Ohio Department of Education

Ohio's State Tests

PRACTICE TEST ANSWER KEY & SCORING GUIDELINES

ALGEBRA I

Table of Contents

Questions 1 – 30: Content Summary and Answer Key	iii
Question 1: Question and Scoring Guidelines	1
Question 1: Sample Responses	5
Question 2: Question and Scoring Guidelines	11
Question 2: Sample Response	13
Question 3: Question and Scoring Guidelines	15
Question 3: Sample Responses	19
Question 4: Question and Scoring Guidelines	25
Question 4: Sample Responses	27
Question 5: Question and Scoring Guidelines	33
Question 5: Sample Responses	37
Question 6: Question and Scoring Guidelines	41
Question 6: Sample Responses	45
Question 7: Question and Scoring Guidelines	49
Question 7: Sample Responses	53
Question 8: Question and Scoring Guidelines	59
Question 8: Sample Responses	63
Question 9: Question and Scoring Guidelines	71
Question 9: Sample Responses	75
Question 10: Question and Scoring Guidelines	81
Question 10: Sample Responses	85
Question 11: Question and Scoring Guidelines	91
Question 11: Sample Response	93
Question 12: Question and Scoring Guidelines	95
Question 12: Sample Response	97
Question 13: Question and Scoring Guidelines	99
Question 13: Sample Response	101
Question 14: Question and Scoring Guidelines Question 14: Sample Responses	

Question 15: Question and Scoring Guidelines Question 15: Sample Responses	111 115
Question 16: Question and Scoring Guidelines Question 16: Sample Response	119 121
Question 17: Question and Scoring Guidelines Question 17: Sample Responses	
Question 18: Question and Scoring Guidelines Question 18: Sample Responses	133 137
Question 19: Question and Scoring Guidelines Question 19: Sample Responses	143 147
Question 20: Question and Scoring Guidelines Question 20: Sample Responses	151 155
Question 21: Question and Scoring Guidelines Question 21: Sample Responses	
Question 22: Question and Scoring Guidelines Question 22: Sample Response	
Question 23: Question and Scoring Guidelines Question 23: Sample Responses	
Question 24: Question and Scoring Guidelines Question 24: Sample Responses	
Question 25: Question and Scoring Guidelines Question 25: Sample Response	
Question 26: Question and Scoring Guidelines Question 26: Sample Response	
Question 27: Question and Scoring Guidelines Question 27: Sample Responses	
Question 28: Question and Scoring Guidelines Question 28: Sample Responses	211
Question 29: Question and Scoring Guidelines Question 29: Sample Responses	
Question 30: Question and Scoring Guidelines Question 30: Sample Responses	231

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
1	Table Item	Understand the concept of a function, and use function notation.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <i>f</i> is a function and <i>x</i> is an element of its domain, then f(x) denotes the output of <i>f</i> corresponding to the input <i>x</i> . The graph of <i>f</i> is the graph of the equation $y = f(x)$. (<i>F.IF.1</i>)		2 points
2	Multiple Choice	Interpret linear models.	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7) \star		1 point
3	Equation Item	Build new functions from existing functions.	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, f(kx), and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F.BF.3)		1 point
4	Equation Item	Create equations that describe numbers or relationships.	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A.CED.2)★		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
5	Graphic Response	Analyze functions using different representations.	Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. <i>a</i> . Graph linear functions and indicate intercepts, maxima, and minima. (<i>F.IF.7a</i>)★		1 point
6	Matching Item	Reason quantitatively and use units to solve problems.	son ntitatively use units to e problems. Define appropriate quantities for the purpose of descriptive modeling. $(N.Q.2)$ *		1 point
7	Graphic Response	Represent and solve equations and inequalities graphically.	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A.REI.12)		1 point
8	Table Item	Solve systems of equations.	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. (A.REI.7)		2 points

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
9	Equation Item	Summarize, represent, and interpret data on two categorical and quantitative variables.	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S.ID.5)★		1 point
10	Multi- Select Item	Summarize, represent, and interpret data on a single count or measurement variable.	In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S.ID.3)★	B, F	1 point
11	Multiple Choice	Summarize, represent, and interpret data on two categorical and quantitative variables.	e, Represent data on two and quantitative variables on a scatter plot, and describe how the variables are related. al and c. Fit a linear function for a scatter plot that suggests a linear association. (S.ID.6c)★		1 point
12	Multiple Choice	Interpret linear models.	Compute (using technology) and interpret the correlation coefficient of a linear fit. (S.ID.8)★	А	1 point
13	Multiple Choice	Perform arithmetic operations on polynomials.	Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1)	D	1 point

Question No.	ltem Type	Content Cluster	ContentContentAClusterStandard		Points
14	Equation Item	Interpret the structure of expressions.	Serpret the ucture of spressions. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. $(A.SSE.1a) \star$		1 point
15	Multi- Select Item	Interpret the structure of expressions.	Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (A.SSE.2)		1 point
16	Multiple Choice	Create equations that describe numbers or relationships.	Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. (A.CED.1)★	D	1 point
17	Equation Item	Solve equations and inequalities in one variable.	Solve quadratic equations in one variable. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (A.REI.4b)		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
18	Equation Item	Analyze functions using different representations.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (F.IF.8a)		1 point
19	Multi- Select Item	Represent and solve equations and inequalities graphically.	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)	B, E	1 point
20	Equation Item	Understand the concept of a function, and use function notation.	Inderstand the concept of a unction, and use unction notation.Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F.IF.2)		1 point
21	Equation Item	Interpret functions that arise in applications in terms of the context.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)★		1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
22	Multiple Choice	Interpret functions that arise in applications in terms of the context.	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)★	С	1 point
23	Equation Item	Build a function that models a relationship between two quantities.	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (F.BF.2)★		1 point
24	Equation Item	Construct and compare linear, quadratic, and exponential models, and solve problems.	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (F.LE.2)★		1 point
25	Multiple Choice	Summarize, represent, and interpret data on a single count or measurement variable.	In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. (S.ID.2)★	В	1 point

Question No.	ltem Type	Content Cluster	Content Standard	Answer Key	Points
26	Multiple Choice	Interpret expressions for functions in terms of the situation they model.	nterpret expressions for functions in terms of the situation they model. Interpret the parameters in a linear or exponential function in terms of a context. (F.LE.5)★		1 point
27	Equation Item	Solve systems of equations.	olve systems of equations algebraically and graphically. (A.REI.6)		1 point
28	Equation Item	Write expressions in equivalent forms to solve problems.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A.SSE.3b)★		1 point
29	Hot Text Item	Understand solving equations as a process of reasoning and explain the reasoning.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A.REI.1)		1 point
30	Editing Task Choice Item	Represent and solve equations and inequalities graphically.	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)		1 point

Question 1

Question and Scoring Guidelines

Question 1

Complete the first table so that f(x) is a function.

Complete the second table so that g(x) is not a function.

x	f(x)	x	g(x)
-1		-1	
	-8		-8
6		6	

Points Possible: 2

Content Cluster: Understand the concept of a function, and use function notation.

Content Standard: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then f(x) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation y = f(x). (*F*.*IF*.1)

Scoring Guidelines

Exemplar Response

x	<i>f</i> (<i>x</i>)	x	g(x)
-1	2	-1	2
3	-8	6	-8
6	64	6	64

Other Correct Responses

- For *f*(*x*), any input value that is not equal to –1 or 6, or equal to –1 or 6 with the same output value as the other instance in the table.
- For g(x), any input value that is equal to -1 or 6, as long as the output value is different than the other instance in the table.

For this item, a full-credit response includes:

• A correctly completed table for *f*(*x*) (1 point);

AND

• A correctly completed table for g(x) (1 point).

Question 1

Sample Responses

Sample Response: 2 points

Complete the first table so that $f(x)$ is a function.						
Complete the second table so that $g(x)$ is not a function.						
x	f(x)		x	g(x)		
-1	2		-1	2		
3	-8		6	-8		
6	64		6	64		

Notes on Scoring

This response earns full credit (2 points) for two correctly completed tables showing correct relations. For full credit, a response demonstrates an understanding that a function from one set (input) to another set (output) assigns to each input value exactly one output value.

In the first table, f(x) is a function if the input is any value that is not equal to -1 or 6, or equal to -1 or 6 with the same output value. In the second table, g(x) is not a function if the input value is equal to -1 or 6, as long as the output value is different than the other instance in the table.

Sample Response: 2 points

Complete the first table so that $f(x)$ is a function.								
Complete the second table so that $g(x)$ is not a function.								
x	f(x)		x	g(x)				
-1	-8		-1	2				
-1	-8		6	-8				
6	-8		6	14				

Notes on Scoring

This response earns full credit (2 points) for two correctly completed tables showing correct relations. For full credit, a response demonstrates an understanding that a function from one set (input) to another set (output) assigns to each input value exactly one output value.

In the first table, f(x) is a function. The response assigns x = -1 to f(-1) = -8 twice.

