, (4x + 2y), cannot exceed \$30. Therefore, the constraint is $4x + 2y \le 30$.

Kenji has at most \$30 to spend on lily bulbs and tulip bulbs at his local flower store. Lily bulbs cost \$4 each, and tulip bulbs cost \$2 each. Tax is included in the prices of the bulbs.

Create a constraint that can be used to represent all possible numbers of lily bulbs, x, and tulip bulbs, y, Kenji can buy.

4x+2y=	=30										
$\bullet \bullet \bullet$											
1	2	3	x	у							
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0	-	-	<u>-</u>			()		$\sqrt{\Box}$	₽	π	i
			sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect constraint, represented by a linear equation, instead of a linear inequality.

Kenji has at most \$30 to spend on lily bulbs and tulip bulbs at his local flower store. Lily bulbs cost \$4 each, and tulip bulbs cost \$2 each. Tax is included in the prices of the bulbs.

Create a constraint that can be used to represent all possible numbers of lily bulbs, x, and tulip bulbs, y, Kenji can buy.

$4y+2x\leq30$											
1	2	3	x	y							
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0		-				()		$\sqrt{\Box}$	₽Ū	π	i
			sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect constraint, represented by a linear inequality.

Question 20

Question and Scoring Guidelines

Question 20

An equation is shown.

 $y = \frac{1}{2}x + \frac{3}{4}$

Select all of the points that are contained in the graph of the equation.

 $(0, \frac{1}{2})$ $(0, \frac{3}{4})$ $(\frac{3}{4}, 0)$ $(\frac{3}{4}, \frac{1}{2})$ $(\frac{1}{2}, 1)$

Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

Content Standard: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)

Scoring Guidelines

<u>Rationale for First Option</u>: This is incorrect. The student may have changed the order of operations and first multiplied $\frac{3}{4}$ by 0, then added $\frac{1}{2}$, to get $\frac{1}{2}$

<u>Rationale for Second Option</u>: **Key** – The student may have correctly realized that points contained in the graph of the equation must make the equation a true statement. When $(0, \frac{3}{4})$ is substituted into the equation, $\frac{3}{4} = \frac{1}{2} \cdot 0 + \frac{3}{4}$, it makes the equation a correct statement.

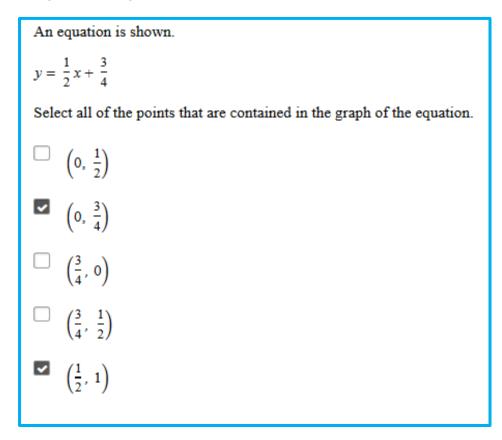
<u>Rationale for Third Option</u>: This is incorrect. The student may have incorrectly used x = 0 and $y = \frac{3}{4}$ instead of $x = \frac{3}{4}$ and y = 0 when substituting values in the equation to get $\frac{1}{2} \cdot 0 + \frac{3}{4} = \frac{3}{4}$

<u>Rationale for Fourth Option</u>: This is incorrect. The student may have incorrectly performed operations with fractions as $\frac{1}{2} \cdot \frac{3}{4} + \frac{3}{4} = \frac{3}{8} + \frac{3}{4} = \frac{6}{12}$, or $\frac{1}{2}$

<u>Rationale for Fifth Option</u>: **Key** – The student correctly multiplied $\frac{1}{2}$ times $\frac{1}{2}$ to get $\frac{1}{4}$, and then added the product to $\frac{3}{4}$ to get 1.

Question 20

Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows both correct answer options, B and E.

An equation is shown.

 $y = \frac{1}{2}x + \frac{3}{4}$

Select all of the points that are contained in the graph of the equation.

 $(0, \frac{1}{2})$ $(0, \frac{3}{4})$ $(3, \frac{3}{4}, 0)$ $(\frac{3}{4}, \frac{1}{2})$ $(\frac{1}{2}, 1)$

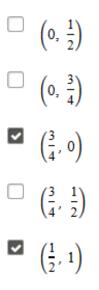
Notes on Scoring

This response earns no credit (0 points) because it shows one incorrect answer option, A, in addition to the two correct answer options, B and E.

