

## Characteristics of Highly Effective Mathematics Teaching and Learning

### Section One: Learning Climate

**Learning Climate:** a safe environment supported by the teacher in which high, clear expectations and positive relationships are fostered; active learning is promoted

#### **Teacher Characteristics:**

**A – Teacher creates learning environments where students are active participants as individuals and as members of collaborative groups. The teacher:**

1) models literacy and numeracy strategies to enable students to communicate and analyze mathematical problems/tasks and solutions.

**B – Teacher motivates students and nurtures their desire to learn in a safe, healthy and supportive environment which develops compassion and mutual respect.**

**C – Teacher cultivates cross cultural understandings and the value of diversity.**

**D – Teacher encourages students to accept responsibility for their own learning and accommodates the diverse learning needs of all students.**

**E – Teacher displays effective and efficient classroom management that includes classroom routines that promote comfort, order and appropriate student behaviors.**

**F – Teacher provides students equitable access to technology, space, tools and time. The teacher:**

1) provides access to the common core curriculum by utilizing differentiated teaching strategies, interventions, manipulatives, calculators, information technology, etc.

**G – Teacher effectively allocates time for students to engage in hands-on experiences, discuss and process content and make meaningful connections**

**H – Teacher designs lessons that allow students to participate in empowering activities in which they understand that learning is a process and mistakes are a natural part of learning.**

**I – Teacher creates an environment where student work is valued, appreciated and used as a learning tool. The teacher:**

1) provides opportunities for students to share mathematical ideas with others and to problem solve.

**Student Characteristics:**

**A – Student accepts responsibility for his/her own learning. The student:**

1) asks for clarifications or additional resources when needed,

2) explains alternate methods to determine solutions and/or

3) analyzes mathematical concepts for understanding.

**B – Student actively participates and is authentically engaged. The student:**

1) utilizes literacy and numeracy skills to communicate and analyze mathematical problems/tasks and solutions.

**C – Student collaborates/teams with other students. The student:**

1) collaborates to examine mathematical ideas and procedures to communicate, reason and problem solve.

**D – Student exhibits a sense of accomplishment and confidence.**

**E – Student takes educational risks in class. The student:**

1) refutes mathematical processes/solutions,

2) listens carefully and

3) asks questions to clarify mathematical thinking.

**F – Student practices and engages in safe, responsible and ethical use of technology.**



[Mathematics Learning Climate PD guide](#)- click here to download a professional development guide on learning climate in mathematics. This module is intended to be used within professional learning communities in your school or district.



[View the video clip for the Mathematics Learning Climate PD guide \(dsl/cable\). \[dialup\]](#) (This video is intended to be a representative 'snapshot' of what may commonly be observed in classrooms throughout the Commonwealth—not necessarily as a benchmark of every characteristic for the associated component. Using the Characteristics of High Quality Teaching and Learning, the video clip and the accompanying facilitator's guide, professional learning communities can engage in conversations around increasing competencies in this component.)

## Section Two: Classroom Assessment and Reflection

**Classroom Assessment and Reflection:** the teacher and student collaboratively gather information and reflect on learning through a systematic process that informs instruction

### **Teacher characteristics:**

**A – Teacher uses multiple methods and systematically gathers data about student understanding and ability.**

**B – Teacher uses student work/data, observations of instruction, assignments and interactions with colleagues to reflect on and improve teaching practice. The teacher:**

1) uses practical applications of current research on classroom assessment as a basis on which to improve mathematics pedagogy.

**C – Teacher revises instructional strategies based upon student's achievement data. The teacher:**

1) makes assessments (formative and summative) an integral component of instructional decision making.

2) utilizes assessment guidelines explicitly for the purpose of improving instruction and increasing student achievement.

**D – Teacher uncovers students’ prior understanding of the concepts to be addressed and addresses students’ misconceptions/incomplete conceptions. The teacher:**

**1) uses this information to determine students’ range of strategies and skills in order to design cutting-edge instruction to stretch student thinking.**

**E – Teacher co-develops scoring guides/rubrics with students and provides adequate modeling to make clear the expectations for quality performance. The teacher:**

1) shares this information for the purpose of demonstrating proficient work in mathematics.

**F – Teacher guides students to apply rubrics to assess their performance and identify improvement strategies.**

**G – Teacher provides regular and timely feedback to students and parents that moves learners forward. The teacher:**

1) gives feedback that is focused, descriptive, and qualitative.