In the second table, g(x) is not a function. The response assigns x = 6 to the two different y values, g(6) = -8 and g(6) = 14.

Sample Response: 1 point

Complete the first table so that $f(x)$ is a function.								
Complete the second table so that $g(x)$ is not a function.								
x	f(x)		x	g(x)				
-1	2		-1	2				
-1	-8		6	-8				
6	2		6	14				

Notes on Scoring

This response earns partial credit (1 point) because it shows one incorrect table. While this response demonstrates a correctly completed table for g(x), the response cannot earn full credit because the table for f(x) shows a relation that is not a function. The same input value x = -1 assigns two different output values, f(-1) = 2 and f(-1) = -8.

Sample Response: 0 points

Complete the first table so that $f(x)$ is a function.								
Complete the second table so that $g(x)$ is not a function.								
x	f(x)		x	g(x)				
-1	2		-1	2				
6	-8		3	-8				
6	64		6	14				

Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect tables.

Question 2

Question and Scoring Guidelines

Question 2



Points Possible: 1

Content Cluster: Interpret linear models.

Content Standard: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7) \star

Scoring Guidelines

<u>Rationale for Option A:</u> **Key** – The student correctly interpreted 1275 as the *y*-intercept of the line of best fit. For each data point, the *x*-coordinate represents the distance from the center of the city to the house and the *y*-coordinate is an amount of rent for the house. For a *y*-intercept, (0, 1275) shows a rent of \$1275 for a house that is 0 miles away from the center of the city.

<u>Rationale for Option B</u>: This is incorrect. The student may have confused the rent of \$1275 with the distance of 1275 miles and thought that 1275 was the *x*-intercept.

<u>Rationale for Option C:</u> This is incorrect. The student may have associated the wording "estimated change in rent" with the slope and misinterpreted 1275 as the slope of the line of best fit.

<u>Rationale for Option D:</u> This is incorrect. The student may have associated the wording "estimated change in distance" with the slope and misinterpreted 1275 as the slope of the line of best fit.

Sample Response: 1 point



Juan wants to rent a house. He gathers data on many similar houses. The distance from the center of the city, *x*, and the monthly rent for each house, *y*, are shown in the scatter plot. Juan models the data with a linear equation.

Based on the scatter plot, what could the number 1275 represent in his equation?

- The estimated rent for a house in the center of the city
- In the estimated minimum rent for a house far from the center of the city
- © The estimated change in rent for each additional mile from the center of the city
- The estimated change in distance from the center of the city for each dollar change in rent

Question 3

Question and Scoring Guidelines

Question 3



Points Possible: 1

Content Cluster: Build new functions from existing functions.

Content Standard: Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(F.BF.3)*

Scoring Guidelines

Exemplar Response

• g(x) = f(x) + 3

Other Correct Responses

• Any equivalent equation.

For this item, a full-credit response includes:

• A correct equation (1 point).

Question 3

Sample Responses

Sample Response: 1 point



Notes on Scoring

This response earns full credit (1 point) because it shows a correct function g(x) in terms of f(x). The response correctly recognizes a single vertical transformation of 3 units up performed on a graph of f(x) to create the graph of g(x). The transformation translating a graph of f(x), by k units up if k is positive and down if k is negative, is represented by the equation g(x) = f(x) + k. When k = 3, g(x) = f(x) + 3.

Sample Response: 1 point



Notes on Scoring

This response earns full credit (1 point) because it shows a correct function g(x) in terms of f(x). The response correctly recognizes a single vertical transformation of 3 units up performed on a graph of f(x) to create the graph of g(x). The transformation translating a graph of f(x), by k units up if k is positive and down if k is negative, is represented by the equation g(x) = f(x) + k. When k = 3, g(x) = f(x) + 3.

The same equation using a commutative property over the addition can be written in the form of g(x) = k + f(x) or g(x) = 3 + f(x).

Sample Response: 0 points



Notes on Scoring

This response earns no credit (0 points) because it shows an equation representing a transformation other than a vertical translation of the original graph f(x) up 3 units. The graph of g(x) = f(3x) will show a horizontal compression of the graph of f(x), not a vertical translation.

Sample Response: 0 points



Notes on Scoring

This response earns no credit (0 points) because it shows an equation representing a transformation other than a vertical translation of the original graph f(x) up 3 units. The graph of g(x) = f(x) - 3 will show a translation of the graph of f(x) down by 3 units.

Question 4

Question and Scoring Guidelines

Question 4

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

$\bullet \bullet \bullet \bullet \bullet \bullet$											
1	2	3	x	y							
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0	-	-				0		$\sqrt{\Box}$	₽□	π	i
			sin	cos	tan	arcsin	arccos	arctan			

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A.CED.2) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• $y = 270000(1.029)^x$

Other Correct Responses

• Any equivalent equation.

For this item, a full-credit response includes:

• The correct model (1 point).
Question 4

Sample Responses

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

$y = 270000(1.029)^{x}$											
1	2	3	x	y							
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0		_				()		√□	V□	π	i
			sin	cos	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation that models a growing population of bats, *y*, after *x* years. The response correctly identifies the exponential model $P(1 + r)^x$, where P = 270,000 is the initial quantity of bats, $(1 + r)^x$ is a factor not depending on *P*, and r = 2.9% is the average annual percent rate, over the period of *x* years. If each year a population of bats grows by a constant factor of (1+.029), then after a period of *x* years, the population of bats, *y*, becomes $270,000(1 + .029)^x$, or $y = 270,000(1.029)^x$.

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation that models a growing population of bats, *y*, after *x* years. The response correctly identifies the exponential model $P(1 + r)^x$, where P = 270,000 is the initial quantity of bats, $(1 + r)^x$ is a factor not depending on *P*, and r = 2.9% is the average annual percent rate, over the period of *x* years. If each year a population of bats grows by a constant factor of (1+.029), then after a period of *x* years, the population of bats, *y*, becomes $270,000(1 + .029)^x$.

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

y=2700	000(.029 • • • •	$\left(\theta \right)^{x}$									
1	2	3	x	y							
4	5	6	+	-	•	÷					
7	8	9	<	<u> </u>	=	<u>≥</u>	>				
0		-				0		$\sqrt{\Box}$	V□	π	i
			sin	cos	tan	arcsin	arccos	arctan			
7 0	8	9	<		=	≥ () arcsin	> arccos	√□ arctan	₽Ū	π	i

Notes on Scoring

This response earns no credit (0 points). The response correctly identifies an exponential model, but uses r^{x} as a factor not depending on *P* instead of $(1 + r)^{x}$. A population modeled by the equation $y = 270,000(.029)^{x}$, where the factor independent from *P* is smaller than 1, represents decay, not growth.

A scientist is studying wildlife. She estimates the population of bats in her state to be 270,000. She predicts the population to grow at an average annual rate of 2.9 percent.

Using the scientist's prediction, create an equation that models the population of bats, y, after x years.

270000	270000(1.029x)										
$\bullet \bullet \bullet$	• • •										
1	2	3	x	у							
4	5	6	+	_	•	÷					
7	8	9	<	≤	=	2	>				
0	-	-			۵	0		√□	₽□	π	i
			sin	COS	tan	arcsin	arccos	arctan			

Notes on Scoring

This response earns no credit (0 points) because it confuses an exponential model with a linear model and uses (1+r)x as a factor not depending on *P*, instead of $(1 + r)^x$. A population modeled by the equation y = 270,000(1.029)x has an average rate of change of 270,000 • 1.029 bats per year, instead of an average annual rate of 2.9 percent per year.

Question 5

Question and Scoring Guidelines

Question 5

A function is shown.

$$f(x) = x^2 + 2x - 3$$

Use the Add Point tool to show the *x*-intercepts and maximum or minimum of the function.



Points Possible: 1

Content Cluster: Analyze functions using different representations.

Content Standard: Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear functions and indicate intercepts, maxima, and minima. (F.IF.7a) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response



Other Correct Responses

• N/A

For this item, a full-credit response includes:

• The correct x-intercepts and minimum of the function (1 point).

Question 5

Sample Responses

A function is shown.

$$f(x) = x^2 + 2x - 3$$

Use the Add Point tool to show the *x*-intercepts and maximum or minimum of the function.



Notes on Scoring

This response earns full credit (1 point) for placing all three points, two *x*-intercepts and one minimum of the function, in the correct locations on a coordinate grid.

The x-intercepts and the minimum point of f(x) can be found by graphing a function on a calculator and locating the vertex (minimum) point at (-1, 4) and the two points of intersection with the x-axis at (-3, 0) and (1, 0).



Notes on Scoring

This response earns no credit (0 points) because one out of three points is plotted incorrectly. The correct x-intercepts are at (-3, 0) and (1, 0). The minimum point (-1, -2) is incorrect.



Notes on Scoring

This response earns no credit (0 points) because all three points are plotted incorrectly. The response shows points at (-2, 3), (-1, 2) and (0, 3), instead of (-3, 0), (1, 0) and (-1, -4).