An equation is shown.

$$y = \frac{1}{2}x + \frac{3}{4}$$

Select all of the points that are contained in the graph of the equation.



Notes on Scoring

This response earns no credit (0 points) because it shows one incorrect answer option, C, and one correct answer option, E.

Question 21

Question and Scoring Guidelines

Question 21

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.

f(n) =										
$\bullet \bullet \bullet \bullet \blacksquare$										
1	2	3	n							
4	5	6	+	- • ÷						
7	8	9	<							
0	-	_		$\Box^{\Box} \Box_{\Box} () \sqrt{\Box} \sqrt{\Box} \pi i$						
			sin	cos tan arcsin arccos arctan						

Points Possible: 1

Content Cluster: Build a function that models a relationship between two quantities.

Content Standard: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (*F.BF.2*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• Explicit: $f(n) = 4 \cdot 3^{n-1}$

Other Correct Responses

• Any equivalent functions.

For this item, a full-credit response includes:

• A correct explicit function (1 point).

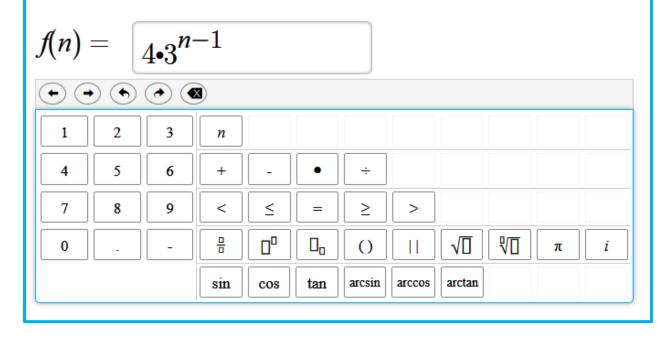
Question 21

Sample Responses

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

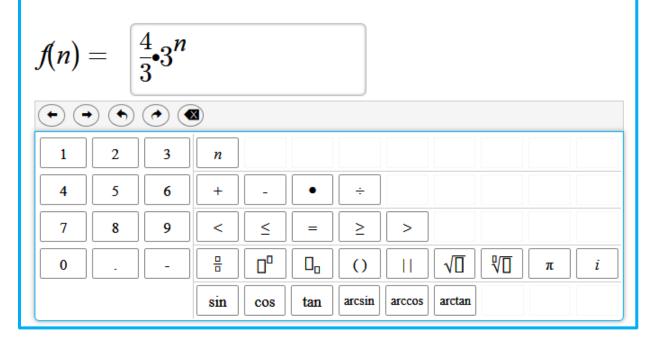
This response earns full credit (1 point) because it shows a correct function $f(n) = 4 \cdot (3)^{n-1}$.

If the situation asks for building a correct exponential function represented by a geometric sequence, where each successive term is a constant multiple, r = 3, of the previous term, the function input is the set of all natural numbers {1, 2, 3, ...} and the function output is the set of all terms of the sequence {4, 12, 36, 108, 324, ...}. The *n*th term of the geometric sequence can be obtained by finding a product of the first term of the geometric sequence and the constant multiple, *r*, raised to the power (n - 1), and can be modeled by a function, $f(n) = 4 \cdot (3)^{n-1}$.

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct exponential function, $f(n) = (\frac{4}{3}) \cdot 3^n$, equivalent to $f(n) = 4 \cdot (3)^{(n-1)}$, because

4 • $3^{(n-1)} = 4 • 3^n • 3^{-1} = 4 • 3^n • \frac{1}{3} = (\frac{4}{3}) • 3^n$.

If the situation asks for building a correct exponential function represented by a geometric sequence, where each successive term is a constant multiple, r = 3, of the previous term, the function input is the set of all natural numbers {1, 2, 3, ...} and the function output is the set of all terms of the sequence {4, 12, 36, 108, 324, ...}. The *n*th term of the geometric sequence can be obtained by finding a product of the first term of the geometric sequence and the constant multiple, *r*, raised to the power (n - 1), and can be modeled by a function, $f(n) = 4 \cdot (3)^{(n-1)}$.

