TONBRIDGE SCHOOL

Scholarship Examination 2021

MATHEMATICS II

Wednesday 28th April 2021
2.00 pm

Time allowed: 1 hour 30 minutes

There are seven questions in this paper.
Answer as many questions as you can.
All the questions carry equal marks.
You may attempt the questions in any order.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. The following equation is used in the manufacture of the lenses of glasses: $\dfrac{P}{n-1} = \dfrac{1}{Q} - \dfrac{1}{R}$,

   where P is the power of the lens, n depends on what the lens is made of, and Q, R are related to the curvature of the two surfaces of the lens.

   Note that any of P, Q, R may be negative numbers, but n is always positive.

   In the questions below, give your answers as decimals correct to 2 decimal places.
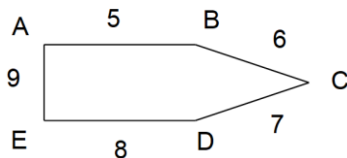
   (a)   If $n = 1.5, Q = 0.4, R = -0.7$, find P.

   (b)   If $P = 2.6, n = 1.7, Q = 0.3$, find R.

   (c)   If $P = 0.2, Q = 2.4, R = 3.7$, find n.

   (d)   If $P = 1.8, n = 1.4$ and $R = 2Q$, find Q.

2. In the following two diagrams, the number on each edge is the sum of the two numbers at the ends of the edge. For example, in (a), we have $A + B = 19$.

   (a)   By forming and solving appropriate equations, find the numbers A, B, C.
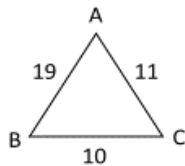
   (b)   In a similar way, find the numbers A, B, C, D, E.

3. [In this question you will need to use the facts that a sphere of radius r has curved surface area $S = 4\pi r^2$ and volume $V = \dfrac{4}{3}\pi r^3$.]

   This question concerns hollow spheres which are the solid shapes formed by removing a small inner sphere of radius r cm from a large solid sphere of radius R cm.

   (a)   If $R = 9$ and $r = 6$, find the volume of the hollow sphere and its total (inner and outer) surface area.

   (b)   If $R = 12$ and the volume of the hollow sphere is 5000 cm³, find r.

   (c)   Find positive integer values of R and r for which the hollow sphere has the same total (inner and outer) surface area as that of a solid sphere of radius 10 cm.

4. The figure below shows a circle centre O with radius 8 cm and angle $\angle AOB = 90°$.
   Region I is the segment bounded by the straight line AB and the arc AB.
   Region II is the sector bounded by the radii OC, OD and the arc CD.
   Region I and Region II have equal areas.
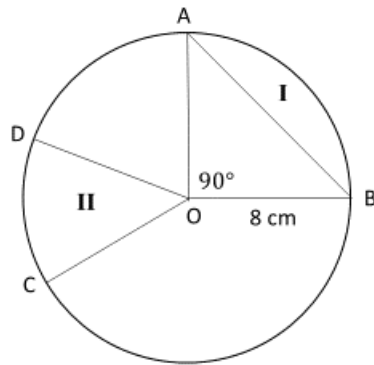
   (a)   Find the area of Region I.

   (b)   By considering what fraction of the whole circle sector COD occupies, find
         angle $\angle COD$.

   (c)   Which of Region I or Region II has the larger perimeter?
         [You need to give calculations to justify your answer.]

5. In the diagram, the triangles ABC, ACD and ADE are isosceles with $\angle BAC = \angle DAE = 40°$.
   Also, angle $\angle CAD = x$ and angle $\angle BFE = y$.
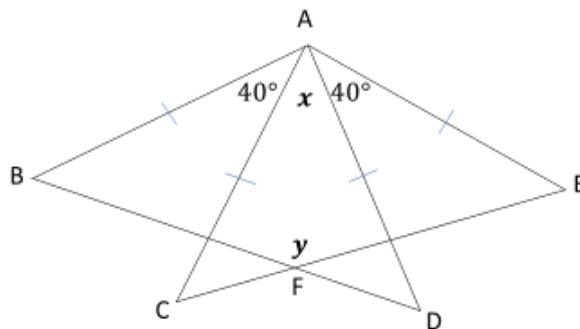
   (a)   If $x = 30°$, find y.

   (b)   Use algebra to show that, whatever the value of x, you always get the same answer for
         y that you got in (a).

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6. This question concerns the graph whose equation is $y = \frac{1}{x} + \frac{5x^2}{x^2+4}$.

    (a)   Make a table of y values (correct to 2 decimal places) first for x values starting at $x = 0.2$ and going to $x = 1$ in steps of 0.2 and also for $x = 2, 4, 6, 8, 10$.

    (b)   Why is $x = 0$ not included in the table of values?

    (c)   Using all of your values in (a) and choosing sensible scales, plot a graph of y against x.

    (d)   Write down the coordinates of two points on the graph whose x and y coordinates are both positive integers with $1 \le x \le 10$. Explain carefully why there are no other such points on the graph.

7. Study the pattern of numbers in the table below carefully. Column B gives the answer to the sum in Column A; Column C gives the number in Column B in a factorised form.

	A	B	C
Row 1	1×4	4	1×4
Row 2	$1 \times 4 + 2 \times 7$	18	2×9
Row 3	$1 \times 4 + 2 \times 7 + 3 \times 10$	48	3×16
Row 4			
Row 5			
Row n			

    (a)   Write down the entries in Columns A, B and C for Row 4 and Row 5.

    (b)   For Row n, find a formula in terms of n for the two numbers at the right-hand end of Column A and for the entry in Column C.

    (c)   (i) If the entry in Column B is 7600, what is the entry in Column C?

           (ii) If the two numbers at the right-hand end of Column A multiply to give 444, what is the entry in Column B?

    (d)   Explain carefully why the entries for Column B in Row n have values that are between n^3 and $(n + 1)^3$.

    (e)   Use (d) to help you find the value of n for which the entry in Column B is 5237748.

END OF PAPER