Question 6

Question and Scoring Guidelines

Question 6

Select the most appropriate unit for each situation.

	<u>feet</u> minute	square feet minute	cubic feet minute
Rate of walking to school			
Rate of painting a bedroom wall			
Rate of filling a bucket with water			
Rate of mopping the kitchen floor			

Points Possible: 1

Content Cluster: Reason quantitatively and use units to solve problems.

Content Standard: Define appropriate quantities for the purpose of descriptive modeling. $(N.Q.2) \star$

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

Select the most appropriate unit for each situation.

	<u>feet</u> minute	square feet minute	cubic feet minute
Rate of walking to school	~		
Rate of painting a bedroom wall			
Rate of filling a bucket with water			~
Rate of mopping the kitchen floor		V	

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• A correct table (1 point).

Question 6

Sample Responses

Select the most appropriate unit for each situation.								
<u>feet</u> square feet <u>cubic feet</u> minute								
Rate of walking to school	~							
Rate of painting a bedroom wall								
Rate of filling a bucket with water								
Rate of mopping the kitchen floor		V						

Notes on Scoring

This response earns full credit (1 point) because it shows four correctly selected units for each situation.

The rate of walking to school is the distance to school divided by the time spent on walking to school. Since in this case the distance is measured in linear units, feet, the correct selection is the first box in the top row.

The rate of painting a bedroom wall is the area of the wall divided by the time spent on painting the wall. Since in this case the area is measured in square feet, the correct selection is the second box in the second row from the top. The rate of filling a bucket with water is measured by the volume of water divided by the time spent on filling the bucket. Since volume is measured in cubic units, or in this case in cubic feet, the correct selection is the third box in the third row from the top.

The rate of mopping the kitchen floor is the area of the floor divided by the time spent on mopping the floor. Since area is measured in square units, or in this case in square feet, the correct selection is the middle box of the bottom row.

Select the most appropriate unit for each situation.							
<u>feet</u> square feet <u>cubic feet</u> minute							
Rate of walking to school							
Rate of painting a bedroom wall							
Rate of filling a bucket with water							
Rate of mopping the kitchen floor							
nate of mopping the interior from		U	_				

Notes on Scoring

This response earns no credit (0 points) because out of four responses it shows one incorrectly selected unit for rate of mopping the kitchen floor. The student may think that because mopping is done with water, and the amount of water is measured in units of volume, the correct response is $\frac{\text{cubic feet}}{\text{minute}}$ instead of $\frac{\text{square feet}}{\text{minute}}$.

Select the most appropriate unit for each situation.								
<u>feet</u> square feet <u>cubic feet</u> minute <u>minute</u>								
Rate of walking to school	~							
Rate of painting a bedroom wall								
Rate of filling a bucket with water			~					
Rate of mopping the kitchen floor								

Notes on Scoring

This response earns no credit (0 points) because it shows one incorrectly selected unit for rate of painting a bedroom wall. The student may think about the volume of the entire bedroom and use the units of volume, $\frac{\text{cubic feet}}{\text{minute}}$, instead of $\frac{\text{square feet}}{\text{minute}}$.

Question 7

Question and Scoring Guidelines

Question 7



Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

Content Standard: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (*A.REI.12*)

Scoring Guidelines

Exemplar Response



Other Correct Responses

- The star can be placed anywhere in the correct region.
- Multiple stars may be placed on the coordinate plane as long as they are all in the correct region.

For this item, a full-credit response includes:

• Two correct lines and at least one correct placement of a star (1 point).

Question 7

Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows two correct lines and one correct placement of a star, indicating a correct portion of a plane.

The graph of $y \ge 5$ includes a horizontal line with the y-intercept at (0, 5) and the half plane above the line. The graph of $y \le 2/3x + 3$ is a line with the y-intercept at (0, 3), a slope of 2/3 and the half plane below the line. The solution set for a system of the two linear inequalities is the intersection of the solution sets (corresponding half planes) for the individual inequalities.



Notes on Scoring

This response earns full credit (1 point) because it shows two correct lines and a correct placement of two stars, indicating a correct portion of a plane.

The graph of $y \ge 5$ includes a horizontal line with the y-intercept at (0, 5) and the half plane above the line. The graph of $y \le 2/3x + 3$ is a line with the y-intercept at (0, 3), a slope of 2/3 and the half plane below the line. The solution set for a system of the two linear inequalities is the intersection of the solution sets (corresponding half planes) for the individual inequalities.



Notes on Scoring

This response earns no credit (0 points) because it shows one correct and one incorrect line, and a correct placement of a star based on the lines graphed.



Notes on Scoring

This response earns no credit (0 points) because it shows one correct and one incorrect line, and a correct placement of a star based on the lines graphed.



Notes on Scoring

This response earns no credit (0 points) because it shows two correct lines, but omits the placement of a star.

Question 8

Question and Scoring Guidelines

Question 8

A system of equations is shown.

$$y = 3x - 2$$
$$y = x^2$$

What are the solutions to the system of equations?

Points Possible: 2

Content Cluster: Solve systems of equations.

Content Standard: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$. (A.REI.7)

(

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Scoring Guidelines

Exemplar Response

(2	,	4)
(1	,	1)

Other Correct Responses

• The order of the ordered pair solutions can be switched.

For this item, a full-credit response includes:

• One correct solution (1 point);

AND

• The other correct solution (1 point).
Question 8

Sample Responses

A system of equations is shown.

$$y = 3x - 2$$
$$y = x^2$$

What are the solutions to the system of equations?

Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions (ordered pairs) to a system of a linear equation and a simple quadratic equation in two variables. There are several methods to solve the system. One of them is by graphing two equations and locating the intersection points of (2, 4) and (1, 1) of the two graphs.

(

(

2

1

4

1

,

)



A system of equations is shown.

$$y = 3x - 2$$
$$y = x^2$$

What are the solutions to the system of equations?

Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions (ordered pairs) to a system of a linear equation and a simple quadratic equation in two variables. There are several methods to solve the system. One of them is by graphing two equations and locating the intersection points of (2, 4) and (1, 1) of the two graphs.

1

2

(||

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1

4

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A system of equations is shown.

$$y = 3x - 2$$
$$y = x^2$$

What are the solutions to the system of equations?

Notes on Scoring

This response earns partial credit (1 point) because it shows only one correct ordered pair (1, 1). The second ordered pair (4, 2) is incorrect because it is not an intersection point of the two graphs.

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1

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1

2

A system of equations is shown.

y = 3x - 2 $y = x^2$

What are the solutions to the system of equations?

Notes on Scoring

This response earns partial credit (1 point) because it shows only one correct ordered pair (2, 4) twice.

A system of equations is shown.

y = 3x - 2 $y = x^2$

What are the solutions to the system of equations?

Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect ordered pairs.

A system of equations is shown.

y = 3x - 2 $y = x^2$

What are the solutions to the system of equations?

Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect ordered pairs.

Question 9

Question and Scoring Guidelines

Question 9

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuc	lear
		Yes	No
Coal	Yes	16	20
	No	2	12

Given that a state does not use nuclear power, what percentage of those states use coal?

		%
$\bullet \bullet \bullet$	•	
1	2	3
4	5	6
7	8	9
	0	
	-	

Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

Content Standard: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. $(S.ID.5) \star$

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 62.5

Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• The correct probability (1 point).

Question 9

Sample Responses

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuc	lear
		Yes	No
Coal	Yes	16	20
	No	2	12

Given that a state does not use nuclear power, what percentage of those states use coal?

62.5		%	
$\bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
-	-	=	

Notes on Scoring

This response earns full credit (1 point) because it shows a correct percentage of states that use coal, given that a state does not use nuclear power, $\frac{20}{(20+12)} = .625$ or 62.5%, as a result of a correct interpretation of relative frequencies for two categories presented in a two-way table.

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuc	lear
		Yes	No
Coal	Yes	16	20
	No	2	12

Given that a state does not use nuclear power, what percentage of those states use coal?

$\frac{2000}{32}$		%	
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
	-	<u><u></u></u>	

Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent form, $\frac{2000}{32} = 62.5$, of a correct percentage of states that use coal, given that a state does not use nuclear power, $\frac{20}{(20+12)} = .625$ or 62.5%, as a result of a correct interpretation of relative frequencies for two categories presented in a two-way table.

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuc	lear
		Yes	No
Coal	Yes	16	20
	No	2	12

Given that a state does not use nuclear power, what percentage of those states use coal?

$\frac{20}{32}$		%	
$\bullet \bullet \bullet$	•		
1	2	3	
4	5	6	
7	8	9	
	0		
· ·	-	<u>-</u>	

Notes on Scoring

This response earns no credit (0 points) because it provides an incorrect percentage of states that use coal, given that a state does not use nuclear power.

An analyst researches the relationship between different energy sources in each state for 2014. The data in the table show the number of states that use coal and nuclear power as an energy source.