The first five terms of a sequence are shown. 4, 12, 36, 108, 324, ... Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4. f(n) =<u>3n</u> ک 📀 • 3 2 1 n 4 5 6 + • ÷ _ 7 8 9 < \leq = \geq > ᄳ 믑 00 0 0 || $\sqrt{1}$ i π _ arcsin arccos arctan sin cos tan

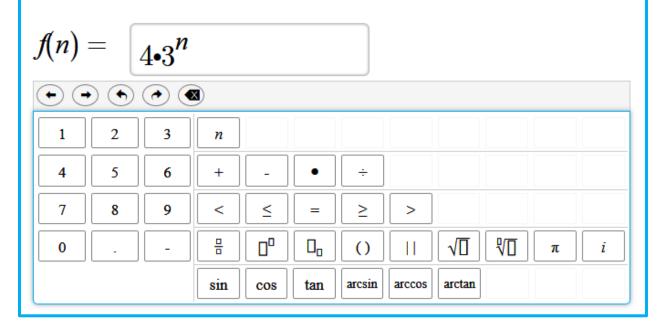
Notes on Scoring

This response receives no credit (0 points) because it shows a linear function instead of an exponential function.

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect exponential function.

Question 22

Question and Scoring Guidelines

Question 22

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time.

Write a function f(x) to model the number of fish in the pond after x months.

f(x) =								
1	2	3	x					
4	5	6	+	-	•	÷		
7	8	9	<	≤	=	2	>	
0		-	<u>-</u>			()		
			sin	cos	tan	arcsin	arccos	arctan

Points Possible: 1

Content Cluster: Construct and compare linear, quadratic, and exponential models, and solve problems.

Content Standard: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (*F.LE.2*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• $f(x) = 5 \cdot 2^x$

Other Correct Responses

• Any equivalent function.

For this item, a full-credit response includes:

• A correct function (1 point).

Question 22

Sample Responses

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time. Write a function f(x) to model the number of fish in the pond after x months. f(x) = $5 \cdot 2^{x}$ ← → ♠ ♠ 🕷 3 1 2 x 4 5 6 + • ÷ _ 7 9 8 < \leq ≥ > = 믐 $\sqrt{\Box}$ ₩П 0 0, ()Ш π i tan arcsin arccos arctan sin cos

Notes on Scoring

This response receives full credit (1 point) because it shows a correct exponential function $f(x) = 5 \cdot 2^x$.

In this situation, the focus is on modeling a verbal situation by an exponential function. If the initial number of fish in the pond is 5, and at end of the first month the number of fish doubles, then the number of fish is $10 = 5 \cdot 2$ or $f(1) = 5 \cdot 2^1$. At the end of the second month, the number of fish doubles again, and is 20 or $10 \cdot 2 = (5 \cdot 2) \cdot 2$ or $f(2) = 5 \cdot 2^2$. At the end of the third month, it doubles again to become 40 or $20 \cdot 2 = ((5 \cdot 2) \cdot 2) \cdot 2$ or $f(3) = 5 \cdot 2^3$. Similarly, at the end of the *x* month, the number of fish is $5 \cdot 2^x$ or $f(x) = 5 \cdot 2^x$.

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time. Write a function f(x) to model the number of fish in the pond after x months. f(x) = $2^{x} \cdot 5$ (+) (+) (+) (*) 1 2 3 x 4 5 6 + _ • ÷ 7 8 9 < < = ≥ > 0 믐 ()Ш $\sqrt{\Box}$ ₩П i _ π tan sin cos arcsin arccos arctan

Notes on Scoring

This response receives full credit (1 point) because it shows a correct exponential function, $f(x) = 2^x \cdot 5$, equivalent to the function $f(x) = 5 \cdot 2^x$.

In this situation, the focus is on modeling a verbal situation by an exponential function. If the initial number of fish in the pond is 5, and at the end of the first month the number of fish doubles, then the number of fish is $10 = 5 \cdot 2$ or $f(1) = 5 \cdot 2^1$. At the end of the second month, the number of fish doubles again, and is 20 or $10 \cdot 2 = (5 \cdot 2) \cdot 2$ or $f(2) = 5 \cdot 2^2$. At the end of the third month, it doubles again to become 40 or $20 \cdot 2 = ((5 \cdot 2) \cdot 2) \cdot 2$ or $f(3) = 5 \cdot 2^3$. Similarly, at the end of the *x* month, the number of fish is $5 \cdot 2^x$ or $f(x) = 5 \cdot 2^x$.