**H – Teacher allows students to use feedback to improve their work before a grade is assigned.**

**I – Teacher facilitates students in self- and peer-assessment.**

**J – Teacher reflects on work and makes adjustments as learning occurs.**

**Student Characteristics:**

**A – Student recognizes what proficient work looks like and determines steps necessary for improving his/her work.**

**B– Student develops and/or uses scoring guides periodically to assess his/her own work or that of peers.**

**C– Student uses teacher feedback to improve his/her work.**

**D – Student reflects on work and makes adjustments as learning occurs.**

**E-Student reflects on work and makes adjustments as learning occurs.**



[Mathematics Learning Targets Video](#)



[Mathematics Summative Assessment Video](#)

(This video is intended to be a representative 'snapshot' of what may commonly be observed in classrooms throughout the Commonwealth—not necessarily as a benchmark of every characteristic for the associated component.)

## Section Three: Instructional Rigor and Student Engagement

**Instructional Rigor and Student Engagement:** a teacher supports and encourages a student's commitment to initiate and complete complex, inquiry-based learning requiring creative and critical thinking with attention to problem solving

### **Teacher Characteristics:**

**A – Teacher instructs the complex processes, concepts and principles contained in state and national standards using differentiated strategies that make instruction accessible to all students.**

**The teacher:**

- 1) plans and integrates appropriate differentiated strategies to ensure all students have access to quality mathematics.
- 2) utilizes current curriculum documents in the development of course outlines/maps in order to ensure instructional rigor.

**B – Teacher scaffolds instruction to help students reason and develop problem-solving strategies.**

**The teacher:**

- 1) scaffolds instruction to align with students' levels of learning for the purpose of helping students reason and solve cognitively challenging mathematical tasks utilizing appropriate problem-solving strategies.

**C – Teacher orchestrates effective classroom discussions, questioning, and learning tasks that promote higher-order thinking skills. The teacher:**

- 1) facilitates effective classroom discussion and learning tasks that promote reasoning higher-order thinking skills that involve reasoning, conjecturing, proof, and validating.
- 2) helps students understand mathematical structures and the connections among them.

**D – Teacher provides meaningful learning opportunities for students. The teacher:**

- 1) provides opportunities for students to be actively engaged in mathematics activities which promote

meaningful learning and discussions among students.

2) introduces appropriate mathematics representations (pictures, objects, symbols) that allow students to communicate and connect mathematical ideas effectively.

**E – Teacher challenges students to think deeply about problems and encourages/models a variety of approaches to a solution. The teacher:**

1) models a variety of approaches for solving mathematics problems and challenges students to reflect on their approaches and the approaches of other students.

**F – Teacher integrates a variety of learning resources with classroom instruction to increase learning options. The teacher:**

1) models and integrates a variety of learning resources (technology including computers, software, calculators, manipulatives, diagrams) in classroom instruction for the purpose of understanding and solving mathematics problems and communicating their solutions.

**G – Teacher structures and facilitates ongoing formal and informal discussions based on a shared understanding of rules and discourse.**

**H – Teacher integrates the application of inquiry skills into learning experiences. The teacher:**

1) requires students to explore and reason about mathematical ideas.

**I – Teacher clarifies and shares with students learning intentions/targets and criteria for success. The teacher:**

1) communicates these using student friendly language so that students have an understanding of mathematical proficiency.

### **Student Characteristics:**

**A – Student articulates and understands learning intentions/targets and criteria for success.**

**B – Student reads with understanding a variety of texts. The student:**

1) reads and solves contextual problems using a variety of strategies.

**C – Student applies and refines inquiry skills. The student:**

1) asks questions and identifies concepts to guide problem solving,  
2) uses appropriate tools (technology and manipulatives) to facilitate mathematical reasoning,  
3) collaborates with other students, and/or  
4) justifies solutions to problems by communicating through symbolic, hands-on, or spoken representations.

## Section Four: Instructional Relevance

**Instructional Relevance:** a teacher's ability to facilitate learning experiences that are meaningful to students and prepare them for their futures.

**Teacher Characteristics:**

**A – Teacher designs learning opportunities that allow students to participate in empowering activities in which they understand that learning is a process and mistakes are a natural part of the learning. The teacher:**

- 1) designs learning opportunities that allow students to participate in relevant activities.
- 2) establishes a learning environment so that students will realize learning is a process and mistakes are a natural part of learning.
- 3) models the characteristics of a lifelong learner in his/her instruction by asking guiding questions of self and students.

**B – Teacher links concepts and key ideas to students' prior experiences and understandings, uses multiple representations, examples and explanations. The teacher:**

- 1) demonstrates how the big ideas in mathematics are connected.
- 2) uses multiple representations, relevant examples and clear explanations to enhance student learning.

**C – Teacher incorporates student experiences, interests and real-life situations in instruction.**

**D – Teacher selects and utilizes a variety of technology that support student learning.**

**E – Teacher effectively incorporates 21st Century Learning Skills that prepare students to meet future challenges.**

**F – Teacher works with other teachers to make connections between and among disciplines. The teacher:**

- 1) poses real-world problems involving other disciplines for students to solve by applying mathematical reasoning.