		Nuc	lear
		Yes	No
Coal	Yes	16	20
	No	2	12

Given that a state does not use nuclear power, what percentage of those states use coal?



Notes on Scoring

This response earns no credit (0 points) because it includes an incorrect percentage of states that use coal, given that a state does not use nuclear power.

Question 10

Question and Scoring Guidelines

Question 10

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

Content Standard: In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (*S.ID.3*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for First Option</u>: This is incorrect. The student may not have realized that since the time value (10) for the outlier point is equal to the mean of the time values (10) in the data set, the mean of the time will not change.

<u>Rationale for Second Option:</u> **Key** – The student correctly calculated the mean of the original data set as 4.06, and the mean of the data set as 3.37 with the outlier removed, and realized that the mean of the distance changed.

<u>Rationale for Third Option:</u> This is incorrect. The student may not have realized that in the original data set, the removed data point was the median point and equaled the average of the fifth and seventh time values, when ordered, so the median of the time will not change.

<u>Rationale for Fourth Option</u>: This is incorrect. The student may have thought that the median of the distances with the outlier included is 3.7. The median of the distances with the outlier omitted is [(3.7 + 3.7) / 2] = 3.7, so there is no change in the medians, because the two middles are both 3.7.

<u>Rationale for Fifth Option:</u> This is incorrect. The student may not have realized that since the time value for the outlier point is within the range of the time of the entire data set, the range of the time will not change.

<u>Rationale for Sixth Option:</u> **Key** – The student correctly recognized that the distance value for the outlier point is much greater than the distance values for other points. Range of the distances of the original data set is 11.0 - 1.0 = 10. Range of the distances when the outlier is removed is 4.9 - 1.0 = 3.9. The range of the distances will change.

Question 10

Sample Responses

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Notes on Scoring

This response earns full credit (1 point) because it shows correct choices B and F.

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Notes on Scoring

This response earns no credit (0 points) because it shows incorrect answer choices A and C, in addition to the correct choices B and F.

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
 - range of the distance

Notes on Scoring

This response earns no credit (0 points) because it shows incorrect answer choices A and C, in addition to the correct choice B.

A group of students measures the distance a toy car has traveled after different amounts of time. A table of the data is shown.

Time (seconds)	Distance (meters)
5	1.0
6	1.2
7	2.4
8	3.3
9	3.7
10	11.0
11	4.2
12	3.7
13	4.8
14	4.5
15	4.9

The students want to perform an analysis of the data set and consider removing the outlier point.

Select all of the quantities that will change if the outlier point is removed from the data set.

- mean of the time
- mean of the distance
- median of the time
- median of the distance
- range of the time
- range of the distance

Notes on Scoring

This response earns no credit (0 points) because it shows incorrect answer choice D, in addition to the correct choices B and F.

Question 11

Question and Scoring Guidelines

Question 11



Ms. Musto opened a new coffee shop. She recorded the number of customers she served between opening and noon for the first 20 days of business. Her results are shown on the graph.

Which line best fits the data?

- (a) y = 3x + 10
- y = 2x + 20
- y = 3x + 30
- (b) y = x + 20

Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

Content Standard: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. c. Fit a linear function for a scatter plot that suggests a linear association. (S.ID.6c) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have set a line of best fit beneath all points and used the point with the highest y-value (20, 65) and the point with the lowest y-value (3, 20) to estimate the slope.

<u>Rationale for Option B:</u> **Key** – The student used the data on the scatter plot to set a line fitting the most points and determined that its slope must be greater than 1 and that the y-intercept is approximately 20.

<u>Rationale for Option C:</u> This is incorrect. The student may have set a line of best fit above all data points and used the point with the highest y-value (20, 65) and the point with the lowest y-value (3, 20) to estimate the slope.

<u>Rationale for Option D:</u> This is incorrect. The student may have chosen a line of best fit with the correct intercept and used the points (0.25, 2.5) and (4, 6.5) to estimate the slope without taking into account the scale used on the graph.



Sample Response: 1 point

Question 12

Question and Scoring Guidelines

Question 12

Bryson collects data on the depth of a river at various points and the velocity of the river at those points. His data have a correlation coefficient of -0.9382.



Which scatterplot could represent Bryson's data?

Points Possible: 1

Content Cluster: Interpret linear models.

Content Standard: Compute (using technology) and interpret the correlation coefficient of a linear fit. (S.ID.8)★

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> **Key** – The student correctly identified that a correlation coefficient of -0.9382 indicates a strong negative linear relationship because the scatterplot shows the set of points lying closely to the imaginary line of best fit with increasing variable *x* and decreasing variable *y*.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that a correlation coefficient of -0.9382 represents a weak negative linear relationship and chose the scatterplot that represented this data.

<u>Rationale for Option C:</u> This is incorrect. The student may have thought that because the absolute value of a correlation coefficient of -0.9382 is close to 1, it represents a strong positive linear relationship and chose the scatterplot that represented this data.

<u>Rationale for Option D:</u> This is incorrect. The student may have thought that a correlation coefficient of –0.9382 represents a weak positive linear relationship and chose the scatterplot that represented this data.

Sample Response: 1 point


Question 13

Which expression is equivalent to $(2x^2 + 3)(x + 4)$?

- (a) $2x^3 + 12$ (b) $2x^2 + 11x + 12$ (c) $2x^3 + 6x^2 + 4x + 12$ (d) $2x^3 + 8x^2 + 3x + 12$

Points Possible: 1

Content Cluster: Perform arithmetic operations on polynomials.

Content Standard: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1)

Scoring Guidelines

<u>Rationale for Option A</u>: This is incorrect. The student may have thought that only variable terms, $2x^2$ and x, could be multiplied together and that only constant terms, 3 and 4, could be multiplied together; as a result, the student may have multiplied the first terms of each binomial and the last terms of each binomial.

<u>Rationale for Option B</u>: This is incorrect. The student may have disregarded the power of 2 on the first term, thus finding the product of 2x + 3 and x + 4 to get $2x^2 + 8x + 3x + 12$ or $2x^2 + 11x + 12$.

<u>Rationale for Option C:</u> This is incorrect. The student may have incorrectly multiplied the constants by the variable terms, $(2x^2) \cdot 3 = 6x^2$ and $x \cdot 4 = 4x$, within the same parentheses.

<u>Rationale for Option D:</u> **Key** – The student used a distributive property to multiply two binomials to get $2x^2 \cdot x = 2x^3$ and $2x^2 \cdot 4 = 8x^2$ and $3 \cdot x = 3x$ and $3 \cdot 4 = 12$. The student realized that the expression did not have like terms and that the product was $2x^3 + 8x^2 + 3x + 12$.

Sample Response: 1 point

Which expression is equivalent to $(2x^2 + 3)$	3)(<i>x</i> + 4)?
(a) $2x^3 + 12$	
(B) $2x^2 + 11x + 12$	
© $2x^3 + 6x^2 + 4x + 12$	
• $2x^3 + 8x^2 + 3x + 12$	

Question 14

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

		pounds
$\bullet \bullet \bullet$		
1	2	3
4	5	6
7	8	9
	0	
	-	=

Points Possible: 1

Content Cluster: Interpret the structure of expressions.

Content Standard: Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients. (A.SSE.1a) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 2.5

Other Correct Responses

• The correct value in any equivalent form.

For this item, a full-credit response includes:

• The correct value (1 point).

Question 14

Sample Responses

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

2.5		pounds
$\bullet \bullet \bullet$		
1	2	3
4	5	6
7	8	9
	0	
· .	-	

Notes on Scoring

This response receives full credit (1 point) because it shows a correct total number of pounds of peaches and grapefruit, 2.5, that Juan buys. In the context of a situation, the first equation models the weight, p + g, of peaches, p pounds, and, grapefruit, g pounds, totaling 2.5 pounds of fruit.

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

$\frac{5}{2}$		pounds
$\bullet \bullet \bullet \bullet$	•	
1	2	3
4	5	6
7	8	9
	0	
	-	<u>-</u>

Notes on Scoring

This response receives full credit (1 point) because it shows an equivalent form for a correct total number of pounds of peaches and grapefruit, ($\frac{5}{2}$ or 2.5) that Juan buys. In the context of a situation, the first equation models the weight, p + g, of peaches, p pounds, and, grapefruit, g pounds, totaling 2.5 pounds of fruit.

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

2.67 pounds						
$\bullet \bullet \bullet \bullet \bullet$						
1	2	3				
4	5	6				
7	8	9				
	0					
-	-	<u>-</u>				

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect total number of pounds of peaches and grapefruit that Juan buys.

Juan buys peaches and grapefruit at the store. He writes the equations shown to model the relationship between the number of pounds of peaches, p, and the number of pounds of grapefruit, g, that he buys.

p + g = 2.5

1.58p + 1.09g = 3.46

What is the total number of pounds of peaches and grapefruit that Juan buys?

5.96 pounds						
$\overline{\bullet \ \bullet \ \bullet \ \bullet}$						
1	2	3				
4	5	6				
7	8	9				
	0					
	-	-				

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect total number of pounds of peaches and grapefruit that Juan buys.