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time. Write a function f(x) to model the number of fish in the pond after x months. $f(x) = \int_{5 \cdot 2}^{x-1}$ $\bullet \bullet \bullet \bullet \blacksquare$ 1 2 3 x 4 5 6 + _ ٠ ÷ 7 8 9 < \leq = \geq > 00 ₽ 믐 $\sqrt{\Box}$ 0 _ 00 ()Ш π i tan arcsin arccos arctan sin COS

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect exponential function that is not equivalent to the function $f(x) = 5 \cdot 2^x$.

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time. Write a function f(x) to model the number of fish in the pond after x months. f(x) =10•x $\bullet \bullet \bullet \bullet \blacksquare$ 2 3 1 x 5 + • 4 6 _ ÷ 7 9 8 < \leq = ≥ > ᄳ 믐 $\sqrt{\Box}$ 0 ()П i _ π sin cos tan arcsin arccos arctan

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect linear function that is not equivalent to the exponential function $f(x) = 5 \cdot 2^x$.

Question 23

Question and Scoring Guidelines

Question 23

A shipping company charges a cost per pound plus a fixed fee to ship a package. The total cost, f(x), in dollars, of shipping x pounds is modeled by the function shown.

f(x) = 4.99x + 5.75

Which part of the function represents the fixed fee?

(A) x

B 4.99

© 5.75

0 4.99x

Points Possible: 1

Content Cluster: Interpret expressions for functions in terms of the situation they model.

Content Standard: Interpret the parameters in a linear or exponential function in terms of a context. (*F.LE.5*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that the variable *x* represents the fixed fee.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that the coefficient of the variable term represents the fixed fee.

<u>Rationale for Option C:</u> **Key** – The student may have correctly recognized that in a linear function, f(x) = mx + b, *m* represents the average rate of change, and *b* is the initial or a fixed value. In the context of this situation, the student correctly identified that a fixed value is the fixed fee represented by the constant term, 5.75.

<u>Rationale for Option D:</u> This is incorrect. The student may have thought that the variable term represents the fixed fee.

Sample Response: 1 point

A shipping company charges a cost per pound plus a fixed fee to ship a package. The total cost, f(x), in dollars, of shipping x pounds is modeled by the function shown.

f(x) = 4.99x + 5.75

Which part of the function represents the fixed fee?

A χ

B 4.99

5.75

0 4.99x

Question 24

Question and Scoring Guidelines

Question 24

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower leve	el tickets:		
Upper leve	el tickets:		
	•		
1	2	3	
4	5	6	
7	8	9	
	0		
	-		

Points Possible: 1

Content Cluster: Solve systems of equations.

Content Standard: Solve systems of linear equations algebraically and graphically. (*A.REI.6*)

Scoring Guidelines

Exemplar Response

Lower seats: 150
 Upper seats: 200

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• The correct values (1 point).

Question 24

Sample Responses

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250. How many tickets of each type were sold? Lower level tickets: 150*Upper level tickets*: 200← → ♠ ⊘ 🛛 2 1 3 4 5 6 7 8 9 0 믂

Notes on Scoring

This response receives full credit (1 point) because it shows a correct number of sold tickets, 150, for the lower level seats and a correct number of sold tickets, 200, for the upper level seats.

By letting x represent the number of lower level tickets sold, and y represent the number of upper level tickets sold, the given situation can be modeled by the system of two linear equations. The first equation, x + y = 350, represents the total number of all tickets sold, and the second equation, 35x + 25y = 10,250, represents the total cost of the tickets sold for all seats. Solving the first equation for y, and then substituting y = 350 - x into the second equation for y, the second equation becomes 35x + 25(350 - x) = 10,250. The solution to the system is x = 150, the number of lower level seat tickets sold, and y = 200, the number of upper level seat tickets sold.

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower leve	el tickets:	200		
Upper level tickets:		150		
	•			
1	2	3		
4	5	6		
7	8	9		
	0			
· .	-			

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect number of tickets sold for the lower level seats, 200, and an incorrect number of tickets sold for the upper level seats, 150.