**G – Teacher makes lesson connections to community, society, and current events. The teacher:**

- 1) poses real-world problems involving community, society and current events for students to solve by applying mathematical reasoning.

**Student Characteristics:**

**A – Student poses and responds to meaningful questions. The student:**

- 1) listens carefully.
- 2) asks questions to clarify mathematical thinking.
- 3) refutes mathematical processes/solutions.
- 4) shows persistence during the process of learning.

**B – Student uses appropriate tools and techniques to gather, analyze, and interpret quantitative and qualitative data. The student:**

- 1) uses multiple representations (e.g., charts, models, graphs, symbols, tables, and diagrams) to communicate mathematically and to uncover different aspects of the problem.

**C – Student develops descriptions, explanations, predictions, and models using evidence. The student:**

1) uses these when communicating about mathematical reasoning.

**D – Student works collaboratively to address complex, authentic problems which require innovative approaches to solve. The student:**

1) works on mathematics that is connected to other content areas.

2) communicates how the big ideas in mathematics are connected.

3) uncovers mathematical connections that may lead to deeper understanding.

**E – Student communicates knowledge and understanding in a variety of real-world forms. The student:**

1) draws from prior learning knowledge to learn new mathematics content.

2) utilizes 21st Century Learning Skills to prepare to meet future challenges.

**F – Student communicates knowledge and understanding for a variety of purposes. The student:**

1) connect his/her personal experiences to mathematics learning and

2) provide examples of how mathematics is used outside the classroom.

## Section Five: Knowledge of Content

**Knowledge of Content:** a teacher's understanding and application of the current theories, principles, concepts and skills of a discipline.

### **Teacher Characteristics:**

**A- Teacher demonstrates an understanding and in-depth knowledge of content and maintains an ability to convey this content to students. The teacher:**

1) demonstrates an understanding and in-depth knowledge of mathematics content (mathematical content knowledge for teaching) and uses this knowledge as a basis for developing and applying pedagogical content knowledge.

2) provides opportunities for students to develop profound reasoning and a deeper understanding of content.

3) demonstrates an in-depth knowledge of mathematics content and is able to engage in dialogue about the content with colleagues, students, parents, and others in the community.

**B- Teacher maintains on-going knowledge and awareness of current content developments. The teacher:**

1) maintains awareness of current developments in the field of mathematics and can explain what these developments mean to each group of stakeholders.

2) maintains an awareness of current developments in the field of mathematics education.

3) uses current developments to build pedagogical content knowledge.

4) engages in professional dialogue with colleagues regarding the research implications for sound practice that develops students' advanced thinking, robust fluency and flexible use of mathematical content.



**C- Teacher designs and implements standards-based courses/lessons/units using state and national standards. The teacher:**

- 1) poses tasks that
  - are based on sound and significant mathematics.
  - are based on knowledge of students' understandings, interests, and experiences.
  - are based on knowledge of the range of ways that diverse students learn mathematics.
  - call for problem formulation, problem solving, and mathematical reasoning.

**D- Teacher uses and promotes the understanding of appropriate content vocabulary. The teacher:**

- 1) models the appropriate use of mathematical language and provides opportunities for students to learn content vocabulary and communicate using precise mathematical language.

**E- Teacher provides essential supports for all students. The teacher:**

- 1) provides essential supports by drawing on pedagogical content knowledge in order to provide supports for all students in mathematics (e.g., ESL, students with disabilities, Title I students, G/T).
- 2) differentiates content levels according to evidence of student need.

**F- Teacher accesses a rich repertoire of instructional practices, strategies, resources and applies them appropriately. The teacher:**

- 1) demonstrates that mathematical understandings occur as a result of solving meaningful problems rather than merely performing procedures.
- 2) encourages students to identify the underlying procedures or create procedures using protocols, protocols, properties, and mathematical reasoning.
- 3) asks questions that allow for rich discussion and deep student thinking about the richness of related mathematical ideas.

**Student Characteristics:**

**A- Student demonstrates growth in content knowledge. The student:**

- 1) communicates, orally and in writing, deep understanding, i.e. reflects on connections between concepts, procedures, and connections.

**B- Student uses and seeks to expand appropriate content vocabulary.**

**C- Student connects ideas across content areas. The student:**

- 1) connects mathematical ideas among different content strands (e.g., number , measurement, probability).
- 2) connects related operations/ideas within a content strand (e.g. inverse operations; using related facts; equivalence; functional representations).

**D- Student uses ideas in realistic problem solving situations. The student:**

- 1) applies mathematical concepts to all problem situations including those in practical/authentic contexts or abstract contexts.