Question 15

Select all of the expressions that are equivalent to $9x^4 - y^2$. $(3x^2 - y)^2$ $(3x^2)^2 - (y)^2$ $9(x^2)^2 - (y)^2$ $(9x^2)^2 - (y)^2$ $(3x^2 + y)(3x^2 - y)$

Points Possible: 1

Content Cluster: Interpret the structure of expressions.

Content Standard: Use the structure of an expression to identify ways to rewrite it. For example, to factor 3x(x - 5) + 2(x - 5), students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to (3x + 2)(x - 5); or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (A.SSE.2)

Scoring Guidelines

<u>Rationale for First Option</u>: This is incorrect. The student may have taken the square root of each term, but confused the difference of squares with the square of a difference.

<u>Rationale for Second Option:</u> **Key** – The student may have taken the square root of each monomial, creating an equivalent expression, or the student may have correctly applied the second power to both terms in the option, $(3x^2)^2 = 9x^4$ and $(y)^2 = y^2$, to see if it equaled the given expression, $9x^4 - y^2$.

<u>Rationale for Third Option:</u> **Key** – The student noticed that each variable could be written as a squared term, $(3x^2)^2$ and y^2 , leaving the coefficient unfactored.

<u>Rationale for Fourth Option</u>: This is incorrect. The student may have recognized that each term was a perfect square, but neglected to take the square root of the coefficient of the $9x^4$ term.

<u>Rationale for Fifth Option:</u> **Key** – The student recognized the difference of squared terms, took the square root of each, and wrote it as the product of two binomials, one using addition and one using subtraction.

Question 15

Sample Responses

Select all of the expressions that are equivalent to $9x^4 - y^2$. ($3x^2 - y$)² ($3x^2$)² - (y)² 9(x^2)² - (y)² ($9x^2$)² - (y)² ($3x^2 + y$)($3x^2 - y$)

Notes on Scoring

This response earns no credit (0 points) because it shows only two correct options, B and E, and one incorrect option, A, instead of three correct options, B, C and E.

Select all of the expressions that are equivalent to $9x^4 - y^2$. $(3x^2 - y)^2$ $(3x^2)^2 - (y)^2$ $9(x^2)^2 - (y)^2$ $(9x^2)^2 - (y)^2$ $(3x^2 + y)(3x^2 - y)$

Notes on Scoring

This response earns no credit (0 points) because it shows only two correct options, B and C, instead of three correct options, B, C and E.

Select all of the expressions that are equivalent to $9x^4 - y^2$. ($3x^2 - y$)² ($3x^2$)² - (y)² 9(x²)² - (y)² ($9x^2$)² - (y)² ($3x^2 + y$)($3x^2 - y$)

Notes on Scoring

This response earns no credit (0 points) because it shows only two correct options, B and E, instead of three correct options, B, C and E.

Question 16

The population of rabbits on a large island doubles every year. On January 1, the population is 150 rabbits.

Which equation can be used to find the number of years, x, it will take for the population to reach 4,800?

- (A) 4,800 = 2x + 150
- (a) $4,800 = 2 \cdot 150^{x}$
- (c) 4,800 = 2^x + 150
- (b) $4,800 = 150 \cdot 2^{x}$

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* (A.CED.1)★

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have recognized that the initial population of 150 rabbits doubles each year, but confused an exponential model where a different population size doubles each year with a linear model where a constant population size doubles each year.

<u>Rationale for Option B:</u> This is incorrect. The student may have correctly recognized that the situation should be modeled by an exponential model, but confused the initial population, 150, with the growth rate, 2.

<u>Rationale for Option C:</u> This is incorrect. The student may have correctly recognized that the situation should be modeled by an exponential model and recognized that the initial population is 150 rabbits, but incorrectly used addition instead of multiplication between factors of the exponential model $a \bullet b^x$.

<u>Rationale for Option D:</u> **Key** – The student correctly recognized that if the initial population of rabbits, 150, doubles at the end of the first year, the number of rabbits is $150 \cdot 2$. At the end of the second year, it is $150 \cdot 2 \cdot 2$ or $150 \cdot 2^2$. At the end of x years, the population of rabbits is $150 \cdot 2^x$. When the population reaches 4800 rabbits, the situation can be modeled by the exponential equation $4800 = 150 \cdot 2^x$.

Sample Response: 1 point

The population of rabbits on a large island doubles every year. On January 1, the population is 150 rabbits.

Which equation can be used to find the number of years, x, it will take for the population to reach 4,800?

(a)
$$4,800 = 2x + 150$$

- (a) $4,800 = 2 \cdot 150^{x}$
- (c) 4,800 = 2^x + 150

4,800 = $150 \cdot 2^{x}$

Question 17



Points Possible: 1

Content Cluster: Solve equations and inequalities in one variable.

Content Standard: Solve quadratic equations in one variable. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (A.REI.4b)

Scoring Guidelines

Exemplar Response

• $x = -\frac{1}{2}$ and x = 3

Other Correct Responses

• Any pair of equivalent values in either order is accepted.

For this item, a full-credit response includes:

• A correct pair of values (1 point).

Question 17

Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows two correct values, x = 3 and x = -0.5 or -1/2, that make the quadratic equation true.

While there are several methods for solving a quadratic equation in the form of $ax^2 + bx + c = 0$, the use of a graphing utility may be a way to find *x*-intercepts or *x* values that make the equation true.



Notes on Scoring

This response earns full credit (1 point) because it shows two correct values, x = -1/2 and x = 3, that make the quadratic equation true.

While there are several methods for solving a quadratic equation in the form of $ax^2 + bx + c = 0$, the use of a graphing utility may be a way to find *x*-intercepts or *x* values that make the equation true.



Notes on Scoring

This response receives no credit (0 points) because it shows values opposite of the two correct *x* values. The sign switch makes both values incorrect.



Notes on Scoring

This response receives no credit (0 points) because it shows one correct *x* value and one incorrect *x* value.

Question 18

The graph of a quadratic function $f(x)$ intersects the x-axis at -3 and 5.												
What is a possible equation for $f(x)$?												
f(x)	= [
\bullet	•											
1	2	3	x									
4	5	6	+	-	•	÷						
7	8	9	<	≤	=	2	>					
0		-				0		$\sqrt{1}$	₩	π	i]
			sin	cos	tan	arcsin	arccos	arctan				

Points Possible: 1

Content Cluster: Analyze functions using different representations.

Content Standard: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (F.IF.8a)
Scoring Guidelines

Exemplar Response

• $f(x) = x^2 - 2x - 15$

Other Correct Responses

• Any function equivalent to f(x) = b(x+3)(x-5), where b is a real number.

For this item, a full-credit response includes:

• A correct function (1 point).

Question 18

Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent form, $y = x^2 - 2x - 15$, for a correct quadratic function y = b(x + 3)(x - 5), where b = 1.

The intercept form for a quadratic function is y = b(x - r)(x - s), where r and s are the points where the function intersects the x-axis, and b is a non-zero real number. Working backwards by substituting both x-intercepts in the intercept form for a quadratic function creates an equation representing a family of quadratic functions with the same x-intercepts. In this situation, for x = -3 and x = 5, working backwards creates a function that is y = b(x + 3)(x - 5), where b is any non-zero real number.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation, y = 6(x + 3)(x - 5), in the form of y = b(x + 3)(x - 5), where b = 6.

The intercept form for a quadratic function is y = b(x-r)(x-s), where r and s are the points where the function intersects the x-axis and b is a non-zero real number. Working backwards by substituting both x-intercepts in the intercept form for a quadratic function creates an equation representing a family of quadratic functions with the same x-intercepts. In this situation, for x = -3 and x = 5, working backwards creates a function that

x = -3 and x = 5, working backwards creates a function that is y = b(x + 3)(x - 5), where b is any non-zero real number.



Notes on Scoring

This response earns no credit (0 points) because it shows a quadratic function that is not equivalent to y = 1(x + 3)(x - 5).



Notes on Scoring

This response receives no credit (0 points) because it shows a quadratic function that is not equivalent to y = 1(x + 3)(x - 5).

Question 19

Question and Scoring Guidelines

Question 19

An equation is shown.

 $y = \frac{1}{2}x + \frac{3}{4}$

Select all of the points that are contained in the graph of the equation.

 $(0, \frac{1}{2})$ $(0, \frac{3}{4})$ $(\frac{3}{4}, 0)$ $(\frac{3}{4}, \frac{1}{2})$ $(\frac{1}{2}, 1)$

Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

Content Standard: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)

Scoring Guidelines

<u>Rationale for First Option</u>: This is incorrect. The student may have changed the order of operations and first multiplied $\frac{3}{4}$ by 0, then added $\frac{1}{2}$, to get $\frac{1}{2}$.