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower leve	el tickets:	175
Upper leve	el tickets:	175
	•	
1	2	3
4	5	6
7	8	9
	0	
	-	

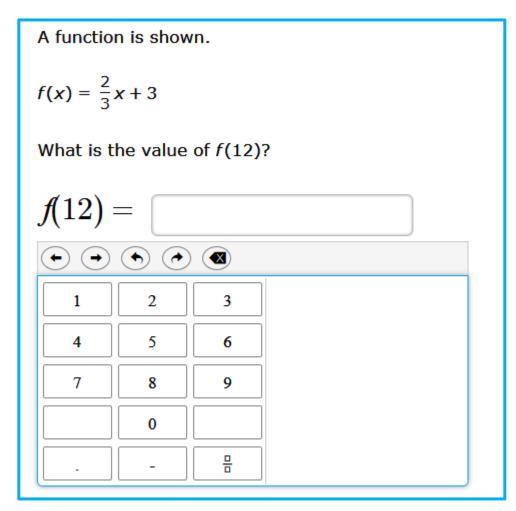
Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect number of tickets sold for the lower level seats, 175, and an incorrect number of tickets sold for the upper level seats, 175.

Question 25

Question and Scoring Guidelines

Question 25



Points Possible: 1

Content Cluster: Understand the concept of a function, and use function notation.

Content Standard: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (*F.IF.2*)

Scoring Guidelines

Exemplar Response

• *f*(12) = 11

Other Correct Responses

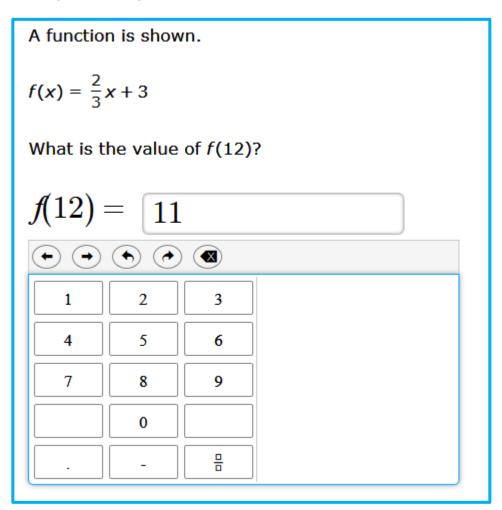
• Any equivalent form.

For this item, a full-credit response includes:

• The correct value (1 point).

Question 25

Sample Responses



Notes on Scoring

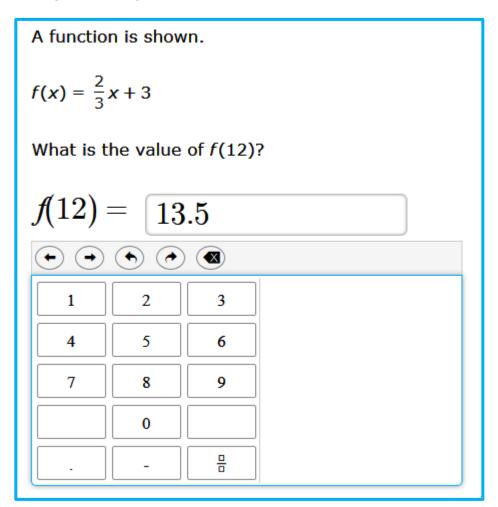
This response receives full credit (1 point) for showing a correct answer of f(12) = 11. If a given function is $f(x) = \frac{2}{3}x + 3$, then f(12) denotes the output of a function f(x)corresponding to the input x = 12, and calculates as $f(12) = (\frac{2}{3}) \cdot 12 + 3 = 8 + 3 = 11$, or f(12) = 11

A function is shown. $f(x) = \frac{2}{3}x + 3$ What is the value of f(12)? 33 f(12) =3 ۱ 🔿 ٠ + 1 2 3 4 5 6 7 8 9 0 믐 _

Notes on Scoring

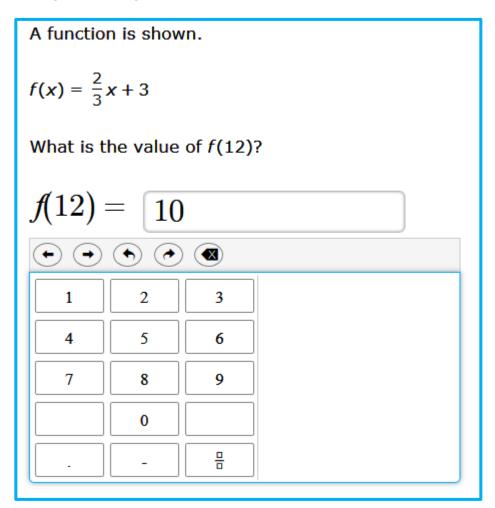
This response receives full credit (1 point) for showing a correct answer of f(12) = 11 in its equivalent form $f(12) = \frac{33}{3}$