<u>Rationale for Second Option</u>: **Key** – The student may have correctly realized that points contained in the graph of the equation must make the equation a true statement. When $(0, \frac{3}{4})$ is substituted into the equation, $\frac{3}{4} = \frac{1}{2} \cdot 0 + \frac{3}{4}$, it makes the equation a correct statement.

<u>Rationale for Third Option</u>: This is incorrect. The student may have incorrectly used x = 0 and $y = \frac{3}{4}$ instead of $x = \frac{3}{4}$ and y = 0 when substituting values in the equation to get $\frac{1}{2} \cdot 0 + \frac{3}{4} = \frac{3}{4}$.

<u>Rationale for Fourth Option</u>: This is incorrect. The student may have incorrectly performed operations with fractions as $\frac{1}{2} \cdot \frac{3}{4} + \frac{3}{4} = \frac{3}{8} + \frac{3}{4} = \frac{6}{12}$, or $\frac{1}{2}$.

<u>Rationale for Fifth Option</u>: **Key** – The student correctly multiplied $\frac{1}{2}$ times $\frac{1}{2}$ to get $\frac{1}{4}$, and then added the product to $\frac{3}{4}$ to get 1.

Question 19

Sample Responses



Notes on Scoring

This response earns full credit (1 point) because it shows both correct answer options, B and E.

An equation is shown.

 $y = \frac{1}{2}x + \frac{3}{4}$

Select all of the points that are contained in the graph of the equation.

 $(0, \frac{1}{2})$ $(0, \frac{3}{4})$ $(\frac{3}{4}, 0)$ $(\frac{3}{4}, \frac{1}{2})$ $(\frac{1}{2}, 1)$

Notes on Scoring

This response earns no credit (0 points) because it shows one incorrect answer option, A, in addition to the two correct answer options, B and E.

An equation is shown. $y = \frac{1}{2}x + \frac{3}{4}$ Select all of the points that are contained in the graph of the equation. $(0, \frac{1}{2})$ $(0, \frac{3}{4})$ $(0, \frac{3}{4})$ $(\frac{3}{4}, 0)$ $(\frac{3}{4}, \frac{1}{2})$ $(\frac{1}{2}, 1)$

Notes on Scoring

This response earns no credit (0 points) because it shows one incorrect answer option, C, and one correct answer option, E.

Question 20

Question and Scoring Guidelines

Question 20



Points Possible: 1

Content Cluster: Understand the concept of a function, and use function notation.

Content Standard: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (*F.IF.2*)

Scoring Guidelines

Exemplar Response

• *f*(12) = 11

Other Correct Responses

• Any equivalent form.

For this item, a full-credit response includes:

• The correct value (1 point).

Question 20

Sample Responses



Notes on Scoring

This response receives full credit (1 point) for showing a correct answer of f(12) = 11. If a given function is $f(x) = \frac{2}{3}x + 3$, then f(12) denotes the output of a function f(x)corresponding to the input x = 12, and calculates as $f(12) = \left(\frac{2}{3}\right) \cdot 12 + 3 = 8 + 3 = 11$, or f(12) = 11.

A function is shown. $f(x) = \frac{2}{3}x + 3$ What is the value of f(12)? 33*f*(12) • + 1 2 3 5 4 6 7 8 9 0 믐

Notes on Scoring

This response receives full credit (1 point) for showing a correct answer of f(12) = 11 in its equivalent form $f(12) = \frac{33}{3}$.

If a given function is $f(x) = \frac{2}{3}x + 3$, then f(12) denotes the output of a function f(x) corresponding to the input x = 12, and calculates as $f(12) = (\frac{2}{3}) * 12 + 3 = 8 + 3 = 11$, or f(12) = 11, or f(12) = 11.



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect value for f(12).



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect value for f(12).

Question 21

Question and Scoring Guidelines

Question 21

A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after *t* seconds is modeled by the function shown.

$$h(t) = -t^2 + \frac{4}{3}t + \frac{1}{4}$$

After how many seconds will the grasshopper land on the ground?

$\bullet \bullet \bullet \bullet$		
1	2	3
4	5	6
7	8	9
	0	
	_	<u>-</u>

Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

Content Standard: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)*

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 1.5

Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• A correct value (1 point).

Question 21

Sample Responses

A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after *t* seconds is modeled by the function shown.

$$h(t) = -t^2 + \frac{4}{3}t + \frac{1}{4}$$

After how many seconds will the grasshopper land on the ground?

$\frac{3}{2}$		
$\bullet \bullet \bullet \bullet \bullet \bullet \bullet$		
1	2	3
4	5	6
7	8	9
	0	
	-	

Notes on Scoring

This response receives full credit (1 point) because it shows a correct time in seconds, $t = \frac{3}{2}$, that is equivalent to t = 1.5.

The height of the grasshopper, in feet, above the ground after t seconds is modeled by the quadratic function $h(t) = -t^2 + (\frac{4}{3})t + (\frac{1}{4})$. When the grasshopper lands on the ground, its height above the ground is 0 feet. By letting h(t) = 0 and solving the equation $0 = -t^2 + (\frac{4}{3})t + (\frac{1}{4})$, the time the grasshopper is on the ground is t = 1.5 sec. While there are several methods for solving a quadratic equation, the use of a graphing utility may be the quickest for finding solutions represented by the horizontal intercepts ("x-intercepts"). Because a variable t represents time, the correct solution is only a positive horizontal intercept of the graph of h(t).



A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after *t* seconds is modeled by the function shown.

$$h(t) = -t^2 + \frac{4}{3}t + \frac{1}{4}$$

After how many seconds will the grasshopper land on the ground?

1.5			
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
-	-	<u>-</u>	

Notes on Scoring

This response receives full credit (1 point) because it shows a correct time in seconds, $t = \frac{3}{2}$, that is equivalent to t = 1.5.

The height of the grasshopper, in feet, above the ground after t seconds is modeled by the quadratic function $h(t) = -t^2 + (\frac{4}{3})t + (\frac{1}{4})$. When the grasshopper lands on the ground, its height above the ground is 0 feet. By letting h(t) = 0 and solving the equation $0 = -t^2 + (\frac{4}{3})t + (\frac{1}{4})$, the time the grasshopper is on the ground is t = 1.5 sec. While there are several methods for solving a quadratic equation, the use of a graphing utility may be the quickest for finding solutions represented by the horizontal intercepts ("x-intercepts"). Because a variable t represents time, the correct solution is only a positive horizontal intercept of the graph of h(t).



A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after *t* seconds is modeled by the function shown.

$$h(t) = -t^2 + \frac{4}{3}t + \frac{1}{4}$$

After how many seconds will the grasshopper land on the ground?

$-\frac{1}{6}$		
$\bullet \bullet \bullet \bullet \bullet \\ \bullet \bullet$		
1	2	3
4	5	6
7	8	9
	0	
-	-	

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect negative value for the time, t = -1/6.
Sample Response: 0 points

A grasshopper jumps off of a tree stump. The height, in feet, of the grasshopper above the ground after *t* seconds is modeled by the function shown.

$$h(t) = -t^2 + \frac{4}{3}t + \frac{1}{4}$$

After how many seconds will the grasshopper land on the ground?



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect value for the time, t = 6.

Question 22



Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

Content Standard: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)*

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have understood that this graph represents a function, but confused the domain, the set of *x*-coordinates, of all real numbers, with the range, the set of *y*-coordinates, of all non-negative real numbers.

<u>Rationale for Option B:</u> This is incorrect. The student may have understood that the domain is the set of non-negative *x*-values, but overlooked the fact that this is not the graph of a function.

<u>Rationale for Option C:</u> **Key** – The student correctly understood that the graph satisfies the requirements of a function where each *x*-coordinate corresponds to only one *y*-coordinate and that the domain is represented by the set of non-negative *x*-coordinates.

<u>Rationale for Option D:</u> This is incorrect. The student may have understood that this graph represents a function, but overlooked the fact that the domain, the set of *x*-coordinates, is all non-positive real numbers and the range is all non-negative real numbers.

Sample Response: 1 point



Question 23

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Points Possible: 1

Content Cluster: Build a function that models a relationship between two quantities.

Content Standard: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (*F.BF.2*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• Explicit: $f(n) = 4 \cdot 3^{n-1}$

Other Correct Responses

• Any equivalent functions.

For this item, a full-credit response includes:

• A correct explicit function (1 point).

Question 23

Sample Responses

Sample Response: 1 point

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct function $f(n) = 4 \cdot (3)^{n-1}$.

If the situation asks for building a correct exponential function represented by a geometric sequence, where each successive term is a constant multiple, r = 3, of the previous term, the function input is the set of all natural numbers $\{1, 2, 3, ...\}$ and the function output is the set of all terms of the sequence $\{4, 12, 36, 108, 324, ...\}$. The *n*th term of the geometric sequence can be obtained by finding a product of the first term of the geometric sequence and the constant multiplier, *r*, raised in the power (n - 1), and can be modeled by a function, $f(n) = 4 \cdot (3)^{n-1}$.