If a given function is $f(x) = \frac{2}{3}x + 3$, then f(12) denotes the output of a function f(x) corresponding to the input x = 12, and calculates as $f(12) = (\frac{2}{3}) \cdot 12 + 3 = 8 + 3 = 11$



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect value for f(12).



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect value for f(12).

Question 26

Question and Scoring Guidelines

Question 26

The ideal gas law is represented by the equation PV = nRT. The variables and their units are defined as

- P for pressure in Pascals (Pa),
- V for volume in meters cubed (m³),
- *n* for the number of moles (mol),
- R for the gas constant (?), and
- *T* for temperature in degrees Kelvin (K).

In terms of the units for the defined variables, what are the units for *R*, the gas constant?

- A Pa m³ mol ● K
- B mol K Pa • m³
- © Pa•m³
- Pa m³ mol K

Points Possible: 1

Content Cluster: Reason quantitatively and use units to solve problems.

Content Standard: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (*N.Q.1*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> **Key** – The student correctly determined the units for the gas constant, *R*, by solving the formula for *R* and getting $R = \frac{PV}{nT}$ to relate each variable to its unit as $R = \frac{Pa*m^3}{mol*K}$.

<u>Rationale for Option B:</u> This is incorrect. The student may have correctly associated each unit with its corresponding variable, but made an error when solving the equation for R to get $R = \frac{nT}{(PV)}$, instead of $R = \frac{PV}{nT}$.

<u>Rationale for Option C:</u> This is incorrect. The student may have only considered units associated with the left side.

<u>Rationale for Option D</u>: This is incorrect. The student may have correctly associated each unit with its corresponding variable, but made an error when solving the equation for R to get R = nTPV instead of $R = \frac{PV}{(nT)}$.

Sample Response: 1 point

The ideal gas law is represented by the equation PV = nRT. The variables and their units are defined as

- P for pressure in Pascals (Pa),
- V for volume in meters cubed (m³),
- n for the number of moles (mol),
- R for the gas constant (?), and
- T for temperature in degrees Kelvin (K).

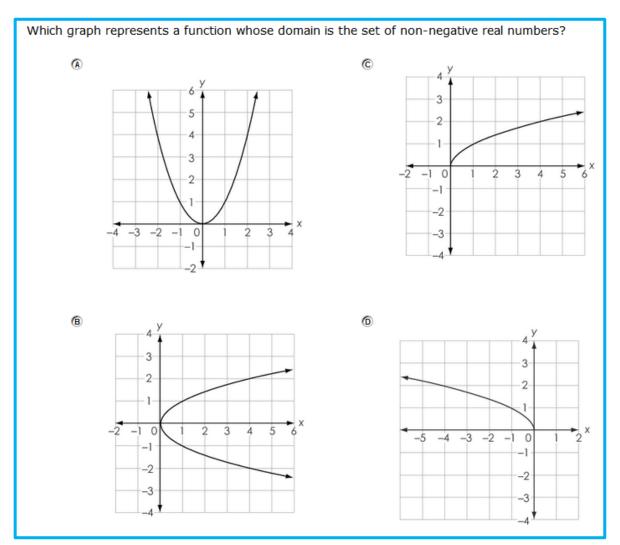
In terms of the units for the defined variables, what are the units for R, the gas constant?

- Pa•m³ mol•K
- © Pa∙m³
- Pa m³ mol K

Question 27

Question and Scoring Guidelines

Question 27



Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

Content Standard: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5) \star

 (\star) indicates that modeling should be incorporated into the standard.