Sample Response: 1 point

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

This response earns full credit (1 point) because it shows a correct exponential function, $f(n) = (\frac{4}{3}) \cdot 3^n$, equivalent to $f(n) = 4 \cdot (3)^{(n-1)}$, because $4 \cdot 3^{(n-1)} = 4 \cdot 3^n \cdot 3^{-1} = 4 \cdot 3^n \cdot \frac{1}{3} = (\frac{4}{3}) \cdot 3^n$.

If the situation asks for building a correct exponential function represented by a geometric sequence, where each successive term is a constant multiple, r = 3, of the previous term, the function input is the set of all natural numbers $\{1, 2, 3, ...\}$ and the function output is the set of all terms of the sequence $\{4, 12, 36, 108, 324, ...\}$. The *n*th term of the geometric sequence can be obtained by finding a product of the first term of the geometric sequence and the constant multiple, *r*, raised in the power (n - 1), and can be modeled by a function, $f(n) = 4 \cdot (3)^{(n-1)}$.

Sample Response: 0 points

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

This response receives no credit (0 points) because it shows a linear function instead of an exponential function.

Sample Response: 0 points

The first five terms of a sequence are shown.

4, 12, 36, 108, 324, ...

Write an explicit function to model the value of the *n*th term in the sequence such that f(1) = 4.



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect exponential function.

Question 24

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time.

Write a function f(x) to model the number of fish in the pond after x months.

f(x) =				
$\bullet \bullet \bullet \bullet \bullet \bullet$				
1 2 3	x			
4 5 6	+ - • ÷			
7 8 9				
0	\square \square \square \square \square \square \square \square i			
	sin cos tan arcsin arccos arctan			

Points Possible: 1

Content Cluster: Construct and compare linear, quadratic, and exponential models, and solve problems.

Content Standard: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (*F.LE.2*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• $f(x) = 5 \cdot 2^x$

Other Correct Responses

• Any equivalent function.

For this item, a full-credit response includes:

• A correct function (1 point).

Question 24

Sample Responses

Sample Response: 1 point

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time.

Write a function f(x) to model the number of fish in the pond after x months.



Notes on Scoring

This response receives full credit (1 point) because it shows a correct exponential function $f(x) = 5 \cdot 2^x$.

In this situation, the focus is on modeling a verbal situation by an exponential function. If the initial number of fish in the pond is 5, and at end of the first month the number of fish doubles, then the number of fish is $10 = 5 \cdot 2$ or $f(1) = 5 \cdot 2^1$. At the end of the second month, the number of fish doubles again, and is 20 or $10 \cdot 2 = (5 \cdot 2) \cdot 2$ or $f(2) = 5 \cdot 2^2$. At the end of the third month, it doubles again to become 40 or $20 \cdot 2 = ((5 \cdot 2) \cdot 2) \cdot 2$ or $f(3) = 5 \cdot 2^3$. Similarly, at the end of the x month, the number of fish is $5 \cdot 2^x$ or $f(x) = 5 \cdot 2^x$.

Sample Response: 1 point

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time.

Write a function f(x) to model the number of fish in the pond after x months.



Notes on Scoring

This response receives full credit (1 point) because it shows a correct exponential function, $f(x) = 2^x \cdot 5$, equivalent to the function $f(x) = 5 \cdot 2^x$.

In this situation, the focus is on modeling a verbal situation by an exponential function. If the initial number of fish in the pond is 5, and at the end of the first month the number of fish doubles, then the number of fish is $10 = 5 \cdot 2$ or $f(1) = 5 \cdot 2^1$. At the end of the second month, the number of fish doubles again, and is 20 or $10 \cdot 2 = (5 \cdot 2) \cdot 2$ or $f(2) = 5 \cdot 2^2$. At the end of the third month, it doubles again to become 40 or $20 \cdot 2 = ((5 \cdot 2) \cdot 2) \cdot 2$ or $f(3) = 5 \cdot 2^3$. Similarly, at the end of the x month, the number of fish is $5 \cdot 2^x$ or $f(x) = 5 \cdot 2^x$.

Sample Response: 0 points

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time.

Write a function f(x) to model the number of fish in the pond after x months.



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect exponential function that is not equivalent to the function $f(x) = 5 \cdot 2^x$.

Sample Response: 0 points

A landscaper puts 5 fish into a new pond. The number of fish doubles each month over a period of time. Write a function f(x) to model the number of fish in the pond after x months. f(x) =10•x $\bullet \to \bullet \mathrel{\textcircled{}} \blacksquare$ 1 2 3 x 4 5 6 + • ÷ 7 8 9 < \leq = ≥ > 0 믐 0, $\sqrt{\Box}$ ᄳ 0 Ш i π _ arcsin arccos arctan sin cos tan

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect linear function that is not equivalent to the exponential function $f(x) = 5 \cdot 2^x$.

Question 25

Which statistical measure changes when every number in a data set is increased by 10?

- A range
- mean
- © standard deviation
- interquartile range

Points Possible: 1

Content Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

Content Standard: In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. (*S.ID.2*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that since each value was increased by 10, the range would also increase, but neglected to consider that the increases of 10 would cancel each other when subtracting the minimum from the maximum to calculate the range.

<u>Rationale for Option B:</u> **Key** – The student correctly determined that the mean of the data set would increase because if every data point increases by 10 and the number of data points remains the same, the average increases.

<u>Rationale for Option C:</u> This is incorrect. The student may have thought that since each value increased by 10, the standard deviation would also increase, but because the mean would increase by 10 and each data point increases by 10, the increases cancel each other when finding the difference between each data point and the mean to calculate the deviation.

<u>Rationale for Option D:</u> This is incorrect. The student may have thought that since all data points increased by 10, the interquartile range would also increase but neglected to consider that the increases of 10 to the first and third quartiles would cancel each other when finding their difference to calculate the interquartile range.

Sample Response: 1 point

Which statistical measure changes when every number in a data set is increased by 10?

A range

- mean
- © standard deviation
- interquartile range

Question 26

A shipping company charges a cost per pound plus a fixed fee to ship a package. The total cost, f(x), in dollars, of shipping x pounds is modeled by the function shown.

f(x) = 4.99x + 5.75

Which part of the function represents the fixed fee?

A x

B 4.99

© 5.75

0 4.99x

Points Possible: 1

Content Cluster: Interpret expressions for functions in terms of the situation they model.

Content Standard: Interpret the parameters in a linear or exponential function in terms of a context. (*F.LE.5*) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that the variable *x* represents the fixed fee.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that the coefficient of the variable term represents the fixed fee.

<u>Rationale for Option C:</u> **Key** – The student may have correctly recognized that in a linear function, f(x) = mx + b, *m* represents the average rate of change, and *b* is the initial or a fixed value. In the context of this situation, the student correctly identified that a fixed value is the fixed fee represented by the constant term, 5.75.

<u>Rationale for Option D:</u> This is incorrect. The student may have thought that the variable term represents the fixed fee.

Sample Response: 1 point

A shipping company charges a cost per pound plus a fixed fee to ship a package. The total cost, f(x), in dollars, of shipping x pounds is modeled by the function shown.

f(x) = 4.99x + 5.75

Which part of the function represents the fixed fee?

A x

8 4.99



0 4.99x

Question 27

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower leve	el tickets:			
Upper level tickets:				
$\bullet \bullet \bullet \bullet \bullet \bullet$				
1	2	3		
4	5	6		
7	8	9		
	0			
	-	<u>-</u>		

Points Possible: 1

Content Cluster: Solve systems of equations.

Content Standard: Solve systems of linear equations algebraically and graphically. (A.REI.6)

Scoring Guidelines

Exemplar Response

• Lower seats: 150 Upper seats: 200

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• The correct values (1 point).
Question 27

Sample Responses

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower level tickets:		150	
Upper level tickets:		200	
$\bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
•	-		

Notes on Scoring

This response receives full credit (1 point) because it shows a correct number of sold tickets, 150, for the lower level seats and a correct number of sold tickets, 200, for the upper level seats.

By letting x represent the number of lower level tickets sold, and y represent the number of upper level tickets sold, the given situation can be modeled by the system of two linear equations. The first equation, x + y = 350, represents the total number of all tickets sold, and the second equation, 35x + 25y = 10,250, represents the total cost of the tickets sold for all seats. Solving the first equation for y, and then substituting y = 350 - x into the second equation for y, the second equation becomes 35x + 25(350 - x) = 10,250. The solution to the system is x = 150, the number of lower level seat tickets sold, and y = 200, the number of upper level seat tickets sold.

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower level tickets: Upper level tickets:		200
		150
	•	
1	2	3
4	5	6
7	8	9
	0	
	-	8

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect number of tickets sold for the lower level seats, 200, and an incorrect number of tickets sold for the upper level seats, 150.

A theater sells tickets for a concert. Tickets for lower-level seats sell for \$35 each, and tickets for upper-level seats sell for \$25 each. The theater sells 350 tickets for \$10,250.

How many tickets of each type were sold?