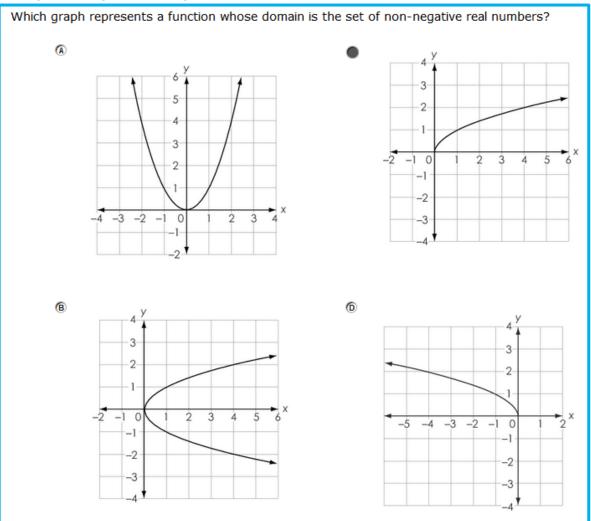
Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have understood that this graph represents a function, but confused the domain, the set of *x*-coordinates, of all real numbers, with the range, the set of *y*-coordinates, of all non-negative real numbers.

<u>Rationale for Option B:</u> This is incorrect. The student may have understood that the domain is the set of non-negative *x*-values, but overlooked the fact that this is not the graph of a function.

<u>Rationale for Option C:</u> **Key** – The student correctly understood that the graph satisfies the requirements of a function where each *x*-coordinate corresponds to only one *y*-coordinate and that the domain is represented by the set of non-negative *x*-coordinates.

<u>Rationale for Option D:</u> This is incorrect. The student may have understood that this graph represents a function, but overlooked the fact that the domain, the set of *x*-coordinates, is all non-positive real numbers and the range is all non-negative real numbers.



Question 28

Question and Scoring Guidelines

Question 28

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property	
3(x-2) = x + 4	Given	
3x - 6 = x + 4		
2x - 6 = 4		
2x = 10		
<i>x</i> = 5		

Addition property of equality Multiplication property of equality Distributive property Associative property Commutative property

Points Possible: 1

Content Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

Content Standard: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (*A.REI.1*)

Scoring Guidelines

Exemplar Response

Step	Property
3(x-2) = x+4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Addition property of equality
2x = 10	Addition property of equality
x = 5	Multiplication property of equality

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• A correctly completed table (1 point).

Question 28

Sample Responses

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property
3(x-2) = x + 4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Addition property of equality
2x = 10	Addition property of equality
<i>x</i> = 5	Multiplication property of equality

Notes on Scoring

This response earns full credit (1 point) because it shows a viable argument to a solution method.

- The student noticed that 3 was multiplied by (x 2) on the left side of the equation from the first step to the second step using the Distributive property.
- The student noticed that -x was added to both sides of the equation from the second step to the third step using the Addition property of equality.
- The student noticed that 6 was added to both sides of the equation from the third step to the fourth step using the Addition property of equality.
- The student noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step using the Multiplication property of equality.

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

StepProperty3(x-2) = x + 4Given3x - 6 = x + 4Multiplication property of equality2x - 6 = 4Addition property of equality2x = 10Addition property of equalityx = 5Distributive property

Move a property to each blank box to show the reason for each step.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect argument to a solution method.

- The student noticed that 3 was multiplied by (x 2) on the left side of the equation from the first step to the second step, but confused using the Distributive property with the Multiplication property of equality.
- The student noticed that -x was added to both sides of the equation from the second step to the third step using the Addition property of equality.
- The student may have noticed that 6 was added to both sides of the equation from the third step to the fourth step using the Addition property of equality.
- The student may have noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step, but confused the distributive property with the Multiplication property of equality.

Students need to get all four properties correct in order to earn full credit for this item.

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property
3(x-2) = x + 4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Associative property
2x = 10	Associative property
<i>x</i> = 5	Multiplication property of equality

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect argument to a solution method.

- The student noticed that 2 was multiplied by (x 2) on the left side of the equation from the first step to the second step using the Distributive property.
- The student may have noticed that x was added to both sides of the equation from the second step to the third step, but confused the Associative property with the Addition property of equality.
- The student may have noticed that 6 was added to both sides of the equation from the third step to the fourth step, but confused the Associative property with the Addition property of equality.
- The student noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step using the Multiplication property of equality.

Students need to get all four properties correct in order to earn full credit for this item.

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property	
3(x-2) = x + 4	Given	
3x - 6 = x + 4	Distributive property	
2x - 6 = 4	Addition property of equality	
2 <i>x</i> = 10	Associative property	
<i>x</i> = 5	Multiplication property of equality	

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect argument to a solution method.

- The student noticed that 3 was distributed to (x 2) on the left side of the equation from the first step to the second step using the Distributive property.
- The student noticed that -x was added to both sides of the equation from the second step to the third step using the Addition property of equality.
- The student may have noticed that 6 was added to both sides of the equation from the third step to the fourth step, but confused the Associative property with the Addition property of equality.
- The student noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step using the Multiplication property of equality.

Students need to get all four properties correct in order to earn full credit for this item.

Question 29

Question and Scoring Guidelines

Question 29

Complete the statement about the equation $9x - 4y = -18$.				
The graph of the equation contains the point	▼ which is	■ to the equation.		

Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

Content Standard: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)

Scoring Guidelines

Exemplar Response

(-4, -4.5)
 One of many possible solutions

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• The correct ordered pair and phrase (1 point).

Question 29

Sample Responses

Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (-4, -4.5), v which is one of many possible solutions v to the equation.

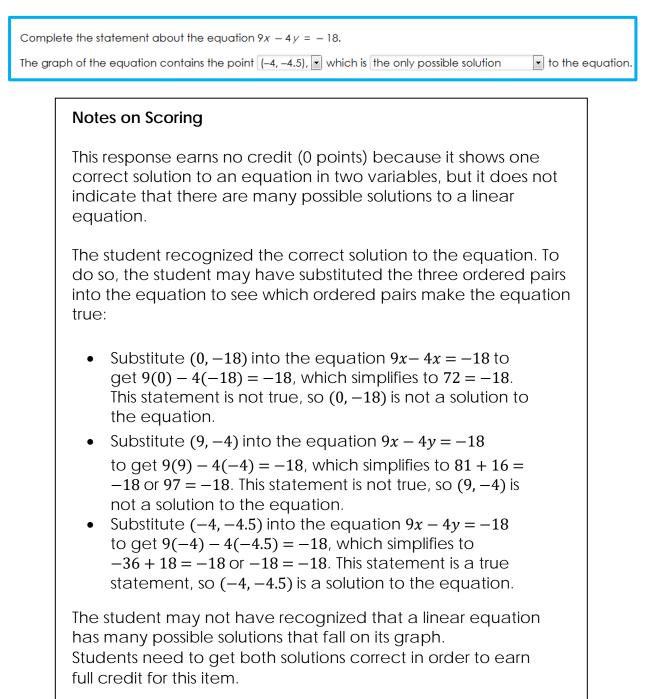
Notes on Scoring

This response earns full credit (1 point) because it shows one correct solution out of many possible solutions to an equation in two variables.

The student recognized the correct solution to the equation. To do so, the student may have substituted the three ordered pairs into the equation to see which ordered pairs make the equation true:

- Substitute (0, -18) into the equation 9x 4x = -18 to get 9(0) 4(-18) = -18, which simplifies to 72 = -18. This statement is not true, so (0, -18) is not a solution to the equation.
- Substitute (9, -4) into the equation 9x 4y = −18 to get 9(9) - 4(-4) = −18, which simplifies to 81 + 16 = −18 or 97 = −18. This statement is not true, so (9, -4) is not a solution to the equation.
- Substitute (-4, -4.5) into the equation 9x 4y = -18to get 9(-4) - 4(-4.5) = -18, which simplifies to -36 + 18 = -18 or -18 = -18. This statement is a true statement, so (-4, -4.5) is a solution to the equation.

The student recognized that 9x - 4y = -18 was a linear equation in standard form, and a linear equation has many possible solutions that fall on its graph.



Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (0, -18), \bullet which is one of many possible solutions \bullet to the equation.

Notes on Scoring

This response earns no credit (0 points) because it indicates that there are many possible solutions to a linear equation in two variables, but finds an incorrect solution to the given equation.

The student selected an incorrect choice for the solution to the equation.

The student recognized that 9x - 4y = -18 was a linear equation in standard form, and a linear equation has many possible solutions that fall on its graph.

Students need to get both choices correct in order to earn full credit for this item.

Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (9, -4), \bullet which is one of two possible solutions \bullet to the equation.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect solution out of many possible solutions to an equation in two variables.

The student selected an incorrect choice for the solution to the equation.

The student may not have recognized that a linear equation has many possible solutions that fall on its graph.

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