Lower leve	el tickets:	175
Upper level tickets:		175
$\bullet \bullet \bullet \bullet$		
1	2	3
4	5	6
7	8	9
	0	
	-	

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect number of tickets sold for the lower level seats, 175, and an incorrect number of tickets sold for the upper level seats, 175.

Question 28

Question and Scoring Guidelines

Question 28

A function is shown.

$$f(x) = 5(x-2)^2 + 3$$

What is the minimum value of the function?



Points Possible: 1

Content Cluster: Write expressions in equivalent forms to solve problems.

Content Standard: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A.SSE.3b) \star

 (\star) indicates that modeling should be incorporated into the standard.

Scoring Guidelines

Exemplar Response

• 3

Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• The correct value (1 point).

Question 28

Sample Responses



Notes on Scoring

This response receives full credit (1 point) because it shows a correct minimum value, 3, for the function f(x).

While there are several equivalent forms of the quadratic function, the vertex form $f(x) = a(x - h)^2 + k$ reveals coordinates (h, k) of the vertex point, where k is the minimum value of the function, if a is positive. In this situation, a function $f(x) = 5(x - 2)^2 + 3$ opens up, and has a vertex at (2, 3) with a minimum value at 3.



Notes on Scoring

This response receives full credit (1 point) because it shows a correct minimum value, -3 or 3, for the function f(x).

While there are several equivalent forms of the quadratic function, the vertex form $f(x) = a(x - h)^2 + k$ reveals coordinates (h, k) of the vertex point, where k is the minimum value of the function, if a is positive. In this situation, a function $f(x) = 5(x - 2)^2 + 3$ opens up, and has a vertex at (2, 3) with a minimum value at 3.

A function is shown. $f(x) = 5(x-2)^2 + 3$ What is the minimum value of the function? -3• • • • • 2 3 1 4 5 6 7 8 9 0 믐 _

Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect minimum value for the function f(x).



Notes on Scoring

This response receives no credit (0 points) because it shows an incorrect minimum value for the function f(x).

Question 29

Question and Scoring Guidelines

Question 29

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property
3(x-2) = x + 4	Given
3x - 6 = x + 4	
2x - 6 = 4	
2x = 10	
<i>x</i> = 5	

Addition property of equality	Multiplication property of equality	Distributive property	Associative property
Commutative property			

Points Possible: 1

Content Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

Content Standard: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (*A.REI.1*)

Scoring Guidelines

Exemplar Response

Step	Property
3(x-2) = x+4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Addition property of equality
2x = 10	Addition property of equality
x = 5	Multiplication property of equality

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• A correctly completed table (1 point).

Question 29

Sample Responses

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property
3(x-2) = x + 4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Addition property of equality
2x = 10	Addition property of equality
<i>x</i> = 5	Multiplication property of equality

Notes on Scoring

This response earns full credit (1 point) because it shows a viable argument to a solution method.

- The student noticed that 3 was multiplied by (x 2) on the left side of the equation from the first step to the second step using the Distributive property.
- The student noticed that x was added to both sides of the equation from the second step to the third step using the Addition property of equality.
- The student noticed that 6 was added to both sides of the equation from the third step to the fourth step using the Addition property of equality.
- The student noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step using the Multiplication property of equality.

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

StepProperty3(x-2) = x + 4Given3x - 6 = x + 4Multiplication property of equality2x - 6 = 4Addition property of equality2x = 10Addition property of equalityx = 5Distributive property

Move a property to each blank box to show the reason for each step.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect argument to a solution method.

- The student noticed that 3 was multiplied by (x 2) on the left side of the equation from the first step to the second step, but confused the Distributive property with the Multiplication property of equality.
- The student noticed that x was added to both sides of the equation from the second step to the third step using the Addition property of equality.
- The student may have noticed that 6 was added to both sides of the equation from the third step to the fourth step using the Addition property of equality.
- The student may have noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step, but confused the Distributive property with the Multiplication property of equality.

Students need to get all four properties correct in order to earn full credit for this item.

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property
3(x-2) = x+4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Associative property
2x = 10	Associative property
<i>x</i> = 5	Multiplication property of equality

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect argument to a solution method.

- The student noticed that 2 was multiplied by (x 2) on the left side of the equation from the first step to the second step using the Distributive property.
- The student may have noticed that x was added to both sides of the equation from the second step to the third step, but confused the Associative property with the Addition property of equality.
- The student may have noticed that 6 was added to both sides of the equation from the third step to the fourth step, but confused the Associative property with the Addition property of equality.
- The student noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step using the Multiplication property of equality.

Students need to get all four properties correct in order to earn full credit for this item.

A sequence of steps for solving the equation 3(x - 2) = x + 4 is shown.

Move a property to each blank box to show the reason for each step.

Step	Property
3(x-2) = x + 4	Given
3x - 6 = x + 4	Distributive property
2x - 6 = 4	Addition property of equality
2x = 10	Associative property
<i>x</i> = 5	Multiplication property of equality

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect argument to a solution method.

- The student noticed that 3 was distributed to (x 2) on the left side of the equation from the first step to the second step using the Distributive property.
- The student noticed that -x was added to both sides of the equation from the second step to the third step using the Addition property of equality.
- The student may have noticed that 6 was added to both sides of the equation from the third step to the fourth step, but confused the Associative property with the Addition property of equality.
- The student noticed that $\frac{1}{2}$ was multiplied by both sides of the equation from the fourth step to the fifth step using the Multiplication property of equality.

Students need to get all four properties correct in order to earn full credit for this item.

Question 30

Question and Scoring Guidelines

Question 30

The graph of the equation contains the point v which is to the equat	on.

Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

Content Standard: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)

Scoring Guidelines

Exemplar Response

(-4, -4.5)
One of many possible solutions

Other Correct Responses

• N/A

For this item, a full-credit response includes:

• The correct ordered pair and phrase (1 point).

Question 30

Sample Responses

Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (-4, -4.5), view which is one of many possible solutions view to the equation.

Notes on Scoring

This response earns full credit (1 point) because it shows one correct solution out of many possible solutions to an equation in two variables.

The student recognized the correct solution to the equation. To do so, the student may have substituted the three ordered pairs into the equation to see which ordered pairs make the equation true:

- Substitute (0, -18) into the equation 9x 4y = -18 to get 9(0) 4(-18) = -18, which simplifies to 72 = -18. The statement is not true, so (0, -18) is not a solution to the equation.
- Substitute (9, -4) into the equation 9x 4y = -18 to get 9(9) - 4(-4) = -18, which simplifies to 81 + 16 =-18 or 97 = -18. The statement is not true, so (9, -4) is not a solution to the equation.
- Substitute (-4, -4.5) into the equation 9x 4y = -18to get 9(-4) - 4(-4.5) = -18, which simplifies to -36 + 18 = -18 or -18 = -18. The statement is a true statement, so (-4, -4.5) is a solution to the equation.

The student recognized that 9x - 4y = -18 was a linear equation in standard form, and a linear equation has many possible solutions that fall on its graph.

Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (-4, -4.5), 🔽 which is the only possible solution

to the equation.

Notes on Scoring This response earns no credit (0 points) because it shows one correct solution to an equation in two variables, but it does not indicate that there are many possible solutions to a linear equation. The student recognized the correct solution to the equation. To do so, the student may have substituted the three ordered pairs into the equation to see which ordered pairs make the equation true: • Substitute (0, -18) into the equation 9x - 4y = -18 to get 9(0) - 4(-18) = -18, which simplifies to 72 = -18. This statement is not true, so (0, -18) is not a solution to the equation. Substitute (9, -4) into the equation 9x - 4y = -18 to get 9(9) - 4(-4) = -18, which simplifies to 81 + 16 =-18 or 97 = -18. This statement is not true, so (9, -4) is not a solution to the equation. Substitute (-4, -4.5) into the equation 9x - 4y = -18to get 9(-4) - 4(-4.5) = -18, which simplifies to -36 + 18 = -18 or -18 = -18. This statement is a true statement, so (-4, -4.5) is a solution to the equation. The student may not have recognized that a linear equation has many possible solutions that fall on its graph. Students need to get both solutions correct in order to earn full credit for this item.

Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (0, –18), 💌 which is one of many possible solutions 💌 to the equation.

Notes on Scoring

This response earns no credit (0 points) because it indicates that there are many possible solutions to a linear equation in two variables, but finds an incorrect solution to the given equation.

The student selected an incorrect choice for the solution to the equation.

The student recognized that 9x - 4y = -18 was a linear equation in standard form, and a linear equation has many possible solutions that fall on its graph.

Students need to get both choices correct in order to earn full credit for this item.

Complete the statement about the equation 9x - 4y = -18.

The graph of the equation contains the point (9, -4), which is one of two possible solutions to the equation.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect solution out of many possible solutions to an equation in two variables.

The student selected an incorrect choice for the solution to the equation.

The student may not have recognized that a linear equation has many possible solutions that fall on its graph.

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